TRAINING COURSE

ON

COMPUTER APPLICATIONS IN HYDROLOGY

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 13

Estimation of

Aquifer Parameters

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ESTIMATION OF AQUIFER PARAMETERS

1.0 INTRODUCTION

The transmissivity and storage coefficient of an aquifer can be determined analysing aquifer test data. The ratio of transmissivity to storage coefficient, known as hydraulic diffusivity, can be ascertained from observation of stream stage during passage of a flood wave and the corresponding water level rise in an observation well in the vicinity of the stream.

2.0 DETERMINATION OF HYDRAULIC DIFFUSIVITY

The estimation of hydraulic diffusivity from the observation of stream stage and consequent water table fluctuations in an adjacent aquifer is an inverse problem. An inverse problem can not be solved unless the corresponding direct problem has been solved a priori. The stream-aquifer interaction problem has been solved by several investigators (Todd 1955; Rowe 1960; Cooper and Rorabaugh 1963; Hall and Moench 1972; Morel-Seytoux and Daly 1975; Halek and Svec 1979) and the stream-aquifer equations have been applied by various other investigators to estimate the aquifer diffusivity (Rowe 1960; Ferris 1962; Pinder et al. 1969; Brown et al. 1972; and Singh and Sagar 1977). An alternate approach is presented here to determine the diffusivity using the measurements of stream stage during passage of a flood wave and the consequent water level fluctuations in a piezometer in the vicinity of the stream.

The unit step response function that relates rise in piezometric surface in an initially steady state semi-infinite homogeneous and isotropic confined aquifer, bounded by a fully penetrating straight stream, to a step rise in stream stage has been derived by Carslaw and Jaeger (1959) for an analogous heat conduction problem. The unit step response function is

$$K(x,t) = erfc\{x/\sqrt{(4\beta t)}\} \tag{1}$$

where x = distance from the bank of the stream; t = time measured since the onset of change in stream stage; β = the hydraulic diffusivity of the aquifer defined as ratio of transmissivity to storage coefficient or as ratio of saturated hydraulic conductivity to specific storage; and $erfc\{.\}$ = complementary error function.

In nature, a stream partially penetrates an aquifer. For a partially penetrating stream, the flow is two-dimensional near the stream. To use eq. (1) in such case, it is necessary to install two observation wells on one side of the stream in a line perpendicular to the stream. The first one should be installed from the stream bank at a distance equal to the thickness of aquifer below the stream bed beyond which the flow is one-dimensional (Streltsova, 1974). Thickness of the aquifer below the stream bed can be ascertained from the study of lithologs in the vicinity of the stream. The water level fluctuations in this well can be considered to represent fluctuations in a fully penetrating stream (Reynolds, 1987). The water level fluctuations in the other well represent the aquifer response. Eq. (1) is a good approximation for an unconfined aquifer if the changes in water level are small in comparison to the saturated thickness of the aquifer (Cooper and Rorabaugh 1963).

For varying stream stage, $\sigma(t)$, the rise in piezometric surface, $s(x,t)$, according to Duhamel's integral (Thomson, 1950) is given by:

$$s(x,t) = \sigma_0 K(x,t) + \int_0^t K(x,t-\tau) \frac{d\sigma(\tau)}{d\tau} d\tau \tag{2}$$

in which σ_0 is the initial sudden rise in the stream stage and τ is a time variable. Duhamel's integral, which can be expressed in two different forms (Thomson, 1950), has been used extensively for solving stream-aquifer interaction problems (Pinder et. al., 1969; Venetis 1970; Hall and Moench 1972; Abdulrazzak and Morel-Seytoux 1983; Morel-Seytoux 1988).

The hydraulic diffusivity can be determined using the fluctuations in stream stage during the passage of a flood and the consequent changes in water level recorded in an observation well near the stream either by applying a Laplace transform technique or using a least squares optimization method. These methods are discussed below.

2.1 Laplace Transform Method

The Laplace transform, which transforms one class of function into another, has the advantage that under certain circumstances, it replaces complicated functions by simpler ones (Widder 1961). An approach based on Laplace transform for determining the hydraulic diffusivity is described below.

Taking the Laplace transform of the terms on both sides of (2)

$$\int_0^\infty s(x,t)\, e^{-\alpha t} dt = \int_0^\infty \sigma_0 K(x,t)\, e^{-\alpha t} dt + \int_0^\infty \Big[\int_0^t \frac{d\sigma(\tau)}{d\tau} K(x,t-\tau) d\tau\Big] e^{-\alpha t} dt \tag{3}$$

in which α is the Laplace transform parameter. Applying Faltung theorem, i.e.,

$$L\left[\int_0^t F_1(t-\tau) F_2(\tau) d\tau\right] = L[F_1(t)]\, L[F_2(t)]$$

eq. (3) reduces to

$$\int_0^{\infty} s(x,t)\, e^{-\alpha t}\, dt = \sigma_0 L[K(x,t)] + L[\frac{d\sigma}{dt}]\, L[K(x,t)] \tag{4}$$

Substituting the Laplace transform of the complementary error function, K(x,t), into (4) (Abramowitz and Stegun 1970)

$$\int_0^{\infty} s(x,t)\, e^{-\alpha t}\, dt = [\sigma_0 + L(\frac{d\sigma}{dt})]\, \exp[-\sqrt{(\alpha x^2/\beta)}\,]/\alpha \tag{5}$$

Discretizing the time domain in steps of uniform size Δt, and assuming that the rate of change of stream stage is a constant within a time-step, (5) is expressed as (Gustav 1961)

$$\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} [s^0_{\gamma-1/2} \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} e^{-\alpha t} dt\,]$$

$$= [\sigma_0 + \sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} \{\frac{\sigma_\gamma - \sigma_{\gamma-1}}{\Delta t} \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} e^{-\alpha t} dt\}\,]\, \exp\{-\sqrt{(\alpha l^2/\beta)}\}/\alpha \tag{6}$$

in which $s^0_{\gamma-1/2} = [s^0_{\gamma-1} + s^0_{\gamma}]/2$ and s^0_{γ} is the observed water level rise at time $\gamma\Delta t$ in a piezometer located at a distance l from the stream bank. Integrating, (6) reduces to

$$\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} [s^0_{\gamma-\frac{1}{2}} \frac{\{e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}\}}{\alpha}\,]$$

$$= [\sigma_0 + \sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} \frac{(\sigma_\gamma - \sigma_{\gamma-1})}{\Delta t} \frac{(e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t})}{\alpha}\,] \frac{\exp\{-\sqrt{\dfrac{\alpha l^2}{\beta}}\}}{\alpha} \tag{7}$$

Equation (7) simplifies to

$$\exp\{-\sqrt{(\alpha l^2/\beta)}\} = \frac{\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} [s^0_{\gamma-1/2}\{e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}\}]}{[\sigma_0 + \sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} \{(\frac{\sigma_\gamma - \sigma_{\gamma-1}}{\Delta t})(\frac{e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}}{\alpha})\}]} \tag{8}$$

Taking the natural logarithm of terms on either side and solving for β yields

$$\beta = \alpha l^2 / \left[\ln \frac{\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} [s^{0}_{\gamma-1/2}(e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t})]}{\sigma_0 + \sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} \{ (\frac{\sigma_\gamma - \sigma_{\gamma-1}}{\Delta t}) (\frac{e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}}{\alpha}) \}} \right]^2 \tag{9}$$

In practice, an observation period is always finite. Therefore, the summations can be truncated beyond a finite observation period. Introducing truncation errors in the summations of the series, (9) is modified to

$$\beta = \alpha l^2 / \left[\ln \frac{\sum_{\gamma=1}^{n} \{ s^{0}_{\gamma-1/2}(e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}) \} + \epsilon_1}{\sigma_0 + \sum_{\gamma=1}^{n} \{ (\frac{\sigma_\gamma - \sigma_{\gamma-1}}{\Delta t}) (\frac{e^{-\alpha(\gamma-1)\Delta t} - e^{-\alpha\gamma\Delta t}}{\alpha}) \} + \epsilon_2} \right]^2 \tag{10}$$

The two series containing the average water level rise at the piezometer and the change in stream stage will converge as they contain exponentials of negative terms. The rate of convergence will depend upon the value of α and Δt. For example, for a step rise in water level in the observation well, for α = 0.06 hour^{-1}, Δt = 1 hour, and n=240, the truncation error ϵ_1 will be about 5%. Moreover, the stream stage and the water level rise at the piezometer after some lag will become smaller after recession of the flood wave. The truncation errors, ϵ_1 and ϵ_2 would, therefore, tend to zero with increasing observation period. Hence, the hydraulic diffusivity, β, can be computed with reasonable accuracy using (10).

2.2 Least Squares Optimization Method

The hydraulic diffusivity, β, can be determined minimizing the objective function

$$\sum_{n=1}^{N} \left[s^{0}_{n} - \{ s(l, n\Delta t)|_{\beta^*} + \frac{\partial s(l, n\Delta t)}{\partial \beta}|_{\beta^*} \Delta\beta \} \right]^2 \tag{11}$$

with respect to $\Delta\beta$, where N is the number of observations, β^* is an initial guess of the hydraulic diffusivity, and $\Delta\beta$ is an increment in β. $s(l, n\Delta t)$ is given by

$$s(l, n\Delta t) = \sigma_0\, erfc \left\{ \frac{l}{\sqrt{(4\beta n\Delta t)}} \right\} + \sum_{\gamma=1}^{n} [(\sigma_\gamma - \sigma_{\gamma-1})\, \delta_r(l, \Delta t, n-\gamma+1)] \tag{12}$$

The discrete kernel coefficient, $\delta_r(x, \Delta t, m)$, be defined as:

$$\delta_r(x, \Delta t, m) = \frac{1}{\Delta t} \int_0^{\Delta t} erfc \{ \frac{x}{\sqrt{4\beta(m\Delta t - \tau)}} \} d\tau \tag{13}$$

in which m is an integer. Performing the integration

$$\begin{aligned}\delta_r(x,\Delta t,m) = 1 &+ \{(m-1)+ x^2/(2\beta\ \Delta t)\}\ erf[x/\sqrt{\{4\beta\ \Delta t\ (m-1)\}}] \\ &- \{m + x^2/(2\beta\ \Delta t)\}\ erf\{x/\sqrt{(4\beta\ \Delta t\ m)}\} \\ &+ x\sqrt{\{(m-1)/(\beta\ \Delta t\ \pi)\}}\ exp[-x^2/\{4\beta\ \Delta t\ (m-1)\}] \\ &- x\sqrt{\{m/(\beta\ \Delta t\ \pi)\}}\ exp\{-x^2/(4\beta\ \Delta t\ m)\}\end{aligned} \tag{14}$$

Following a numerical method, the derivative $\partial s(1,n\Delta t)/\partial\beta$ at $\beta=\beta^*$ can be computed from (12). The well known Marquardt algorithm (Marquardt 1963) is a technique of estimation of nonlinear parameters by linearization and minimization of the least squares. The Marquardt algorithm has been applied by several investigators to determine transmissivity and storage coefficient of a confined aquifer from pumping test data (Chander et al. 1981; Johns et al. 1992).

EXAMPLE

The proposed methods for determining the aquifer parameter from the response of the stream-aquifer system have been tested using synthetic data. The water level rise in a piezometer has been generated for different values of β, for a flood wave that follows

$$\sigma(t) = \begin{cases} NH_0\ (1 - \cos\omega t)\ e^{-\delta t} & \text{for } 0 \le t \le t_d \\ 0 & \text{for } t > t_d \end{cases} \tag{15}$$

where $\sigma(t)$ = the rise above initial water level in the stream; H_0 = height of peak flood stage above initial water level; t_d = the period of the flood wave; t_c = the time of flood peak; $\omega=2\pi/t_d$, the frequency of oscillation; $N=exp(\delta t_c)/(1 - cos\ \omega\ t_c)$; and $\delta=\omega\ cot(0.5\omega\ t_c)$.
Let the flood have the following characteristics:

Time of flood peak, t_c	: 24 hours
Duration of the flood wave, t_d	: 120 hours
Maximum rise in stream stage, H_0	: 2m
Time-step size or sampling period, Δt	: { 1, 0.5, 0.25 hour }
Duration of observation, $n\Delta t$	: { 120, 240, 360, 600 hours}
Distance of the piezometer from the stream, l	: { 50, 200m }
Hydraulic diffusivity, β	: {25.0, 50000.0 m^2 per hour}

The standard deviation, s_d, of the error free set of rises in water level in a piezometer was computed. Random errors with zero mean and a prescribed percentage of s_d as standard deviation have been added to the piezometric levels. These piezometric levels containing random errors have been regarded as observed piezometric levels and have been used for testing the proposed methods of identification of parameter.

(a) Laplace Transform Technique

For solving a problem by Laplace transformation, measurements of input and output are made in the time domain. There is a limit on how fine a spacing one can take near the origin and there is also a limit on how long a time one can make measurement. Thus the transform is not as well determined as one would wish. However, in practice the transform is applied. Gustav (1961) has suggested an approach for choosing the time-step size. According to him, selection of time-step should be based upon the fact that both the perturbation and the response should change on an average by about 10% of their maximum values within successive intervals of the chosen time-step.

The error in the estimated diffusivity is linked to the duration of observation, time-step size, value of Laplace transform parameter α, error due to numerical integration and the random error in the observation.

The diffusivity evaluated by the Laplace transform technique is presented in Tables 1(a) and 1(b) for different durations of observation and Laplace transform parameter α. A time-step size of 1 hour, which satisfies Gustav's condition, has been chosen. For the assumed flood wave and sampling period of 1 hour, the changes in flood stage and piezometric level within two successive sampling periods are contained within 6.9 and 6.4 percent of their respective maximum fluctuations. Two sets of observations containing normally distributed random errors with 5 and 10 % of s_d as standard deviation have been considered. Table 1(a) shows that when the duration of observation equals the duration of flood wave, the error in estimation of hydraulic diffusivity is 26%. Table 1(b) shows that for high hydraulic diffusivity, the error is 13%. When the duration of observation is twice the duration of flood wave, the error is less than 4%. **Thus the duration of observation should be at least twice the duration of flood wave.** From data containing random error, whose standard deviation is 10% of s_d, low hydraulic diffusivity can be computed with 98.5% accuracy if the duration of observation is twice the duration of flood. The corresponding accuracy for higher diffusivity, when the data contain random error at 10% of s_d, is 84%. The accuracy can be further improved using a smaller time-step.

For determining the hydraulic diffusivity, a time-step size for which the maximum change in the perturbation within two successive time-steps does not exceed by 2% of the maximum stream stage rise, is preferable. Once the time step size is selected, α should be assigned a value such that $0.02 < \alpha \Delta t < 0.06$. With this α, diffusivity can be estimated with at least 87% accuracy.

b) Least Squares Optimization

Using the rate of change of stream stage during passage of the flood and corresponding changes in piezometric levels at a piezometer, the parameter β was estimated applying the Marquardt algorithm to (12) and is presented in Table 2. In a low diffusivity aquifer, the fluctuation in piezometric level at an observation well is gradual. Therefore, in such an aquifer, for many initial sampling periods, the rises in piezometric level are insignificant in comparison to the random error that may occur. In an aquifer having high diffusivity, the response to a flood wave is quick. Hence, in such type of aquifer, the rises in piezometric level during the first few and the last several sampling periods are insignificant. It is found that if the periods corresponding to the insignificant water level rises are excluded from the evaluation of the objective function, the parameter β is estimated accurately. Accordingly, the objective function chosen was the sum of squares of difference between the observed and predicted rises for a period during which the observed rise at the well is more than 20% of the maximum rise in it. For the case of high diffusivity aquifer, observations during 5th to 60th hour have been included in the objective function. In the case of a low diffusivity aquifer, the observations up to 20th hour have been excluded. The value of the objective function at the optimum point is given in column 4 of Table 2. The results presented in Table 2 show that if the data are free from random error, high hydraulic diffusivity of several thousand m^2/hour magnitude as well as low hydraulic diffusivity of the order of 25 m^2/hour can be estimated with 99.99% accuracy. If the data contain random error with standard deviation of 20% s_d, low hydraulic diffusivity is computed with 99.5 % accuracy. The accuracy for high diffusivity is 97.5%. **The hydraulic diffusivity can thus be estimated very accurately using the Marquardt algorithm using selected part of the observed data.**

The transmissivity and storage coefficient both control the unsteady state response of an aquifer to any boundary perturbation whereas the transmissivity alone controls the near steady state response of an aquifer. Therefore, for determining the aquifer diffusivity, the unsteady response of an aquifer should be given more weight than the steady state response. The weighting factor in Laplace transform is $e^{-\alpha t}$. Thus, Laplace transform technique automatically gives more weight to the unsteady response (fluctuating part) of the aquifer which occurs earlier than to the near steady state response which occurs later. This is the major strength and motivation for using the Laplace transform approach.

The least squares optimization gives equal weight to all input data and thereby also to the random error contained in it. When the response of the aquifer over a threshold level was considered, the results improved. Thus, this approach requires careful screening of the input data.

3.0 AQUIFER PARAMETER DETERMINATION

The partial differential equation describing unsteady radially symmetric flow in a nonleaky homogeneous confined aquifer of constant thickness can be written as :

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{16}$$

in which h is the piezometric head at a radius r from the pumped well at a time t since the start of pumping. T and S are the transmissivity and storage coefficient, respectively. Equation (16) assumes that the pumping well fully penetrates the aquifer layer during the test. A solution to equation (16) which satisfies the initial condition $h(r,0) = h_o$; and the boundary conditions $h(\infty,t) = h_o$ and $\lim_{r=rw\to 0} 2\pi rT\partial h/\partial r = Q_w$ as given by Theis (1935) is

$$h_0 - h = \frac{Q_w}{4\pi T}\int_{u=r^2S/4Tt}^{\infty} (e^{-u}/u)\,du \tag{17}$$

where h_o is the initial piezometric head, and Q_w is the constant rate of pumping.

Equation (17) is usually written as

$$s = \frac{Q_w}{4\pi T}W(u) \tag{18}$$

where s is the drawdown ($=h_o$ - h) and W(u) is the Theis well function representing the exponential integral of equation (17).

Equation (18) is nonlinear in T and S. These parameters can be estimated by nonlinear regression analysis. The algorithm used for determining T and S is the one due to Marquardt (1963) as explained below. Let s_i* be the drawdown at any instant computed by substinuting the initial trial values of parameters T* and S* in equation (18). Let ΔT and ΔS be the respective increments in T* and S* to yield the improved estimates T and S at the end of each trial and let s_i be the corresponding drawdown value given as :

$$s_i = f(T, S)$$

Expanding s_i by Taylor series about the trial values.

$$s_i = f(T^* + \Delta T, S^* + \Delta S)$$

or

$$s_i = f(T^*, S^*) + \frac{\delta f(T^*, S^*)}{\delta T^*}\Delta T + \frac{\delta f(T^*, S^*)}{\delta S^*}\Delta S$$

or

$$s_i = s_i^* + \frac{\delta s_i^*}{\delta T^*}\Delta T + \frac{\delta s_i^*}{\delta S^*}\Delta S \tag{19}$$

The increments ΔT and ΔS are determined such that the sum of squares of the difference between the observed and the calculated drawdown is minimum.

$$\underset{\Delta T, \Delta S}{Min}\ sum = \sum_{i=1}^{N} (s_i^0 - s_i)^2 \tag{20}$$

where s_i^0 is the observed drawdown at any instant.

The linearized model given by equation (19) is substituted in equation (20) and normal equations are formed by setting the partial derivatives of the objective function given by equation (20) with respect to ΔT and ΔS equal to zero.

i.e.

$$\frac{\delta\ sum}{\delta T^*} = 0$$

and

$$\frac{\delta\ sum}{\delta S^*} = 0$$

These equations will take the form

$$(A^T A)\ \underline{\Delta A} = A^T (s^0 - s^*) \tag{21}$$

where

$$A = \begin{bmatrix} \frac{\delta s_1^*}{\delta T^*} & \frac{\delta s_1^*}{\delta S^*} \\ \frac{\delta s_2^*}{\delta T^*} & \frac{\delta s_2^*}{\delta S^*} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{\delta s_N^*}{\delta T^*} & \frac{\delta s_N^*}{\delta S^*} \end{bmatrix}$$

and $\Delta A = [\ \Delta T \quad \Delta S]^T$

A^T is the transpose of the matrix A; s^o and s^* are the vectors of observed and calculated drawdowns respectively.

The normal equations (21) are solved for ΔA and the new drawdowns are calculated by substituting the improved estimates (T, S) of the parameters in equation (18). The error criterion is checked and if the same is not satisfied, the process is repeated with the updated estimates of the parameters.

In order to ensure convergence with relatively poor starting values, equation (21) is modified as

$$(A^T A + \lambda I)\,\underline{\Delta A} = A^T(s^0 - s^*)$$

where λ is the convergence factor and I is the identity matrix. Initial values of λ are large and decrease towards zero as convergence is reached.

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TABLE 1(a)- Estimation of Hydraulic Diffusivity by Laplace Transform Technique (time to peak=24 hour, duration of the flood wave=120 hour, maximum rise in stream stage= 2m and distance of observation well from the stream=50m, sampling period=1 hour)

β assumed for generation of synthetic data (m^2/hour)	Duration of observation (hour)	Laplace parameter α $(hour)^{-1}$	Standard deviation of random error/ standard deviation of error free data (%)	Computed β (m^2/hour)
25	120	0.01	0	18.6
			5	18.8
			10	19.0
		0.05	0	24.9
			5	25.3
			10	25.6
		0.10	0	25.0
			5	26.4
			10	27.7
	240	0.01	0	24.1
			5	24.3
			10	24.5
		0.05	0	25.0
			5	25.3
			10	25.7
		0.10	0	25.0
			5	26.3
			10	27.5
	360	0.01	0	24.8
			5	25.0
			10	25.2
		0.05	0	25.0
			5	25.3
			10	25.6
		0.10	0	25.0
			5	25.2
			10	27.4
	600	0.01	0	25.0
			5	25.2
			10	25.4
		0.05	0	25.0
			5	25.3
			10	25.6
		0.10	0	25.0
			5	26.1
			10	27.2

TABLE 1(b). Estimation of Hydraulic Diffusivity by Laplace Transform Technique (time to peak=24 hour, duration of the flood wave=120 hour, maximum rise in stream stage= 2m and distance of observation well from the stream=200m, sampling period= 1 hour)

β assumed for generation of synthetic data (m^2/hour)	Duration of observation (hour)	Laplace parameter $(hour)^{-1}$	Standard deviation of random error/ standard deviation of error free data (%)	Computed β m^2\hour)
50000	120	0.01	0	43418
			5	51106
			10	60949
		0.05	0	50182
			5	54448
			10	59243
		0.10	0	50599
			5	57877
			10	66667
	240	0.01	0	49215
			5	56001
			10	64241
		0.05	0	50210
			5	53879
			10	57936
		0.10	0	50599
			5	56822
			10	64142
	360	0.01	0	49880
			5	56105
			10	63535
		0.05	0	50210
			5	53355
			10	56782
		0.10	0	50599
			5	55910
			10	62013
	600	0.01	0	50014
			5	55093
			10	60963
		0.05	0	50210
			5	52727
			10	55425
		0.10	0	50599
			5	54828
			10	59556

TABLE 2. Estimation of Hydraulic Diffusivity by Marquardt Algorithm (time to peak=24 hour, duration of the flood wave=120 hour, maximum rise in stream stage= 2m, distance of observation well from the stream=50m, sampling period= 1hour, duration of observation = 120 hour)

Assumed β for generation of synthetic data (m^2\hour)	Standard deviation of error free data s_d (m)	Standard deviation of random error/s_d (%)	Minimum objective function attained (m^2)	Number of function calls	Estimated β (m^2/hour)
25	0.158	0	8.43×10^{-8}	17	25
		5	7.56×10^{-3}	30	25
		10	3.02×10^{-2}	33	25
		20	1.21×10^{-1}	36	25
50000	0.682	0	0.30×10^{-5}	38	49942
		5	6.76×10^{-2}	45	49656
		10	2.71×10^{-1}	48	49426
		20	1.08	48	48772