TRAINING COURSE

ON

RESERVOIR OPERATION

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 5

Reservoir Routing

BY

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RESERVOIR ROUTING

1.0 INTRODUCTION

Flood routing forms an integral part of the surface water modelling and analysis. Its other major real world applications include: reservoir design, design flood estimation, design of flood control structures, flood forecasting, and dam break flood wave analysis. The passage of flood hydrograph through a reservoir is an unsteady flow phenomenon. The equation of continuity is used in all hydrologic routing methods as primary equation. According to this equation, the difference between the inflow and outflow is equal to the rate of change of storage, i.e.:

$$I - Q = dS/dt \qquad \ldots(1)$$

where, I = inflow, Q = outflow, S = storage, and t = time.

For the sake of clarity, it is necessary to introduce the frequently used terms e.g. translation and attenuation characteristics of the flood wave propagation. The translation is taken as the time difference between the occurrence of inflow and outflow peak discharges, and the attenuation as the difference between the inflow and outflow peak discharges. Having known the peak discharge of the outflow hydrograph, the attenuation is computed as

Attenuation = Inflow peak discharge - Outflow peak discharge

and the translation as

Translation = Time-to-peak of inflow - Time-to-peak of outflow

Over a small time interval Δt, the difference between the total inflow volume and total outflow volume in reach is equal to the change in storage in that reach. Hence, the equation (1) can be written as:

$$I_m \Delta t - Q_m \Delta t = \Delta S \qquad \ldots(2)$$

where, I_m, Q_m, and ΔS denote average inflow, average outflow and change in storage during time period Δt respectively.

2.0 ROUTING TECHNIQUES

The routing can be broadly classified into two categories: (i) reservoir routing; and (ii) channel routing. In the following, only reservoir routing techniques will be discussed.

The reservoirs can be either controlled or uncontrolled. The controlled reservoirs have spillway with gates operated for making releases at the time of demand of water. The uncontrolled reservoirs are those whose spillway is not controlled by the gate operation. Reservoir routing requires the relationship between the reservoir elevation, storage and discharge to be known. This relationship is a function of the topography of reservoir site and the characteristics of the outlet facility. It is important to view the relationship between elevation, storage and discharge as a single function because changes can take place in the topography and the elevation-discharge characteristics will get affected. McCuen(1989) has discussed this aspect in detail.

Using the basic equation (1), several methods for routing a flood wave through a reservoir have been developed, namely:

- ⇨ The Mass Curve Method,
- ⇨ The Puls Method,
- ⇨ The Modified Puls Method,
- ⇨ The Wisler-Brater Method,
- ⇨ The Goodrich Method,
- ⇨ The Steinberg Method, and
- ⇨ The Coefficient Method.

A brief description of each of these methods follows. The schematic representation of reservoir routing is given in Fig. 1.

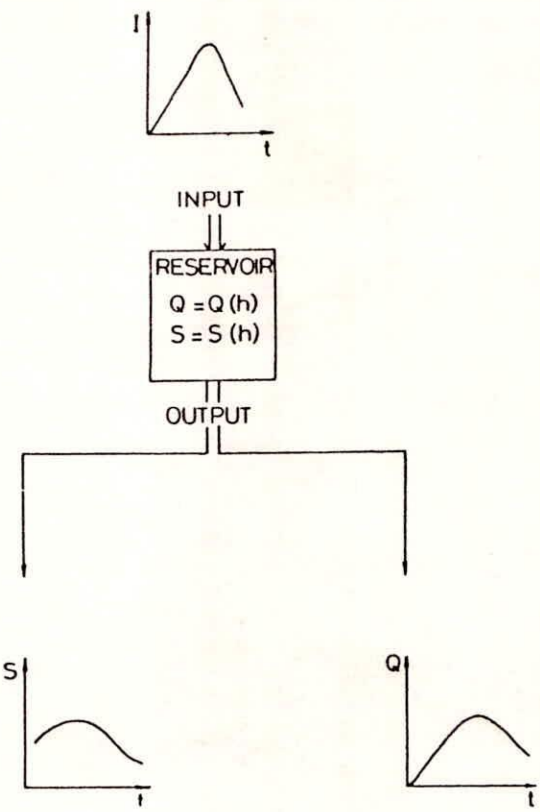

Fig. 1 Schematic Representation of Reservoir Routing

3.0 THE MASS CURVE METHOD

This is one of the most versatile methods of reservoir routing, various versions of which include: (i) direct, (ii) trial and error, and (iii) graphical. Here the trial and error version is being described in detail.

For solution by trial and error method, equation (2) can be rewritten as:

$$M_2 - (V_1 + Q_m \Delta t) = S_2 \qquad ...(3)$$

where, M is the accumulated mass inflow, and V is the accumulated mass outflow.

A storage-discharge relationship and the mass curve of inflow should be plotted before obtaining trial and error solution. Necessary adjustments are made to show zero storage at the beginning elevation and, correspondingly, spillway discharge is obtained. Now, the following steps are involved in trial and error solution :

a) A time is chosen and Δt is computed. Mass inflow is also computed.
b) Mass outflow is assumed. As a guideline, it is a function of accumulated mass inflow.
c) Reservoir storage is computed by deducting mass outflow from mass inflow.
d) The instantaneous and average spillway discharges are calculated.
e) Outflow for the time period Δt is computed by multiplying Δt with average discharge. Then the mass outflow is computed.
f) Now, computed mass outflow is compared with assumed mass outflow. If the two values agree within an acceptable degree of accuracy, then the routing is complete. If this agreement is not acceptable, then another mass outflow is assumed and the above procedure is repeated.

4.0 THE MODIFIED PULS METHOD

The basic law used in the Modified Puls method states: *The inflow minus outflow is equal to the rate of change in storage*. This is also referred to as the Storage-Indication method. Assuming $I_m = (I_1 + I_2)/2$, $Q_m = (Q_1 + Q_2)/2$ and $\Delta S = S_2 - S_1$, equation (2) is written as:

$$(I_1 + I_2)\Delta t/2 - (Q_1 + Q_2)\Delta t/2 = S_2 - S_1 \qquad ...(4)$$

where, suffixes 1 and 2 denote the beginning and end of time interval Δt and Q may incorporate controlled discharge as well as uncontrolled discharge. Here the time interval Δt must be sufficiently small so that the inflow and outflow hydrographs can be assumed to be linear in that time interval. Further, Δt must be shorter than the time of transit of flood wave through the reservoir. Separating the known quantities from the unknown ones and rearranging:

$$(I_1 + I_2) + (2S_1/\Delta t - Q_1) = (2S_2/\Delta t + Q_2) \qquad ...(5)$$

Here, the known quantities are I_1 (inflow at time 1), I_2 (inflow at time 2), Q_1 (outflow at time 1) and S_1 (storage in the reservoir at time 1), and the unknown quantity are S_2 and Q_2. Since one equation with two unknowns can not be solved, therefore, one must have another relation that relates between storage, S, and outflow, Q. As the outflow from the reservoir takes place through the spillway, the discharge passing through the spillway can be conveniently related with the reservoir elevation which, in turn, can be related to the reservoir storage. Such a relationship is invariably available for any reservoir. Also, it can be computed from the following relation:

$$Q = C_d \, L \, H^{1.5} \qquad \ldots(6)$$

where, Q is the outflow discharge (cumec); C_d is the coefficient of discharge (=1.70 in metric unit); L is the length of spillway (m); and H is the depth of flow above the spillway crest (m).

Thus, the left side of equation (5) contains the known terms and the right side is unknown. The inflow hydrograph is known. The discharge Q, which may pass through the turbines, outlet works, or over the spillway is also known. The uncontrolled discharge goes freely over the spillway. It depends upon the depth of flow over the spillway and the spillway geometry. Further, the depth of flow over the spillway depends upon the level of water in the reservoir. Therefore:

$$S = S\,(Y)$$
$$Q = Q\,(Y)$$

where, Y represents the water surface elevation. The right side of equation (5) can be written as:

$$2S/\Delta t + Q = f(Y)$$

Adding the crest elevation with the depth of flow, the elevation for which storage in the reservoir is known can be computed. Therefore, one can develop a relation between storage and outflow. This storage outflow relation is used to develop the storage indication $[(2S/\Delta t) + Q]$ vs. outflow relation. To develop this relation, it is necessary to select a time interval such that the resulting linearisation of the inflow hydrograph remains a close approximation of the actual non-linear (continuous time varying) shape of the hydrograph. For smoothly rising hydrographs, a minimum value of $t_p/\Delta t = 5$ is recommended, in which t_p is the time to peak of the inflow hydrograph. In practice, a computer aided calculation would normally use a much greater ratio, say 10 to 20.

In order to utilize equation (5), the elevation storage and elevation-discharge relationship must be known. Before routing, the curves of $(2S/\Delta t \pm Q)$ versus Q are constructed. The routing is now very simple and can be performed using the above equation.

The computations are performed as follows. At the starting of flood routing, the initial storage and outflow discharge are known. In equation (5) all the terms in the left hand side are known at the beginning of time step Δt. Hence the value of $(S_2 + Q_2\Delta t/2)$ at the end of the time step is calculated by equation (5). Since the relation $S = S(h)$ and $Q = Q(h)$ are known, $(S_2 + Q_2\Delta t/2)$ will enable one to determine the reservoir elevation and hence the discharge at the end of the time step. This procedure is repeated to cover the full inflow hydrograph.

5.0 THE WISLER-BRATER METHOD

In this method, storage is expressed as a function of sum of inflow and outflow and storage curves for $(I+Q)$ versus $(2S/\Delta t + I + Q)$ are constructed. The basic equation of reservoir routing can be expressed as :

$$2S_1/\Delta t + I_1 + 2I_2 - Q_1 = 2S_2/\Delta t + I_2 + Q_2 \qquad ...(7)$$

In the above equation all terms on left hand side are known and hence the right side can be computed. Then the value of $(I_2 + Q_2)$ can be read from the storage curves. Since, I_2 is known, Q_2 is obtained. This procedure is repeated for subsequent routing periods. This procedure can also be extended to the case where storage is a function of weighted sum of inflow and outflow.

6.0 THE GOODRICH METHOD

In this method, the continuity equation is expressed as:

$$2S_1/\Delta t + I_1 + I_2 - Q_1 = 2S_2/\Delta t + Q_2 \qquad ...(8)$$

The Goodrich method involves construction of a family of routing curves for $[(2S/\Delta t) \pm Q]$ against Q for various values of I. As all the terms on the left side of the above equation are known, the right side can be obtained for a routing period Δt. The value of Q_2 can now be read from the routing curves against $[2S_2/\Delta t + Q_2]$ and then S_2 can be computed. The routing can be carried out for subsequent time periods in a similar manner.

7.0 THE STEINBERG METHOD

The Steinberg method expresses equation (1) as :

$$\Delta t(I_1 + I_2 - Q_1)/2 + S_1 = \Delta t/2\, Q_2 + S_2 + K \qquad ...(9)$$

The term $K = S\,(Q\Delta t/2)$ is called the storage factor. Curves are plotted for storage factor which are termed as K-curves. The storage curves, showing storage as a function of inflow and outflow, are superimposed over K-curves. All the left side terms of above equation are known and so the right side is obtained. Then Q_2 and S_2 are read from the superimposed curves. The routing is similarly carried out for subsequent time periods.

8.0 THE COEFFICIENT METHOD

In the coefficient method, the reservoir is represented by a single conceptual storage element assuming storage S to be directly proportional to outflow Q :

$$S = K\,Q \qquad ...(10)$$

where K is a proportionality factor equal to the reciprocal of the slope of the storage curve that can be a constant or a variable function of outflow. If K is constant, then the reservoir is linear, otherwise the reservoir is non-linear.

For flood routing, a finite difference approximation is normally employed. Equation (1) and (2) can be combined and written as:

$$\Delta t(I_1 + I_2)/2 - (Q_1 + Q_2)\Delta t/2 = K(Q_2 - Q_1)$$

or

$$Q_2 = Q_1 + C(I_1 - Q_1) + C(I_2 - I_1)/2 \qquad ...(11)$$

in which,

$$C = \Delta t/\ (\ K + 0.5\Delta t) \qquad ...(12)$$

If K is variable, then C can be derived and plotted as a function of Q. For each routing period, the appropriate value of C must be obtained corresponding to the outflow under consideration. Then, by using equation (13) flood routing can be performed.

9.0 RESERVOIR ROUTING WITH CONTROLLED OUTFLOW

Most of the big dams have gated spillways and the gates of the spillway can be raised or lowered to control the outflow from the dam. The dam may also have undersluices to control the outflow for purposes such as irrigation, water supply etc. The operation of the spillway gates and undersluices depends on the state of the reservoir, level of demands, and the operation policy.

In case of gated dams, the reservoir outflow can be either a) controlled, b) uncontrolled, and c) a combination of these two. The continuity equation can be written as:

$$(I_1 + I_2)/2 - (Q_1 + Q_2)/2 - Q_c = (S_2 - S_1)/\Delta t \qquad ...(13)$$

where, Q_c is the mean controlled outflow from the reservoir during the time interval Δt. Rearranging the terms, the equation (15) can be written as follows:

$$2S_2/\Delta t + Q_2 = I_1 + I_2 + 2S_1/\Delta t - Q_1 - 2Q_c \qquad ...(14)$$

When the controlled outflow Q_c is known, the solution can be obtained as explained above. The solution of the eq. (16) is simple if the entire outflow is controlled. The spillway rating chart can be used to determine the outflow if the reservoir elevation and the gate opening are known.

10.0 GENERAL COMMENTS

Selection of a proper routing time interval Δt in all flood routing problems is very important. Its value should be neither too long nor too short. If it is too long and exceeds the travel time through the reservoir, then the crest segment of outflow containing the peak discharge could pass through the reservoir between time intervals and would, therefore, not be computed. If on the other hand, it is too short, then it takes longer to perform flood routing. Further, Δt is assumed so that the inflow is approximately linear during this period. As a guideline, Δt should be one-third to one-half of the travel time through the reservoir. Furthermore, the routing interval Δt can be either variable or constant. However, it is more realistic to use a variable Δt, keeping it small for a large change in mass inflow and large for a small change therein.

The merits and demerits of different methods are enumerated below.

The routing operation performed by trial and error solution of the Mass Curve method is simple and easily done. This can be efficiently adapted to complex routing problems.

The Puls method and the Modified Puls method, both have two shortcomings. First, the assumption that the outflow begins at the same time as the inflow implies that the inflow passes through the reservoir instantaneously regardless of its length. Second, it is difficult to choose an appropriate Δt since negative outflow occurs during recession whenever $\Delta t > 2S_2/Q_2$ or $Q_2/2 > S_2/\Delta t$. The former drawback is not a serious one if the ratio of T_t/T_m is less than or equal to 1/2, where T_m denotes time to peak of inflow hydrograph and T_t denotes travel time. T_t is defined as L/u, with L being the length of the reach and u being average steady state velocity. The latter weakness can be circumvented by plotting discharge versus $[(2S/\Delta t)+Q]$ curve on a log-log paper and comparing the plot with the line of equal values. If the plotted values lie above the line of equal values, drawn figure must be abandoned and a new value of Δt must be selected. Further negative outflow can be avoided usually by taking Δt less than T_t.

The Wisler-Brater method requires observed basis for routing computations and so this can best be simulated in controlled conditions. Hence, its use is less for practical purposes.

The Steinberg method requires K-curves and their superimposition over storage curves, thereby, it involves a lot of graphical work before actual routing computations are carried out.

11.0 DATA REQUIREMENTS

For obtaining solution of a reservoir routing problem, the following data are needed :

(a) Storage volume vs. elevation curve for the reservoir,
(b) Water surface elevation vs. outflow discharge curve,
(c) Inflow hydrograph,
(d) Initial values of storage, inflow and outflow,

(e) For the coefficient method, the value of proportionality constant K, which is the reciprocal of the slope of the storage curve, is also needed.

12.0 EXAMPLE PROBLEM

The spillway of a dam is a broad-crested weir of length 10.0 m; rating coefficient C_d is 1.70. The spillway crest is at elevation 101 m. Above this level, the reservoir walls are vertical, with a surface area of 100 ha. The dam crest is at elevation 107 m. Baseflow is 17 m^3/s, and initially the reservoir level is at elevation 102 m. Route the following hydrograph through the reservoir.

TABLE 1. INFLOW HYDROGRAPH

Time (h)	1	2	3	4	5	6	7	8
Inflow (m^3/s)	20	50	100	130	150	140	110	90
Time (h)	9	10	11	12	13	14	15	16
Inflow (m^3/s)	70	50	30	20	17	17	17	17

Carry out the flood routing through this reservoir.

Solution: The calculations for the storage indication function are shown in Table 2 in which

- Column 1 shows water surface elevations, from 101 m to 107 m.
- Column 2 shows the head above spillway crest.
- Column 3 shows the outflows, calculated by Eq. 3.
- Column 4 shows the storage volume in cubic meters above spillway crest elevation, calculated as the product of reservoir surface area (100 ha) times head above spillway crest (Column 2).
- Column 5 shows storage volume in (cumec-hour). A time interval $\Delta t = 1$ h is appropriate for this example. Computed by dividing the values in Column 4 by 3600.
- Column 6 shows the storage indication quantities $[(2S/\Delta t + Q]$, in m^3/s.

TABLE 2. STORAGE INDICATION VERSUS OUTFLOW RELATION

(1)	(2)	(3)	(4)	(5)	(6)
Elevation (m)	Head (m)	Outflow (m^3/s)	Storage (m^3)	Storage (m^3/s)-h	$[(2S/\Delta t)+Q]$ (m^3/s)
101	0	0	0	0.0	0.0
102	1	17.00	1000,000	277.78	572.56
103	2	48.08	2000,000	555:55	1159.18
104	3	88.33	3000,000	833.33	1754.99
105	4	136.00	4000,000	1111.11	2358.22
106	5	190.07	5000,000	1388.89	2967.85
107	6	249.85	6000,000	1666.66	3583.17

The routing is summarized in Table 3 in which column 1 shows time; column 2 shows the inflow hydrograph; column 3 shows [(2S/Δt) - Q]; column 4 shows the storage indication quantities [(2S/Δt + Q]; and column 5 shows the calculated outflow.

Here, the initial outflow is 17 m^3/s; the initial storage indication value is 572.56 m^3/s (it corresponds to initial elevation 102 m in Table 2); the initial value of Column 3 is 538.56 m^3/s (=572.56 - 2 X 17.0). The next storage indication value is 17 + 20 + 538.56 = 575.56 m^3/s. It leads to an outflow of 17.1 m^3/s. The recursive procedure continues until the outflow has substantially reached baseflow magnitude. Sample calculations are shown in Table 3.

TABLE 3: STORAGE INDICATION METHOD

(1)	(2)	(3)	(4)	(5)
Time (h)	Inflow (m^3/s)	[2S/Δt)-Q] (m^3/s)	[2S/Δt)+Q] (m^3/s)	Outflow (m^3/s)
0	100	300.0	500.0	100.0
1	150	330.0	550.0	110.0
2	250	438.0	730.0	146.0
3	400	652.8	1088.0	217.6
4	800	1111.6	1852.8	370.6
5	1000	1747.0	2911.6	582.3
6	900	2188.2	3647.0	729.4
7	700	2273.0	3788.2	757.6
8	550	2113.8	3523.0	704.6
Remaining lines are deleted.				

12.0 REFERENCES

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