TRAINING COURSE

ON

RESERVOIR OPERATION

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 9

Reservoir Sizing

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Linear Programming

BY

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RESERVOIR SIZING - LINEAR PROGRAMMING

1.0 INTRODUCTION

A reservoir is constructed to change the temporal and spatial availability of water of a stream. Since the natural flow in a stream varies in quantity with time and it seldom follows the demand pattern, it is essential to store the water when the availability is more than the requirements and release it from storage when the situation reverse.

The objective of this chapter is to present a few simple Linear Programming based formulations to determine the optimal size of a reservoir given inflows, demands and other relevant data. It is assumed that the reader is familiar with the basic concepts of LP. After understanding this chapter, the reader would be able to apply the LP technique to the reservoir sizing problems.

2.0 LINEAR PROGRAMMING TECHNIQUES FOR RESERVOIR SIZING

Let us consider a situation in which a reservoir is to be constructed at a particular site. Monthly inflow data for past n months is available. The projected demand of water during a critical year is known along with its distributions among each month. The losses from the reservoir are neglected for the time being. The problem is to find out the minimum capacity of reservoir which will supply the required quantity of water without failure. Let X be the annual water demand from the reservoir and α_i, i = 1,2..... 12 be its fractions for different months. Hence the demand in a particular month will be $\alpha_i X$. Let I_i be the inflow to the reservoir during the i^{th} month and R_i be the water actually released from the reservoir.

Representing by S_i the storage content of the reservoir at the beginning of month i, the continuity equation is :

$$S_i + I_i - R_i = S_{i+1} \qquad i=1,...n \qquad ...(1)$$

This equation has to be satisfied for each of the n months and hence we shall have n such equations which will be constraints in the formulation. The value of S_1 is given as input.

It is also required that the amount of water actually released from the reservoir must be more than or equal to the amount demanded. This can be mathematically expressed as

$$R_i \geq \alpha_i X \qquad i = 1,....n \qquad ...(2)$$

Since this condition also must hold for each month, there will be n such constraints.

If the capacity of the required reservoir is C then in any month, from physical point of view, the storage content of the reservoir must be equal to or less than this value. Hence

$$C \geq S_i \quad i = 1,2....n \qquad ...(3)$$

Moreover, the storage S_i capacity C and release R_i can take only positive values. This completes the problem formulation. The problem is quite easy to solve particularly due to availability of standard package programs.

3.0 LINEAR DECISION RULE FOR RESERVOIR DESIGN

The Linear Decision Rule (LDR) for reservoir design was proposed by Revelle, Joeres and Kirkby (1969). The simplest form of LDR is

$$R = S - b \qquad \ldots(4)$$

where R is the release during a period of reservoir operation (say a month), S is the storage at the end of the previous period and b is a decision parameter chosen to optimize some criterion function. This rule is to be interpreted as an aid to the reservoir operator's judgement in selecting a release commitment to be honored under normal conditions. In exceptional cases, however, the actual release during the time period might have to differ from the commitment R. For example, the optimal value of the decision parameter b might be negative, so that commitments might be made to release more than was in storage at the beginning of the time period. Under normal circumstances, this commitment might be perfectly feasible, but the it may become infeasible in the event of insufficient inflow during the period.

The LDR is intuitively appealing in its structure and can be easily applied in practice. However, a linear decision rule might not be the best rule for any given system. A power rule, a fractional rule, or some combination thereof with different rules for each period might yield a better value of the criterion function. But such rules frequently lead to unwieldy problems that are exceedingly difficult to solve. Formulations utilizing the linear decision rule have been examined for mathematical tractability and have been found in many cases to lead to linear programming problems.

The linear decision rules can be applied in two frameworks : (1) the deterministic framework where the magnitude of each input in a sequence is specified in advance, either from historic records or from synthetic generation based on the statistical properties of the streamflow process, and (2) the stochastic framework where the magnitudes of reservoir inputs are treated as random variables unknown in advance.

3..1 Reservoir Design - Deterministic Approach

A dam is to be built to provide a regulated outflow for irrigation, waste dilution, water supply, and other uses and to provide pools for recreation and flood control. The intent of the dam builder is to provide a dependable supply for the downstream users. At the beginning of each time period, he will make a commitment to release a total volume of exactly R_i during the ith time period, as far as reasonably possible. The downstream users can consider this release commitment in planning their activities for the time period.

The projected requirements of the downstream users are expressed by minimum acceptable releases M_i to be supplied in period i. To prevent excessive channel erosion, flooding and other damage that would occur if the release were too large, the release during period i should not exceed the volume f_i. Another set of requirements is imposed by the other uses of the reservoir. For hydropower generation, recreational and esthetic purposes, it is desirable to maintain the storage in the reservoir above a lower limit S_{min} during all time periods. An additional requirement imposed by flood control considerations is that a freeboard of at least v_i be available at the end of each period for storing floods that might occur in the next period.

The problem is to find an operating policy (a guideline for the release R_i) that causes the requirements to be satisfied while minimizing the size, and hence the cost, of the dam required.

Let a sufficiently long time-series of monthly inputs is available. It is required to find twelve linear decision rule parameters, one for each month of the year, that minimize the reservoir capacity required to meet the specified performance characteristics with the postulated input sequence. The linear decision rule is

$$R_t = S_{t-1} - b_i \qquad \text{...(5)}$$

where, R_t release during the month t of operation, S_t storage at the end of the month t of operation, and b_i is the linear decision rule parameter for the ith month of the year. These are to be determined by the linear decision rule. The continuity equation for the reservoir is

$$S_t = S_{t-1} - R_t + I_t \qquad \text{...(6)}$$

where I_t is the expected reservoir input in the month t of operation. All variables are expressed in volumetric units. Substitution of the linear decision rule into the continuity equation yields

$$S_t = b_i + I_t \qquad \text{...(7)}$$

Substituting a similar equation for S_{t-1} into the decision rule yields the following expression for the release during period t :

$$R_t = I_{t-1} + b_{i-1} - b_i \qquad \text{...(8)}$$

The engineering specifications on release commitments and storage utilization can be expressed mathematically by treating them as limitations on the range of decisions acceptable at each point in time at which decisions are to be made. The constraints take the following form :

The freeboard $C - S_t$ at the end of period t must be greater than V_i.

$$C - S_t \geq V_i \quad (t = 1,...n) \qquad ...(9)$$

where V_i is the flood storage capacity required at the end of the ith month of the year and C is the reservoir capacity, to be determined. In the deterministic sense this is equivalent to saying that the decision at the beginning of time period t should not lead to insufficient freeboard at the end of t, given the extremes of the hydrologic record. The longer the record in general the more severe the observed extremes and the lower the probability of violating the constraints in practice.

The storage at the end of period t must be greater than or equal to the minimum storage required.

$$S_t \geq S_{min} \quad (t = 1,...,n) \qquad ...(10)$$

In terms of the decision to be made at the beginning of period t, the constraint limits the control function to those linear rules which lead to storages exceeding S_{min}, given the extremes of the hydrologic record. The longer the record the smaller the likelihood of observing a worse extreme and hence violating the constraint.

The release in period t must exceed M_i, the minimum release for the ith month.

$$R_t \geq M_i \quad (t = 1,...,n) \qquad ...(11)$$

The release in period t must be less than f_i which is the maximum allowable release in the ith month of the year.

$$R_t \leq f_i \quad (t = 1,...,n) \qquad ...(12)$$

These last two constraints further limit the range of decision rules which can be considered. Substitution of equations 7 and 8 into 9 to 12 yields

$$C - b_i \geq V_i + I_t \qquad ...(13)$$
$$b_i \geq S_{min} - I_t \qquad ...(14)$$
$$b_{i-1} - b_i \geq M_i - I_{t-1} \qquad ...(15)$$
$$b_{i-1} - b_i \leq f_i - I_{t-1} \quad (t = 1,2,...,n) \qquad ...(16)$$

The variables M, f, V, and b are indexed by a parameter $i = 1, ..., 12$ because their values in the ith month are the same from year to year. The variables I, R, and S, however, do not follow a regular cyclic pattern and therefore are indexed by the parameter $t = 1,...,n$ where n is the number of periods for which data are available. The correspondence between i and t is, $i = t(\text{mod } 12)$.

This problem has a number of constraints; if twenty years of monthly data are considered, there will be 960 constraints. A noteworthy property is that each constraint appears in the same form twenty times, except for a different stipulation on the right-hand side. Of each constraint's twenty appearances then, one occurrence should be more restrictive than any other. Only this dominant constraint need be retained.

In the final constraint set, the term S_{min} is set equal to some fraction of the total capacity and the term S_0, the initial storage, is some different and larger fraction of the capacity

$$S_{min} = a_m \cdot C$$
$$S_0 = a_0 \cdot C \qquad (a_0 \geq a_m)$$

The constraint set now becomes

$$C - b_i \geq \max_n (I_{i+12n}) + V_i \qquad (i = 1,...,12)$$
$$a_m C - b_i \leq \min_n (I_{i+12n}) \qquad (i = 1,....,12)$$
$$b_{i-1} - b_i \geq M_i - \min (r_{i-1+12n}) \qquad (i = 2,....,12)$$
$$b_{12} - b_1 \geq M_1 - \min (r_{12+12n})$$
$$a_0 C - b_1 \geq M_1$$
$$b_{i-1} - b_i \leq f_i - \max (I_{i-1+12n}) \qquad (i = 2,...,12)$$
$$b_{12} - b_1 \leq f_1 - \max (I_{12+12n})$$
$$a_0 C - b_1 \leq f_1$$

The total number of constraints is now 50 rather than n*8 encountered earlier. The number of unknowns is 13. The objective is to minimize the size of the reservoir.

Minimize C

The problem is finding the smallest reservoir that will deliver flows in the specified range over the entire record under the added constraint of a linear decision rule. The results of solution will be the required reservoir capacity and the twelve decision parameters constituting the decision rule for management of the reservoir.

The above constraints ensure that the release and storage requirements would be met if this optimal linear decision rule were applied to the postulated input sequence. In practice, however, future reservoir inflows are not known with certainty, so there is no absolute assurance that this policy will yield the desired releases and storages in the future. On the contrary one may estimate that in each month i there is a probability of 2/21 that the input will lie outside the 20-year recorded range of inputs. Consequently in the absence of any information to the contrary one might expect that in each future month the probability of violating some of the constraints also would be 2/21.

These observations indicate two shortcomings of the deterministic formulation. First the deterministic formulation yields no explicit statement of the reliability with which the reservoir will meet the specified performance objectives in the future. Second the reservoir's reliability is fixed by the specific postulated input sequence and is not under the direct control of the designer. Chance-constrained programming can be used to eliminate these deficiencies in the deterministic formulation of the reservoir management problem.

3.2 Reservoir Design - Chance-constrained Formulation

The above problem will now be presented in the stochastic environment. Here, the flows in particular periods are not specified and are known only with some probability. Thus the total discharge in the ith month of the year is treated as a random variable X_i having the cumulative probability distribution function

$$F_{Xi}(r) = P[\, X_i \leq I]$$

In addition the constraints are now expressed as limitations on the allowable risk of violating the performance requirements.

Although formally identical to the deterministic formulation, the probabilistic representation of the flood freeboard requirement has several advantages. Most important the chance-constrained formulation comes squarely to grips with the impossibility of absolutely ensuring the specific performance of a reservoir fed by random inputs. In a way, this formulation attaches a statement of reliability to the mathematical representation of each performance requirement. Moreover, the level of reliability at which each requirement is satisfied is under the direct control of the designer.

A related advantage of the probabilistic formulation is that it clarifies the operational significance of the decision rule. In the deterministic formulation, one might interpret the linear decision rule as a specification of the actual reservoir outflow during the next month. In practice, however, this interpretation may lead to confusion when it is recognized that excessively large or small inflows during a month may make it physically impossible to release a specified volume. The probabilistic formulation on the other hand emphasizes that the linear decision rule is merely an aid to the operator's judgement in deciding how much to release during a month. If the rule is followed, the release commitment will be compatible with the reservoir performance requirements with a specified degree of reliability. When a conflict does arise, however, the operator has the ability to adjust the actual release in the light of the specific conditions of the case.

Finally the chance-constrained formulation of the performance requirements seems to permit more direct economic interpretation of the constraints than the deterministic formulation. For example, it might be asked if there would be any advantage in changing the flood control performance requirement. The form of this requirement suggests that the specified freeboard V_i is based on hydrologic analysis of a standard design flood and that more detailed physical and economic data on the relation between flood damages and the flood freeboard are not readily available. Thus the designer cannot immediately interpret the marginal costs that the deterministic formulation would associate with changes in the freeboard specification V_i. In the probabilistic formulation, on the other hand, the marginal costs are associated with changes in the reliability with which the specified freeboard is made available and hence with changes in the reliability of protection against the design flood. The economic consequences of changes in the reliability appear clearer than those of changes in the freeboard specification.

The fundamentals of the linear decision rules have been in the above. The LDR has been the subject of intense debates. It has been modified, extended and criticised by a number of investigators. For further details, the reader may refer to the text by Loucks et. al. (1981).

4.0 REFERENCES

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