TRAINING COURSE

ON

COMPUTER APPLICATIONS IN HYDROLOGY

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 10

Groundwater Modelling

BY

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GROUNDWATER MODELLING

1.0 INTRODUCTION

Saturated groundwater flow may be confined or unconfined and steady or unsteady; the flow may be one dimensional two dimensional or three dimensional. If the flow domain boundaries are known a priori, the flow is known as confined flow. In a confined flow domain the flow is linearly proportional to the potential difference across the boundaries that causes the flow to take place. When a phreatic line or an interface of fresh water and saline water forms the boundary of the flow domain, the flow is termed as unconfined flow. In an unconfined flow the flow and the potential difference causing the flow are nonlinearly dependent. If the boundary heads change with time and there is a change in the hydrologic stress, the resulting flow becomes unsteady.

A model is a device that represents an approximation of a field situation. Physical models such as laboratory sand tanks simulate groundwater flow directly. A mathematical model simulates ground water flow indirectly by solving the governing differential equation satisfying the prevailing boundary and the initial conditions. Mathematical models can be solved analytically or numerically.

Groundwater models have been divided broadly into two categories: groundwater flow models and solute transport models. Groundwater flow models solve for the distribution of head, where as solute transport models solve for the concentration of solute as affected by advection, dispersion and chemical reaction.

A mathematical model describing the groundwater system must incorporate the following aspects (Keshari, 1994):

i. geometry of the aquifer boundary and its physical domain under investigation;

ii. nature of the porous medium (heterogeneity, anisotrophy, porosity, hydraulic conductivity, storativity);

iii. mode of flow in the aquifer (3-dimensional, 2-dimensional or 1-dimensional);

iv. flow regime (laminar or turbulent);

v. relevant state variables (hydraulic head, temperature, solute concentration);

vi. sources and sinks of water and of relevant pollutants within the aquifer domain and on its boundaries (point or distributed sources and sinks);

vii. boundary conditions of the aquifer domain being investigated (Dirichlet, Neumann, Cauchy);

viii. initial conditions for the flow and transport process (in transient cases);

ix. nature of pollutant (conservative, radioactive, degradable or adsorbent);

x. dispersivities.

2.0 AQUIFER

An aquifer is a saturated bed, formation, or group of formations which yields water in sufficient quantity to be economically useful. Water-bearing formations and groundwater reservoirs are synonyms for the word aquifer. To be an aquifer, a geologic formation must contain pores or open spaces (both of these are often called interstices) that are filled with water. These interstices must be large enough to transmit water toward wells at a useful rate.

Aquifers have two main functions in the underground phase of the water cycle. They store water for varying periods in the underground reservoir, and they act as pathways or conduits to pass water along through the reservoir. Although some are more efficient as pipelines (e.g., cavernous limestones) and some are more effective as storage reservoirs (e.g., sandstones), most aquifers perform both functions continuously.

Aquifers may be classified as unconfined or confined depending on the presence or absence of a water table. For an unconfined aquifer a water table serves as the upper surface of the zone of saturation. The water table is defined as "that surface in the groundwater body at which the water pressure is atmospheric". Unconfined groundwater, then, is water in an aquifer that has a water table in contact with the atmosphere through pores in the unsaturated soil above. Unconfined aquifers are sometimes called water table aquifers. Confined groundwater, on the other hand, is water under pressure greater than atmospheric pressure. The upper boundary of a confined aquifer is an essentially impermeable formation that "traps" or "confines" water in the aquifer, sealing it off from the atmosphere.

Both the size of pores and the total number of pores in a formation can vary remarkably, depending on the types of material and the geologic and chemical history. Individual pores in a fine-grained sediment such as clay are extremely small, but the combined volume of the pores can be unusually large. Subsequent compaction of clay reduces the pore space considerably. Although clay has a large water holding capacity, water cannot move readily through the tiny open spaces. This means that a clay formation under normal conditions will not yield water to wells, and therefore it is not an aquifer even though it may be water-saturated.

Ordinarily a clay or shale formation is nearly impermeable and is called an aquiclude, or a formation through which virtually no water moves. Formations which do yield some water, but usually not enough to meet even modest demands, are called aquitards. In reality, almost all formations will yield some water, and therefore are classified as either aquifers or aquitards. In water-poor areas, a formation producing small quantities of water may be called an aquifer, whereas the same formation in a water-rich area would be an aquitard.

A non-dimensional parameter, K_c , designated as conductivity class has been defined (Bear and Verruijt, 1987) by the relation

$$K_c = -[2 + \log_{10} K]$$

in which K is the hydraulic conductivity in m/s. K_c is used as an index to classify the aquifers and the porous media as given in Table-1.

Table 1 Classification of porous media/aquifer using conductivity class (K_c)

Porous media/Aquifer	K_c
Pervious	-2 to 2
Semipervious	2 to 6
Impervious	6 to 11
Good aquifer	-2 to 3
Poor aquifer	3 to 7
No aquifer	7 to 11

3.0 MODELLING FROM AQUIFER VIEWPOINT

A complete mathematical description of a model consists of a statement of the governing equation, the boundary conditions and the initial condition. The governing differential equations are either derived **from aquifer view point** (Anderson and Woessner, (1992)) considering the groundwater system to be comprised of aquifers, aquitard, aquiclude and bed sources or derived **from flow system view point** considering the entire flow domain as a nonhomogeneous unit. In the aquifer view point the hydraulic conductivity is integrated in the vertical dimension to give an average transmission characteristic known as transmissivity, or hydraulic conductivity times the aquifer's saturated thickness. The transmissivity of a confined aquifer is constant if the aquifer is homogeneous and of uniform thickness, but the transmissivity of an unconfined aquifer always varies spatially because saturated thickness depends on the elevation of the water table. Although assumed to be constants in the analytical solution used in well hydraulics, hydraulic conductivity and transmissivity, vary spatially in field situations because aquifers are always heterogeneous.

The aquifer viewpoint is used to simulate two-dimensional horizontal flow in confined and unconfined aquifers. Leaky confined aquifers can be simulated using a quasi three-dimensional approach whereby vertical flow through confining beds is represented by a leakage term that adds or extracts water from the aquifers overlying and underlying the confined leaky aquifer. The amount of leakage depends on the hydraulic gradient across the confining beds and the thickness and vertical hydraulic conductivity of the confining beds. Confining beds are not explicitly modeled and heads in confining beds are not calculated.

In the aquifer viewpoint, while modelling a three layered ground water system, where the upper layer is an unconfined aquifer, the second layer is a clay layer, and the third layer is a confined aquifer, heads in the confining bed are not of interest and are not calculated during the simulation. A one-layer model may be used to represent the system if the head in the unconfined aquifer is at steady state and will not be affected by changes in the confined aquifer. Attention is then focused on the confined aquifer; heads in the unconfined aquifer are not calculated but are used to calculate the hydraulic gradient across the confining bed. A numerical model of the system based on the flow system viewpoint would have at least three layers and heads would be calculated in each layer. In the flow system viewpoint equipotential lines pass through all geologic units, both aquifers and confining beds.

A general forms of the governing equation from the aquifer viewpoint is

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} + Q - L$$

where

$$L = K_z' \frac{h_{source} - h}{b'}$$

S is the storage coefficient; Q is the withdrawal per unit area (meter/sec) and L is the leakage per unit area (meter/sec). The terms on the left-hand side of above differential equation represent horizontal flow through the aquifer where h is head and T_x and T_y are transmissivities in x and y direction.

When above equation is applied to an unconfined aquifer, the Dupuit assumptions are used:(1) flow lines are horizontal and equipotential lines are vertical and (2) the horizontal hydraulic gradient is equal to the slope of the free surface and is invariant with depth. It is understood that $T_x = K_x h$ and

$T=K_y h$ where h is the elevation of the water table above the bottom of the aquifer, i.e., the saturated thickness; h may vary in both time and space. S is understood to be the specific yield. The leakage terms is typically zero unless there is leakage to or from a unit located below the unconfined aquifer.

4.0 MODELLING FROM FLOWSYSTEM VIEWPOINT

In the flow system viewpoint, one is not concerned with identifying aquifers and confining beds per se but in constructing the three-dimensional distribution of heads, hydraulic conductivities, and storage properties everywhere in the system. The flowsystem viewpoint allows for both vertical and horizontal components of flow throughout the system and thereby allows treatment of flow in two-dimensional profile or in three dimensions. A general forms of the governing equation is

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t} - R^*$$

where K_x, K_y, and K_z are hydraulic conductivities in x y and z directions which are the principal directions. S_s is specific storage; R^* is a general source term which is the volume of inflow to the system per unit volume of aquifer per unit of time. To simulate outflow $R^* = -W^*$.

5.0 GENERALISED DARCY'S LAW

If the properties of the soil that are responsible for the resistance to flow are independent of the direction, such a material is said to be isotropic with regard to permeability. Not every soil possesses that property, however. In many soil deposits the resistance to flow in the vertical direction is considerably larger than the resistance to horizontal flow, due to the presence of layered structure in the soil, generated by its geological history. For such anisotropic porous media Darcy's law has to be generalised. The proper generalisation is, in terms of hydraulic conductivity (Verrujit, 1982)

$$q_x = -k_{xx}\frac{\partial \phi}{\partial x} - k_{xy}\frac{\partial \phi}{\partial y} - k_{xz}\frac{\partial \phi}{\partial z}$$

$$q_y = -k_{yx}\frac{\partial \phi}{\partial x} - k_{yy}\frac{\partial \phi}{\partial y} - k_{yz}\frac{\partial \phi}{\partial z} \qquad (1)$$

$$q_z = -k_{zx}\frac{\partial \phi}{\partial x} - k_{zy}\frac{\partial \phi}{\partial y} - k_{zz}\frac{\partial \phi}{\partial z}$$

The quantities q_x, q_y and q_z are the three components of the specific discharge vector q, where specific discharge vector denotes the discharge through a certain area of soil divided by that area. $\partial\phi/\partial x$, $\partial\phi/\partial y$ and $\partial\phi/\partial z$ are components of the hydraulic head gradient in x, y and z direction respectively. These equations express the most general linear relationship between the specific discharge vector and the gradient of the groundwater head. The coefficients k_{xx} ,........,k_{zz} are said to be components of a second-order tensor. It is usually assumed, on the basis of thermodynamic considerations, that this is a symmetric tensor (that is, $k_{xy}=k_{yx}$,$k_{yz}=k_{zy}$,$k_{zx}=k_{xz}$). It can be shown that this means that there exist three mutually orthogonal directions, the so-called principal directions of permeability, in which the cross-components vanish. Physically speaking this means that a gradient of the groundwater head in one of these directions leads to a flow in that same direction only.

Consider the two-dimensional case of an anisotropic porous medium consisting of two orthogonal systems of channels of different cross-section, and hence of different resistance (Fig.1)

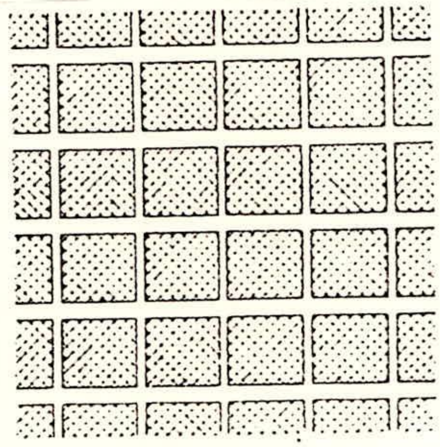

Fig. 1 Anisotropic Porous Medium

In this case the principal directions coincide with the direction of the channels, and one may write

$$q_x = -k_{xx} \frac{\partial \phi}{\partial x} \qquad (2)$$

$$q_y = -k_{yy} \frac{\partial \phi}{\partial y}$$

Where, $\phi = p/\gamma_w + z$ = the groundwater head

p = pressure

γ_w = unit weight of water

z = elevation head (the vertical z-axis is positive upwards)

If the channels in the x-direction are wider than those in the y-direction, the permeability k_{xx} will be greater than k_{yy}. Now consider the same situation being described with respect to coordinates ξ and η, which are obtained from x and y by rotation through an angle α.

$$\xi = x \cos \alpha + y \sin \alpha$$

$$\eta = y \cos \alpha - x \sin \alpha \qquad (3)$$

A vector q with components q_x and q_y can also be decomposed into components q_ξ and q_η , where

$$q_\xi = q_x \cos \alpha + q_y \sin \alpha$$

$$q_\eta = q_y \cos \alpha - q_x \sin \alpha \qquad (4)$$

Incorporating equation (2) in (4)

$$q_\xi = -k_{xx} \cos \alpha \frac{\partial \phi}{\partial x} - k_{yy} \sin \alpha \frac{\partial \phi}{\partial y}$$

$$q_\eta = -k_{yy} \cos \alpha \frac{\partial \phi}{\partial y} + k_{xx} \sin \alpha \frac{\partial \phi}{\partial x} \qquad (5)$$

The partial derivatives $\partial\phi/\partial x$ and $\partial\phi/\partial y$ can be related to $\partial\phi/\partial\xi$ and $\partial\phi/\partial\eta$ with the aid of equation 3. This gives

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial \phi}{\partial \xi}\cos \alpha - \frac{\partial \phi}{\partial \eta}\sin \alpha$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial \phi}{\partial \xi}\sin \alpha + \frac{\partial \phi}{\partial \eta}\cos \alpha \qquad (6)$$

Using these expressions the equations (5) can be written as

$$q_\xi = -k_{\xi\xi}\frac{\partial \phi}{\partial \xi} - k_{\xi\eta}\frac{\partial \phi}{\partial \eta}$$

$$q_\eta = -k_{\eta\xi}\frac{\partial \phi}{\partial \xi} - k_{\eta\eta}\frac{\partial \phi}{\partial \eta} \qquad (7)$$

where,

$$k_{\xi\xi} = k_{xx}\cos^2\alpha + k_{yy}\sin^2\alpha = \frac{1}{2}(k_{xx} + k_{yy}) - \frac{1}{2}(k_{xx} - k_{yy})\cos 2\alpha$$

$$k_{\eta\eta} = k_{yy}\cos^2\alpha + k_{xx}\sin^2\alpha = \frac{1}{2}(k_{xx} + k_{yy}) + \frac{1}{2}(k_{yy} - k_{xx})\cos 2\alpha \qquad (8)$$

$$k_{\eta\xi} = k_{\xi\eta} = (k_{yy} - k_{xx})\sin\alpha\cos\alpha = \frac{1}{2}(k_{yy} - k_{xx})\sin 2\alpha$$

Equations (7) are the description of a general flow, using the coordinates ξ and η. It is interesting to note the appearance of the cross-coefficients $k_{\xi\eta}$ and $k_{\eta\xi}$ vanish only if $k_{xx}=k_{yy}$(when the soil is isotropic) or if $\alpha=0$, π etc. (when the ξ, η-coordinates coincide with x and y). It now follows that a gradient of the groundwater head in ξ-direction not only leads to a flow in that direction, but also to a flow in η-direction. This can be realised physically by noting (see Fig.1) that in that case the channels in x-direction will transport much more water than the narrow channels in y-direction (because $k_{xx} > k_{yy}$). This means that the resultants flow will always have a tendency towards the most permeable direction. It also means that in general anisotropy cannot be formulated by simply using different coefficients in the three directions in which Darcy's law is formulated. In general the anisotrophy law should be of the form of equation (1), with six independent coefficients. Fortunately in engineering practice it is usually acceptable to distinguish only between the permeability in vertical direction and one in horizontal direction, assuming that this difference has been created during the geological process of deposition of the soil. Then it may assumed that the x,y and z-directions are principal directions (if the z-axis is vertical), with $k_{xx} = k_{yy} = k_h$ and $k_{zz} = k_v$. Darcy's law can then be used in the form

$$q_x = -k_n \frac{\partial\phi}{\partial x}$$

$$q_y = -k_n \frac{\partial\phi}{\partial y} \qquad (9)$$

$$q_z = -k_v \frac{\partial\phi}{\partial z}$$

which involves only two coefficients. They must be measured by doing two independent tests.

6.0 SCIENCE OF MODELLING

For the steady state flow of viscous, incompressible fluids (at small Reynolds numbers), the Navier-Stokes equations of motion, the most general equation governing fluid flow, in which conservation of momentum is embedded, reduce in form to generalised statement of Darcy's law. The mass balance equation (continuity equation) when combined with Darcy's law yields the equation of flow.

The equations of flow are partial differential equations with respect to time and space. They are of parabolic type. Both initial conditions in the whole modelled domain and boundary conditions on the boundary must be given for their solution. The initial conditions consist of the known head distribution at an initial time t, from which the simulation is supposed to start. There are three possible types of boundary conditions which may apply to any part of the boundary of the modelled domain (Kinzelbach, 1986).

- Boundary conditions of the first kind (Dirichlet type) prescribe the head value. In a modelled domain there should be at least one point that constitutes a first-kind boundary. This is necessary to guarantee the uniqueness of the solution.

- Boundary conditions of the second kind (Neumann type) specify the boundary flux, which means the head gradient normal to the boundary. A special case of this type of boundary is the impervious boundary where the flux is zero. If streamlines form boundaries of the modelled domain they are treated as impervious boundaries. Wells can be viewed as inner boundaries of the second kind by cutting out a circle around the well and specifying the flow across the circle. In view of the discretization in mathematical models, wells are considered as distributed recharge and discharge sources and not as boundaries.

- Boundary conditions of the third kind (semipervious boundary, mixed boundary conditions) specify a linear combination of head and flux at a boundary. They are used at semipervious (leakage) boundaries. If a leaky river forms the boundary the appropriate condition is of the form:

$$Q = \Gamma_r (h_r - h) \tag{10}$$

While the equation of the confined aquifer is linear, the equation of the phreatic aquifer is nonlinear. This does not present major problems for the numerical solution. In cases where the spatial and temporal variations of head are small compared to the thickness of saturated flow, the equation can be linearized. The phreatic aquifer may in that case be described by the confined aquifer equation.

7.0 DERIVATION OF THE FINITE DIFFERENCE FORM

The finite difference form of the linearized Boussinesq's equation can be derived through mass balance and application of Darcy's law as given below:

The harmonic mean of transmissivity values at interfaces 2,3,4,5 of node i,j (fig.2) are respectively given by

$$T_{hm2}(i,j) = \frac{\Delta X_j + \Delta X_{j+1}}{\dfrac{\Delta X_j}{T(i,j)} + \dfrac{\Delta X_{j+1}}{T(i,j+1)}} \quad ; \quad T_{hm3}(i,j) = \frac{\Delta y_i + \Delta y_{i+1}}{\dfrac{\Delta y_i}{T(i,j)} + \dfrac{\Delta y_{i+1}}{T(i+1,j)}}$$

$$T_{hm4}(i,j) = \frac{\Delta X_j + \Delta X_{j-1}}{\dfrac{\Delta X_j}{T(i,j)} + \dfrac{\Delta X_{j-1}}{T(i,j-1)}} \quad ; \quad T_{hm5}(i,j) = \frac{\Delta y_i + \Delta y_{i-1}}{\dfrac{\Delta y_i}{T(i,j)} + \dfrac{\Delta y_{i-1}}{T(i-1,j)}}$$

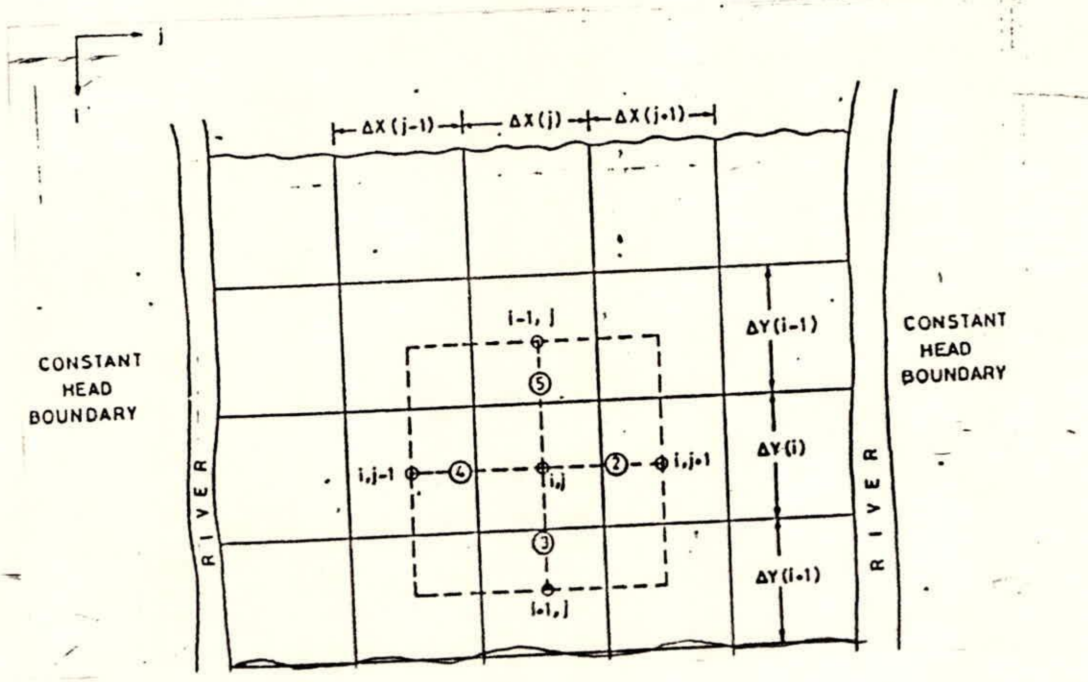

Fig. 2 Discretisation of the study area

Applying mass balance equation for the control volume containing node i,j for a time period of Δt during kth time step i.e Inflow-outflow = Change in storage we get

$$w\left[T_{hm2}(i,j)\left\{\frac{h^k(i,j+1)-h^k(i,j)}{\dfrac{\Delta x_{j+1}+\Delta x_j}{2}}\right\}\Delta y_i + T_{hm3}(i,j)\left\{\frac{h^k(i+1,j)-h^k(i,j)}{\dfrac{\Delta y_i+\Delta y_{i+1}}{2}}\right\}\Delta x_j\right.$$

$$+T_{hm4}(i,j)\left\{\frac{h^k(i,j-1)-h^k(i,j)}{\frac{\Delta x_j+\Delta x_{j-1}}{2}}\right\}\Delta y_i+T_{hm5}(i,j)\left\{\frac{h^k(i-1,j)-h^k(i,j)}{\frac{\Delta y_{i-1}+\Delta y_i}{2}}\right\}\Delta x_j\Bigg]\Delta t$$

$$+(1-w)\Bigg[T_{hm2}(i,j)\left\{\frac{h^{k-1}(i,j+1)-h^{k-1}(i,j)}{\frac{\Delta x_{j+1}+\Delta x_i}{2}}\right\}\Delta y_i+T_{hm3}(i,j)\left\{\frac{h^{k-1}(i+1,j)-h^{k-1}(i,j)}{\frac{\Delta y_i+\Delta y_{i+1}}{2}}\right\}\Delta x_j$$

$$+T_{hm4}(i,j)\left\{\frac{h^{k-1}(i,j-1)-h^{k-1}(i,j)}{\frac{\Delta x_j+\Delta x_{j-1}}{2}}\right\}\Delta y_i+T_{hm5}(i,j)\left\{\frac{h^{k-1}(i-1,j)-h^{k-1}(i,j)}{\frac{\Delta y_{i-1}+\Delta y_i}{2}}\right\}\Delta x_j\Bigg]\Delta t$$

$$-Q_P^k(i,j)\,\Delta t+Q_R^k(i,j)\,\Delta t=S(i,j)\,\Delta x_j\,\Delta y_i\,\{h^k(i,j)-h^{k-1}(i,j)\} \qquad (11)$$

Dividing terms on either sides by $\Delta x_j \Delta y_i \Delta t$ and simplifying equation (11) reduces to

$$F_1(i,j)\,h^k(i,j)+F_2(i,j)\,h^k(i,j+1)+F_3(i,j)\,h^k(i+1,j)$$

$$+F_4(i,j)\,h^k(i,j-1)+F_5(i,j)\,h^k(i-1,j)=F_6^k(i,j) \qquad (12)$$

In which $F_2(i,j)$, $F_3(i,j)$ etc. are given by:

$$F_2(i,j)=\frac{wT_{hm2}(i,j)}{\left(\frac{\Delta x_j+\Delta x_{j+1}}{2}\right)\Delta x_j} \quad ; \quad F_3(i,j)=\frac{wT_{hm3}(i,j)}{\left(\frac{\Delta y_i+\Delta y_{i+1}}{2}\right)\Delta y_i}$$

$$F_4(i,j)=\frac{wT_{hm4}(i,j)}{\left(\frac{\Delta x_{j-1}+\Delta x_j}{2}\right)\Delta x_j} \quad ; \quad F_5(i,j)=\frac{wT_{hm5}(i,j)}{\left(\frac{\Delta y_{i-1}+\Delta y_i}{2}\right)\Delta y_i}$$

$$F_1(i,j)=\frac{-S(i,j)}{\Delta t}-\{F_2(i,j)+F_3(i,j)+F_4(i,j)+F_5(i,j)\}$$

$$F_6^k(i,j) = -\frac{S(i,j)\,h^{k-1}(i,j)}{\Delta t} + \frac{Q_p^k(i,j)}{\Delta x_j\,\Delta y_i} - \frac{Q_r^k(i,j)}{\Delta x_j\,\Delta y_i}$$

$$+\frac{(1-w)}{w}\left[F_2(i,j)+F_3(i,j)+F_4(i,j)+F_5(i,j)\right]h^{k-1}(i,j)$$

$$-\frac{(1-w)}{w}\left[F_2(i,j)\,h^{k-1}(i,j+1)+F_3(i,j)\,h^{k-1}(i+1,j)\right.$$

$$\left.+F_4(i,j)\,h^{k-1}(i,j-1)+F_5(i,j)\,h^{k-1}(i-1,j)\right]$$

An equation can be written for each interior node. For nodes adjacent to river boundary, mass balance equation can be written considering the influent or effluent seepage which is governed by the boundary head, unknown head at the node under consideration, distance of the node from the boundary, hydraulic conductivity and river cross section. The influent seepage is given by

$$Q_r^k(i) = \Gamma_r(i)\left[h_r^k - h^k(i,j)\right] \tag{13}$$

in which $h_r^k(i)$ = head at the river during time $k\Delta t$. $\Gamma_r(i)$= reach transmissivity or the constant of proportionality between seepage and potential difference [$h_r^k(i)$ - $h^k(i,j)$]. For a partially penetrating river of large width, the variation of reach transmissivity constant Γ_r with distance of the node from the river bank is shown in figure 3. The reach transmissivity constant Γ_r is equal to $Q_r/(K\Delta h)$. K is the hydraulic conductivity and Δh is the head different between the river and the grid adjacent to the river. D_1 is the thickness of aquifer below the river bed and D_2 is the thickness of aquifer far away from the river.

For a node adjacent to a river which forms the boundary on the left side, equation (11) reduces to

$$F_1(i,j)\,h^k(i,j) + F_2(i,j)\,h^k(i,j+1) + F_3(i,j)\,h^k(i+1,j)$$

$$+F_5(i,j)\,h^k(i-1,j) = F_6^k(i,j) \tag{14}$$

In which $F_1(i,j)$ is given by:

$$F_1(i,j) = \frac{-S(i,j)}{\Delta t} - \left\{F_2(i,j) + F_3(i,j) + w\frac{\Gamma_r(i)}{\Delta x_j} + F_5(i,j)\right\}$$

$$F_6^k(i,j) = -\frac{S(i,j)\,h^{k-1}(i,j)}{\Delta t} + \frac{Q_p^k(i,j)}{\Delta x_j\,\Delta y_i} - \frac{Q_r^k(i,j)}{\Delta x_j\,\Delta y_i}$$

$$+\frac{(1-w)}{w}\left[F_2(i,j) + F_3(i,j) + F_5(i,j)\right] h^{k-1}(i,j)$$

$$-\frac{(1-w)}{w}\left[F_2(i,j)\,h^{k-1}(i,j+1) + F_3(i,j)\,h^{k-1}(i+1,j) + F_5(i,j)\,h^{k-1}(i-1,j)\right]$$

$$\frac{-w\Gamma_r(i)\,k}{\Delta x_j}\,h_r^k(i) - (1-w)\,\frac{\Gamma_r(i)\,k}{\Delta x_j}\left[h_r^{k-1}(i) - h^{k-1}(i,j)\right]$$

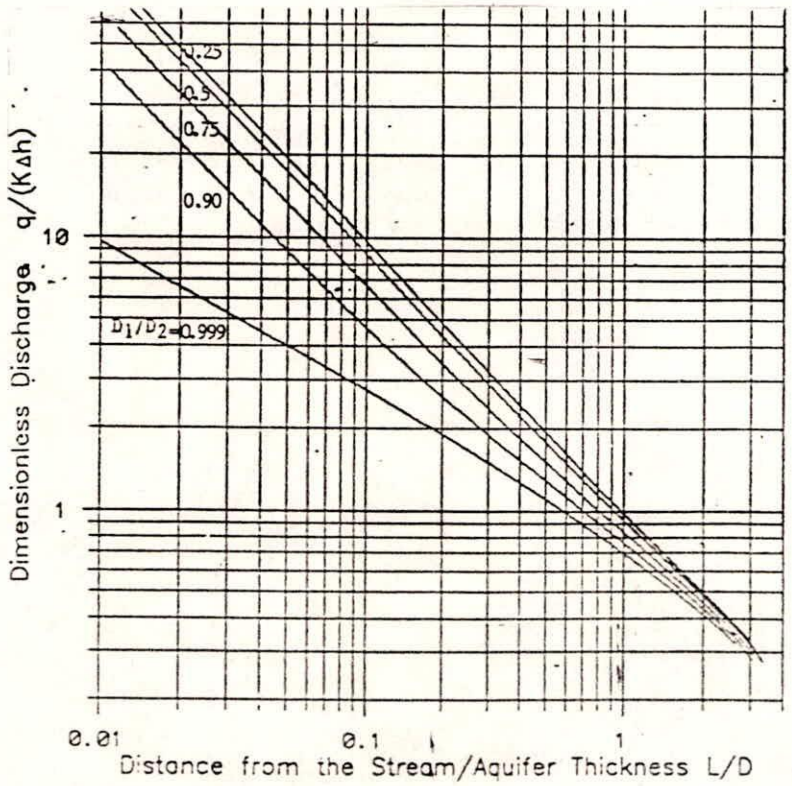

Fig. 3 Variation of $Q_r/(K\Delta h)$ with distance from the river bank for different penetration of the river

Applying the observed initial and the prevailing boundary conditions, for known withdrawal and distributed percolation losses and rainfall recharge, the unknown heads at the grid points can be solved by matrix inversion method. The simulated water table fluctuation is then to be compared with the observed groundwater level fluctuation. The influent seepage, the subsurface inflow and subsurface outflow and the change in storage can be ascertained by the above groundwater flow model.

8.0 CONCLUSIONS

A groundwater flow model has been described to account for exchange of flow between a partially penetrating river and an aquifer. The procedure to account influent seepage from partially penetrating stream has been described.

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