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DEVELOPMENT OF A SOIL MOISTURE PREDICTION MODEL

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PREFACE

The theory for transient isothermal flow of water into nonswelling unsaturated soil is well understood and has been developed to a large extent in terms of solutions of the nonlinear Richards' equation. In the field the description of infiltration is highly complicated since the initial and boundary conditions are usually not constant while the soil characteristics may vary with time and space. In view of this, most efforts in recent past, have been concentrated on seeking numerical solutions. There exist quite a variety of finite difference solutions employing different forms of the nonlinear Richards' equation and different ways of discretization.

This report entitled 'Development of a Soil Moisture Prediction Model' is a part of the research activities of 'Ground Water Assessment' division of the Institute. The purpose of this study is to develop a soil moisture prediction model using various discretization schemes. The study has been carried out by Mr. Chandra Prakash Kumar, Scientist 'C' and Dr. G. C. Mishra, Scientist 'F'

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ABSTRACT

Flow of water through unsaturated porous media is common in nature. The basic equation of flow in the unsaturated zone of a porous medium is Richards' equation. The exact solution to the Richards' equation is not yet known. Therefore finite difference methods are widely used for solving the partial differential equation describing one-dimensional water transfer in unsaturated soil. The purpose of this study was to develop a numerical model of transient, one-dimensional water flow through the unsaturated porous medium. Seven models, employing different ways of discretization of the nonlinear infiltration equation, were compared with Philip's quasi-analytical solution. All models yielded good agreement with water content profiles at various times in a sand column.

1.0 INTRODUCTION

Subsurface formations containing water may be divided vertically into several horizontal zones according to how large a portion of the pore space is occupied by water. Essentially, we have a zone of saturation in which all the pores are completely filled with water, and an overlaying zone of aeration in which the pores contain both gases (mainly air and water vapour) and water.The latter zone is called the unsaturated zone. Sometimes the term soil water is used for the water in this zone.

The water movements in the unsaturated zone are, together with the water holding capacity of this zone, very important for the water demand of the vegetation, as well as for the recharge of the ground water storage. A fair description of the flow in the unsaturated zone is crucial for predictions of the movement of pollutants into ground water aquifers.

The vertical movement of soil moisture in the liquid phase between the surface and the water table can be subdivided into the following three categories according to predominant forces involved.

i. Infiltration and exfiltration

Alternate wetting and drying of soil surface during consecutive storm and interstorm periods will cause a penetration of the medium by an unsteady wave like diffusion of liquid soil moisture into the soil during wet surface (storm) periods under the complementary effects of capillarity and gravity and out of the soil during dry surface (interstorm) periods when capillarity opposes gravity. With increasing depth of penetration, diffusion reduces the soil moisture gradients and thus reduces the effect of capillarity until moisture movement becomes dominated by gravity. The depth at which surface induced capillary forces become negligible determines the penetration depth of the surface process and is used to define the thickness of the zone of soil moisture. The presence of transpiring vegetation adds another mechanism for moisture extraction distributed over a depth which is related to root structure.

ii. Percolation

Liquid soil moisture moves out of the bottom of the zone of soil moisture and percolates downward under the domination of gravity forces until it encounters the increasing soil moisture gradients lying above the water table. At some depth upward capillary forces will be prominent defining the bottom of this intermediate zone.

iii. Capillary rise

Between the water table and the intermediate zone there is a capillary fringe in which gravity and capillarity again jointly govern the liquid soil moisture movement.

For analytical studies on soil moisture regime, critical review and accurate assessment of the different controlling factors is necessary . The controlling factors of soil moisture may be classified under two main groups viz. climatic factors and soil factors. Climatic factors include precipitation data containing rainfall intensity, storm duration, interstorm period, temperature of soil surface, relative humidity, radiation, evaporation, and evapotranspiration. The soil factors include soil matric potential and water content relationship, hydraulic conductivity and water content relationship of the soil, saturated hydraulic conductivity, and effective medium porosity. Besides these factors, the information about depth to water table is also required.

In the present study, a model has been developed to simulate the soil moisture profile in an initially unsaturated soil during infiltration. A number of discretization schemes are used for the one-dimensional Richards' equation and the results are compared with the quasi-analytical solution of Philip (1957).

2.0 REVIEW

Water in soil moves from points where it has a high energy status to points where it has a lower one. If we consider the origin of Z at the soil surface and positive in downward direction, then the hydraulic head, H may be defined as

$$H = h - Z \qquad \ldots(1)$$

where, h is the soil water pressure head (relative to the atmosphere) expressed in cm of water and Z is the gravitational head (cm). In unsaturated soil, h is negative because work is needed to withdraw water against the soil matric forces.

For one- dimensional vertical flow in the unsaturated soil, Darcy's law is given by

$$v_w = -K(\theta) \frac{\partial H}{\partial Z} \qquad \ldots(2)$$

where, $K(\theta)$ is the hydraulic conductivity (cm/h) which depends on the soil moisture content, θ. Substitution of equation (1) into equation (2) yields

$$v_w = -K(\theta) \frac{\partial}{\partial Z} (h - Z)$$

or

$$v_w = -K(\theta) \left(\frac{\partial h}{\partial Z} - 1\right) \qquad \ldots(3)$$

Applying the continuity principle (law of conservation of matter) for a soil mass of unit area and height δZ,

$$\theta_t \delta Z + v_w \delta t = \theta_{t+\delta t} \delta Z + \left(v_w + \frac{\partial v_w}{\partial Z} \delta Z\right)\delta t$$

or

$$\frac{\partial \theta}{\partial t} + \frac{\partial v_w}{\partial Z} = 0 \qquad \ldots(4)$$

Substitution of equation (3) into equation (4) yields the partial differential equation to describe the flow of water in soil systems.

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial Z} \left[-K(\theta) \left(\frac{\partial h}{\partial Z} - 1\right)\right] = 0$$

or $$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z} \left[K(\theta) \left(\frac{\partial h}{\partial Z} - 1 \right) \right] \qquad ...(5)$$

Equation (5) is a second order, parabolic, nonlinear, partial differential equation, known as Richards' equation.

With the soil water diffusivity defined as

$$D(\theta) = \frac{K(\theta)}{C(\theta)} \qquad ...(6)$$

where,

$$C(\theta) = \frac{d\theta}{dh} \qquad ...(7)$$

is the specific water capacity, equation (5) can also be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z} \left\{ D(\theta) \frac{\partial \theta}{\partial Z} \right\} - \frac{\partial K(\theta)}{\partial Z} \qquad ...(8)$$

Equation (8) is only valid if one assumes the $h(\theta)$ relationship to be unique.

For a unique solution of θ with respect to time and space, initial and boundary conditions must be applied. As initial condition either h as a function of Z must be given

$$h\ (Z, t = 0) = h_o(Z) \qquad ...(9)$$

or θ as a function of Z

$$\theta\ (Z, t = 0) = \theta_o(Z) \qquad ...(10)$$

must be applied.

Boundary conditions at the top and the bottom of the unsaturated zone can be specified in three different ways:

(i) Dirichlet condition: the pressure head is specified as a function of time.

$$h\ (Z = Z_B, t) = h_B\ (t) \qquad ...(11)$$

(ii) Neumann condition: the flux is specified as function of time.

$$q\ (Z = Z_B\ ,\ t) = q_B\ (t) \qquad ...(12)$$

(iii) Cauchy condition: the flux is a function of the dependent variable h at $Z = Z_B$.

$$q\ (Z = Z_B,\ t) = f\ (\ h_B\ ,t) \qquad ...(13)$$

In general at the top and at the bottom of the unsaturated zone, different types of boundary conditions can be used at the same time. For the unsaturated zone, the boundaries are constituted by the soil surface and the phreatic surface. Through these boundaries, relations can be established with the atmosphere and the saturated zone.

Due to the strong nonlinearity of equation (5) and (8), there exists no general analytical solution. However, a specific solution of equation (8) was first obtained by Philip (1957) in the case of infiltration in an homogeneous semi-infinite column satisfying the boundary conditions:

$$\left.\begin{array}{lll} t < 0 & Z \geq 0 & \theta = \theta_n \\ t \geq 0 & Z = 0 & \theta = \theta_u \end{array}\right\} \qquad ...(14a)$$

In a later paper (Philip, 1958), equation (5) was solved for the conditions:

$$\left.\begin{array}{lll} t < 0 & Z \geq 0 & h = h_n \\ t \geq 0 & Z = 0 & h = h_u \end{array}\right\} \qquad ...(14b)$$

where h_u could take positive values corresponding to an infiltration experiment with submersion. Philip's method led to a solution in the form of a power series in $t^{1/2}$. Since the series converges only for finite t, the solution becomes unreliable as $t \to$ infinity; the t-range of convergence is depending upon the characteristics of soil and the initial and boundary conditions.

A serious limitation of the use of quasi-analytical solutions for practical cases is imposed by the representativity of the initial and boundary conditions (14a) and (14b). The soil column is considered to be semi-infinite and with an initial uniform water content, the boundary conditions are constant in time, and heterogeneity can not be taken into account. Most of these conditions are scarcely met in practice. Consequently, numerical solutions without such restrictions were developed; they mostly differ in the way of discretization or in the method of linearization used to solve equation (5) or (8).

If both saturated and non-saturated regions in the soil profile are of interest, it is better to consider h instead of θ as the independent variable (Philip, 1958). Using the specific water capacity, equation (5) is then transformed into

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial Z} \left[K(h) \left(\frac{\partial h}{\partial Z} - 1\right)\right] \qquad \ldots(15)$$

In the saturated zone, equation (15) becomes Laplace's equation, provided the soil is homogeneous and isotropic. With both saturated and non-saturated regions, h varies from positive values in the saturated zone to negative values in the unsaturated zone.

3.0 PROBLEM DEFINITION

There exist quite a variety of finite difference solutions employing different forms of the nonlinear Richards' equation and different ways of discretization. It is the purpose of this study to develop a soil moisture prediction model and using seven finite difference schemes. For each of the finite difference scheme, a comparison is made between calculated water content profiles at various times in a sandy soil and as calculated with the quasi-analytical solution of Philip, which was obtained by solving the equation (8) subject to condition of a constant pressure at the soil surface (equation 14b).

4.0 METHODOLOGY

Seven models, employing different ways of discretization of the nonlinear Richards' equation were compared with the Philip's quasi-analytical solution. Haverkamp et al. (1977) has presented the water content profiles at various times obtained quasi-analytically with the solution of Philip for infiltration in the sand. The following functional relations were used for characterizing the hydraulic properties of the soil (figure 1):

$$K = K_s \frac{A}{A + |h|^{\beta_1}} \; ; \qquad \ldots(16)$$

$K_s = 34$ cm/h,

$A = 1.175 \times 10^6$,

$\beta_1 = 4.74$.

and

$$\theta = \frac{\alpha\,(\theta_s - \theta_r)}{\alpha + |h|^{\beta_2}} + \theta_r \; ; \qquad \ldots(17)$$

$\theta_s = 0.287$,

$\theta_r = 0.075$,

$\alpha = 1.611 \times 10^6$,

$\beta_2 = 3.96$.

where, subscript s refers to saturation, i.e. the value of θ for which $h = 0$, and the subscript r to residual water content. The initial and boundary conditions for infiltration of water in the sand were taken as

$t<0 \quad Z \geq 0 \quad \theta_n = 0.10\ cm^3/cm^3$

$t\geq 0 \quad Z = 0 \quad \theta_u = 0.267\ cm^3/cm^3$
(0.268 in some cases)

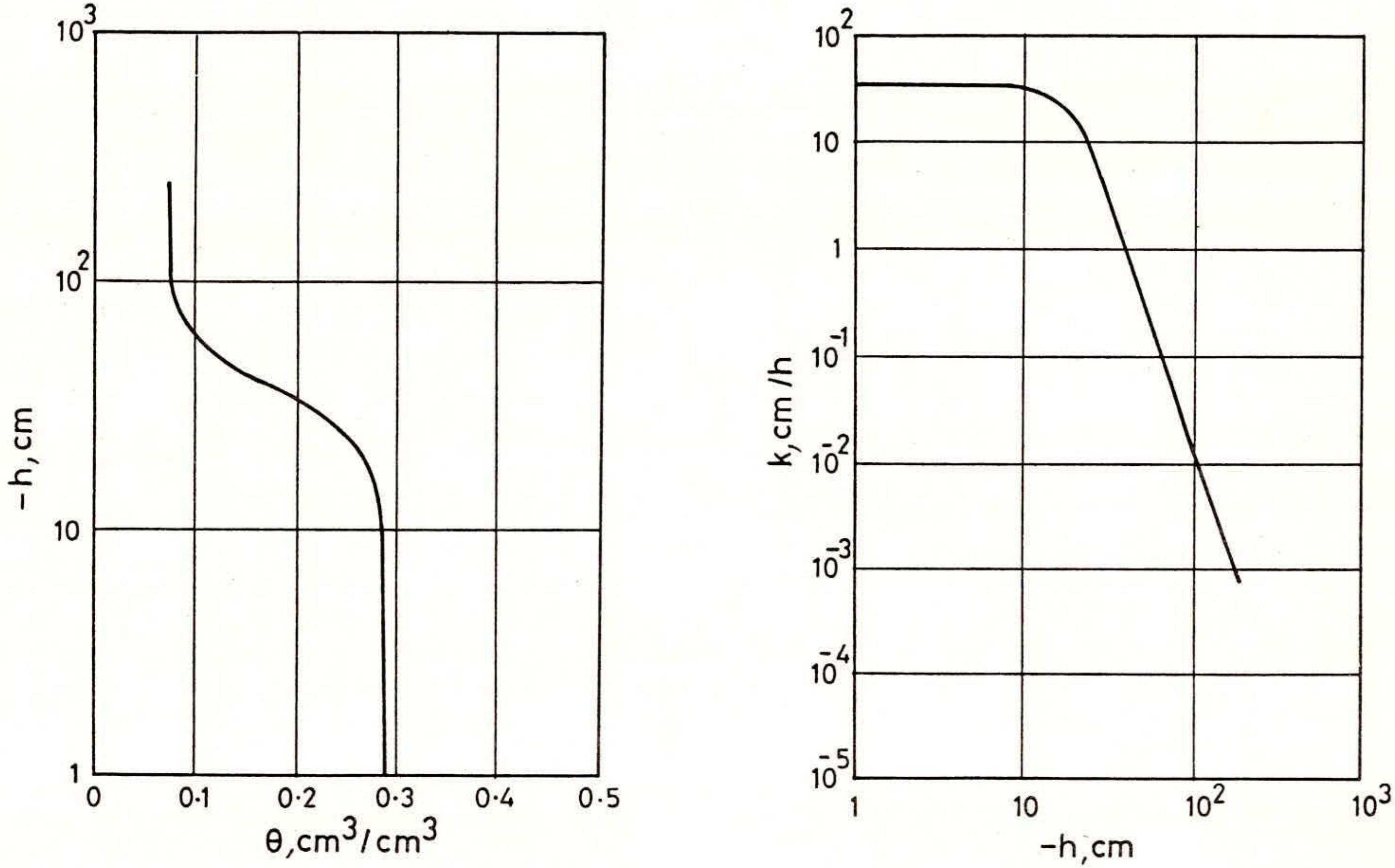

FIG.1– RELATIONSHIPS BETWEEN THE SOIL WATER PRESSURE h, THE WATER CONTENT θ AND THE HYDRAULIC CONDUCTIVITY K FOR THE SOIL USED IN THE STUDY

From the analytical expressions for K and θ (equation 16 and 17), which were obtained by a least square fit through all data points of a series of infiltration experiments in the laboratory (Haverkamp et al., 1977), the expression for soil water diffusivity, D (θ) may be derived as follows:

From equation (17),

$$|h| = \left[\frac{\alpha (\theta_s - \theta)}{(\theta - \theta_r)} \right]^{1/\beta_2} \qquad ...(18)$$

or

$$\frac{dh}{d\theta} = - \frac{1}{\beta_2} (\alpha)^{1/\beta_2} (\theta_s - \theta_r)(\theta - \theta_r)^{-1/\beta_2 - 1} (\theta_s - \theta)^{1/\beta_2 - 1} \qquad ...(19)$$

From equation (16) and (18),

$$K = K_s \frac{A (\theta - \theta_r)^{\beta_1/\beta_2}}{A (\theta - \theta_r)^{\beta_1/\beta_2} + \alpha^{\beta_1/\beta_2} (\theta_s - \theta)^{\beta_1/\beta_2}} \qquad ...(20)$$

Now, D(θ) can be found out from equation (19) and (20) as

$$D(\theta) = K \frac{dh}{d\theta} \qquad ...(21)$$

A total of seven models are compared in this study. Different discretization schemes are used for the variousmodels, using explicit or implicit methods. In the explicit method, a series of linearized independent equations is solved directly, while in the implicit method, a system of simultaneous linear algebraic equations (involving tridiagonal coefficient matrix with zero elements outside the diagonals) has to be solved. For a given grid point at a given time, the values of the coefficients K(h) or K(θ) and C(h) or D(θ) can be expressed either from their values at the preceding time step (explicit linearization) or from a prediction at time (t + 1/2 Δt) using a method described by Douglas and Jones, 1963 (implicit linearization).

The following discretization schemes were used for the various models:

Model 1: Equation (15) - Explicit scheme solved directly

$$h_i^{j+1} = h_i^j + \frac{\Delta t}{C_i^j \, \Delta Z}\left[K_{i+1/2}^j \left(\frac{h_{i+1}^j - h_i^j}{\Delta Z} - 1 \right) - K_{i-1/2}^j \left(\frac{h_i^j - h_{i-1}^j}{\Delta Z} - 1 \right) \right] \qquad \ldots(22)$$

where, j refers to time , and i refers to depth and

$$K_{i+1/2}^j = \frac{K_{i+1}^j + K_i^j}{2} \; ;$$

$$K_{i-1/2}^j = \frac{K_i^j + K_{i-1}^j}{2} \; .$$

Defining D_{max} as the maximum value of the soil water diffusivity in the soil profile at time t, the scheme is stable when (Haverkamp et al., 1977)

$$\Delta t < \frac{r \, (\Delta Z)^2}{D_{max}} \qquad \ldots(23)$$

where, ΔZ is the layer thickness and r an arbitrary chosen coefficient equal to 0.5 for the sand of figure 1.

The method is limited by extensive use of computer time when the water content approaches saturation and Δt becomes very small (D_{max} becomes large).

Model 2 : Equation (15) - Implicit scheme with explicit linearization (in terms of h)

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta Z} \left[K_{i+1/2}^j \left(\frac{h_{i+1}^{j+1} - h_i^{j+1}}{\Delta Z} - 1 \right) - K_{i-1/2}^j \left(\frac{h_i^{j+1} - h_{i-1}^{j+1}}{\Delta Z} - 1 \right) \right]$$

or

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta Z} \left[\left(\frac{K_{i+1}^j + K_i^j}{2} \right) \left(\frac{h_{i+1}^{j+1} - h_i^{j+1}}{\Delta Z} - 1 \right. \right. - \left(\frac{K_i^j + K_{i-1}^j}{2} \right) \left(\frac{h_i^{j+1} - h_{i-1}^{j+1}}{\Delta Z} - 1 \right.$$

Rearranging the terms , we get

$$\left[- F_2 \frac{\Delta t}{(\Delta Z)^2} \right] h_{i-1}^{j+1} + \left[C_i^j + (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2} \right] h_i^{j+1} - \left[F_1 \frac{\Delta t}{(\Delta Z)^2} \right] h_{i+1}^{j+1} = C_i^j h_i^j + (F_2 - F_1) \frac{\Delta t}{\Delta Z} \quad \ldots(24)$$

where,

$$F_1 = K_{i+1/2}^j = \frac{K_{i+1}^j + K_i^j}{2} \quad ;$$

$$F_2 = K_{i-1/2}^j = \frac{K_i^j + K_{i-1}^j}{2}$$

Model 3 : Equation (15) - Implicit scheme with implicit linearization (Prediction -correction)

From equation (15), we have

$$C \frac{\partial h}{\partial t} = \frac{\partial}{\partial Z} \left[K \left(\frac{\partial h}{\partial Z} - 1 \right) \right]$$

or

$$C \frac{\partial h}{\partial t} = \frac{\partial K}{\partial Z} \left(\frac{\partial h}{\partial Z} - 1 \right) + K \frac{\partial^2 h}{\partial Z^2}$$

or

$$\frac{C}{K} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial Z^2} + \frac{1}{K} \frac{\partial K}{\partial Z} \left(\frac{\partial h}{\partial Z} - 1 \right) \qquad \ldots(25)$$

Prediction (estimation of C_i^j and K_i^j):

From equation (25), by taking time step as $\frac{\Delta t}{2}$, we have

$$\frac{2C_i^j}{K_i^j} \frac{h_i^{j+1/2} - h_i^j}{\Delta t} = \frac{h_{i+1}^{j+1/2} - 2h_i^{j+1/2} + h_{i-1}^{j+1/2}}{(\Delta Z)^2}$$

$$+ \frac{1}{K_i^j} \frac{K_{i+1}^j - K_{i-1}^j}{2\,\Delta Z} \left[\frac{h_{i+1}^j - h_{i-1}^j}{2\,\Delta Z} - 1 \right]$$

Rearranging the terms, we get

$$- \frac{\Delta t}{(\Delta Z)^2} h_{i-1}^{j+1/2} + \left[\frac{2C_i^j}{K_i^j} + \frac{2\Delta t}{(\Delta Z)^2} \right] h_i^{j+1/2} - \frac{\Delta t}{(\Delta Z)^2} h_{i+1}^{j+1/2}$$

$$= \frac{2C_i^j}{K_i^j} h_i^j + \frac{1}{2} \frac{K_{i+1}^j - K_{i-1}^j}{K_i^j} \frac{\Delta t}{\Delta Z} \left[\frac{h_{i+1}^j - h_{i-1}^j}{2\Delta Z} - 1 \right]$$

...(26)

Correction (estimation of h_i^j) :

From equation (25) , by taking time step as Δt, we have

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}} \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{2} \left[\frac{h_{i+1}^{j+1} - 2h_i^{j+1} + h_{i-1}^{j+1}}{(\Delta Z)^2} + \frac{h_{i+1}^j - 2h_i^j + h_{i-1}^j}{(\Delta Z)^2} \right]$$

$$+ \frac{1}{K_i^{j+1/2}} \frac{K_{i+1}^{j+1/2} - K_{i-1}^{j+1/2}}{2\Delta Z} \left[\frac{h_{i+1}^{j+1/2} - h_{i-1}^{j+1/2}}{2\Delta Z} - 1 \right]$$

Rearranging the terms, we get

$$- \frac{1}{2} \frac{\Delta t}{(\Delta Z)^2} h_{i-1}^{j+1} + \left[\frac{C_i^{j+1/2}}{K_i^{j+1/2}} + \frac{\Delta t}{(\Delta Z)^2} \right] h_i^{j+1} - \frac{1}{2} \frac{\Delta t}{(\Delta Z)^2} h_{i+1}^{j+1}$$

$$= \frac{C_i^{j+1/2}}{K_i^{j+1/2}} h_i^j + \frac{1}{2} \frac{\Delta t}{(\Delta Z)^2} \left[h_{i+1}^j - 2h_i^j + h_{i-1}^j \right]$$

$$+ \frac{1}{2} \frac{K_{i+1}^{j+1/2} - K_{i-1}^{j+1/2}}{K_i^{j+1/2}} \frac{\Delta t}{\Delta Z} \left[\frac{h_{i+1}^{j+1/2} - h_{i-1}^{j+1/2}}{2\,\Delta Z} - 1 \right] \quad ...(27)$$

Model 4 : Equation (15) - Crank-Nicolson scheme (in terms of h)

$$C_i^{j+1/2} \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta Z} \left[K_{i+1/2}^{j+1/2} \left(\frac{h_{i+1}^{j+1/2} - h_i^{j+1/2}}{\Delta Z} - 1 \right) - K_{i-1/2}^{j+1/2} \left(\frac{h_i^{j+1/2} - h_{i-1}^{j+1/2}}{\Delta Z} - 1 \right) \right]$$

where,

$$h_i^{j+1/2} = \frac{h_i^j + h_i^{j+1}}{2} \quad ;$$

$$K_{i-1/2}^{j+1/2} = F_1 = \left(\frac{K_i^j \; K_{i-1}^j}{K_i^j + K_{i-1}^j} \right) + \left(\frac{K_i^{j+1} \; K_{i-1}^{j+1}}{K_i^{j+1} + K_{i-1}^{j+1}} \right);$$

$$K_{i+1/2}^{j+1/2} = F_2 = \left(\frac{K_i^j \; K_{i+1}^j}{K_i^j + K_{i+1}^j} \right) + \left(\frac{K_i^{j+1} \; K_{i+1}^{j+1}}{K_i^{j+1} + K_{i+1}^{j+1}} \right); \text{ and}$$

$$C_i^{j+1/2} = F_3 = \frac{C_i^j + C_i^{j+1}}{2} .$$

Rearranging the terms, we get

$$- \frac{1}{2} F_1 \frac{\Delta t}{(\Delta Z)^2} h_{i-1}^{j+1} + \left[F_3 + \frac{1}{2} (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2} \right] h_i^{j+1}$$

$$-\frac{1}{2} F_2 \frac{\Delta t}{(\Delta Z)^2} h_{i+1}^{j+1} = \frac{1}{2} F_1 \frac{\Delta t}{(\Delta Z)^2} h_{i-1}^{j} + \left[F_3 - \frac{1}{2} (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2} \right] h_i^j$$

$$+ \frac{1}{2} F_2 \frac{\Delta t}{(\Delta Z)^2} h_{i+1}^{j} + \left[(F_1 - F_2) \frac{\Delta t}{\Delta Z} \right] . \quad \ldots (28)$$

Model 5 : Equation (8) - Implicit scheme with explicit linearization (in terms of θ)

$$\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} = \frac{1}{\Delta Z} \left[D_{i+1/2}^{j+1/2} \left(\frac{\theta_{i+1}^{j+1} - \theta_i^{j+1}}{\Delta Z} \right) - D_{i-1/2}^{j+1/2} \left(\frac{\theta_i^{j+1} - \theta_{i-1}^{j+1}}{\Delta Z} \right) \right] - \left(\frac{K_{i+1/2}^{j+1/2} - K_{i-1/2}^{j+1/2}}{\Delta Z} \right)$$

where,

$$D_{i+1/2}^{j+1/2} = F_1 = \left(\frac{D_i^j \, D_{i+1}^j}{D_i^j + D_{i+1}^j} \right) + \left(\frac{D_i^{j+1} \, D_{i+1}^{j+1}}{D_i^{j+1} + D_{i+1}^{j+1}} \right) \; ;$$

$$D_{i-1/2}^{j+1/2} = F_2 = \left(\frac{D_i^j \, D_{i-1}^j}{D_i^j + D_{i-1}^j} \right) + \left(\frac{D_i^{j+1} \, D_{i-1}^{j+1}}{D_i^{j+1} + D_{i-1}^{j+1}} \right) \; ;$$

$$K_{i+1/2}^{j+1/2} = F_3 = \left(\frac{K_i^j \, K_{i+1}^j}{K_i^j + K_{i+1}^j} \right) + \left(\frac{K_i^{j+1} \, K_{i+1}^{j+1}}{K_i^{j+1} + K_{i+1}^{j+1}} \right) \; ; \text{ and}$$

$$K_{i-1/2}^{j+1/2} = F_4 = \left(\frac{K_i^j \, K_{i-1}^j}{K_i^j + K_{i-1}^j} \right) + \left(\frac{K_i^{j+1} \, K_{i-1}^{j+1}}{K_i^{j+1} + K_{i-1}^{j+1}} \right)$$

Rearranging the terms, we get

$$- F_2 \frac{\Delta t}{(\Delta Z)^2} \, \theta_{i-1}^{j+1} + \left[1 + (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2} \right] \theta_i^{j+1} - F_1 \frac{\Delta t}{(\Delta Z)^2} \, \theta_{i+1}^{j+1}$$

$$= \theta_i^j - (F_3 - F_4) \frac{\Delta t}{\Delta Z} \qquad \ldots (29)$$

Model 6 : Equation (8) - Crank-Nicolson scheme (in terms of θ)

$$\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} = \frac{1}{\Delta Z} \left[D_{i+1/2}^{j+1/2} \left(\frac{\theta_{i+1}^{j+1/2} - \theta_i^{j+1/2}}{\Delta Z} \right) \right.$$

$$\left. - D_{i-1/2}^{j+1/2} \left(\frac{\theta_i^{j+1/2} - \theta_{i-1}^{j+1/2}}{\Delta Z} \right) \right] - \left(\frac{K_{i+1/2}^{j+1/2} - K_{i-1/2}^{j+1/2}}{\Delta Z} \right)$$

where,

$$\theta_i^{j+1/2} = \frac{\theta_i^j + \theta_i^{j+1}}{2} ;$$

$$D_{i+1/2}^{j+1/2} = F_1 = \left(\frac{D_i^j \, D_{i+1}^j}{D_i^j + D_{i+1}^j} \right) + \left(\frac{D_i^{j+1} \, D_{i+1}^{j+1}}{D_i^{j+1} + D_{i+1}^{j+1}} \right) ;$$

$$D_{i-1/2}^{j+1/2} = F_2 = \left(\frac{D_i^j \, D_{i-1}^j}{D_i^j + D_{i-1}^j}\right) + \left(\frac{D_i^{j+1} \, D_{i-1}^{j+1}}{D_i^{j+1} + D_{i-1}^{j+1}}\right) ;$$

$$K_{i+1/2}^{j+1/2} = F_3 = \left(\frac{K_i^j \, K_{i+1}^j}{K_i^j + K_{i+1}^j}\right) + \left(\frac{K_i^{j+1} \, K_{i+1}^{j+1}}{K_i^{j+1} + K_{i+1}^{j+1}}\right) ; \text{ and}$$

$$K_{i-1/2}^{j+1/2} = F_4 = \left(\frac{K_i^j \, K_{i-1}^j}{K_i^j + K_{i-1}^j}\right) + \left(\frac{K_i^{j+1} \, K_{i-1}^{j+1}}{K_i^{j+1} + K_{i-1}^{j+1}}\right)$$

Rearranging the terms, we get

$$- \frac{1}{2} F_2 \frac{\Delta t}{(\Delta Z)^2} \theta_{i-1}^{j+1} + \left[1 + \frac{1}{2} (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2}\right] \theta_i^{j+1} - \frac{1}{2} F_1 \frac{\Delta t}{(\Delta Z)^2} \theta_{i+1}^{j+1}$$

$$= \frac{1}{2} F_2 \frac{\Delta t}{(\Delta Z)^2} \theta_{i-1}^j + \left[1 - \frac{1}{2} (F_1 + F_2) \frac{\Delta t}{(\Delta Z)^2}\right] \theta_i^j + \frac{1}{2} F_1 \frac{\Delta t}{(\Delta Z)^2} \theta_{i+1}^j$$

$$- (F_3 - F_4) \frac{\Delta t}{\Delta Z} \qquad \ldots (30)$$

Model 7 : Equation (8) - Implicit scheme with explicit linearization (in terms of θ) and Miller and Bresler relation for soil water diffusivity

The following relation for soil water diffusivity was reported by Miller and Bresler (1977):

$$D = \alpha' \, m^2 \, c^{\beta' S_r} \qquad \ldots (31)$$

in which α' and β' appear to be 'universal constants' both dimensionless; S_r is the dimensionless water content given by

$$S_r = \frac{\theta - \theta_r}{\theta_s - \theta_r} \qquad \ldots (32)$$

where,

θ is the actual water content ,
θ_r is the residual water content, and
θ_s is the water content at saturation.

The parameter, m in equation (31) is a unique constant for each soil. Its value can be estimated from observations of the visual wetting front by infiltration in an air dry soil :

$$m = \frac{X_f}{\sqrt{t}} \qquad \ldots (33)$$

where X_f is the distance of the wetting front at the time t.

According to Green and Ampt (1911), the position of wetting front at time t is given by

$$\frac{X_f}{\sqrt{t}} = \sqrt{\frac{2\ (H + H_f)\ K_s}{\theta_s - \theta_i}} \qquad \ldots (34)$$

where,

H = ponded depth of water ;
H = the capillary drive head ;
K = saturated hydraulic conductivity ; and
θ_i = initial water content.

The capillary drive head can be found from the available soil moisture and capillary pressure relationship in the following manner :

$$H_f = \int_0^{h_i} K_{rw}\ dh \qquad \ldots (35)$$

where,

$$K_{rw} = \frac{K}{K_s} = \text{relative hydraulic conductivity ;}$$

h = capillary pressure head; and

h_i = capillary pressure head corresponding to the initial water content θ_i, prevailing before the onset of infiltration.

By using equation (16) and numerical integration of equation (35), the value of H_f was found to be 20.489 . And by taking $H = 0$ in equation (34), the value of m $(= x_\xi / \sqrt{t})$ was obtained as 81.06680 .

The soil water diffusivity was calculated from equation (21) and substituted in equation (31) for different values of θ. The best fitted values of the constants α' and β' were arrived at 0.0054 and 4.7 respectively.

For this model, the same discretization scheme, as in model 5, was used (equation 29) and the values of diffusivity were computed from equation (31) as described above.

Remarks

The implicit methods (model 2, 3, 5 and 7) and Crank-Nicolson approximation (model 4 and 6) generally use much larger time steps than the explicit methods (model1), but their stability conditions have to be determined by trial and error, as they depend upon the nonlinearity of the equations. Also, the programming is more involved than for the explicit method. A tridiagonal system of equations results, which can be solved by direct elimination using Thomas' algorithm (Remson et al., 1971). Implicit evaluation of the coefficients at time ($t+1/2\ \Delta t$) (model 3, a method described by Douglas and Jones, 1963) requires that the tridiagonal system of equations be solved twice for each time step : first at time ($t + 1/2\ \Delta t$) to obtain values for K and C, then at time ($t+\Delta t$) to evaluate the pressure distribution ; on the other hand, the implicit models with explicit evaluation of the coefficient (model 2, 5 and 7) and Crank-Nicolson approximation (model 4 and 6) use only about half the computer time. The main advantages of using implicit methods and Crank-Nicolson approximation are their stability, even for fairly large time steps (5 seconds - model 2, 4, 5, 6 and 7 ; 2.5 seconds - model 3 instead of 0.4 second - model 1 in our case for the explicit model), and their flexibility for solving flow problems when saturated and unsaturated zones have to be considered simultaneously, since for C = 0 one simply has to solve the Laplace's equation.

The computer code, for discretization schemes used for the various models, has been written in FORTRAN IV and presented in appendix.

5.0 RESULTS

The seven different models were tested by comparing water content profiles calculated at given times by each of the models with results obtained from quasi-analytical solution of Philip. Using the functional relations given in equation (16) and (17) for characterizing the hydraulic properties of the soil, the water content profiles subject to the conditions

$$t<0 \quad Z\geq 0 \quad \theta_n = 0.10\ cm^3/cm^3$$

$$t\geq 0 \quad Z=0 \quad \theta_u = 0.267\ cm^3/cm^3 \text{ (model 1,2,3 and 7) or}$$

$$0.268\ cm^3/cm^3 \text{ (model 4,5 and 6)}$$

were determined with the seven models. The numerical computations were made with a depth interval ΔZ =1 cm, and a time step varying from 0.4 second to 5 seconds, the total simulation period being 0.8 hour. Table 1 presents the details for the various models.

Haverkamp et al. (1977) has reported the infiltration profiles at various times for infiltration in the sand (under consideration) obtained by quasi-analytical solution of Philip. In order to compare each method, numerical data of Philip's solution are given in table 2. For 'limited times' Philip's method gives at each time, t the depth, Z(t) which reaches a given water content, θ_i according to;

$$Z(t,\theta_i) = \emptyset(\theta_i)\, t^{1/2} + \aleph(\theta_i)t + \psi(\theta_i)\, t^{3/2}$$

$$+ \omega(\theta_i)\, t^2 + \ldots\ldots\ldots\ldots + f_n(\theta_i)\, t^{n/2} \qquad \ldots(36)$$

Table 1 - Basic Set-up for the Various Models

Reference	Type of discretization	Linearization	Independent variable in the governing equation	Depth interval (cm)	Time step (seconds)	Upper boundary condition (θ_u)
Model 1	Explicit	Explicit	h	1.0	0.4	0.267
Model 2	Implicit	Explicit	h	1.0	5.0	0.267
Model 3	Implicit	Implicit with prediction-correction	h	1.0	2.5	0.267
Model 4	Crank-Nicolson Scheme		h	1.0	5.0	0.268
Model 5	Implicit	Explicit	θ	1.0	5.0	0.268
Model 6	Crank-Nicolson Scheme		θ	1.0	5.0	0.268
Model 7	Implicit (Miller and Bresler relation for diffusivity)	Explicit	θ	1.0	5.0	0.267

Table 2 - Water Content Profiles determined with the Solution of Philip

Water content (θ)	Depth (Z)		
	t = 0.1 hour	t = 0.2 hour	t = 0.8 hour
0.2523	9.4	17.7	65.2
0.2356	12.0	20.7	69.2
0.2189	13.2	22.1	71.1
0.2021	14.1	23.1	72.3
0.1854	14.8	23.8	73.2
0.1686	15.3	24.5	74.0
0.1519	15.9	25.2	74.8
0.1351	16.5	25.9	75.7
0.1184	17.3	26.8	76.8
0.1016	19.5	29.5	78.6

The numerical models, on the other hand, calculates θ for a given value of Z. As a result, interpolations are necessary ,at a given stage of calculations to compare the results. The prediction of the water content profiles using Philip's method is only valid within the domain of convergence of the series (equation 36). To calculate the time for which the series would converge, Philip (1969, pp. 250) introduced a characteristic time of infiltration, t_{grav}, as :

$$t_{grav} = \left[\frac{S}{K_u - K_n} \right]^2 \qquad ...(37)$$

where, K_u is the hydraulic conductivity corresponding with θ_u and S is the sorptivity, defined as

$$S = \int_{\theta_n}^{\theta_u} \emptyset \, d\theta \qquad ...(38)$$

For the sand material, S was found to be 5.441 $cm/h^{1/2}$, and the characteristic time was t_{grav} = 0.16hour. Consequently for $t \leq 0.2$ hour, the water content profile could be calculated with equation (36). In our calculations the series was limited to four terms. To use more terms of the series would, according to Philip, 'extend the range of accurate results only by small amounts quite disproportionate to the extra labour involved'. For $t \geq 0.3$ hour, the profiles were calculated by an approximation of the 'infinite' profile, as proposed by Philip (1957, pp. 444). The power series solution (equation 36) and the asymptotic solution of the profile at infinity are expected to overlap.

Tables 3,4 and 5 present the comparison between water content profiles determined with the solution of Philip and the seven models at t = 0.1 hour, 0.2 hour and 0.8 hour respectively. In all cases the rate of advance of the water front is particularly well described. Some discrepancies are found between numerical water content profiles and quasi-analytical solution in the low water content domain. However, all numerical models yield comparable results, which are not significantly different from the quasi-analytical solution.

Table 3 - Comparison between Water Content Profiles at t = 0.1 hour

Depth (Z)	Water Content (θ)							
	Philip	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
10	0.2484	0.2484	0.2481	0.2491	0.2477	0.2479	0.2492	0.2391
11	0.2420	0.2423	0.2416	0.2434	0.2404	0.2404	0.2426	0.2324
12	0.2356	0.2338	0.2325	0.2354	0.2295	0.2291	0.2329	0.2243
13	0.2217	0.2215	0.2189	0.2240	0.2126	0.2111	0.2181	0.2143
14	0.2040	0.2039	0.1990	0.2076	0.1853	0.1833	0.1949	0.2017
15	0.1787	0.1796	0.1719	0.1847	0.1484	0.1503	0.1629	0.1852
16	0.1491	0.1514	0.1427	0.1566	0.1204	0.1244	0.1325	0.1633
17	0.1247	0.1270	0.1206	0.1302	0.1075	0.1103	0.1137	0.1366
18	0.1130	0.1118	0.1086	0.1131	0.1026	0.1040	0.1052	0.1154
19	0.1054	0.1046	0.1033	0.1050	0.1008	0.1015	0.1018	0.1051

Table 4 - Comparison between Water Content Profiles at t = 0.2 hour

Depth (Z)	Water Content (θ)							
	Philip	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
18	0.2506	0.2480	0.2479	0.2487	0.2491	0.2489	0.2499	0.2378
19	0.2451	0.2432	0.2430	0.2442	0.2442	0.2438	0.2452	0.2327
20	0.2395	0.2369	0.2364	0.2382	0.2375	0.2367	0.2389	0.2266
21	0.2320	0.2284	0.2273	0.2302	0.2281	0.2267	0.2301	0.2195
22	0.2201	0.2166	0.2146	0.2193	0.2146	0.2120	0.2174	0.2110
23	0.2038	0.2006	0.1972	0.2044	0.1951	0.1911	0.1993	0.2008
24	0.1806	0.1799	0.1746	0.1847	0.1686	0.1646	0.1747	0.1884
25	0.1567	0.1560	0.1498	0.1611	0.1403	0.1388	0.1476	0.1730
26	0.1332	0.1338	0.1285	0.1377	0.1198	0.1202	0.1255	0.1542
27	0.1172	0.1178	0.1144	0.1199	0.1088	0.1096	0.1121	0.1336
28	0.1109	0.1084	0.1067	0.1094	0.1037	0.1043	0.1053	0.1166
29	0.1047	0.1038	0.1030	0.1042	0.1015	0.1019	0.1023	0.1068

Table 5 - Comparison between Water Content Profiles at t = 0.8 hour

Depth (Z)	Water Content (θ)							
	Philip	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
66	0.2490	0.2470	0.2478	0.2482	0.2542	0.2533	0.2539	0.2354
67	0.2448	0.2427	0.2437	0.2442	0.2515	0.2503	0.2511	0.2308
68	0.2406	0.2372	0.2383	0.2392	0.2481	0.2465	0.2475	0.2254
69	0.2364	0.2300	0.2312	0.2327	0.2437	0.2414	0.2429	0.2193
70	0.2286	0.2204	0.2217	0.2240	0.2379	0.2346	0.2368	0.2121
71	0.2198	0.2076	0.2088	0.2124	0.2300	0.2253	0.2285	0.2038
72	0.2063	0.1909	0.1918	0.1972	0.2193	0.2122	0.2171	0.1940
73	0.1891	0.1707	0.1709	0.1781	0.2043	0.1943	0.2012	0.1824
74	0.1686	0.1493	0.1489	0.1566	0.1840	0.1716	0.1803	0.1686
75	0.1482	0.1305	0.1298	0.1361	0.1594	0.1477	0.1563	0.1525
76	0.1305	0.1170	0.1164	0.1205	0.1358	0.1280	0.1342	0.1354
77	0.1165	0.1088	0.1085	0.1107	0.1190	0.1150	0.1185	0.1202
78	0.1072	0.1044	0.1043	0.1053	0.1094	0.1076	0.1093	0.1099

Considering Philip's solution as standard, the average relative error in the water content distributions for various times was found to vary from 2% to 6% for all the numerical schemes. The best performance was obtained with model 3 (implicit solution with implicit linearization - 2.31%) and model 1 (explicit solution - 2.70%).

The agreement between infiltration profiles calculated with Philip's method and model 3 is very good . Model 1 (direct explicit) has the advantage that it is simple and easy to program. However, for reasons of stability the time step should be adjusted with equation (23). Since D is large in wet soil, this method is too slow when the soil is quite wet, notably during the infiltration. Another limitation is that when the soil becomes saturated, the right hand side of equation (15) is divided by very small C-values. In general the agreements obtained through methods solved in terms of h (model 1, 2 and 3) are better than the methods solved in terms of θ (model 5, 6 and 7) with the exception of model 4 (Crank-Nicolson scheme solved in terms of h). The discrepancies obtained through model 7, however, can be attributed to the use of empirical relationship for soil water diffusivity.

But the results could not be obtained with $\theta_u = \theta_s = 0.287$ since $dh/d\theta$ becomes infinity at saturation and the value of diffusivity is not properly represented by equations (19) and (20) for this situation. Haverkamp et al. (1977) compared the water content profiles with quasi-analytical solution of Philip, subject to the condition $\theta_u = 0.267$. However in the present study, better agreements were obtained with $\theta_u = 0.268$ in some cases (model 4, 5 and 6). Need arises, therefore, to test the validity of the presented numerical models under saturation conditions at the surface.

6.0 CONCLUSIONS

The close agreement between water content profiles obtained through quasi-analytical solution of Philip and those computed with seven different numerical schemes, as well as the close agreement among these seven schemes, indicate that numerical models are a reliable tool for predicting infiltration of water into soil. Considering computer time and stability problems, it appears that the implicit finite difference approximation with implicit or explicit evaluation of the hydraulic conductivity and water capacity functions has the widest range of applicability for predicting water movement in soil with both saturated and nonsaturated regions. For specific cases explicit models may be preferred, mainly because they are easy to program.

The excellent agreement between water content distributions obtained with the implicit model with implicit linearization of the hydraulic conductivity and water capacity functions and Philip's quasi-analytical solution shows that numerical solutions can yield very accurate results.

```
C        SOIL MOISTURE PREDICTION MODEL
C
C        EXPLICIT SCHEME SOLVED DIRECTLY
C        (MODEL 2 OF HAVERKAMP ET AL., 1977)
C
         DIMENSION THETA(90,2),HYDCON(90),CCC(90),H(90,2)
         OPEN(UNIT=1,FILE='EXPLIC.DAT',STATUS='OLD')
         OPEN(UNIT=2,FILE='EXPLIC.OUT',STATUS='NEW')
C
C        J REFERS TO TIME
C        I REFERS TO DEPTH
C        Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C        THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C        H = SOIL WATER PRESSURE (RELATIVE TO THE ATMOSPHERE)
C            EXPRESSED IN CM OF WATER
C        THETAR = RESIDUAL MOISTURE CONTENT
C        THETAS = MOISTURE CONTENT AT SATURATION
C        THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                (UPPER BOUNDARY CONDITION)
C        BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                       AND SOIL WATER PRESSURE RELATIONSHIP
C        BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                       SOIL WATER PRESSURE RELATIONSHIP
C        HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C        AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C        DELT = TIME STEP (HOURS)
C        DELZ = DEPTH INTERVAL (CM)
C        NTIME = NUMBER OF TIME STEPS
C        NNODE = NUMBER OF NODES
C        CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C
         READ(1,11)THETAR,THETAS,THETAU
11       FORMAT(3F12.3)
         READ(1,12)BETA1,BETA2
12       FORMAT(2F12.3)
         READ(1,13)CONA,ALPHA
13       FORMAT(2F12.3)
         READ(1,14)AKS
14       FORMAT(F12.3)
         READ(1,15)DELT,DELZ
15       FORMAT(F12.8,F12.3)
         READ(1,16)NTIME,NNODE
16       FORMAT(I4,6X,I4)
C
C        READING OF INITIAL CONDITIONS
C
         READ(1,17)(THETA(I,1),I=1,NNODE)
17       FORMAT(5F10.4)
C
         WRITE(2,18)
18       FORMAT(2X,'Soil Moisture Prediction Model (EXPLIC)')
         WRITE(2,19)
19       FORMAT(2X,'Explicit Scheme Solved Directly')
         WRITE(2,21)
21       FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
         WRITE(2,31)THETAR,THETAS,THETAU
31       FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
         WRITE(2,22)
22       FORMAT(2X,'BETA1',10X,'BETA2')
```

```
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I4,9X,I4)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
      DO 100 I=1,NNODE
      H(I,1)=-(ALPHA*(THETAS-THETA(I,1))/(THETA(I,1)
     1 -THETAR))**(1./BETA2)
100   CONTINUE
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      THETA(1,1)=THETAU
      THETA(1,2)=THETA(1,1)
      H(1,1)=-(ALPHA*(THETAS-THETA(1,1))/(THETA(1,1)
     1 -THETAR))**(1./BETA2)
      H(1,2)=H(1,1)
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      THETA(NNODE,2)=THETA(NNODE,1)
      H(NNODE,2)=H(NNODE,1)
C
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
      DO 200 JJ=2,NTIME
C
      DO 300 I=1,NNODE
      TERM1=(THETA(I,1)-THETAR)/E2
      HYDCON(I)=E4*TERM1**E1/(CONA*TERM1**E1+E3*(1.-TERM1)**E1)
      CCC(I)=1./(E5*E2)*(THETAS-THETA(I,1))**(-1./BETA2+1.)*
     1 (THETA(I,1)-THETAR)**(1./BETA2+1.)
300   CONTINUE
C
```

```
      DO 400 I=2,NNODE-1
      TERM2=DELT/DELZ*( (HYDCON(I+1)+HYDCON(I)) *.5*
     1  ( (H(I+1,1)-H(I,1))/DELZ -1.)-
     2  (HYDCON(I)+HYDCON(I-1)) *.5*
     3  ( (H(I,1)-H(I-1,1))/DELZ -1.))
      H(I,2)=TERM2/CCC(I)+H(I,1)
      THETA(I,2)=ALPHA*(THETAS-THETAR)/(ALPHA+ABS(H(I,2))**BETA2)+
     1  THETAR
400   CONTINUE
C
      IF(JJ.EQ.2) GO TO 500
      IF(JJ.EQ.450) GO TO 500
      IF(JJ.EQ.900) GO TO 500
      IF(JJ.EQ.1800) GO TO 500
      IF(JJ.EQ.2700) GO TO 500
      IF(JJ.EQ.3600) GO TO 500
      IF(JJ.EQ.7200) GO TO 500
      GO TO 600
500   CONTINUE
      WRITE(2,41)JJ
41    FORMAT(/2X,'TIME STEP = ',I4)
      WRITE(2,42)(THETA(I,2),I=1,NNODE)
42    FORMAT(5F10.4)
600   CONTINUE
      DO 700 I=2,NNODE-1
      H(I,1)=H(I,2)
700   THETA(I,1)=THETA(I,2)
200   CONTINUE
      STOP
      END
```

```
C        SOIL MOISTURE PREDICTION MODEL
C
C        IMPLICIT SCHEME WITH EXPLICIT LINEARIZATION
C        (MODEL 3 OF HAVERKAMP ET AL., 1977)
C
         DIMENSION SUB(90),SUP(90),DIAG(90),B(90)
         DIMENSION H(90,577),CCC(90,577)
         DIMENSION THETA(90,577),HYDCON(90,577)
         OPEN(UNIT=1,FILE='IMPLIC1.DAT',STATUS='OLD')
         OPEN(UNIT=2,FILE='IMPLIC1.OUT',STATUS='NEW')
C
C        J REFERS TO TIME
C        I REFERS TO DEPTH
C        Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C        THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C        H = SOIL WATER PRESSURE (RELATIVE TO THE ATMOSPHERE)
C            EXPRESSED IN CM OF WATER
C        THETAR = RESIDUAL MOISTURE CONTENT
C        THETAS = MOISTURE CONTENT AT SATURATION
C        THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                 (UPPER BOUNDARY CONDITION)
C        BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                       AND SOIL WATER PRESSURE RELATIONSHIP
C        BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                       SOIL WATER PRESSURE RELATIONSHIP
C        HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C        AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C        DELT = TIME STEP (HOURS)
C        DELZ = DEPTH INTERVAL (CM)
C        NTIME = NUMBER OF TIME STEPS
C        NNODE = NUMBER OF NODES
C        CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C
         READ(1,11)THETAR,THETAS,THETAU
11       FORMAT(3F12.3)
         READ(1,12)BETA1,BETA2
12       FORMAT(2F12.3)
         READ(1,13)CONA,ALPHA
13       FORMAT(2F12.3)
         READ(1,14)AKS
14       FORMAT(F12.3)
         READ(1,15)DELT,DELZ
15       FORMAT(F12.8,F12.3)
         READ(1,16)NTIME,NNODE
16       FORMAT(I3,8X,I3)
C
C        READING OF INITIAL CONDITIONS
C
         READ(1,17)(THETA(I,1),I=1,NNODE)
17       FORMAT(5F10.4)
C
         WRITE(2,18)
18       FORMAT(2X,'Soil Moisture Prediction Model (IMPLIC1)')
         WRITE(2,19)
19       FORMAT(2X,'Implicit Scheme with Explicit Linearization')
         WRITE(2,21)
21       FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
         WRITE(2,31)THETAR,THETAS,THETAU
31       FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
```

```
      WRITE(2,22)
22    FORMAT(2X,'BETA1',10X,'BETA2')
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I3,12X,I3)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
      DO 100 I=1,NNODE
      H(I,1)=-(ALPHA*(THETAS-THETA(I,1))/(THETA(I,1)
     1 -THETAR))**(1./BETA2)
100   CONTINUE
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      DO 200 J=1,NTIME
      THETA(1,J)=THETAU
      H(1,J)=-(ALPHA*(THETAS-THETA(1,J))/(THETA(1,J)
     1 -THETAR))**(1./BETA2)
200   CONTINUE
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      DO 300 J=1,NTIME
      THETA(NNODE,J)=THETA(NNODE,1)
      H(NNODE,J)=-(ALPHA*(THETAS-THETA(NNODE,J))/(THETA(NNODE,J)
     1 -THETAR))**(1./BETA2)
300   CONTINUE
C
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
      DO 400 J=2,NTIME
C
      DO 500 I=1,NNODE
      TERM1=(THETA(I,J-1)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*((1.-TERM1))**E1)
      CCC(I,J-1)=1./(E5*E2)*(THETAS-THETA(I,J-1) )**(-1./BETA2+1.)*
     1  ( THETA(I,J-1)-THETAR ) **(1./BETA2+1.)
500   CONTINUE
C
```

```
      DO 600 I=2,NNODE-1
      F1=(HYDCON(I,J-1)+HYDCON(I+1,J-1))*0.5
      F2=(HYDCON(I,J-1)+HYDCON(I-1,J-1))*0.5
      DIAG(I-1)=CCC(I,J-1)+(F1+F2)*DELT/DELZ**2
      SUB(I-1)=-F2*DELT/DELZ**2
      SUP(I-1)=-F1*DELT/DELZ**2
      B(I-1)=CCC(I,J-1)*H(I,J-1)+(F2-F1)*DELT/DELZ
600   CONTINUE
C
      B(1)=B(1)-SUB(1)*H(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*H(NNODE,J)
      DO 700I=1,NNODE-3
700   SUB(I)=SUB(I+1)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      DO 800 I=1,NNODE-2
800   H(I+1,J)=B(I)
      DO 900 I=2,NNODE-1
      THETA(I,J)=ALPHA*(THETAS-THETAR)/(ALPHA+ABS(H(I,J))**BETA2)+
     1 THETAR
900   CONTINUE
      IF (J.EQ.2) GO TO 111
      IF (J.EQ.36) GO TO 111
      IF (J.EQ.72) GO TO 111
      IF (J.EQ.144) GO TO 111
      IF (J.EQ.216) GO TO 111
      IF (J.EQ.288) GO TO 111
      IF (J.EQ.576) GO TO 111
      GO TO 222
111   CONTINUE
      WRITE(2,41)J
41    FORMAT(/2X,'TIME STEP = ',I4)
      WRITE(2,42)(THETA(I,J),I=1,NNODE)
42    FORMAT(5F10.4)
222   CONTINUE
400   CONTINUE
      STOP
      END
C
      SUBROUTINE TRID(M,SUP,SUB,DIAG,B)
      DIMENSION SUP(90),SUB(90),DIAG(90),B(90)
      N=M
      NN=N-1
      SUP(1)=SUP(1)/DIAG(1)
      B(1)=B(1)/DIAG(1)
      DO 51 I=2,N
      II=I-1
      DIAG(I)=DIAG(I)-SUP(II)*SUB(II)
      IF (I.EQ.N) GO TO 51
      SUP(I)=SUP(I)/DIAG(I)
51    B(I)=(B(I)-SUB(II)*B(II))/DIAG(I)
      DO 52 K=1,NN
      I=N-K
52    B(I)=B(I)-SUP(I)*B(I+1)
      RETURN
      END
```

```
C       SOIL MOISTURE PREDICTION MODEL
C
C       IMPLICIT SCHEME WITH IMPLICIT LINEARIZATION (PREDICTION - CORRECTION)
C       (MODEL 4 OF HAVERKAMP ET AL., 1977)
C
        DIMENSION SUB(90),SUP(90),DIAG(90),B(90)
        DIMENSION H(90,1153),CCC(90,1153)
        DIMENSION THETA(90,1153),HYDCON(90,1153)
        DIMENSION HP(90,1153),THETAP(90,1153)
        OPEN(UNIT=1,FILE='IMPLIC2.DAT',STATUS='OLD')
        OPEN(UNIT=2,FILE='IMPLIC2.OUT',STATUS='NEW')
C
C       J REFERS TO TIME
C       I REFERS TO DEPTH
C       Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C       THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C       H = SOIL WATER PRESSURE (RELATIVE TO THE ATMOSPHERE)
C           EXPRESSED IN CM OF WATER
C       THETAR = RESIDUAL MOISTURE CONTENT
C       THETAS = MOISTURE CONTENT AT SATURATION
C       THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                (UPPER BOUNDARY CONDITION)
C       BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                      AND SOIL WATER PRESSURE RELATIONSHIP
C       BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                      SOIL WATER PRESSURE RELATIONSHIP
C       HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C       AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C       DELT = TIME STEP (HOURS)
C       DELZ = DEPTH INTERVAL (CM)
C       NTIME = NUMBER OF TIME STEPS
C       NNODE = NUMBER OF NODES
C       CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C
        READ(1,11)THETAR,THETAS,THETAU
11      FORMAT(3F12.3)
        READ(1,12)BETA1,BETA2
12      FORMAT(2F12.3)
        READ(1,13)CONA,ALPHA
13      FORMAT(2F12.3)
        READ(1,14)AKS
14      FORMAT(F12.3)
        READ(1,15)DELT,DELZ
15      FORMAT(F12.8,F12.3)
        READ(1,16)NTIME,NNODE
16      FORMAT(I4,6X,I4)
C
C       READING OF INITIAL CONDITIONS
C
        READ(1,17)(THETA(I,1),I=1,NNODE)
17      FORMAT(5F10.4)
C
        WRITE(2,18)
18      FORMAT(2X,'Soil Moisture Prediction Model (IMPLIC2)')
        WRITE(2,19)
19      FORMAT(2X,'Implicit Scheme with Implicit Linearization')
        WRITE(2,20)
20      FORMAT(2X,'(Prediction - Correction)')
        WRITE(2,21)
21      FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
```

```
      WRITE(2,31)THETAR,THETAS,THETAU
31    FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
      WRITE(2,22)
22    FORMAT(2X,'BETA1',10X,'BETA2')
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I4,9X,I4)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
      DO 100 I=1,NNODE
      H(I,1)=-(ALPHA*(THETAS-THETA(I,1))/(THETA(I,1)
     1 -THETAR))**(1./BETA2)
100   CONTINUE
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      DO 200 J=1,NTIME
      THETA(1,J)=THETAU
      THETAP(1,J)=THETAU
      H(1,J)=-(ALPHA*(THETAS-THETA(1,J))/(THETA(1,J)
     1 -THETAR))**(1./BETA2)
      HP(1,J)=H(1,J)
200   CONTINUE
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      DO 300 J=1,NTIME
      THETA(NNODE,J)=THETA(NNODE,1)
      THETAP(NNODE,J)=THETA(NNODE,1)
      H(NNODE,J)=-(ALPHA*(THETAS-THETA(NNODE,J))/(THETA(NNODE,J)
     1 -THETAR))**(1./BETA2)
      HP(NNODE,J)=H(NNODE,J)
300   CONTINUE
C
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
```

```
      DO 400 J=2,NTIME
C
      DO 500 I=1,NNODE
      TERM1=(THETA(I,J-1)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*((1.-TERM1))**E1)
      CCC(I,J-1)=1./(E5*E2)*(THETAS-THETA(I,J-1) )**(-1./BETA2+1.)*
     1 ( THETA(I,J-1)-THETAR ) **(1./BETA2+1.)
500   CONTINUE
C
      DO 600 I=2,NNODE-1
      DIAG(I-1)=2.*CCC(I,J-1)/HYDCON(I,J-1)+2.*DELT/DELZ**2
      SUB(I-1)=-DELT/DELZ**2
      SUP(I-1)=-DELT/DELZ**2
      B(I-1)=2.*CCC(I,J-1)/HYDCON(I,J-1)*H(I,J-1)+DELT/DELZ*.5
     1 *(HYDCON(I+1,J-1)-HYDCON(I-1,J-1))/HYDCON(I,J-1)*((H(I+1,J-1)-
     2 H(I-1,J-1))/(2.*DELZ)-1.)
600   CONTINUE
C
      B(1)=B(1)-SUB(1)*H(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*H(NNODE,J)
      DO 700 I=1,NNODE-3
700   SUB(I)=SUB(I+1)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      DO 800 I=1,NNODE-2
800   HP(I+1,J)=B(I)
      DO 900 I=2,NNODE-1
      THETAP(I,J)=ALPHA*(THETAS-THETAR)/(ALPHA+ABS(HP(I,J))**
     1 BETA2)+THETAR
900   CONTINUE
C
      DO 1000 I=1,NNODE
      TERM1=(THETAP(I,J)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*((1.-TERM1))**E1)
      CCC(I,J-1)=1./(E5*E2)*(THETAS-THETAP(I,J) )**(-1./BETA2+1.)*
     1 ( THETAP(I,J)-THETAR ) **(1./BETA2+1.)
1000  CONTINUE
C
      DO 1100 I=2,NNODE-1
      DIAG(I-1)=CCC(I,J-1)/HYDCON(I,J-1)+DELT/DELZ**2
      SUB(I-1)=-DELT/DELZ**2*.5
      SUP(I-1)=-DELT/DELZ**2*.5
      B(I-1)=CCC(I,J-1)/HYDCON(I,J-1)*H(I,J-1)+DELT/DELZ*.5
     1 *(HYDCON(I+1,J-1)-HYDCON(I-1,J-1))/HYDCON(I,J-1)*((HP(I+1,J)-
     2 HP(I-1,J))/(2.*DELZ)-1.)+DELT/DELZ**2*.5*(H(I+1,J-1)-2.*
     3 H(I,J-1)+H(I-1,J-1))
1100  CONTINUE
C
      B(1)=B(1)-SUB(1)*H(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*H(NNODE,J)
      DO 1200 I=1,NNODE-3
1200  SUB(I)=SUB(I+1)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      DO 1300 I=1,NNODE-2
1300  H(I+1,J)=B(I)
      DO 1400 I=2,NNODE-1
      THETA(I,J)=ALPHA*(THETAS-THETAR)/(ALPHA+ABS(H(I,J))**BETA2)+
     1 THETAR
1400  CONTINUE
```

```
      IF (J.EQ.2) GO TO 111
      IF (J.EQ.72) GO TO 111
      IF (J.EQ.144) GO TO 111
      IF (J.EQ.288) GO TO 111
      IF (J.EQ.432) GO TO 111
      IF (J.EQ.576) GO TO 111
      IF (J.EQ.1152) GO TO 111
      GO TO 222
111   CONTINUE
      WRITE(2,41)J
41    FORMAT(/2X,'TIME STEP = ',I4)
      WRITE(2,42)(THETA(I,J),I=1,NNODE)
42    FORMAT(5F10.4)
222   CONTINUE
400   CONTINUE
      STOP
      END
C
      SUBROUTINE TRID(M,SUP,SUB,DIAG,B)
      DIMENSION SUP(90),SUB(90),DIAG(90),B(90)
      N=M
      NN=N-1
      SUP(1)=SUP(1)/DIAG(1)
      B(1)=B(1)/DIAG(1)
      DO 51 I=2,N
      II=I-1
      DIAG(I)=DIAG(I)-SUP(II)*SUB(II)
      IF (I.EQ.N) GO TO 51
      SUP(I)=SUP(I)/DIAG(I)
51    B(I)=(B(I)-SUB(II)*B(II))/DIAG(I)
      DO 52 K=1,NN
      I=N-K
52    B(I)=B(I)-SUP(I)*B(I+1)
      RETURN
      END
```

```
C        SOIL MOISTURE PREDICTION MODEL
C
C        CRANK-NICOLSON SCHEME
C
         DIMENSION SUB(90),SUP(90),DIAG(90),B(90)
         DIMENSION H(90,577),CCC(90,577)
         DIMENSION THETA(90,577),HYDCON(90,577)
         OPEN(UNIT=1,FILE='CRANK.DAT',STATUS='OLD')
         OPEN(UNIT=2,FILE='CRANK.OUT',STATUS='NEW')
C
C        J REFERS TO TIME
C        I REFERS TO DEPTH
C        Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C        THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C        H = SOIL WATER PRESSURE (RELATIVE TO THE ATMOSPHERE)
C            EXPRESSED IN CM OF WATER
C        THETAR = RESIDUAL MOISTURE CONTENT
C        THETAS = MOISTURE CONTENT AT SATURATION
C        THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                 (UPPER BOUNDARY CONDITION)
C        BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                       AND SOIL WATER PRESSURE RELATIONSHIP
C        BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                       SOIL WATER PRESSURE RELATIONSHIP
C        HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C        AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C        DELT = TIME STEP (HOURS)
C        DELZ = DEPTH INTERVAL (CM)
C        NTIME = NUMBER OF TIME STEPS
C        NNODE = NUMBER OF NODES
C        CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C
         READ(1,11)THETAR,THETAS,THETAU
11       FORMAT(3F12.3)
         READ(1,12)BETA1,BETA2
12       FORMAT(2F12.3)
         READ(1,13)CONA,ALPHA
13       FORMAT(2F12.3)
         READ(1,14)AKS
14       FORMAT(F12.3)
         READ(1,15)DELT,DELZ
15       FORMAT(F12.8,F12.3)
         READ(1,16)NTIME,NNODE
16       FORMAT(I3,8X,I3)
C
C        READING OF INITIAL CONDITIONS
C
         READ(1,17)(THETA(I,1),I=1,NNODE)
17       FORMAT(5F10.4)
C
         WRITE(2,18)
18       FORMAT(2X,'Soil Moisture Prediction Model (CRANK)')
         WRITE(2,19)
19       FORMAT(2X,'Crank-Nicolson Scheme')
         WRITE(2,21)
21       FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
         WRITE(2,31)THETAR,THETAS,THETAU
31       FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
```

```
      WRITE(2,22)
22    FORMAT(2X,'BETA1',10X,'BETA2')
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I3,12X,I3)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
      DO 100 I=1,NNODE
      H(I,1)=-(ALPHA*(THETAS-THETA(I,1))/(THETA(I,1)
     1 -THETAR))**(1./BETA2)
100   CONTINUE
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      DO 200 J=1,NTIME
      THETA(1,J)=THETAU
      H(1,J)=-(ALPHA*(THETAS-THETA(1,J))/(THETA(1,J)
     1 -THETAR))**(1./BETA2)
200   CONTINUE
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      DO 300 J=1,NTIME
      THETA(NNODE,J)=THETA(NNODE,1)
      H(NNODE,J)=-(ALPHA*(THETAS-THETA(NNODE,J))/(THETA(NNODE,J)
     1 -THETAR))**(1./BETA2)
300   CONTINUE
C
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
      DO 400 J=2,NTIME
C
      DO 500 I=2,NNODE-1
      THETA(I,J)=THETA(I,J-1)
      ITER=1
500   CONTINUE
600   CONTINUE
C
```

```
      DO 700 I=1,NNODE
      TERM1=(THETA(I,J-1)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*((1.-TERM1))**E1)
      TERM1=(THETA(I,J)-THETAR)/E2
      HYDCON(I,J)=E4*TERM1**E1/(CONA*TERM1**E1+E3*((1.-TERM1))**E1)
      CCC(I,J-1)=1./(E5*E2)*(THETAS-THETA(I,J-1) )**(-1./BETA2+1.)*
     1 ( THETA(I,J-1)-THETAR ) **(1./BETA2+1.)
      CCC(I,J)=1./(E5*E2)*(THETAS-THETA(I,J) )**(-1./BETA2+1.)*
     1 ( THETA(I,J)-THETAR ) **(1./BETA2+1.)
700   CONTINUE
C
      DO 800 I=2,NNODE-1
      F1=(HYDCON(I,J-1)*HYDCON(I-1,J-1)/(HYDCON(I,J-1)+HYDCON(I-1,J-1)))+
     1 (HYDCON(I,J)*HYDCON(I-1,J)/(HYDCON(I,J)+HYDCON(I-1,J)))
      F2=(HYDCON(I,J-1)*HYDCON(I+1,J-1)/(HYDCON(I,J-1)+HYDCON(I+1,J-1)))+
     1 (HYDCON(I,J)*HYDCON(I+1,J)/(HYDCON(I,J)+HYDCON(I+1,J)))
      F3=0.5*(CCC(I,J-1)+CCC(I,J))
      DIAG(I-1)=F3+0.5*(F1+F2)*DELT/DELZ**2
      SUB(I-1)=-0.5*F1*DELT/DELZ**2
      SUP(I-1)=-0.5*F2*DELT/DELZ**2
      B(I-1)=(0.5*F1*DELT/DELZ**2)*H(I-1,J-1)+(F3-0.5*(F1+F2)*DELT/DELZ**2)*
     1 H(I,J-1)+(0.5*F2*DELT/DELZ**2)*H(I+1,J-1)+(F1-F2)*DELT/DELZ
800   CONTINUE
C
      B(1)=B(1)-SUB(1)*H(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*H(NNODE,J)
      DO 900I=1,NNODE-3
900   SUB(I)=SUB(I+1)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      SUM=0.
      DO 1000 I=1,NNODE-2
1000  SUM=SUM+(H(I+1,J)-B(I))**2
      DO 1100 I=1,NNODE-2
1100  H(I+1,J)=B(I)
      ITER=ITER+1
      IF(ITER.GT.10)GO TO 1200
      IF(SUM.GT.0.0001)GO TO 600
1200  CONTINUE
      DO 1300 I=2,NNODE-1
      THETA(I,J)=ALPHA*(THETAS-THETAR)/(ALPHA+ABS(H(I,J))**BETA2)+
     1 THETAR
1300  CONTINUE
      IF (J.EQ.2) GO TO 111
      IF (J.EQ.36) GO TO 111
      IF (J.EQ.72) GO TO 111
      IF (J.EQ.144) GO TO 111
      IF (J.EQ.216) GO TO 111
      IF (J.EQ.288) GO TO 111
      IF (J.EQ.576) GO TO 111
      GO TO 222
111   CONTINUE
      WRITE(2,41)
41    FORMAT(/2X,'TIME STEP',7X,'ITERATION')
      WRITE(2,42)J,ITER
42    FORMAT(2X,I5,14X,I2)
      WRITE(2,43)(THETA(I,J),I=1,NNODE)
43    FORMAT(5F10.4)
222   CONTINUE
400   CONTINUE
      STOP
      END
C
```

```
      SUBROUTINE TRID(M,SUP,SUB,DIAG,B)
      DIMENSION SUP(90),SUB(90),DIAG(90),B(90)
      N=M
      NN=N-1
      SUP(1)=SUP(1)/DIAG(1)
      B(1)=B(1)/DIAG(1)
      DO 51 I=2,N
      II=I-1
      DIAG(I)=DIAG(I)-SUP(II)*SUB(II)
      IF (I.EQ.N) GO TO 51
      SUP(I)=SUP(I)/DIAG(I)
51    B(I)=(B(I)-SUB(II)*B(II))/DIAG(I)
      DO 52 K=1,NN
      I=N-K
52    B(I)=B(I)-SUP(I)*B(I+1)
      RETURN
      END
```

```
C         SOIL MOISTURE PREDICTION MODEL
C
C         RICHARDS EQUATION SOLVED IN TERMS OF THETA
C         (IMPLICIT SCHEME AND CRANK-NICOLSON SCHEME)
C         FOR THE VALUES OF HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY :
C         HARMONIC MEANS (WITH RESPECT TO SPACE) AND ARITHMETIC MEANS
C         (WITH RESPECT TO TIME) HAVE BEEN TAKEN.
C
          DIMENSION SUB(90),SUP(90),DIAG(90),B(90)
          DIMENSION DTHETA(90,577)
          DIMENSION THETA(90,577),HYDCON(90,577)
          OPEN(UNIT=1,FILE='IMPLIC4.DAT',STATUS='OLD')
          OPEN(UNIT=2,FILE='IMPLIC4.OUT',STATUS='NEW')
C
C         W = 1.0 INDICATES IMPLICIT SCHEME
C         W = 0.5 INDICATES CRANK-NICOLSON SCHEME
C         J REFERS TO TIME
C         I REFERS TO DEPTH
C         Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C         THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C         THETAR = RESIDUAL MOISTURE CONTENT
C         THETAS = MOISTURE CONTENT AT SATURATION
C         THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                  (UPPER BOUNDARY CONDITION)
C         BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                        AND SOIL WATER PRESSURE RELATIONSHIP
C         BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                        SOIL WATER PRESSURE RELATIONSHIP
C         HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C         AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C         DELT = TIME STEP (HOURS)
C         DELZ = DEPTH INTERVAL (CM)
C         NTIME = NUMBER OF TIME STEPS
C         NNODE = NUMBER OF NODES
C         CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C         DTHETA = SOIL WATER DIFFUSIVITY DEFINED AS HYDCON/CCC
C
          READ(1,10)W
10        FORMAT(F3.1)
          READ(1,11)THETAR,THETAS,THETAU
11        FORMAT(3F12.3)
          READ(1,12)BETA1,BETA2
12        FORMAT(2F12.3)
          READ(1,13)CONA,ALPHA
13        FORMAT(2F12.3)
          READ(1,14)AKS
14        FORMAT(F12.3)
          READ(1,15)DELT,DELZ
15        FORMAT(F12.8,F12.3)
          READ(1,16)NTIME,NNODE
16        FORMAT(I3,8X,I3)
C
C         READING OF INITIAL CONDITIONS
C
          READ(1,17)(THETA(I,1),I=1,NNODE)
17        FORMAT(5F10.4)
C
```

```
      WRITE(2,18)
18    FORMAT(2X,'Soil Moisture Prediction Model (IMPLIC4)')
      WRITE(2,19)
19    FORMAT(2X,'Richards Equation solved in terms of Theta')
      WRITE(2,20)W
20    FORMAT(2X,'W = ',F3.1)
      WRITE(2,21)
21    FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
      WRITE(2,31)THETAR,THETAS,THETAU
31    FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
      WRITE(2,22)
22    FORMAT(2X,'BETA1',10X,'BETA2')
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I3,11X,I3)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      DO 100 J=1,NTIME
100   THETA(1,J)=THETAU
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      DO 200 J=1,NTIME
200   THETA(NNODE,J)=THETA(NNODE,1)
C
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
      DO 300 J=2,NTIME
C
      DO 400 I=2,NNODE-1
      THETA(I,J)=THETA(I,J-1)
      ITER=1
400   CONTINUE
500   CONTINUE
C
```

```
      DO 600 I=1,NNODE
      TERM1=(THETA(I,J-1)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*(1.-TERM1)**E1)
      TERM1=(THETA(I,J)-THETAR)/E2
      HYDCON(I,J)=E4*TERM1**E1/(CONA*TERM1**E1+E3*(1.-TERM1)**E1)
      TERM2=E5*E2*(THETAS-THETA(I,J-1))**(1./BETA2-1.)*(THETA(I,J-1)
     1 -THETAR)**(-1./BETA2-1.)
      DTHETA(I,J-1)=HYDCON(I,J-1)*TERM2
      TERM2=E5*E2*(THETAS-THETA(I,J))**(1./BETA2-1.)*(THETA(I,J)
     1 -THETAR)**(-1./BETA2-1.)
      DTHETA(I,J)=HYDCON(I,J)*TERM2
600   CONTINUE
C
      DO 700 I=2,NNODE-1
      F11=2.*DTHETA(I+1,J)*DTHETA(I,J)/(DTHETA(I+1,J)+DTHETA(I,J))
      F12=2.*DTHETA(I+1,J-1)*DTHETA(I,J-1)/(DTHETA(I+1,J-1)+
     1 DTHETA(I,J-1))
      F1=(F11+F12)*0.5
      F21=2.*DTHETA(I,J)*DTHETA(I-1,J)/(DTHETA(I,J)+DTHETA(I-1,J))
      F22=2.*DTHETA(I,J-1)*DTHETA(I-1,J-1)/(DTHETA(I,J-1)+
     1 DTHETA(I-1,J-1))
      F2=(F21+F22)*0.5
      F31=2.*HYDCON(I+1,J)*HYDCON(I,J)/(HYDCON(I+1,J)+HYDCON(I,J))
      F32=2.*HYDCON(I+1,J-1)*HYDCON(I,J-1)/(HYDCON(I+1,J-1)+
     1 HYDCON(I,J-1))
      F3=(F31+F32)*0.5
      F41=2.*HYDCON(I,J)*HYDCON(I-1,J)/(HYDCON(I,J)+HYDCON(I-1,J))
      F42=2.*HYDCON(I,J-1)*HYDCON(I-1,J-1)/(HYDCON(I,J-1)+
     1 HYDCON(I-1,J-1))
      F4=(F41+F42)*0.5
      DIAG(I-1)=1.+W*(F1+F2)*DELT/DELZ**2
      SUB(I-1)=-W*F2*DELT/DELZ**2
      SUP(I-1)=-W*F1*DELT/DELZ**2
      B(I-1)=THETA(I,J-1)-DELT/DELZ*(F3-F4)+(1.-W)*(F1*DELT/DELZ**2*
     1 (THETA(I+1,J-1)-THETA(I,J-1)))-(1.-W)*(F2*DELT/DELZ**2*
     2 (THETA(I,J-1)-THETA(I-1,J-1)))
700   CONTINUE
C
      B(1)=B(1)-SUB(1)*THETA(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*THETA(NNODE,J)
      DO 800 I=2,NNODE-1
800   SUB(I-1)=SUB(I)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      SUM=0.
      DO 900 I=1,NNODE-2
900   SUM=SUM+(THETA(I+1,J)-B(I))**2
      DO 1000 I=1,NNODE-2
1000  THETA(I+1,J)=B(I)
      ITER=ITER+1
      IF(ITER.GT.10) GO TO 1100
      IF(SUM.GT.0.0001) GO TO 500
1100  CONTINUE
      IF (J.EQ.2) GO TO 111
      IF (J.EQ.36) GO TO 111
      IF (J.EQ.72) GO TO 111
      IF (J.EQ.144) GO TO 111
      IF (J.EQ.216) GO TO 111
      IF (J.EQ.288) GO TO 111
      IF (J.EQ.576) GO TO 111
      GO TO 222
111   CONTINUE
```

```
      WRITE(2,41)
41    FORMAT(/2X,'TIME STEP',7X,'ITERATION')
      WRITE(2,42)J,ITER
42    FORMAT(2X,I5,14X,I2)
      WRITE(2,43)(THETA(I,J),I=1,NNODE)
43    FORMAT(5F10.4)
222   CONTINUE
300   CONTINUE
      STOP
      END
C
      SUBROUTINE TRID(M,SUP,SUB,DIAG,B)
      DIMENSION SUP(90),SUB(90),DIAG(90),B(90)
      N=M
      NN=N-1
      SUP(1)=SUP(1)/DIAG(1)
      B(1)=B(1)/DIAG(1)
      DO 51 I=2,N
      II=I-1
      DIAG(I)=DIAG(I)-SUP(II)*SUB(II)
      IF (I.EQ.N) GO TO 51
      SUP(I)=SUP(I)/DIAG(I)
51    B(I)=(B(I)-SUB(II)*B(II))/DIAG(I)
      DO 52 K=1,NN
      I=N-K
52    B(I)=B(I)-SUP(I)*B(I+1)
      RETURN
      END
```

MODEL 7

```
C         SOIL MOISTURE PREDICTION MODEL
C
C         RICHARDS EQUATION SOLVED IN TERMS OF THETA (IMPLICIT SCHEME)
C
C         USING MILLER AND BRESLER RELATIONSHIP FOR SOIL WATER DIFFUSIVITY
C         AND BOUWER'S CAPILLARY DRIVE
C
C         FOR THE VALUES OF HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY :
C         HARMONIC MEANS (WITH RESPECT TO SPACE) AND ARITHMETIC MEANS
C         (WITH RESPECT TO TIME) HAVE BEEN TAKEN.
C
          DIMENSION SUB(90),SUP(90),DIAG(90),B(90)
          DIMENSION DTHETA(90,577)
          DIMENSION THETA(90,577),HYDCON(90,577)
          OPEN(UNIT=1,FILE='BCD.DAT',STATUS='OLD')
          OPEN(UNIT=2,FILE='BCD.OUT',STATUS='NEW')
C
C         J REFERS TO TIME
C         I REFERS TO DEPTH
C         Z = DEPTH (CM), ORIENTED POSITIVELY DOWNWARD
C         THETA = VOLUMETRIC MOISTURE CONTENT (CUBIC CM / CUBIC CM)
C         THETAR = RESIDUAL MOISTURE CONTENT
C         THETAS = MOISTURE CONTENT AT SATURATION
C         THETAU = MOISTURE CONTENT AT THE SURFACE NODE
C                  (UPPER BOUNDARY CONDITION)
C         BETA1, CONA =  PARAMETERS IN THE HYDRAULIC CONDUCTIVITY
C                        AND SOIL WATER PRESSURE RELATIONSHIP
C         BETA2, ALPHA = PARAMETERS IN THE MOISTURE CONTENT AND
C                        SOIL WATER PRESSURE RELATIONSHIP
C         HYDCON = HYDRAULIC CONDUCTIVITY OF THE SOIL (CM/HOUR)
C         AKS = HYDRAULIC CONDUCTIVITY AT SATURATION (CM/HOUR)
C         DELT = TIME STEP (HOURS)
C         DELZ = DEPTH INTERVAL (CM)
C         NTIME = NUMBER OF TIME STEPS
C         NNODE = NUMBER OF NODES
C         CCC = SPECIFIC WATER CAPACITY (/CM) DEFINED AS d(theta)/dh
C         DTHETA = SOIL WATER DIFFUSIVITY DEFINED AS HYDCON/CCC
C         HF = BOUWER'S CAPILLARY DRIVE
C
          READ(1,11)THETAR,THETAS,THETAU
11        FORMAT(3F12.3)
          READ(1,12)BETA1,BETA2
12        FORMAT(2F12.3)
          READ(1,13)CONA,ALPHA
13        FORMAT(2F12.3)
          READ(1,14)AKS
14        FORMAT(F12.3)
          READ(1,15)DELT,DELZ
15        FORMAT(F12.8,F12.3)
          READ(1,16)NTIME,NNODE
16        FORMAT(I3,8X,I3)
C
C         READING OF INITIAL CONDITIONS
C
          READ(1,17)(THETA(I,1),I=1,NNODE)
17        FORMAT(5F10.4)
C
```

```
      WRITE(2,18)
18    FORMAT(2X,'Soil Moisture Prediction Model (BCD)')
      WRITE(2,19)
19    FORMAT(2X,'Richards Equation solved in terms of Theta')
      WRITE(2,20)
20    FORMAT(2X,'using Miller and Bresler relation for diffusivity')
      WRITE(2,21)
21    FORMAT(/2X,'THETAR',9X,'THETAS',9X,'THETAU')
      WRITE(2,31)THETAR,THETAS,THETAU
31    FORMAT(2X,F5.3,10X,F5.3,10X,F5.3)
      WRITE(2,22)
22    FORMAT(2X,'BETA1',10X,'BETA2')
      WRITE(2,32)BETA1,BETA2
32    FORMAT(2X,F5.3,10X,F5.3)
      WRITE(2,23)
23    FORMAT(2X,'CONA',11X,'ALPHA')
      WRITE(2,33)CONA,ALPHA
33    FORMAT(2X,F11.3,4X,F11.3)
      WRITE(2,24)
24    FORMAT(2X,'AKS')
      WRITE(2,34)AKS
34    FORMAT(2X,F6.3)
      WRITE(2,25)
25    FORMAT(2X,'DELT',11X,'DELZ')
      WRITE(2,35)DELT,DELZ
35    FORMAT(2X,F10.8,5X,F5.3)
      WRITE(2,26)
26    FORMAT(2X,'NTIME',10X,'NNODE')
      WRITE(2,36)NTIME,NNODE
36    FORMAT(2X,I3,11X,I3)
      WRITE(2,27)
27    FORMAT(/2X,'SOIL MOISTURE AT DIFFERENT NODES')
      WRITE(2,28)
28    FORMAT(/2X,'INITIAL CONDITIONS')
      WRITE(2,38)(THETA(I,1),I=1,NNODE)
38    FORMAT(5F10.4)
C
C     GENERATION OF UPPER BOUNDARY CONDITION
C
      DO 100 J=1,NTIME
100   THETA(1,J)=THETAU
C
C     GENERATION OF LOWER BOUNDARY CONDITION
C
      DO 200 J=1,NTIME
200   THETA(NNODE,J)=THETA(NNODE,1)
C
      HF=20.489
      THETAI=0.100
      AM=SQRT((2.0*AKS*HF)/(THETAS-THETAI))
      E1=BETA1/BETA2
      E2=(THETAS-THETAR)
      E3=ALPHA**E1
      E4=CONA*AKS
      E5=1./BETA2*ALPHA**(1./BETA2)
C
      DO 300 J=2,NTIME
C
```

```
      DO 400 I=2,NNODE-1
      THETA(I,J)=THETA(I,J-1)
      ITER=1
400   CONTINUE
500   CONTINUE
C
      DO 600 I=1,NNODE
      TERM1=(THETA(I,J-1)-THETAR)/E2
      HYDCON(I,J-1)=E4*TERM1**E1/(CONA*TERM1**E1+E3*(1.-TERM1)**E1)
      DTHETA(I,J-1)=0.0054*AM**2*EXP(4.7*((THETA(I,J-1)-THETAR)
     1 /(THETAS-THETAR)))
600   CONTINUE
700   CONTINUE
C
      DO 800 I=1,NNODE
      TERM1=(THETA(I,J)-THETAR)/E2
      HYDCON(I,J)=E4*TERM1**E1/(CONA*TERM1**E1+E3*(1.-TERM1)**E1)
      DTHETA(I,J)=0.0054*AM**2*EXP(4.7*((THETA(I,J)-THETAR)
     1 /(THETAS-THETAR)))
800   CONTINUE
C
      DO 900 I=2,NNODE-1
      F11=2.*DTHETA(I+1,J)*DTHETA(I,J)/(DTHETA(I+1,J)+DTHETA(I,J))
      F12=2.*DTHETA(I+1,J-1)*DTHETA(I,J-1)/(DTHETA(I+1,J-1)+
      DTHETA(I,J-1))
      F1=(F11+F12)*0.5
      F21=2.*DTHETA(I,J)*DTHETA(I-1,J)/(DTHETA(I,J)+DTHETA(I-1,J))
      F22=2.*DTHETA(I,J-1)*DTHETA(I-1,J-1)/(DTHETA(I,J-1)+
     1 DTHETA(I-1,J-1))
      F2=(F21+F22)*0.5
      F31=2.*HYDCON(I+1,J)*HYDCON(I,J)/(HYDCON(I+1,J)+HYDCON(I,J))
      F32=2.*HYDCON(I+1,J-1)*HYDCON(I,J-1)/(HYDCON(I+1,J-1)+
     1 HYDCON(I,J-1))
      F3=(F31+F32)*0.5
      F41=2.*HYDCON(I,J)*HYDCON(I-1,J)/(HYDCON(I,J)+HYDCON(I-1,J))
      F42=2.*HYDCON(I,J-1)*HYDCON(I-1,J-1)/(HYDCON(I,J-1)+
     1 HYDCON(I-1,J-1))
      F4=(F41+F42)*0.5
      DIAG(I-1)=1.+(F1+F2)*DELT/DELZ**2
      SUB(I-1)=-F2*DELT/DELZ**2
      SUP(I-1)=-F1*DELT/DELZ**2
      B(I-1)=THETA(I,J-1)-DELT/DELZ*(F3-F4)
900   CONTINUE
C
      B(1)=B(1)-SUB(1)*THETA(1,J)
      B(NNODE-2)=B(NNODE-2)-SUP(NNODE-2)*THETA(NNODE,J)
      DO 1000 I=2,NNODE-1
1000  SUB(I-1)=SUB(I)
      M=NNODE-2
      CALL TRID(M,SUP,SUB,DIAG,B)
      SUM=0.
      DO 1100 I=1,NNODE-2
1100  SUM=SUM+(THETA(I+1,J)-B(I))**2
      DO 1200 I=1,NNODE-2
1200  THETA(I+1,J)=B(I)
      ITER=ITER+1
      IF(ITER.GT.10) GO TO 1300
      IF(SUM.GT.0.0001) GO TO 700
1300  CONTINUE
```

```
      IF (J.EQ.2) GO TO 111
      IF (J.EQ.36) GO TO 111
      IF (J.EQ.72) GO TO 111
      IF (J.EQ.144) GO TO 111
      IF (J.EQ.216) GO TO 111
      IF (J.EQ.288) GO TO 111
      IF (J.EQ.576) GO TO 111
      GO TO 222
111   CONTINUE
      WRITE(2,41)
41    FORMAT(/2X,'TIME STEP',7X,'ITERATION')
      WRITE(2,42)J,ITER
42    FORMAT(2X,I5,14X,I2)
      WRITE(2,43)(THETA(I,J),I=1,NNODE)
43    FORMAT(5F10.4)
222   CONTINUE
300   CONTINUE
      STOP
      END
C
      SUBROUTINE TRID(M,SUP,SUB,DIAG,B)
      DIMENSION SUP(90),SUB(90),DIAG(90),B(90)
      N=M
      NN=N-1
      SUP(1)=SUP(1)/DIAG(1)
      B(1)=B(1)/DIAG(1)
      DO 51 I=2,N
      II=I-1
      DIAG(I)=DIAG(I)-SUP(II)*SUB(II)
      IF (I.EQ.N) GO TO 51
      SUP(I)=SUP(I)/DIAG(I)
51    B(I)=(B(I)-SUB(II)*B(II))/DIAG(I)
      DO 52 K=1,NN
      I=N-K
52    B(I)=B(I)-SUP(I)*B(I+1)
      RETURN
      END
```

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