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COMPARATIVE STUDY OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV-1 DISTRIBUTION

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ABSTRACT

Design of water resources structures needs estimation of hydrologic and meteorologic events for different return periods. This can be obtained through frequency analysis approach. The extreme value type 1 (EV-1) distribution is widely used for frequency analysis of extreme events in hydrology and meteorology. It's successful application depends upon the accuracy with which its parameters can be estimated. However, there is no universal agreed method of estimating its parameters.

This report explains and statistically compares various parameter estimation techniques cited in literature using data of different sample sizes generated by Monte Carlo simulation. These include (i) method of moments (MOM), (ii) Method of maximum likelihood (MLE), (iii) Method of probability weighted moments (PWM), (iv) Method based on principle of maximum entropy (POME), (v) Method of mixed moments (MIX), (vi) Method of least squares (LEAS) and (vii) Incomplete means procedure (ICM). All the above methods have been compared on the basis of (i) bias in location parameter (ii) bias in scale parameter, (iii) standard deviation of location parameter, (iv) standard deviation of scale parameter, (v) efficiency in estimating location paramater, (vi) efficiency in estimating scale parameter, (vii) bias in quantile estimates, (viii) standard deviation of quantile estimates, (ix) efficiency of quantile estimates, (x) average of relative deviations between computed and expected value

of reduced variates (DA) and (xi) average of squares of relative deviations between computed and expected value of reduced variates.

Based on criteria (i) to (ix) it is difficult to arrive at any definite conclusion as no method is the best according to all the criteria. However MIX and ICM are the least accurate methods. PWM, POME and MLE give nearly similar results and can be recommended for use. Based on criteria (x) and (xi) the ranks of different methods in descending orderare as follows:

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

1.0 INTRODUCTION

Design of water resources structures needs estimation of hydrologic and meteorologic events for different return periods. This can be achieved through frequency analysis approach, in which, in general a past record is fitted with a statistical distribution which is then used to make inferences about future flows. Many probability distributions have been employed for at site flood frequency analysis. At the same time several parameter estimation techniques have been proposed for each distribution. The selection of appropriate parameter estimation technique for the distribution can be as important as selecting the distribution itself, because several estimation techniques based on the same distribution can lead to widely differing parameters and quantile estimates. This variability in estimation, as reflected by the bias and mean square error, tends to be more pronounced as the sample size decreases.

It is therefore, important to study the properties of various estimators of each probability distribution, especially for small samples of hydrological data encountered in practice. This would ensure that the best available estimator extracting maximum useful information from the sample is utilized in engineering practice. The present study deals with assessment of the behaviour of several estimators available for the Gumbel's extreme value type 1 (EV1) distribution which is widely used for frequency analysis of extremes in hydrology (Gumbel,1941 a, 1941 b, 1942, 1958; Kaczmarek 1957; Majumdar and Sawhney, 1965; Lowery and Nash, 1970;

Cicioni et al. 1973; Karr, 1976, Bardsley, 1977; Kite, 1977; Otten and Van Mont fort, 1978, 1980; Landwehr, et al. 1979; Phien and Arbhabhirama, 1980 ; Lettenmaier and Burges, 1982). The popularity of EV 1 distribution may be due to the following reasons (Phien and Arbhabhirama, 1980; Lettenmaier and Burges, 1982, Jain and Singh, 1987):

a) The EV1 distribution results from an initial unlimited distribution of exponential type which converges to an exponential function,

b) Under certain assumptions the extreme values in a sample follow this distribution.

c) It is simple and has only two parameters,

d) From statistical view point, it may be preferable to apply when the sample size is small.

e) Because it is available in closed form, it is easier to determine the extreme value for a specified value of probability or return period.

f) In a Monte Carlo study, the EV1 random variables can be easily generated.

Despite extensive use of EV 1 distribution, there is no generally accepted method for estimating its parameters. Method of moments, method of maximum likelihood, method of least squares, method of probability weighted moments, method based on principle of maximum entropy, incomplete mean procedure and method of mixed moments are some of the methods which have been used and advocated for the estimation of parameters of this distribution by various researchers. The primary objective of the study is to compare various estimation techniques for EV 1 distribution.

Keeping in view the objective of the study, various parameter estimation techniques are reviewed besides the properties of the EV 1 distribution.

2.0 REVIEW

2.1 EV 1 Distribution

The EV1 distribution is commponly defined by its distribution function (CDF)

$$F(x) = \exp(-\exp(-(x-b)a))$$

$$\exp(-\exp(-y)) \qquad \ldots(1)$$

Where, a and b are respectively scale and location parameters and y is the reduced variate defined by

$$y = a(x - b) \qquad \ldots(2)$$

The density function (PDF) is obtained by the differentiating equation 2 with respect to x to give

$$f(x) = a \exp(-y - \exp(-y)) \qquad \ldots(3)$$

For a given return period T, the magnitude X_T of the T year event is obtained from equation 1.

i.e $F(X_T) = \text{Prob} \; (x \geq X_T) = \frac{1}{T}$ as

$$X_T = b + Y_T/a \qquad \ldots(4)$$

where Y_T is the value of the reduced variate corresponding to T ;

$$Y_T = -\ln(-\ln(T-1)/T) \qquad \ldots(5)$$

In flood frequency analysis with EV1 being used as the probability distribution the most important problem is the estimation of the parameters a and b from a given set of data. Once a and b have been estimated the estimate of X_T can readily be obtained:

$$\hat{X}_T = \hat{b} + Y_T/\hat{a} \qquad \ldots(6)$$

Where $\hat{a}$ and $\hat{b}$ are respectively the estimates of a and b.

If μ and σ are the mean and standard deviation of x then it can be shown that :

$$a = \frac{\pi}{\sigma\sqrt{6}} \quad \text{and} \quad b = \mu - \gamma\sigma\sqrt{6}/\pi \qquad \ldots(7)$$

Where, $\gamma = 0.5772\ldots$ is Euler's constant. Consequently equation (4) can be rewritten as

$$X_T = \mu + K_T\,\sigma \qquad \ldots(8)$$

where,

$$K_T = -\frac{\sqrt{6}}{\pi}\,(Y_T - \gamma)$$

$$= \frac{\sqrt{6}}{\pi}(\ln\,(-\ln\,(\frac{T-1}{T}) + \gamma\,) \qquad \ldots(9)$$

is commonly referred to as the frequency factor. Clearly, K_T depends only on the return period T, as pointed out by Lowery and Nash (1970).

Although many methods of estimating the parameters of EV1 distribution are described in the statistical literature, only seven, which are widely used in flood frequency analysis are considered here. A brief description of these methods and solution techniques is given in following sections.

2.2 Method of Moments (MOM)

This is one of the most popular methods of estimating the parameters a and b (Lowery and Nash, 1970; Landwehr et al., 1979). Since there are two parameters, only the first two moments of the distribution are needed. If X and S are the mean and standard deviation (almost unbiassed) of a sample size n,

$$\bar{X} = \sum_{i=1}^{n} x_i/n \qquad \ldots(10)$$

$$S_x = \left(\sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1) \right)^{0.5} \qquad ...(11)$$

then from equation (7) the estimates of a and b are given as:

$$a = 1.28255/S_x \qquad ...(12)$$

$$b = x - 0.450041\ S_x \qquad ...(13)$$

2.3 Method of Maximum Likelihood (MLE)

This method involves finding the values of a and b that maximise the likelihood, or equivalently, the log likelihood function of the observed data. The log likelihood function of a sample $(x_1, x_2 \ ....x_n)$ is :

$$L = n \ln (a) - \Sigma y - \Sigma \exp(-y) \qquad ...(14)$$

where the summations are taken over all the n values of y deduced from those of x according to equation 2. By equating the partial derivatives of L with respect to a and b to zero, the following equations are obtained.

$$P = n - \Sigma y + \Sigma y \exp(-y) = 0 \qquad ...(15)$$

$$Q = -n + \Sigma \exp(-y) = 0 \qquad ...(16)$$

These can be reduced to the following equation containing a only

$$\frac{1}{a} = \frac{\bar{x} - \Sigma x \ \exp(-a\ x)}{\Sigma \exp(-a\ x)} \qquad ...(17)$$

$$b = \frac{1}{a} \ln \left(\frac{n}{\Sigma \exp(-a\ x)} \right) \qquad ...(18)$$

The equation 17 can readily be solved by the Newton-Raphson method. However, as reported by many researchers, the version given by Clarke provides quick convergence.

As evident from work of Lowery and Nash (1970) the

MLE estimates are biassed. A good correction for the bias has been proposed by Fiorentio and Gabriele (1984) and the resulting corrected maximum likelihood estimates are as follows:

$$\frac{1}{a} = \frac{1}{\hat{a} \quad (1 - 0.8/n)} = \frac{n}{\hat{a} \quad (n - 0.8)} \qquad \ldots(19)$$

$$b = \frac{1}{a} \ln \left(\frac{n}{\exp(-a\,x)} \right) - \frac{0.7}{n.a} \qquad \ldots(20)$$

where $\hat{a}$ and $\hat{b}$ are MLE estimates of a and b respectively, and as before, the sum extends over n values of x in the sample. With this correction the bias of the MLE estimates can be reduced significantly.

2.4 Method of Least Squares (LEAS)

The parameters a and b are estimated by considering equation 8 as a linear regression of x on k where μ and σ are treated simply as the parameters of linear relationship between x and k. This requires an apriori estimate of T (and hence of k from equation 9) which is usually done by arranging the data in descending order of magnitude, choosing an appropriate plotting position formula and assigning a return period to each event accordingly. These parameters can be estimated as:

$$\frac{1}{a} = \frac{n \Sigma Z_i x_i - \Sigma x_i \Sigma Z_i}{n \Sigma x_i^2 - (\Sigma x_i)^2} \qquad \ldots(21)$$

and

$$b = \bar{x} - \frac{\Sigma Z_i}{an} \qquad \ldots(22)$$

where

$Z_i = -\ln(-\ln(F(x_i)))$ is obtained for each data point from the plotting position formula.

2.5 Method of Probability Weighted Moments (PWM)

This method requires expressing the distribution function in inverse form which for the EV1 distribution is :

$$x = b - \frac{1}{a} \ln (-\ln. F(x)) \qquad ...(23)$$

Following landwehr et al (1979) the parameters a and b can be given as

$$b = \bar{x} - \frac{0.5772}{a} \qquad ...(24)$$

$$\frac{1}{a} = \frac{\bar{x} - 2 M_{101}}{\ln 2} \qquad ...(25)$$

where M_{101} is the first probability weighted moment defined as

$$M_{101} = \frac{1}{n (n-1)} \sum_{i=1}^{n} x_i (n-i) \qquad ...(26)$$

2.6 Method Based on Principle of Maximum Entropy

This method is based on maximizing the information (entropy) content inbued in the EV1 distribution subject to the constraints

$$\int_0^\infty f(x)\, dx = 1 \qquad ...(27)$$

$$\int_0^\infty x f(x)\, dx = \bar{x} \qquad ...(28)$$

$$\int_0^\infty f(x) \exp(-a(x-b))\, dx = E(\bar{e}^{a(x-b)}) = 1 \qquad ...(29)$$

Jowitt (1979) used this method to obtain a and b as:

$$E(y) = 0.5772 \qquad ...(30)$$

$$E(\exp(-y)) = 1 \qquad ...(31)$$

Where $y = a(x-b)$. Clearly a and b have to be estimated using an interative procedure.

2.7 Incomplete Mean Procedure (ICM)

The ICM method uses means calculated over only parts of the data range. By arranging the sample in ascending order $x_1, x_2, \ldots x_n$, first the sample mean ($\bar{x}$) is calculated. $\bar{x}$ is then used to devide the sample into two disjointed sets. The mean of the upper set having values greater than $\bar{x}$ is calculated and called first incomplete mean $\bar{x}_1$. Similarly, the mean of all observations above $\bar{x}_1$ is calculated and is the second incomplete mean $\bar{x}_2$, and so on. For the EV1 distribution, the first two incomplete means are

$$\bar{x}_i = b - \frac{n}{a(n-n_i)} \left(J \ln J - \frac{J^2 \ln J}{2} + \frac{J^3 \ln J}{6} - \frac{J^4 \ln J}{24} - J + \frac{J^2}{4} - \frac{J^3}{18} + \frac{J^4}{96} \right), i = 1,2$$

The sum of terms containing ln J can be simplified as $\ln J\ (1 - e^{-J})$. Therefore,

$$\bar{x}_i = b - \frac{n}{a(n-n_i)} \left(\ln J\ (1-e^{-J}) - J + \frac{J^2}{4} - \frac{J^3}{18} + \frac{J^4}{96}\right) \qquad \ldots(32)$$

where $J = \ln (n/n_i)$, n is the sample size and n_i the number of observations corresponding to the lower limit of the range on which the incomplete mean is calculated. Jain and Singh (1987) have given the derivation of equation 32.

2.8 Method of Mixed Moments (MIX)

This method uses the first moment of the EV1 distribution and the first moment of its logarithmic version. The parameters a and b are given by

$$a = \frac{1.28255}{S_x} \quad \ldots(33)$$

$$\text{and } \exp(a\,b) = 1 + a\,\bar{x} + \frac{a^2}{2}\,(S_x^2 + \bar{X}^2) \quad \ldots(34)$$

Equation 34 has been derived by Jain and Singh (1987).

2.9 Other Methods

Raynal and Salas (1986) use Best Linear Combination of Order Statistics to estimate the parameters of EV1 distribution. The table of coefficients used in the method is only upto sample size 20. The efficiency of BLCOS and PWM is almost same. Both the methods operate on the principle that good estimates of parameters may be found by linear combinations of order statistics. Raynal and Salas (1986) also use mode and inter quantile range method and conclude that the method gives very large bias as compared to others.

2.10 Conclusions Drawn by Other Researchers

(1) Raynal and Salas (1986) analyse and compare six methods viz. i) Method of moments using biased estimator of variance, ii) Method of moments using unbiased estimator of variance, iii) Method of maximum likelihood, iv) Mode and Interquantile range method, v) Least squares method and vi) Best linear combination of order statistics, using data generating techniques. Considering criteria of bias, variance and mean square error of estimates of parameters and quantile points and ease of obtaining estimators, they conclude that the best linear combination of order statistics compares favourably with the other methods for sample sizes smaller than 20. For larger samples the probability weighted

moments method is preferred. Likewise considering all factors of comparison, the least squares and mode and inter quantile range methods should not be used for fitting the Gumbel distribution.

(2) Phien (1987) recommends suitable solution procedures for MOM, MLE, POM and PWM for EV1 distribution and discusses related problems. On the basis of simulation study to evaluate the performance of these methods in terms of commonly used criteria, i.e. the bias, root mean square error and goodness of fit statistics he concludes that

i) MOM is not as good as the remaining three methods

ii) If one is interested in unbiassed estimates then PWM is obviously the best choice.

iii) If one is interested in having minimum values for the RMSE, then the MLE is the most appropriate method.

iv) With respect to both the bias and the RMSE, the POME has almost the same performance as the MLE. As such one may choose the POME instead of MLE also.

(3) Jain and Singh (1987) estimated parameters of EV1 distribution for 55 annual flood data sets by seven methods viz method of (1) moments, (2) probability weighted moments, (3) mixed moments (4) maximum likelihood estimation, (5) incomplete means (6) principle of maximum entropy and (7) least squares. They conclude:

i) The MLE method was the most accurate of all followed by the POME, PWM, MOM and LEAS methods.

ii) The ICM and MIX methods were the least accurate of all and could not be recommended.

iii) The MLE, POME, MOM and PWM methods may be recommended for general use. However the POME method would be preferable, as convergence in parameter estimation is guaranteed in each case.

(4) Arora and Singh (1987) estimate and inter compare statistically the parameters and quantiles of EV1 distribution using synthetically generated data for sample lengths 5 to 1000. They compare seven parameter estimation techniques for two sampling cases i.e. (i) purely random process and (ii) serially correlated process. They also give a bias correction for the method of moments-quantile estimator. With regard to the intercomparison of parameters and quantile estimators some of their conclusions are as follows:

i) The methods of mixed moments and incomplete means resulted in poor estimation of the parameters and the quantiles.

ii) The method of least squares provided minimum bias and maximum efficiency estimate of the scale parameter (a) for very small samples and also provided competetive estimate of the location parameter.

iii) The maximum likelihood method generally provided most efficient quantile estimates followed closely by the entropy method. In fact POME performed practically in the same manner as MLE and was relatively easier to solve.

(iv) For small samples the method of probability weighted moments and the method of moments performed comparably in efficiency of estimating the quantiles. However efficiency of PWM improved relative to MLE with increasing sample size. PWM also resulted in nearly unbiassed quantile estimates.

(v) The incorporation of serial correlation in sample resulted in deterioration of the performance of all estimators. However all the methods performed much more similarly in this case.

3.0 PROBLEM DEFINITION

The main objective of the study is to statistically compare various methods for estimating the parameters and quantiles of the Gumbel's Extreme Value type 1 or EV1distribution. Some of these methods have been widely known and used in hydrologic analysis and some others are relatively less used.

4.0 PROPOSED METHODOLOGY

The following methods to estimate the parameters and quantiles of EV 1 distribution have been analysed and statistical intercomparison made.

(i) Method of moments (MOM)

(ii) Method of maximum likelihood (MLE).

(iii) Method of probability weighted moments (PWM).

(iv) Method based on principle of maximum entropy (POME).

(v) Method of mixed moments (MIX).

(vi) Method of least squares (LEAS).

(vii) Incomplete mean procedure (ICM).

All the above methods have been intercompared. The criteria of intercomparison used in present study are as follows:

(i) Bias in location parameter b.

(ii) Bias in scale parameter a.

(iii) Standard deviation of location parameter b.

(iv) Standard deviation of scale parameter a.

(v) Efficiency of the method in estimating the location parameter as compared to maximum likelihood method.

(vi) Efficiency of the method in estimating the scale parameter as compared to maximum likelihood method.

(vii) Bias of quantile estimates.

(viii) Standard deviation of quantile estimates.

(ix). Efficiency of quantile estimates.

(x) Average of relative deviations between computed and expected value of reduced variates.

(xi) Average of squares of relative deviations between computed and expected value of reduced variates.

In the study synthetic data has been used as this sort of studies for a particular distribution are possible only with the synthetic data.

The study has been carried out for different length of samples. The sample lengths have been taken as 10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100. Various steps involved are explained in details as follows:

1. Generate 50,000 EV1 distributed randum numbers 50,000 has been chosen in order to have sufficient samples of different lengths. The location parameter b and scale parameter a have been taken as 0.0 and 1.0 respectively. Value of b and a being 0.0 and 1.0 respectively, the EV1 distributed random numbers are directly the reduced variates. The algorithm used for generation comprises of :

i Generate 50,000 uniformly distributed random numbers between 0.0 and 1.0

ii Transform uniformly distributed random numbers to EV1 distributed random numbers by the following formula.

$$X = b + \frac{1}{a} (- \ln (-\ln (Z))) \qquad \ldots(34)$$

where,

$b = 0.0$

$a = 1.0$

X = EV1 distributed random numbers and

z = uniformly distributed random numbers

2. Get samples of different lengths (10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100) from the generated EV1 distributed reduced variates e.g. if the sample length is 10 then one

will be getting 50,000/10 i.e 5000 samples.

3. Arrange the samples in descending order.

4. Estimate the parameters (a and b) of the EV1 distribution by the following methods.

(i) Method of maximum likelihood (MLE)

(ii) Method of moments (MOM)

(iii) Method of probability weighted moments (PWM).

(iv) Method based on principle of maximum entropy (POME).

(v) Method of mixed moments (MIX).

(vi) Method of least squares (LEAS)

(vii) Incomplete mean procedure (ICM).

The equations used to estimate the parameters of the distribution by the above methods are given in section 2.0.

5. Calculate quantiles $x_{(F)}$ for F = 0.001, 0.01, 0.02, 0.05, 0.1, 0.250, 0.5, 0.75, 0.9, 0.95, 0.98, 0.99 and 0.999 by the following equation.

$$x\ (F) = b - \frac{\ln\ (-\ln\ (F)}{a} \qquad \ldots(35)$$

6. Calculate bias, standard deviation, and efficiency of location parameter b, scale parameter a and quantiles by the following procedure.

Let $\hat{\theta}$ denote an estimate of $\theta \ \varepsilon\ (a, b, x\ (f))$. $\hat{\theta}$ is a random variable whose distribution function depends upon sample size, the method of parameter estimation, and of course, the distribution of the sample itself. The performance statistics of $\hat{\theta}$ are given as follows :

Bias, BIAS $(\hat{\Theta}) = \Theta - E(\hat{\Theta})$ ...(36)

Variance, $STD^2(\hat{\Theta}) = E(\hat{\Theta} - E(\hat{\Theta}))^2$ ...(37)

Mean Square Error, MSE $(\hat{\Theta}) = E(\Theta - \hat{\Theta})^2$

$= BIAS^2(\hat{\Theta}) + STD^2(\hat{\Theta})$ ...(38)

$$E(\Theta) = \mu(\hat{\Theta}) = \sum_{i=1}^{N} \frac{\hat{\Theta}_i}{N} \quad ...(39)$$

$$STD^2(\hat{\Theta}) = \sum_{i=1}^{N} \left(\frac{(\hat{\Theta}_i - \mu(\hat{\Theta}))^2}{N-1} \right) \quad ...(40)$$

$$BIAS(\hat{\Theta}) = \Theta - \mu(\hat{\Theta}) \quad ...(41)$$

$$MSE(\hat{\Theta}) = BIAS^2(\hat{\Theta}) + STD^2(\hat{\Theta}) \quad ...(42)$$

$$EFF(\hat{\Theta}) = \frac{MSE(\hat{\Theta} \mid MLE)}{MSE(\hat{\Theta} \mid \text{other method})} \quad ...(43)$$

A value of $EFF(\hat{\Theta}) < 1$ implies that the method under consideration is less efficient (i.e has higher mean square error) compared to MLE and vice versa.

7. Calculate average of relative deviation between computed and expected value of reduced variates for various methods.

$$DA = \sum_{i=1}^{N} \left| \frac{X_{ci} - X_{Ei}}{X_{Ei}} \right| \times 100/N \quad ...(44)$$

where,

N = sample size

X_{ci} = Computed value of ith reduced variate using esimated value of a and b.

X_{Ei} = expected value of ith reduced variate.

$$= \sum_{j=1}^{N} Y_{i,j}/N \quad ...(45)$$

$y_{i,j}$ = ith reduced variate for the j th sample

The number of reduced variates will be equal to the size of the sample.

8. Calculate average of squares of relative deviations between computed and expected value of reduced variates by following equation.

$$DR = \sum_{i=1}^{N} \left(\frac{X_{ci} - X_{Ei}}{X_{Ei}} \right)^2 \times 100/N \qquad \ldots(46)$$

9. Compare various methods of parameter estimation on the basis of statistics calculated above.

5.0 ANALYSIS AND RESULTS

Seven parameter estimation techniques for EV 1 distribution have been inter compare on the basis of above mentioned ten criteria. The analysis of results is being given in two parts i.e. (i) parameter estimates and (ii) quantile estimates, as follows:

5.1 Parameter Estimates

Bias (BIAS), standard deviation (STD) and efficiency (EFF) of scale parameter a and location parameter b is given in Table 1 for sample size 10 to 100. The ranking of different parameter estimation techniques on the basis of bias in b, bias in a, standard deviation of b, standard deviation of a efficiency of b and efficiency of a is given in Table 2-7.

The MIX and ICM can be prime facic rejected as unreliable estimators of Gumbel parameters. For sample size 10 the results are not consistent and it is difficult to arrive at definite conclusion. For the sake of simplification, the ranking of remaining five methods is as follows:

		Ranking of			
Basis	PWM	MOM	MLE	POME	LEAS
Bias in b	1	2	4	3	5
Bias in a	1	4	3	2	5
STD of b	1	4	2	3	5
STD of a	3	4	1	2	5
EFF of b	1	4	2	3	5
EFF of a	3	4	1	2	5

From the above table it is seen that PWM, MLE and POME are comparable and it is difficult to arrive at any difinite conclusion regarding superiority of any one of them.

TABLE 1

PERFORMANCE OF STATISTICS OF PARAMETERS

Method	Sample Size	BIAS(A)	STD(A)	EFF(A)	BIAS(B)	STD(B)	EFF(B)
MLE	10	-0.162	0.344	1.00	-0.038	0.330	1.000
MOM		-0.152	0.377	0.876	-0.027	0.333	0.988
PWM		-0.088	0.344	1.147	+0.001	-0.329	1.023
POME		-0.154	0.343	1.026	-0.039	0.331	0.997
LEAS		-0.078	0.346	1.154	-0.025	0.512	0.420
MIX		-0.152	0.377	0.876	-0.919	0.368	0.113
ICM		-0.193	0.832	0.199	0.277	0.772	0.164
MLE	20	-0.071	0.200	1.000	-0.019	0.231	1.000
MOM		-0.074	0.239	0.719	-0.015	0.235	0.973
PWM		-0.040	0.216	0.937	0.000	0.231	1.008
POME		-0.067	0.203	0.987	-0.019	0.232	0.996
LEAS		-0.056	0.243	0.722	-0.024	0.366	0.401
MIX		-0.074	0.239	0.719	-0.930	0.267	0.057
ICM		-0.209	0.460	0.177	-0.068	0.420	0.297
MLE	30	-0.045	0.151	1.000	-0.013	0.188	1.000
MOM		-0.049	0.189	0.653	-0.010	0.192	0.964
PWM		-0.025	0.168	0.860	-0.001	0.188	1.004
POME		-0.042	0.154	0.973	-0.013	0.189	0.996
LEAS		-0.045	0.197	0.612	-0.022	0.299	0.397
MIX		-0.049	0.189	0.653	-0.935	0.218	0.039
ICM		-0.038	0.316	0.246	0.090	0.374	0.241
MLE	40	-0.034	0.131	1.000	-0.010	0.163	1.00
MOM		-0.037	0.164	0.648	-0.008	0.167	0.957
PWM		-0.019	0.146	0.849	-0.001	0.164	1.001
POME		-0.032	0.134	0.971	-0.010	0.164	0.995
LEAS		-0.037	0.170	0.603	-0.019	0.263	0.385
MIX		-0.037	0.164	0.648	-0.937	0.192	0.029
ICM		-0.098	0.280	0.209	-0.039	0.301	0.291

Method	Sample Size	BIAS(A)	STD(A)	EFF(A)	BIAS(B)	STD(B)	EFF(B)
MLE	50	-0.026	0.115	1.000	-0.008	0.145	1.00
MOM		-0.031	0.148	0.607	-0.007	0.148	0.965
PWM		-0.015	0.130	0.805	-0.001	0.145	1.004
POME		-0.025	0.118	0.954	-0.008	0.146	0.992
LEAS		-0.032	0.154	0.562	-0.017	0.236	0.379
MIX		-0.031	0.148	0.607	-0.938	0.173	0.023
ICM		-0.022	0.231	0.258	0.051	0.278	0.265
MLE	60	-0.022	0.102	1.000	-0.007	0.133	1.00
MOM		-0.024	0.131	0.613	-0.006	0.136	0.961
PWM		-0.012	0.115	0.814	-0.001	0.133	1.002
POME		-0.020	0.104	0.959	-0.007	0.134	0.993
LEAS		-0.029	0.137	0.547	-0.016	0.213	0.390
MIX		-0.024	0.131	0.613	-0.939	0.156	0.020
ICM		-0.062	0.202	0.202	-0.029	0.242	0.301
MLE	70	-0.019	0.095	1.00	-0.007	0.126	1.00
MOM		-0.022	0.123	0.60	-0.006	0.129	0.950
PWM		-0.011	0.108	0.795	-0.001	0.126	0.998
POME		-0.018	0.098	0.953	-0.007	0.126	0.996
LEAS		-0.026	0.129	0.542	-0.015	0.199	0.399
MIX		-0.022	0.123	0.60	-0.939	0.146	0.018
ICM		-0.015	0.186	0.270	0.033	0.234	0.283
MLE	75	-0.018	0.094	1.000	-0.006	0.120	1.000
MOM		-0.021	0.120	0.609	-0.006	0.124	0.945
PWM		-0.010	0.106	0.802	-0.002	0.121	0.999
POME		-0.017	0.096	0.961	-0.006	0.121	0.996
LEAS		-0.025	0.125	0.559	-0.014	0.196	0.375
MIX		-0.021	0.120	0.609	-0.940	0.143	0.016
ICM		-0.018	0.185	0.263	-0.011	0.230	0.275

Method	Sample Size	BIAS(A)	STD(A)	EFF(A)	BIAS(B)	STD(B)	EFF(B)
MLE	80	0.017	0.092	1.00	-0.006	0.117	1.000
MOM		-0.020	0.118	0.603	-0.005	0.120	0.944
PWM		-0.010	0.103	0.803	-0.001	0.117	0.998
POME		-0.016	0.094	0.959	-0.006	0.117	0.997
LEAS		-0.023	0.123	0.553	-0.013	0.190	0.375
MIX		-0.020	0.118	0.603	-0.940	0.139	0.015
ICM		-0.050	0.182	0.242	-0.024	0.216	0.290
MLE	90	-0.015	0.083	1.000	-0.005	0.109	1.000
MOM		-0.017	0.110	0.582	-0.005	0.111	0.970
PWM		-0.008	0.095	0.782	-0.001	0.109	1.003
POME		-0.014	0.086	0.948	-0.006	0.110	0.989
LEAS		-0.022	0.114	0.531	-0.013	0.177	0.381
MIX		-0.017	0.110	0.582	-0.940	0.130	0.013
ICM		-0.013	0.163	0.269	+0.023	0.197	0.303
MLE	100	-0.013	0.079	1.000	-0.005	0.104	1.000
MOM		-0.016	0.106	0.569	-0.004	0.107	0.951
PWM		-0.007	0.092	0.766	-0.001	0.104	0.998
POME		-0.012	0.082	0.937	-0.005	0.104	0.996
LEAS		-0.021	0.109	0.529	-0.012	0.169	0.375
MIX		-0.016	0.106	0.569	-0.940	0.124	0.012
ICM		-0.040	0.159	0.240	-0.021	0.191	0.293

TABLE - 2

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV1 DISTRIBUTION ON THE BASIS ON BIAS IN LOCATION PARAMETER B

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	4	4	4	4	4	4	4	4	4	3	4
MOM	3	2	2	2	2	2	2	2	2	2	2
PWM	1	1	1	1	1	1	1	1	1	1	1
POME	5	3	3	3	3	3	3	3	3	4	3
LEAS	2	5	5	5	5	5	5	6	5	5	5
MIX	7	7	7	7	7	7	7	7	7	7	7
ICM	6	6	6	6	6	6	6	5	6	6	6

TABLE - 3

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV1 DISTRIBUTION ON THE BASIS OF BIAS IN SCALE PARAMETER A

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	6	4	5	3	4	3	4	3	3	4	3
MOM	3	5	6	4	5	4	5	5	4	5	4
PWM	2	1	1	1	1	1	1	1	1	1	1
POME	5	3	3	2	3	2	3	2	2	3	2
LEAS	1	2	4	5	7	6	7	7	6	7	6
MIX	4	6	7	6	6	5	6	6	5	6	5
ICM	7	7	2	7	2	7	2	4	7	2	7

TABLE - 4

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV1 DISTRIBUTION ON THE BASIS OF STANDARD DEVIATION OF LOCATION PARAMETER B

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	2	2	2	1	2	2	2	1	2	2	2
MOM	4	4	4	4	4	4	4	4	4	4	4
PWM	1	1	1	2	1	1	1	2	1	1	1
POME	3	3	3	3	3	3	3	3	3	3	3
LEAS	6	6	6	6	6	6	6	6	6	6	6
MIX	5	5	5	5	5	5	5	5	5	5	5
ICM	7	7	7	7	7	7	1	7	7	7	7

TABLE 5

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV1 DISTRIBUTION ON THE BASIS OF STANDARD DEVIATION OF SCALE PARAMETER A

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	3	1	1	1	1	1	1	1	1	1	1
MOM	5	4	4	4	4	4	4	4	4	4	4
PWM	2	3	3	3	3	3	3	3	3	3	3
POME	1	2	2	2	2	2	2	2	2	2	2
LEAS	4	6	6	6	6	6	6	6	6	6	6
MIX	6	5	5	5	6	5	5	6	5	5	5
ICM	7	7	7	7	7	7	7	7	7	7	7

TABLE - 6

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV1 DISTRIBUTION ON THE BASIS OF EFFICIENCY OF THE METHOD IN ESTIMATION LOCATION PARAMETER B AS COMPARED TO MAXIMUM LIKELIHOOD METHOD

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	2	2	2	2	2	2	1	1	1	2	1
MOM	4	4	4	4	4	4	4	4	4	4	4
PWM	1	1	1	1	1	1	2	2	2	1	2
POME	3	3	3	3	3	3	3	3	3	3	3
LEAS	5	5	6	6	6	5	5	6	6	5	5
MIX	7	7	7	7	7	7	7	7	7	7	7
ICM	6	6	6	6	6	6	6	6	6	6	6

TABLE 7

COMPARISON OF DIFFERENT PARAMETER ESTIMATION TECHNIQUE FOR EV1 DISTRIBUTION ON THE BASIS OF EFFICIENCY OF THE METHOD IN ESTIMATION SCALE PARAMETER AS A COMPARED TO MAXIMUM LIKELIHOOD METHODS

METHOD	RANK OF DIFFERENT METHODS FOR SAMPLE SIZE										
	10	20	30	40	50	60	70	75	80	90	100
MLE	4	1	1	1	1	1	1	1	1	1	1
MOM	5	5	4	4	4	4	4	4	4	4	4
PWM	2	3	3	3	3	3	3	3	3	3	3
POME	3	2	2	2	2	2	2	2	2	2	2
LEAS	1	4	6	6	6	6	6	6	6	6	6
MIX	6	6	5	6	6	6	6	6	6	5	5
ICM	7	7	7	7	7	7	7	7	7	7	7

5.2 Quantile Estimates

The quantiles corresponding to probability of non-exceedence (F) equal to 0.001, 0.01, 0.02, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.98, 0.99 & 0.999 have been estimated by various methods. The bias, standard deviation and efficiency of quantiles for various methods and various sample size are given in Table 8-10. The ranking of different methods on this basis is given in table 11-13.

5.2.1 Bias of quantile estimates

MIX and ICM can be rejected as unreliable estimators of quantiles.

PWM provided unbiassed quantile estimates for all n and F. MOM provided lesser bias than MLE and POME. POME resulted in slightly less bias than MLE. The bias of LEAS does not show any consistent trend for all the F values.

5.2.2 Standard deviation of quantile estimates

ICM can be rejected on the basis of standard deviation of quantiles. For the lower six quantiles i.e. for F = 0.001, 0.01, 0.02, 0.05, 0.1, 0.25 MIX gives lowest standard deviation of quantiles while for upper six quantiles i.e. for F = 0.75, 0.9, 0.95, 0.98, 0.99 and 0.999 LEAS is giving lowest standard deviation nearly for all the sample sizes. MLE is better than MOM, PWM and POME. In general, the ranking can be:

(1) MLE, (2) POME, (3) PWM and (4) MOM

5.2.3 Efficiency of quantile estimates

ICM and MIX can be rejected as unreliable estimates. LEAS gives maximum efficiency for the top six quantiles i.e

TABLE-8

BIAS IN QUANTILE ESTIMATES

Method	Sample Size	F = 0.001	0.010	0.020	0.050	0.10	0.250	0.50	0.75	0.90	0.95	0.98	0.99	0.999
		x = -1.933	-1.527	-1.364	-1.057	-0.834	-0.327	0.367	1.246	2.25	2.97	3.902	4.6	6.907
MLE	10	-0.1802	-0.1504	-0.1384	-0.1189	-0.0995	0.0623	-0.0115	0.0531	0.1268	0.1796	0.2479	0.2992	0.4684
MOM		-0.119	-0.0932	-0.0861	-0.0745	-0.0630	-0.0409	-0.0107	0.0277	0.0715	0.1029	0.1435	0.1740	0.2746
PWM		0.0079	0.0064	0.0058	0.0048	0.0038	0.0019	-0.0001	-0.004	-0.0077	-0.0104	-0.0139	-0.0165	-0.0251
POME		-0.1641	-0.1378	-0.1273	-0.1100	-0.0929	-0.0600	-0.0151	0.0419	0.1070	0.1536	0.2140	0.2592	0.04087
LEAS		0.0199	0.0104	0.0066	0.0004	-0.0057	-0.0175	-0.0336	-0.0419	-0.0774	-0.0941	-0.1158	-0.1320	-0.1857
MIX		-1.0033	-0.9856	-0.9785	-0.9669	-0.9554	-0.9333	-0.9031	-0.0540	-0.8209	-0.7825	-0.7489	-0.7184	-0.6179
ICM		0.5440	0.4880	0.4654	0.4286	0.39226	0.3222	0.2264	-0.8647	-0.0337	-0.1331	-0.2618	-0.3582	-0.6768
MLE	20	-0.0873	-0.0730	-0.0672	-0.0578	-0.0486	-0.0307	-0.0062	0.0248	0.0602	0.0856	0.1185	0.1431	0.2248
MOM		-0.0592	-0.0498	-0.0461	-0.0400	-0.0339	-0.0222	-0.0063	0.0139	0.0370	0.0536	0.0750	0.0910	0.1441
PWM		0.0035	0.0027	0.0024	0.0019	0.0014	0.0003	-0.0010	-0.0028	-0.0048	-0.0062	-0.0081	-0.0095	-0.0141
POME		-0.0785	-0.0661	-0.0611	-0.0529	-0.0448	-0.0292	-0.0079	0.0191	0.0499	0.0720	0.1006	0.1221	0.1929
LEAS		-0.0339	-0.0319	-0.0311	-0.0298	-0.0285	-0.0260	-0.0226	-0.01833	-0.0133	-0.0098	-0.0053	-0.0018	0.0095
MIX		-0.9747	-0.9654	-0.9616	-0.9555	-0.94947	-0.9378	-0.9218	-0.9016	-0.8785	-0.8620	-0.8405	-0.8245	-0.7715
ICM		-0.1931	-0.1669	-0.1563	-0.1391	-0.1221	-0.0892	-0.0444	0.0125	0.0774	0.1240	0.1842	0.2294	0.3786
MLE	30	-0.0578	-0.0484	-0.0446	0.0385	-0.0324	-0.0206	-0.0046	0.0157	0.0383	0.0555	0.0771	0.09325	0.1965
MOM		-0.0395	-0.0334	-0.0309	-0.0269	-0.0229	-0.0153	-0.0048	0.0085	0.0236	0.0345	0.0485	0.0591	0.0939
PWM		0.0023	0.0016	0.0014	0.0010	0.0006	-0.0002	-0.0013	0.0027	-0.0042	-0.0053	0.0068	-0.0077	-0.0114
POME		-0.0515	-0.0434	-0.0402	-0.0349	-0.097	-0.0196	-0.0058	0.0117	0.0316	0.0459	0.0644	0.0783	0.1241
LEAS		-0.0430	-0.0386	-0.0369	-0.0340	-0.0311	-0.0256	-0.0180	0.0085	0.0025	0.0103	0.0204	0.0280	0.0531
MIX		-0.9640	-0.9579	-0.9554	-0.9514	-0.9474	-0.9398	-0.9293	-0.9160	-0.9009	-0.8900	0.8760	-0.8654	-0.8306
ICM		0.1758	0.1577	0.1504	0.1385	0.1268	0.1041	0.0731	0.0339	-0.0110	-0.0431	0.0847	-0.1159	-0.2190

MLE	40	-0.0441	-0.0369	-0.0341	-0.0294	-0.0248	-0.0159	-0.0037	0.0117	0.9293	0.0420	0.0583	0.0705	0.1110
MOM		-0.0313	0.0265	-0.0246	-0.0214	-0.0183	-0.0122	0.0040	0.0065	0.0184	0.0270	0.0381	0.0464	0.0739
PWM		0.0010	0.0006	0.0004	0.0002	-0.0001	-0.0006	-0.0013	-0.0021	-0.0031	-0.0038	-0.0097	-0.0054	-0.0076
POME		-0.0393	-0.0332	-0.0307	-0.0267	-0.0227	-0.0151	-0.0046	0.0086	0.0238	0.0346	0.0487	0.0592	0.0940
LEAS		-0.0400	-0.0356	-0.0338	-0.0308	-0.0280	-0.0224	-0.0148	-0.0051	0.0059	0.0138	0.0240	0.0317	0.0590
MIX		-0.9596	-0.9547	-0.9528	-0.9496	-0.9465	-0.9404	-0.9322	-0.9217	-0.7098	-0.9012	-0.8901	-0.8818	-0.8544
ICM		-0.1069	-0.0926	-0.0869	-0.0775	-0.0682	-0.0504	-0.0260	0.0050	0.0403	0.0657	0.0985	0.1231	0.2043
MLE	50	-0.0350	-0.0293	-0.0271	-0.0234	-0.0198	-0.0127	-0.0032	0.0090	0.0229	0.0322	0.0458	0.0554	0.0873
MOM		-0.0258	-0.0219	-0.0203	-0.0177	-0.0152	-0.0102	-0.0035	0.0050	0.0148	0.0217	0.0308	0.0376	0.0599
PWM		0.0013	0.0009	0.0007	0.0004	0.0001	-0.0005	0.0012	-0.0022	-0.0033	-0.0041	-0.0051	-0.0059	-0.0085
POME		-0.0309	-0.0261	-0.0242	-0.0211	-0.0180	-0.0121	-0.0039	0.0064	0.0181	0.0266	0.0375	0.0457	0.0727
LEAS		-0.0370	-0.0327	-0.0310	-0.0282	-0.0254	-0.0200	-0.0127	-0.0035	0.0071	0.0147	0.0245	0.0317	0.0562
MIX		-0.9565	-0.9526	-0.9510	-0.9484	-0.9459	-0.9410	-0.9342	-0.9257	-0.9160	-0.9090	-0.8999	-0.8932	-0.8708
ICM		0.0997	0.0894	0.0853	0.0785	0.0718	0.0590	0.0414	0.0191	-0.0063	-0.0246	-0.0482	-0.0659	-0.1294
MLE	60	-0.0294	-0.0248	-0.0229	-0.0198	-0.0168	-0.0110	-0.0030	0.0071	0.0186	0.0269	0.0376	0.0456	0.0721
MOM		-0.0208	-0.0177	-0.0164	-0.0144	-0.0124	-0.0085	-0.0032	0.0035	0.0111	0.0166	0.0237	0.0291	9.0467
PWM		0.0010	0.0005	0.0004	0.0001	-0.0002	-0.0007	-0.0014	-0.0023	-0.0034	-0.0091	-0.0051	-0.0058	-0.0082
POME		-0.0261	-0.0222	-0.0206	-0.0180	-0.0154	-0.0104	-0.0037	0.0049	0.0147	0.0217	0.0308	0.0376	0.0601
LEAS		-0.0390	-0.0392	-0.0323	-0.0291	-0.0260	-0.0200	-0.0118	-0.0014	0.0104	0.0189	0.300	0.0382	0.0655
MIX		-0.9533	-0.9507	-0.9495	-0.9475	-0.9954	-0.9416	-0.9363	-0.9296	-0.9449	-0.9164	-0.9093	-0.9040	-0.8864
ICM		-0.0759	-0.0660	-0.0620	-0.0554	-0.0490	-0.0365	-0.0195	0.0021	0.0267	0.0444	0.0673	0.0844	0.1410

MLE	70	-0.0268	-0.0225	-0.0208	-0.0180	-0.0153	-0.0100	-0.0027	0.0065	0.0170	0.0245	0.0342	0.0415	0.0657
MOM		-0.0191	-0.0163	-0.0151	-0.0133	-0.0115	-0.0079	0.0031	0.0030	0.0100	0.0150	0.0215	0.0264	0.0424
PWM		0.0000	-0.0002	-0.0003	-0.0005	-0.0007	-0.0010	-0.0015	-0.0021	-0.0027	-0.0032	-0.0038	-0.0043	-0.0058
POME		-0.0236	-0.0200	-0.0186	-0.0163	-0.0140	-0.0095	-0.0035	0.0043	0.0130	0.0193	0.0274	0.0335	0.0437
LEAS		-0.0352	-0.0308	-0.0291	-0.0262	-0.0234	-0.0280	-0.0106	-0.0012	0.0095	-0.0172	0.0271	0.0346	0.0592
MIX		-0.9529	-0.9501	-0.9489	-0.9971	-0.9453	-0.9417	-0.9369	-0.9308	-0.9238	-0.9188	-0.9123	-0.9074	-0.8914
ICM		0.0650	0.0583	0.0556	0.0512	0.0468	0.0384	0.0269	0.0124	-0.0043	-0.0162	-0.0316	-0.0432	-0.0814
MLE	75	-0.0239	-0.0202	-0.0187	-0.0163	-0.0139	-0.0093	-0.0030	0.0050	0.0141	0.0207	0.0291	0.0355	0.0564
MOM		-0.0185	-0.0158	-0.0147	-0.0130	0.0112	-0.0078	-0.0032	0.0027	0.0094	0.0142	0.0204	0.0251	0.0405
PWM		-0.0006	-0.0008	-0.0009	-0.0010	-0.0011	-0.0014	-0.0017	-0.0021	-0.0025	-0.0029	-0.0033	-0.0036	-0.0047
POME		-0.0213	-0.0181	-0.0169	-0.0148	-0.0127	-0.0088	-0.0034	0.0034	0.0112	0.0163	0.0241	0.0295	0.0474
LEAS		-0.0338	-0.0296	-0.0280	-0.0252	-0.0225	-0.0174	-0.0103	-0.0013	0.0090	0.0163	0.0258	0.0330	0.0566
MIX		-0.2526	-0.9999	-0.9988	-0.9470	-0.9153	-0.9419	-0.9373	-0.9314	-0.9247	-0.9194	-0.9136	-0.9090	-0.8936
ICM		0.0369	0.0312	0.0291	0.0256	0.0222	0.01156	0.0066	-0.0048	-0.0178	-0.0272	-0.0393	-0.0483	-0.0783
MLE	80	-0.0229	-0.0193	-0.0179	-0.0155	-0.0132	-0.0087	-0.0025	0.0053	0.0142	0.0205	0.0288	0.0350	0.0559
MOM		-0.0179	-0.0152	-0.0142	-0.0124	-0.0107	-0.0074	-0.0028	0.0029	0.0075	-0.0142	0.0202	0.0249	0.0400
PWM		-0.0004	-0.0006	-0.0006	-0.0007	-0.0009	-0.0011	-0.0014	-0.0018	-0.0022	-0.0025	-0.0029	-0.0032	-0.0042
POME		-0.0203	-0.0173	-0.0161	-0.0140	-0.0121	-0.0033	-0.0030	0.0036	0.0111	0.0165	0.0235	0.0288	0.0461
LEAS		-0.0307	-0.0269	-0.0254	-0.0230	-0.0205	-0.0158	-0.0094	-0.0013	0.0080	0.0147	0.0233	0.0276	0.0510
MIX		-0.9522	-0.9496	-0.9485	-0.9468	-0.9450	-0.9417	-0.9372	-0.9314	-0.7248	-0.9291	-0.9140	-0.9095	-0.8943
ICM		-0.0636	-0.0554	-0.0531	-0.0467	-0.0413	-0.0310	-0.0170	0.0009	0.0213	0.0359	0.0548	0.0690	0.1158

MLE	90	−0.0206	−0.0174	−0.0161	−0.0140	−0.0120	−0.0080	−0.0025	0.0044	0.0125	0.0179	0.0252	0.0307	0.0438
MOM		−0.0151	−0.0130	−0.0121	−0.0107	−0.0093	−0.0066	−0.0029	0.0018	0.0071	0.0109	0.0159	0.0195	0.03182
PWM		0.0003	−0.0001	−0.0002	−0.0004	−0.0006	−0.0010	−0.0016	−0.0023	−0.0031	−0.0037	−0.0045	−0.0056	−0.0069
POME		−0.0180	−0.0154	−0.0143	−0.0126	−0.0109	−0.0076	−0.0031	0.0025	0.0080	0.0137	0.0197	0.0242	0.0392
LEAS		−0.0324	−0.0284	−0.0267	−0.0240	−0.0214	−0.0163	−0.0093	−0.0005	0.0096	0.0169	0.0263	0.0333	0.0565
MIX		−0.9507	−0.9486	−0.9477	−0.9463	−0.9442	−0.9422	−0.9385	−0.9339	−0.9285	−0.9247	−0.9198	−0.9161	−0.9038
ICM		0.0466	0.0417	0.0398	0.0366	0.0334	0.0273	0.0190	0.0084	−0.0037	−0.0123	−0.0235	−0.0319	−0.0596
MLE	100	−0.0181	−0.0153	−0.0142	−0.0124	−0.0106	−0.0070	−0.0023	0.0038	0.0108	0.0158	0.0222	0.0270	0.0430
MOM		−0.0140	−0.0120	−0.0112	−0.0098	−0.0085	−0.0060	−0.0025	0.0019	0.0069	0.0105	−0.0152	0.0187	0.0302
PWM		−0.0001	−0.0002	−0.0003	−0.0004	−0.0006	−0.0009	−0.0013	−0.0019	−0.0025	−0.0029	− 0.0035	−0.0039	−0.0053
POME		−0.0160	−0.0137	−0.0127	−0.0112	−0.0096	−0.0067	−0.0027	0.0024	0.0082	0.0124	0.0178	0.0218	0.0352
LEAS		−0.0295	−0.0257	−0.0242	−0.0218	−0.0194	−0.0147	−0.0083	−0.0002	0.0090	0.0156	0.0242	0.0306	0.0518
MIX		−0.9501	−0.9481	−0.9473	−0.9460	−0.9446	−0.9421	−0.9386	−0.9342	−0.9292	−0.9256	− 0.9209	−0.9174	0.9059
ICM		−0.0538	−0.0470	−0.0443	−0.0398	−0.0353	−0.0368	−0.0151	−0.0002	0.0167	0.0288	0.0446	0.0563	0.0952

TABLE-9

STANDARD DEVIATION OF QUANTILE ESTIMATES

Method	Sample Size	F = 0.001 X = -1.933	0.010 -1.527	0.020 -1.364	0.050 -1.097	0.10 -0.834	0.250 -0.327	0.50 0.367	0.75 1.246	0.90 2.25	0.95 2.97	0.98 3.902	0.99 4.6	0.999 6.907
MLE	10	0.4941	0.4236	0.3984	0.3628	0.3364	0.3175	0.3651	0.5064	0.7137	0.8743	1.0888	1.2525	1.8016
MOM		0.6291	0.5295	0.4920	0.4358	0.3887	0.3342	0.3654	0.5356	0.7961	0.9976	1.2659	1.4701	2.1533
PWM		0.6021	0.5055	0.4696	0.4161	0.3722	0.3250	0.3643	0.5356	0.7918	0.9890	1.2513	1.4508	2.1181
POME		0.5059	0.4311	0.4044	0.3665	0.3382	0.3175	0.3674	0.5159	0.7323	0.8792	1.1219	1.2916	1.8606
LEAS		1.0801	0.9531	0.9026	0.8211	0.7424	0.5976	0.4292	0.3436	0.5021	0.6968	0.9789	1.1994	1.9477
MIX		0.3354	0.2472	0.2210	0.1980	0.2059	0.2898	0.4652	0.7142	1.0086	1.2221	1.4999	1.7087	2.4007
ICM		1.8295	1.5953	1.5020	1.3508	1.2042	0.9228	0.6165	0.5106	0.9011	1.2876	1.8199	2.2283	3.5982
MLE	20	0.3369	0.2876	0.2703	0.2462	0.2290	0.2195	0.2585	0.3620	0.5099	0.6236	0.7751	0.8906	1.2775
MOM		0.4514	0.3782	0.3507	0.3094	0.2748	0.2349	0.2590	0.3850	0.5756	0.7223	0.9174	1.0657	1.5616
PWM		0.4034	0.3384	0.3145	0.2794	0.2514	0.2247	0.2589	0.3800	0.5568	0.6923	0.8725	1.0094	1.4673
POME		0.3440	0.2921	0.2738	0.2482	0.2299	0.2194	0.2605	0.3691	0.5232	0.6413	0.7984	0.9180	1.3185
LEAS		0.7495	0.6609	0.6273	0.5730	0.5205	0.4234	0.3079	0.2350	0.3202	0.4439	0.6283	0.7736	1.2691
MIX		0.2407	0.1766	0.1577	0.1414	0.1478	0.2100	0.3379	0.5188	0.7324	0.8873	1.0888	1.2402	1.7419
ICM		0.9225	0.8123	0.7649	0.6890	0.6168	0.4890	0.3637	0.3743	0.5876	0.7880	1.0665	1.2816	2.0076
MLE	30	0.2676	0.2288	0.2153	0.1968	0.1839	0.1780	0.2108	0.2944	0.4129	0.5039	0.6252	0.7176	1.0273
MOM		0.3713	0.3112	0.2887	0.2547	0.2262	0.1927	0.2112	0.3134	0.4690	0.5889	0.7485	0.3697	1.2753
PWM		0.3194	0.2686	0.2499	0.2228	0.2015	0.1822	0.2112	0.3077	0.4480	0.5556	0.6987	0.8074	1.1713
LEAS		0.6067	0.5383	0.5110	0.4671	0.4245	0.3457	0.2516	0.1904	0.2569	0.3565	0.5055	0.6230	1.0241
MIX		0.8557	0.7456	0.7020	0.6318	0.5644	0.4427	0.3137	0.3015	0.4912	0.6732	0.9258	1.1204	1.7758
ICM		0.8557	0.7456	0.7020	0.6318	0.5644	0.4427	0.3137	0.3015	0.4912	0.6732	0.9258	1.1204	1.7758

MLE	40	0.2329	0.1983	0.1862	0.1696	0.1582	0.1535	0.1841	0.2597	0.3656	0.4467	0.5545	0.6366	0.9114
MOM		0.3241	0.2713	0.2514	0.2215	0.1964	0.1673	0.1846	0.2755	0.4128	0.5185	0.6589	0.7656	1.1223
PWM		0.2746	0.2303	0.2142	0.1907	0.1725	0.1571	0.1645	0.2702	0.3933	0.4873	0.6123	0.7072	1.0246
POME		0.2369	0.2007	0.1881	0.1707	0.1586	0.1535	0.1854	0.2639	0.3733	0.4568	0.5678	0.6521	0.9347
LEAS		0.5349	0.4746	0.4506	0.4118	0.3742	0.3046	0.2210	0.1656	0.2234	0.3112	0.4424	0.5460	0.8991
MIX		0.1730	0.1268	0.1131	0.1023	0.1059	0.1506	0.2427	0.3728	0.5265	0.6379	0.7828	0.8917	1.2525
ICM		0.6608	0.5761	0.5428	0.4994	0.4386	0.3489	0.2613	0.2683	0.4172	0.5581	0.7542	0.9059	1.4181
MLE	50	0.2084	0.1777	0.1670	0.1522	0.1418	0.1371	0.1631	0.2292	0.3226	0.3942	0.4895	0.5621	0.8053
MOM		0.2953	0.2465	0.2282	0.2004	0.1768	0.1488	0.1636	0.2470	0.3728	0.4694	0.5976	0.6951	1.0205
PWM		0.2480	0.2077	0.1930	0.1715	0.1546	0.1398	0.1638	0.2411	0.3521	0.4371	0.5498	0.6354	0.9217
POME		0.2126	0.1803	0.1689	0.1532	0.1422	0.1371	0.1648	0.2345	0.3320	0.4065	0.5055	0.5808	0.8329
LEAS		0.4837	0.4288	0.4069	0.3716	0.3373	0.2738	0.1972	0.1462	0.2001	0.2806	0.4004	0.4945	0.8155
MIX		0.1574	0.1145	0.1016	0.0901	0.0939	0.1348	0.2191	0.3381	0.4783	0.5800	0.7121	0.8115	1.1405
ICM		0.6413	0.5576	0.5245	0.4713	0.4203	0.3289	0.2350	0.2359	0.3855	0.5252	0.7182	0.8667	1.3668
MLE	60	0.1885	0.1611	0.1516	0.1385	0.1296	0.1258	0.1497	0.2093	0.2935	0.3581	0.4442	0.5097	0.7293
MOM		0.2649	0.2218	0.2056	0.1812	0.1607	0.1366	0.1501	0.2238	0.3354	0.4214	0.5357	0.6226	0.9130
PWM		0.2211	0.1858	0.1730	0.1545	0.1402	0.1282	0.1502	0.2186	0.3170	0.3924	0.4925	0.5686	0.8233
POME		0.1920	0.1633	0.1533	0.1395	0.1299	0.1259	0.1510	0.2133	0.3008	0.3676	0.4566	0.5243	0.7509
LEASE		0.4335	0.3847	0.3652	0.3338	0.3034	0.2471	0.1797	0.1352	0.1820	0.2529	0.3591	0.4430	0.7290
MIX		0.1445	0.1038	0.0927	0.0829	0.0864	0.1225	0.1973	0.3032	0.4283	0.5190	0.6370	0.7257	1.0196
ICM		0.5299	0.4621	0.4354	0.3926	0.3520	0.2804	0.2110	0.2177	0.3373	0.4503	0.6077	0.7294	1.1407

MLE	70	0.1905	0.1544	0.1453	0.1327	0.1238	0.1193	0.1406	0.1961	0.2753	0.3362	0.4174	0.4793	0.6867
MOM		0.2551	0.2140	0.1986	0.1732	0.1552	0.1309	0.1409	0.2089	0.3139	0.3951	0.5033	0.5855	0.8605
PWM		0.2122	0.1788	0.1665	0.1488	0.1348	0.1222	0.1409	0.2041	0.2963	0.3672	0.4616	0.5333	0.7735
POME		0.1840	0.1 567	0.1471	0.1337	0.1242	0.1193	0.1415	0.1994	0.2816	0.3445	0.4284	0.4922	0.7061
LEAS		0.4056	0.3596	0.3413	0.3117	0.2831	0.2304	0.1678	0.1292	0.1765	0.2441	0.3446	0.4240	0.6942
MIX		0.1365	0.1004	0.0895	0.0793	0.0815	0.1144	0.1846	0.2847	0.4031	0.4889	0.6006	0.6846	0.9629
ICEM		0.5235	0.4563	0.4298	0.3872	0.3465	0.2741	0.2013	0.2024	0.3198	0.4313	0.5865	0.7063	1.1105
MLE	75	0.1693	0.1436	0.1347	0.1226	0.1145	0.1122	0.1367	0.1943	0.2739	0.3345	0.4150	0.4761	0.6809
MOM		0.2419	0.2022	0.1873	0.1649	0.1459	0.1240	0.1371	0.2056	0.3087	0.3878	0.4930	0.5729	0.8400
PWM		0.1996	0.1671	0.1553	0.1382	0.1251	0.1149	0.1370	0.2015	0.2927	0.3626	0.4550	0.5253	0.7599
POME		0.1722	0.1455	0.1362	0.1235	0.1149	0.1122	0.1376	0.1971	0.2790	0.3412	0.4238	0.4866	0.6964
LEAS		0.4006	0.3554	0.3374	0.3083	0.2801	0.2278	0.1650	0.1230	0.1662	0.2321	0.3305	0.4081	0.6727
MIX		0.1291	0.0944	0.0842	0.0753	0.0788	0.1125	0.1816	0.2791	0.3942	0.4775	0.5860	0.6676	0.9377
ICM		0.5117	0.4458	0.4198	0.3782	0.3385	0.2680	0.1982	0.2021	0.3184	0.4261	0.5804	0.6981	1.0953
MLE	80	0.1678	0.1423	0.1334	0.1212	0.1127	0.1093	0.1321	0.1877	0.2653	0.3245	0.4032	0.4631	0.6633
MOM		0.2412	0.2016	0.1866	0.1639	0.1446	0.1213	0.1324	0.1995	0.3013	0.3797	0.4838	0.5628	0.8270
PWM		0.1980	0.1658	0.1540	0.1369	0.1236	0.1122	0.1323	0.1948	0.2843	0.3526	0.4432	0.5121	0.7423
POME		0.1713	0.1446	0.1353	0.1223	0.1133	0.1093	0.1328	0.1906	0.2708	0.3319	0.4129	0.4745	0.6807
LEAS		0.3911	0.3466	0.3289	0.3003	0.2726	0.2212	0.1594	0.1191	0.1637	0.2291	0.3262	0.4027.	0.6628.
MIX		0.1287	0.0933	0.0832	0.0736	0.0762	0.1090	0.1772	0.2737	0.3875	0.4700	0.5773	0.6579	0.9251
ICM		0.4735	0.4127	0.3888	0.3505	0.3142	0.2500	0.1879	0.1942	0.3017	0.4030	0.5440	0.6530	1.0211

MLE	90	0.1593	0.1360	0.1278	0.1164	0.1082	0.0038	0.1222	0.1711	0.2411	0.2949	0.3665	0.4212	0.6041
MOM		0.2228	0.1961	0.1722	0.1512	0.1333	0.119	0.1226	0.1852	0.2798	0.3524	0.4490	0.5223	0.7672
PWM		0.1857	0.1558	0.1449	0.1290	0.1165	0.1054	0.1229	0.1800	0.2125	0.3256	0.4095	0.4733	0.6265
POME		0.1619	0.1375	0.1287	0.1169	0.1084	0.1038	0.1237	0.1753	0.2482	0.3041	0.3784	0.4349	0.6243
LEAS		0.3645	0.3229	0.3064	0.2796	0.2537	0.2056	0.1479	0.1106	0.1530	0.2144	0.3054	0.3769	0.6201
MIX		0.1188	0.0865	0.0767	0.0679	0.0706	0.1011	0.1695	0.2540	0.3596	0.4360	0.5355	0.6102	0.8579
IC M		0.4498	0.3309	0.3676	0.3303	0.2947	0.2316	0.1696	0.1771	0.2846	0.3837	0.5205	0.6259	0.9808

MLE	100	0.1471	0.1252	0.1176	0.1072	0.1001	0.0975	0.1174	0.1656	0.2331	0.2846	0.3531	0.4053	0.5799
MOM		0.2139	0.1787	0.1654	0.1453	0.1282	0.1077	0.1176	0.1771	0.2675	0.3370	0.4293	0.4994	0.7338
PWM		0.1742	0.1461	0.1359	0.1210	0.1095	0.0999	0.1176	0.1724	0.2508	0.3106	0.3902	0.4506	0.6528
POME		0.1503	0.1273	0.1192	0.1082	0.1005	0.0975	0.1182	0.1685	0.2384	0.2917	0.3625	0.4163	0.5964
LEASE		0.3498	0.3098	0.2939	0.2682	0.2433	0.1971	0.1417	0.1063	0.1477	0.2069	0.2944	0.3632	0.5972
MIX		0.1141	0.0832	0.0738	0.0653	0.0677	0.0968	0.1573	0.2429	0.3439	0.4170	0.5122	0.5837	0.820[illegible]
ICM		0.4212	0.3671	0.3458	0.3116	0.2792	0.2219	0.1661	0.1712	0.2670	0.3572	0.4827	0.5796	0.9071

TABLE-10

EFFICIENCY OF QUANTILE ESTIMATES

Method	Sample Size	F = 0.001 X = -1.933	0.010 -1.527	0.020 -1.364	0.050 -1.097	0.10 -0.834	0.250 -0.327	0.50 0.367	0.75 +1.246	0.90 2.25	0.95 2.97	0.98 3.902	0.99 4.6	0.999 6.907
MLE	10	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MOM		0.6778	0.6990	0.7129	0.7458	0.7941	0.9235	0.9982	0.9016	0.8225	0.7920	0.7682	0.7566	0.7354
PWM		0.7641	0.7906	0.8066	0.8417	0.8886	0.9914	1.0051	0.9037	0.8382	0.8144	0.7963	0.7877	0.7723
POME		0.9796	0.9864	0.9896	0.9953	1.0005	1.0031	0.9867	0.9677	0.9595	0.9573	0.9560	0.9555	0.9549
LEAS		0.2374	0.2224	0.2183	0.2162	0.2234	0.2930	0.7198	2.1432	2.0365	1.6111	1.2833	1.1389	0.9052
MIX		0.2476	0.1957	0.1768	0.1496	0.1289	0.1096	0.1293	0.2062	0.3107	0.3763	0.4437	0.4826	0.5639
ICM		0.0761	0.0726	0.0719	0.0726	0.0767	0.1075	0.3093	0.9541	0.6463	0.4754	0.3689	0.3255	0.2585
MLE	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MOM		0.5842	0.6050	0.6199	0.6571	0.7149	0.8826	0.9957	0.8869	0.7925	0.7552	0.7257	0.7112	0.6841
PWM		0.7441	0.7686	0.7840	0.8191	0.8672	0.9735	0.9975	0.9116	0.8502	0.8264	0.8077	0.7984	0.7813
POME		0.9726	0.9814	0.9855	0.9929	0.9995	1.0028	0.9843	0.9637	0.9543	0.9514	0.9494	0.9487	0.9474
LEAS		0.2175	0.2011	0.1966	0.1943	0.2017	0.2730	0.7014	2.3697	2.5666	2.0090	1.5575	1.3594	1.0445
MIX		0.1202	0.0914	0.0817	0.0686	0.0594	0.0532	0.0694	0.1217	0.2015	0.2589	0.3250	0.3669	0.4635
ICM		0.1336	0.1280	0.1273	0.1295	0.1386	0.1989	0.4981	0.9387	0.7505	0.6225	0.5249	0.4800	0.4031
MLE	30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MOM		0.5377	0.5584	0.5737	0.6127	0.6745	0.8590	0.9964	0.8840	0.7801	0.7385	0.7055	0.6891	0.6586
PWM		0.7344	0.7585	0.7742	0.8099	0.8589	0.9670	0.9973	0.9179	0.8571	0.8326	0.8128	0.8032	0.7847
POME		0.9719	0.9800	0.9842	0.9913	0.9977	1.0014	0.9858	0.9673	0.9581	0.9550	0.9529	0.5520	0.9505
LEAS		0.2026	0.1878	0.1842	0.1833	0.1924	0.2672	0.6990	2.3917	2.6053	2.0203	1.5508	1.3464	1.0240
MIX		0.0774	0.0583	0.0520	0.0438	0.0382	0.0352	0.0473	0.0853	0.1471	0.1951	0.2546	0.2949	0.3963
ICM		0.0982	0.0942	0.0938	0.0961	0.1042	0.1553	0.4285	0.9445	0.7127	0.5647	0.4592	0.4128	0.3363

MLE	40	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MOM		0.5300	0.5474	0.5615	0.5984	0.6589	0.8468	0.9948	0.8895	0.7879	0.7467	0.7137	0.6974	0.6664
PWM		0.7455	0.7668	0.7812	0.8147	0.8614	0.9655	0.9961	0.9252	0.8700	0.8475	0.8294	0.8202	0.8029
POME		0.9749	0.9823	0.9861	0.9927	0.9985	1.0012	0.9860	0.9695	0.9616	0.9591	0.9573	0.9567	0.9553
LEAS		0.1953	0.1795	0.1755	0.1738	0.1821	0.2555	0.6911	2.4613	2.6938	2.0748	1.5836	1.3715	1.0386
MIX		0.0591	0.0438	0.0389	0.0325	0.0283	0.0263	0.0365	0.0683	0.1218	0.1651	0.2213	0.2608	0.3667
ICM		0.1254	0.1194	0.1186	0.1207	0.1301	0.1917	0.4918	0.9385	0.7659	0.6375	0.5374	0.4909	0.4106
MLE	50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.5082	0.5294	0.5452	0.5858	0.6512	0.8516	0.9937	0.8620	0.7516	0.7087	0.6750	0.6585	0.6278
PWM		0.7261	0.7517	0.7682	0.8059	0.8573	0.9694	0.9917	0.9054	0.8436	0.8192	0.7998	0.7903	0.7722
POME		0.9671	0.9775	0.9823	0.9909	0.9981	1.0006	0.9789	0.9565	0.9462	0.9430	0.9407	0.9401	0.9396
LEAS		0.1898	0.1754	0.1718	0.1707	0.1792	0.2514	0.6815	2.4605	2.6104	1.9822	1.5025	1.2992	0.9820
MIX		0.0475	0.0352	0.0313	0.0261	0.0227	0.0210	0.0289	0.0542	0.0980	0.0346	0.1836	0.2191	0.3187
ICM		0.1060	0.1017	0.1013	0.1038	0.1128	0.1697	0.4675	0.9396	0.7038	0.5660	0.4665	0.4223	0.3484
MLE	60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.5153	0.5365	0.5522	0.5927	0.6572	0.8512	0.9943	0.8759	0.7681	0.7252	0.6910	0.6741	0.6425
PWM		0.7444	0.7691	0.7848	0.8207	0.8687	0.9707	0.9935	0.9180	0.8607	0.8378	0.8191	0.8099	0.7922
POME		0.9691	0.9779	0.9826	0.9899	0.9967	0.9996	0.9826	0.9634	0.9540	0.9510	0.9488	0.9477	0.9463
LEAS		0.1921	0.1781	0.1748	0.1744	0.1840	0.2595	0.6912	2.3982	2.6027	2.0052	1.5300	1.3246	1.0025
MIX		0.0391	0.0290	0.0258	0.0217	0.0189	0.0177	0.0245	0.0459	0.0837	0.1163	0.1612	0.1949	0.2942
ICM		0.1270	0.1219	0.1215	0.1246	0.1351	0.1995	0.4992	0.9259	0.7557	0.6300	0.5316	0.4857	0.4065

MLE	70	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.5092	0.5296	0.5432	0.5812	0.6422	0.8328	0.9947	0.8823	0.7712	0.7267	0.6911	0.6737	0.6411
PWM		0.7400	0.7623	0.7767	0.8103	0.8564	0.9601	0.9958	0.9243	0.8662	0.8426	0.8232	0.8135	0.7953
POME		0.9680	0.9765	0.9806	0.9883	0.9955	1.0012	0.9868	0.9677	0.9576	0.9543	0.9518	0.9506	0.9489
LEAS		0.2010	0.1870	0.1836	0.1832	0.1928	0.2684	0.6994	2.3071	2.4363	1.8982	1.4677	1.2790	0.9802
MIX		0.0360	0.0267	0.0237	0.0199	0.0173	0.0159	0.0217	0.0406	0.0749	0.1049	0.1470	0.1791	0.2764
ICM		0.1197	0.1151	0.1147	0.1176	0.1272	0.1870	0.4796	0.9366	0.7439	0.6100	0.5084	0.4622	0.3838
MLE	75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.4967	0.5112	0.5240	0.5597	0.6209	0.8211	0.9941	0.8935	0.7888	0.7456	0.7107	0.6931	0.6600
PWM		0.7336	0.7534	0.7673	0.8010	0.8494	0.9601	0.9972	0.9306	0.8756	0.8542	0.8357	0.8263	0.8083
POME		0.9715	0.9791	0.9830	0.9898	0.9961	1.0007	0.9880	0.9728	0.9650	0.9623	0.9604	0.9594	0.9580
LEAS		0.1809	0.1654	0.1614	0.1599	0.1685	0.2429	0.6849	2.4983	2.7144	2.0742	1.5745	1.3596	1.0244
MIX		0.0316	0.0231	0.0204	0.0170	0.0148	0.0141	0.0205	0.0400	0.0744	0.1046	0.1469	0.1792	0.2782
ICM		0.1111	0.1053	0.1045	0.1065	0.1156	0.1759	0.4758	0.9243	0.7397	0.6104	0.5113	0.4655	0.3871
MLE	80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.4901	0.5048	0.5173	0.5523	0.6124	0.8140	0.9955	0.8866	0.7768	0.7326	0.6969	0.6795	0.6463
PWM		0.7312	0.7502	0.7630	0.7964	0.8437	0.9535	0.9974	0.9292	0.8737	0.8507	0.8319	0.8224	0.8041
POME		0.9634	0.9718	0.9761	0.9844	0.9925	1.0009	0.9887	0.9709	0.9614	0.9579	0.9554	0.9514	0.9519
LEAS		0.1864	0.1706	0.1665	0.1646	0.1724	0.2447	0.6843	2.4856	2.6301	2.0067	1.5278	1.3220	1.0025
MIX		0.0311	0.0227	0.0200	0.0166	0.0143	0.0134	0.0192	0.0374	0.0702	0.0991	0.1398	0.1712	0.2676
ICM		0.1256	0.1189	0.1177	0.1193	0.1283	0.1896	0.4905	0.9352	0.7717	0.6460	0.5467	0.5002	0.4196

MLE	90	1.00	1.00	1.000	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.5171	0.5401	0.5567	0.5982	0.6640	0.8613	0.9929	0.8549	0.7442	0.7019	0.6688	0.6528	0.6230
PWM		0.7476	0.7736	0.7897	0.8255	0.8739	0.9746	0.9884	0.9042	0.8457	0.8229	0.8046	0.7960	0.7794
POME		0.9721	0.9819	0.9865	0.9938	0.9998	0.9992	0.9756	0.9537	0.9445	0.9417	0.9399	0.9398	0.9388
LEAS		0.1926	0.1788	0.1753	0.1744	0.1830	0.2545	0.6798	2.3968	2.4807	1.8864	1.4368	1.2457	0.9474
Mix		0.0281	0.0207	0.0183	0.0153	0.0132	0.0121	0.0164	0.0313	0.0588	0.0835	0.1192	0.1472	0.2366
ICM		0.1261	0.1216	0.1213	0.1244	0.1348	0.1991	0.5125	0.9324	0.7192	0.5921	0.4972	0.4540	0.3805
MLE	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MOM		0.4784	0.4961	0.5103	0.5488	0.6132	0.8219	0.9965	0.8749	0.7603	0.7147	0.6783	0.6604	0.6269
PWM		0.7237	0.7452	0.7598	0.7947	0.8444	0.9574	0.9955	0.9235	0.8656	0.8418	0.8221	0.8124	0.7935
POME		0.9624	0.9713	0.9759	0.9843	0.9928	1.0002	0.9862	0.9672	0.9569	0.9533	0.9505	0.9494	0.9474
LEAS		0.1783	0.1647	0.1613	0.1608	0.1701	0.2448	0.6837	2.4285	2.4877	1.8873	1.4343	1.2419	0.9410
MIX		0.0240	0.0176	0.0155	0.0130	0.0113	0.0107	0.0152	0.0295	0.0555	0.0788	0.1127	0.1395	0.2263
ICM		0.1219	0.1162	0.1155	0.1180	0.1279	0.1914	0.4957	0.9363	0.7607	0.6327	0.5328	0.4864	0.4064

TABLE 11

COMPARISON OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES (RANK) FOR EVI DISTRIBUTION ON THE BASIS OF BIAS IN QUANTILES

Method	Sample	F = .001	.01	0.02	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.98	0.99	0.999
		x = 1.933	-1.527	-1.364	-1.097	-.834	-0.327	0.367	1.246	2.250	2.970	8.902	4.60	6.907
MLE	10	5	5	5	5	5	5	3	4	6	6	5	5	5
MOM		3	3	3	3	3	3	2	2	3	3	3	3	3
PWM		1	1	1	2	1	1	1	1	1	1	1	1	1
POME		4	4	4	4	4	4	4	3	5	5	4	4	4
LEAS		2	2	2	1	2	2	5	5	4	2	2	2	2
MIX		7	7	7	7	7	7	6	7	7	7	6	7	6
ICM		6	6	6	6	6	6	7	6	2	4	7	6	7
MLE	20	5	5	5	5	5	5	2	5	5	5	5	5	5
MOM		3	3	3	3	3	2	3	2	3	3	3	3	3
PWM		1	1	1	1	1	1	1	1	1	1	2	2	2
POME		4	4	4	4	4	4	4	4	4	4	4	4	4
LEAS		2	2	2	2	2	3	5	3	2	2	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	6	6	6	6	6
MLM	30	5	5	5	5	5	4	2	1	6	6	5	5	5
MOM		2	2	2	2	3	2	3	2	4	3	3	3	3
PWM		1	1	1	1	1	1	1	1	2	1	1	1	1
POME		4	4	4	4	2	3	4	13	5	4	4	4	4
LEASE		3	3	3	3	4	5	5	2	1	2	2	2	2
MIX		7	7	7	7	7	7	7	6	7	7	7	7	7
ICM		6	6	6	6	6	6	6	5	3	5	6	6	6

MLE	40	5	5	5	4	4	4	2	5	5	5	5	5	5
MOM		2	2	2	2	2	2	3	3	3	3	3	3	3
PWM		1	1	1	1	1	1	1	1	1	1	1	1	1
POME		3	3	3	3	3	3	4	4	4	4	4	4	4
LEAS		4	4	4	5	5	5	5	2	2	2	2	2	2
MIX		7	7	7	7	7	7	7	6	7	7	7	7	7
ICM		6	6	6	6	6	6	6	7	6	6	6	6	6
MLE	50	4	4	5	4	4	4	2	5	6	6	5	5	5
MOM		2	2	2	2	2	2	3	3	4	3	3	3	3
PWM		1	1	1	1	1	1	1	1	1	1	1	1	1
POME		3	3	3	3	3	3	4	4	5	4	4	4	4
LEAS		5	5	5	5	5	5	5	2	3	2	2	2	2
MIX		9	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	2	5	6	6	6
MLE	60	4	4	4	4	9	4	2	6	5	5	5	5	5
MOM		2	2	2	2	2	2	3	4	3	2	2	2	2
PWM		1	1	1	1	1	1	1	3	1	1	1	1	1
POME		3	3	3	3	3	3	4	5	4	4	4	3	3
LEAS		5	5	5	5	5	5	5	1	2	3	3	4	4
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	2	6	6	6	6	6

MIE	70	4	4	4	4	4	4	2	5	6	6	5	5	5
MOM		2	2	2	2	2	2	3	3	4	2	2	2	2
PWM		1	1	1	1	1	1	1	2	1	1	1	1	1
POME		3	3	3	3	3	3	4	4	5	5	4	3	3
LEAS		5	5	5	5	5	5	5	1	2	4	3	4	4
MIX		7	7	7	7	7	9	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	2	3	6	6	6
MIE	75	3	4	4	4	4	9	2	6	5	5	5	5	4
MOM		2	2	2	2	2	2	3	3	3	2	2	2	2
PWM		1	1	1	1	1	1	1	2	1	1	1	1	1
POME		9	3	3	3	3	3	4	4	4	4	3	3	3
LEAS		5	5	5	5	6	6	6	1	2	3	4	4	5
MIX		7	7	6	7	7	7	7	7	7	7	7	7	7
ICM		6	6	7	6	5	5	5	5	6	6	6	6	6
MIE	80	4	4	4	5	4	9	2	6	5	5	5	5	5
MOM		2	2	2	2	2	2	3	4	1	2	2	2	2
PWM		1	1	1	1	1	1	1	3	3	1	1	1	1
LEAS		3	3	3	4	3	3	4	5	4	4	4	4	3
MIX		5	5	5	3	5	5	5	2	2	3	3	3	4
ICM		7	7	7	7	7	7	7	7	7	7	7	7	7
		6	6	6	6	6	6	6	1	6	6	6	6	6

MLE	90	4	3	4	4	4	4	2	5	6	6	5	4	4
MOM		2	2	2	2	2	2	3	2	3	2	2	2	2
PWM		1	1	1	1	1	1	1	3	1	1	1	1	1
POME		3	4	3	3	3	3	4	4	4	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	5	5	6	6	5
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	2	4	4	5	6
MLE	100	4	4	4	4	4	3	2	4	5	5	4	3	4
MOM		2	2	2	2	2	2	3	2	2	2	3	2	2
PWM		1	1	1	1	1	1	1	2	1	1	1	1	1
POME		3	3	3	3	3	4	4	3	3	3	3	4	3
LEAS		5	5	5	5	5	5	5	1	4	4	5	5	5
MIX		7	7	7	7	7	7	7	5	7	7	7	7	7
ICM		6	6	6	6	6	6	6	1	6	6	6	6	6

TABLE 12

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES (RANKS) FOR EVI DISTRIBUTION ON THE BASIS OF STD DEV. OF QUANTILES

METHODS	Sample size	F=0.001 x=-1.933	0.01 -1.527	0.02 -1.364	0.05 -1.093	0.10 -0.834	0.25 -0.327	0.5 0.367	0.75 1.246	0.90 2.250	0.95 2.970	0.98 3.902	0.99 4.6	0.999 6.907
MLE	10	2	2	2	2	2	3	2	2	2	2	2	2	1
MOM		5	5	5	5	5	5	3	5	5	5	5	5	5
PWM		4	4	4	4	4	4	1	6	4	4	4	4	4
POME		3	3	3	3	3	2	4	4	3	3	3	3	2
LEAS		6	6	6	6	6	6	6	1	1	1	1	1	3
MIX		1	1	1	1	1	1	5	7	7	6	6	6	6
ICM		7	7	7	7	7	7	7	3	6	7	7	7	7
MLE	20	2	2	2	2	2	3	1	2	2	2	2	2	2
MOM		5	5	5	5	5	5	3	5	5	5	5	5	5
PWM		4	4	4	4	4	4	2	4	4	4	4	4	4
POME		3	3	3	3	3	2	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	6	7	7	7	6	6
ICM		7	7	7	7	7	7	7	7	6	6	6	7	7
MLE	30	2	2	2	2	2	2	1	2	2	2	2	2	2
MOM		5	5	6	5	5	5	2	6	5	5	5	5	5
PWM		4	4	4	4	4	4	3	5	4	4	4	4	4
POME		3	3	3	3	3	3	4	3	3	3	3	3	3
LEAS		6	6	5	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	7	7	7	6	6	6
ICM		7	7	7	7	7	7	7	4	6	6	7	7	7

MLE	40	2	2	2	2	2	3	1	2	2	2	2	2	2
MCM		5	5	5	5	5	5	3	5	5	5	5	5	5
PWM		4	4	4	4	4	4	2	4	4	4	4	4	4
PCME		3	3	3	3	3	2	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	7	7	7	7	6	6
ICM		7	7	7	7	7	7	7	6	6	6	6	7	7
MLE	50	2	2	2	2	2	2	1	2	2	2	2	2	1
MCM		5	5	5	5	5	5	2	5	5	5	5	5	5
PWM		4	4	4	4	4	4	3	4	4	4	4	4	4
PCME		3	3	3	3	3	3	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	2
MIX		1	1	1	1	1	1	6	7	7	7	7	7	7
ICM		7	7	7	7	7	7	7	6	6	6	6	6	6
MLE	60	2	2	2	2	2	2	1	2	2	2	2	2	2
MCM		5	5	5	5	5	5	2	5	5	5	5	5	5
PWM		4	4	4	4	4	4	3	4	4	4	4	4	4
PCME		3	3	3	3	3	3	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	7	7	7	7	6	6
ICM		7	7	7	7	7	7	7	6	6	6	6	7	7

MLE	70	2	2	2	2	2	3	1	2	2	2	2	2	1
MQM		5	5	5	5	5	5	2	5	5	5	5	5	5
PWM		4	4	4	4	4	3	3	4	4	4	4	4	4
POME		3	3	3	3	3	2	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	2
MIX		1	1	1	1	1	1	6	7	7	7	7	6	6
ICM		7	7	7	7	7	7	7	6	6	6	6	7	7
MLE	75	2	2	2	2	2	2	1	2	2	2	2	2	1
MQM		5	5	5	5	5	5	3	5	5	5	5	5	5
PWM		4	4	4	4	4	4	2	4	4	4	4	4	4
POME		3	3	3	3	3	1	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	2
MIX		1	1	1	1	1	3	6	7	7	7	7	6	6
ICM		7	7	7	7	7	7	7	6	6	6	6	7	7
MLE	80	2	2	2	2	2	3	1	2	2	2	2	2	2
MOM		5	5	5	5	5	5	3	6	5	5	5	5	5
PWM		4	4	4	4	4	4	2	5	4	4	4	4	4
POME		3	3	3	3	3	2	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	7	7	7	7	7	6
ICM		7	7	7	7	7	7	7	4	6	6	6	6	7

MLE	90	2	2	2	2	2	3	1	2	2	2	2	2	2
MOM		5	5	5	5	5	5	2	6	5	5	5	5	5
PWM		4	4	4	4	4	4	3	5	4	4	4	4	4
PCME		3	3	3	3	3	2	4	3	3	3	3	3	3
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	1
MIX		1	1	1	1	1	1	6	7	7	7	7	6	6
ICM		7	7	7	7	7	7	7	4	6	6	6	7	7
MLE	100	2	2	2	2	2	3	1	2	2	2	2	2	1
MOM		5	5	5	5	5	5	2	6	6	5	5	5	5
PWM		4	4	4	4	4	4	3	5	4	4	4	4	4
PCME		3	3	3	3	3	2	4	3	3	3	3	3	2
LEAS		6	6	6	6	6	6	5	1	1	1	1	1	3
MIX		1	1	1	1	1	1	6	7	7	7	7	7	6
ICM		7	7	7	7	7	7	7	4	5	6	6	6	7

TABLE - 13

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES (RANKS) FOR EVI DISTRIBUTION ON THE BASIS OF EFFICIENCY OF QUANTILE ESTIMATES

Method	Sample Size	F= 0.001	0.010	0.020	0.050	0.10	0.250	0.50	0.75	0.90	0.95	0.98	0.99	0.999
		X= 1.933	-1.527	-1.364	-1.097	-1.834	-0.327	0.367	1.246	2.25	2.97	3.902	4.6	6.907
MLE	10	1	1	1	1	2	2	2	2	2	2	2	2	1
MOM		4	4	4	4	4	4	3	6	5	5	5	5	5
PWM		3	3	3	3	3	3	1	5	4	4	4	4	4
POME		2	2	2	2	1	1	4	3	3	3	3	3	2
LEAS		6	5	5	5	5	5	5	1	1	1	1	1	3
MIX		5	6	6	6	6	6	7	7	7	7	6	6	6
ICM		7	7	7	7	7	9	6	4	6	6	7	7	7
MLE	20	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	4	6	5	5	5	5	5
PWM		3	3	3	3	3	3	2	5	4	4	4	4	4
POME		2	2	2	2	2	1	3	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	4	6	6	6	6	6
MLE	30	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	3	6	5	5	5	5	5
PWM		3	3	3	3	3	3	2	5	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	6	6	6	6	6

MLE	40	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	3	5	5	5	5	5	5
PWM		3	3	3	3	3	3	2	4	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	6	6	6	6	6
MLE	50	1	1	1	1	1	2	1	2	2	2	2	2	1
MOM		4	4	4	4	4	4	2	6	5	5	5	5	5
PWM		3	3	3	3	3	3	3	5	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	2
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	6	6	6	6	6
MLE	60	1	1	1	1	1	1	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	2	6	5	5	5	5	5
PWM		3	3	3	3	3	3	3	5	4	4	4	4	4
POME		2	2	2	2	2	2	4	3	3	3	3	3	3
LEAS		6	6	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		5	5	6	6	6	6	6	4	6	6	6	6	6

MLE	70	1	1	1	1	1	2	1	2	2	2	2	2	1
MOM		4	4	4	4	4	4	3	6	5	5	5	5	5
PWM		3	3	3	3	3	3	2	5	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	2
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	4	6	6	6	6	6
MLE	75	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	3	6	5	5	5	5	5
PWM		3	3	3	3	3	3	2	4	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	5	6	6	6	6	6
MLE	80	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	3	5	5	5	5	5	5
PWM		3	3	3	3	3	3	2	4	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	6	6	6	6	6	6

MLE	90	1	1	1	1	1	1	1	2	2	2	2	2	1
MOM		4	4	4	4	4	4	2	6	5	5	5	5	5
PWM		3	3	3	3	3	3	4	5	4	4	4	4	4
POME		2	2	2	2	2	2	3	3	3	3	3	3	3
LEAS		5	5	5	5	5	5	5	11	1	1	1	1	1
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	4	6	6	6	6	6
MLE	100	1	1	1	1	1	2	1	2	2	2	2	2	2
MOM		4	4	4	4	4	4	2	6	5	5	5	5	5
PWM		3	3	3	3	3	3	3	5	4	4	4	4	4
POME		2	2	2	2	2	1	4	3	3	3	3	3	2
LEAS		5	5	5	5	5	5	5	1	1	1	1	1	3
MIX		7	7	7	7	7	7	7	7	7	7	7	7	7
ICM		6	6	6	6	6	6	6	4	6	6	6	6	6

quantiles corresponding to F = 0.75, 0.9, 0.95, 0.98, 0.99, and 0.999 while MLE is the second best for top six quantiles and best for lower six quantiles nearly for all the sample sizes. In general POME is better than PWM and PWM is better than MOM.

It is difficult to decide about the method which gives minimum bias and standard deviation and maximum efficiency of all the quantiles for all the samaple sizes. As such, it is proper to compare all the methods based an average of relative and average of squares of relative deviations between computed and expected value of reduced variates.

5.2.4 Average of relative deviations between computed and expected values of reduced variates (DA)

The values of DA and its rank are given in Table 14. The various methods can be in general ranked as given below in descending order for all the sample sizes.

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

5.2.5 Average of squares of relative deviations between computed and expected value of reduced variates (DR)

The values of DR and its rank are given in Table 15. The various methods can be in general ranked as given below in descending order for all the sample sizes:

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

TABLE - 14

COMPARISION OF DIFFERENT PARAMETER ESTIMATION FOR EVI DISTRIBUTION TECHNIQUES ON THE BASIS OF AVERAGE OF SQUARES OF RELATIVE DIVIATION BETWEEN COMPUTED AND EXPECTED VALUE OF REDUCED VARIATES (DA)

Method	Sample Size	DR AND RANK FOR SAMPLE SIZE																					
		10		20		30		40		50		60		70		75		80		90		100	
MLE		60.9	4	10.80	3	7.8	3	32.1	3	5.4	3	4.7	3	6.3	3	5.6	3	4.1	3	3.6	3	4.9	3
MOM		52.2	3	10.80	3	8.3	5	37.4	5	6.4	5	5.6	5	7.9	5	7.1	6	5.3	5	4.7	5	6.6	5
PWM		0.90	1	0.2	1	0.3	1	1.7	1	.3	1	.42	1	0.7	1	.8	1	0.5	1	.5	1	.7	1
POME		61.9	5	10.9	4	7.9	4	33.7	4	5.5	4	4.8	4	6.7	4	5.8	4	4.3	4	3.8	4	5.2	4
LEAS		33.4	2	6.8	2	5.5	2	25.9	2	4.3	2	4.0	2	5.5	2	4.8	2	3.5	2	3.3	2	4.6	2
MIX		930.5	7	307.1	6	318.1	7	1864.2	7	360.3	7	364.5	7	565.8	7	523.1	7	412.7	7	393.1	7	616.90	7
ICM		231.1	6	22.9	5	27.7	6	78.6	6	18.2	6	11.0	6	19.0	6	6.2	5	10.6	6	9.3	6	13.9	6

TABLE - 15

COMPARISION OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EVI DISTRIBUTION ON THE BASIS OF AVERAGE OF SQUARES OF RELATIVE DEVIATIONS BETWEEN COMPUTED AND EXPECTED VALUE OF REDUCED VARIATES (DR)

Method	Sample Size	DR AND RANK FOR SAMPLE SIZE																				
	10		20		30		40		50		60		70		75		80		90		100	
MLE	252.1	4	3.4	3	2.4	4	304.7	3	1.4	3	1.1	3	7.1	3	4.9	3	1.3	3	.9	3	6.3	2
MOM	187.3	3	3.5	4	2.5	5	419.2	5	2.1	5	1.8	5	11.6	5	8.2	6	2.4	5	1.6	5	1..8	4
PWM	.03	1	.0008	1	.002	1	.9	1	.005	1	.009	1	0.1	1	.1	1	.02	1	.02	1	.1	1
POME	263.7	5	3.6	5	2.3	3	340.2	4	1.6	4	1.3	4	8.3	4	5.6	4	1.6	4	1.0	4	7.4	3
LEAS	79.8	2	1.6	2	1.2	2	208.1	2	1.0	2	1.0	2	6.0	2	4.2	2	1.1	2	.8	2	6.3	2
MIX	62688.1	7	3384.5	7	4082.6	7	1068572.5	7	7495.3	7	8202.3	7	63065.0	7	48519.4	7	16007.9	7	12062.7	7	107894.2	6
ICM	3951.4	6	18.9	6	33.1	6	1909.7	6	20.08	6	7.8	6	73.8	6	6.7	5	10.9	6	7.1	6	56.2	5

6.0 CONCLUSIONS

The various methods have been statistically inter-compared using synthetically generated samples of EV1 distribution. The sample sizes have been taken as 10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100. Based on this study following conclusions can be drawn:

1. Based on bias, standard deviation and efficiency of parameters and quantiles, it is difficult to arrive at any definit conclusion as no method is the best according to all the criteria. However MIX and ICM are the least accurate methods. PWM, POME and MLE give nearly similar results and can be recommended for use in practice. This is in confirmation with the results of Jain and Singh (1987) and Arora and Singh (1987).

2. Based on average of relative deviations and square of deviations between expected and computed value of reduced variates, the ranking of different methods is as given below in descending order.

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

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