TRAINING COURSE

ON

RESERVOIR OPERATION

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 15

Operation

of a

Multireservoir System

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OPERATION OF A MULTIRESERVOIR SYSTEM

1.0 INTRODUCTION

An assemblage of reservoirs can be treated as a system because of the existence of interaction between the component reservoirs. These interactions are due to the possibilities of the joint use of space in reservoirs, transfer of water from one reservoir to another and the substitution of the releases from one reservoir for the releases from another for meeting a particular demand. Such interactions make it possible for the effectiveness of the system as a whole to be more than the sum of the effectiveness of the individual reservoirs operated independently of each other. In other words, if the system made up of a number of interacting reservoirs is to be replaced by an equal number of non-interacting reservoirs, benefits from operation of such reservoirs have to be at least equal to, and often larger, than the benefits from component reservoirs they replaced. A multi reservoir system can be decomposed into interlinked subsystems for which operating policies are derived by model studies.

The concept of decomposition has been used to decompose lower Betwa system (Fig. 1). The system consisting of two reservoirs Rajghat (A) and Matatila (B) on river Betwa (a tributary of Yamuna) and their service area are considered here. The system is to be optimally operated to meet the irrigation demands of six canal systems (Fig. 1) and to generate hydroelectric power. The canals and their service areas are located in the administrative jurisdiction of the Governments of Uttar Pradesh and Madhya Pradesh which have an agreement on sharing of water.

2.0 SYSTEM MODEL

In the dynamic programming formulation, the problem to be solved is represented by the recursive equation:

$$fn\ (S_A,S_B) = \text{opt}\ \{Rn(xa) + Rn(xb) + Rn(xab) + fn\text{-}1\ (S_A,S_B)]\ xa,\ xb,\ xab \qquad \ldots(1)$$

where:

$fn(S_A,S_B)$	=	the optimal return from the operation of Rajghat (A) and Matatila (B) reservoirs defined by the state variables S_A and S_B when there are n stages (time steps) to the end of the plan (operation plan period).
Rn(xa), Rn(xb), Rn(xab)	=	the returns in the current stage due to the implementation of the decisions xa, xb and xab respectively.
xa	=	the decision vector on the release of water from Rajghat reservoir (A) for meeting the target demands. These target demands are irrigation releases in Jakhlaun canal, UP canal and MP canal.
xb	=	the decision vector on the release of water from Matatila reservoir (B) for meeting the demands of power generation at dam power plant, downstream releases in excess of power releases for irrigation omitted prior to the construction of

Rajghat.

xab = decision vector composed of water transferred from Rajghat reservoir (A) to Matatila reservoir (B) and release from Rajghat reservoir for meeting the irrigation demands of service area downstream of Matatila.

3.0 SYSTEM DECOMPOSITION

We can write the decomposition concept as:

$$fn(S_A,S_B) = Hn\ (S''A) + Kn\ (S''B) \qquad ...(2)$$

where:

Hn(S"A) = the optimal return from the operation of Rajghat reservoir (A) defined by the state variable S"A operated to meet the irrigation demands of Jakhlaun, UP and MP canals only when there are n time steps left to the end of the operation plan period.

Kn (S"B) = the optimal return from the operation of Matatila reservoir (B) defined by the state variable S"B operated to meet the hydro power demand and irrigation demand prior to construction of Rajghat dam only when there are n time steps left to the end of the operation plan period.

S"A and S"B would be different from S_A and S_B and the differences reflect the effect of interaction between the components of the system due to the transfer of water and substitution of release from one component for the release from another.

$$\text{Let } S''A = S_A + \alpha\, S_A \qquad ...(3a)$$

$$\text{and } S''B = S_B + \alpha\, S_B \qquad ...(3b)$$

$$\text{then } fn\ (S_A,S_B) = Hn\ (S_A + \alpha\, S_A) + Kn\ (S_B + \alpha\, S_B) \qquad ...(3c)$$

Because of optimality, the derivative of fn (S_A,S_B) with respect to each vector xa, xb and xab would be zero. Hence,

$$\frac{\delta fn(S^A,S^B)}{\delta xab} = \frac{\delta Hn(S''A)}{\delta xab} + \frac{\delta Kn(S''B)}{\delta xab} \qquad ...(4)$$

$$\frac{\delta fn\ (S_A,S_B)}{\delta xab} = \frac{\delta Hn}{\delta S_A}\frac{\delta S_A}{\delta xab} + \frac{\delta Kn}{\delta S_B}\frac{\delta S_B}{\delta xab} \qquad ...(5)$$

$$\frac{\delta fn\ (S_A,S_B)}{\delta xab} = H'n\ (S_A)\ \frac{\delta(S_A+\alpha S_A)}{\delta xab} + \frac{K'n(S_B)\delta(S_B+\alpha S_B)}{\delta xab} \qquad ...(6)$$

$$\frac{\delta fn\ (S_A,S_B)}{\delta xab} = H'n\ (S_A)\ \frac{\delta(\alpha S_A)}{\delta xab} + \frac{K'n(S_B)\delta(S_B+\alpha S_B)}{\delta xab} \qquad ...(7)$$

As αS_A and αS_B are the measures of the interaction between reservoirs. A and B. only, they are functions of the interacting decision vector xab. If the interactions are mainly due to the transfer of water or the substitution of release from one reservoir for release from another.

$$\alpha S^A = \alpha S^B$$

Therefore,

$$H'n\ (S_A)\ -\ K'n\ (S_B)\ = 0 \qquad ...(8)$$

$$H'n\ (S_A)\ =\ K'n\ (S_B)$$

Expanding H'n (S"A) and K'n (S"B) as Taylor series in αS_A and αS_B and retaining only the first term, we have

$$H'n(S_A)\ =\ K'n(S_B)$$

The preceding relationship means that for optimality, the derivative of the return function from the optimal operation of reservoir A with respect to the state of that reservoir would be equal to the corresponding derivative for reservoir B. In other words, the transfer of water from one reservoir to another or the substitution of the release from one reservoir for that from another would be done until the marginal benefit from such an action, denoted by the derivative on the right hand side of the equation, is equal to the marginal cost indicated by the terms on the left hand side of the equation. The decomposition adopted here is based on the concept of putting a measure on the benefit due to and the cost of implementing the interacting decisions.

Now the problem of optimal operation of the system of two reservoirs A and B can be viewed as an interactive process consisting of two steps, as discussed below. As before, we can write

$$fn\ (S^A\ ,S^B) = Hn\ (S^A+\alpha S^A)\ +K\ (S^B+\alpha S^B) \qquad ...(9)$$

From the earlier discussion, it is seem that when the operation is optimal

$$|H'n\,(S^A)| = |K'n\,(S^B)| \qquad \ldots(10)$$

The iteration is done as follows:

$$\text{Step I} \quad Kn\,(S^B/xab) = Opt\,\{Rn\,(xb) + Kn\text{-}l\,(S'^B)\}\; xb \qquad \ldots(11)$$

$$\text{Step II} \quad Hn\,\{S^A/K'n(S^B)\} = Opt\,\{R(xa) + R(xab, K'n(S^B)\} + Hn\text{-}l\,(S'^A)\}\; xa,\, xab \qquad \ldots(12)$$

In step I, with an assumed value of xab, the operation of the reservoir B is optimized and the value of the derivative K'n (S^B) determined. The derivative obtained in step I can be considered as a measure of the benefit due to the implementation of the interaction decision xab. With this knowledge, the operation of the reservoir A is optimized in step II. The result is the quantity of xab allotted if the measure of the benefit from its implementation were K'n (S^B). This value of xab is compared with that assumed in step I. If they differ, the cycle is continued with the value of xab in step I of any iteration being set equal to the value obtained in step II of the previous iteration. The iterations are continued until convergence is obtained within a set limit of tolerance.

The solution of the problem would be a set of xab. each value being defined for every combination of S^A and S^B at every stage.

4.0 SUB-SYSTEM I - RAJGHAT RESERVOIR

$$\begin{aligned} Hn\,(S^R) = Opt\,\{&(crj^*,drj)+(cru^*dru)+(crm^*drm)+ drj\,,dru,drm,xrc(crc^*xrc) \\ &+prj(Drj\text{-}\,drj)+pru(Dru\text{-}dru)+prm(Drm\text{-}drm)+prc(Xrc\text{-}xrc) \\ &+Hn\text{-}1(S'R)((1+r/12)^{-1})\} \end{aligned} \qquad \ldots(13)$$

subject to:

$$drj \leq Drj \qquad \ldots(14a)$$
$$dru \leq Dru \qquad \ldots(14b)$$
$$drm \leq Drm \qquad \ldots(14c)$$
$$xrc \leq Xrc \qquad \ldots(14d)$$
$$S^R + ir\text{-}drj\text{-}dru\text{-}drm\text{-}Xrc\text{-}xrs\text{-}er = S'^R \qquad \ldots(14e)$$
$$S^a \leq S^R min \qquad \ldots(14f)$$
$$S'^R \leq S^a max \qquad \ldots(14g)$$

where:

SR	=	the state of the Rajghat reservoir when there are n stages to the end of the plan.
drj, dru, drm, xrc	=	releases (MCM) from Rajghat reservoir to Jakhlaum, UP canal,MP canal and for downstream use, respectively
Drj, Dru, Drm, Xrc	=	target releases (MCM) from Raighat reservoir to Jakhlaun, UP canal,

		MP canal and for downstream use, respectively.
crj, cru, crm, cre	=	unit costs (Rs/MCM) of releases from Rajghat reservoir to Jahklaun, UP canal, MP canal and for downstream use, respectively.
prj, pru, prm, prc	=	penalty costs (Rs/MCM) of deficit releases from Rajghat reservoir to Jakhlaun, UP canal, MP canal and for committed downstream use, respectively
ir	=	inflow (MCM) to Raighat reservoir
xrs	=	spillover (MCM) from Rajghat reservoir
er	=	evaporation loss (MCM) from Rajghat reservoir
r	=	interest rate (%)

4.1 Simplified inventory control model for Rajghat reservoir

A dam has two components of cost of diverting water i.e. annual cost per unit water diverted into canal and second is the cost of creating facility for storing water i.e. annual cost of storage per unit water stored. Hence,

$$Hn\ (S^R) = \text{Opt}\ \{(crq*qr) + (crs*S^R) + Hn\text{-}l\ (S'^R)\}\ qr \qquad \ldots(15)$$

subject to:

$$S^R + qr - (Qr+er) = S'^R \qquad \ldots(16a)$$

$$S^R \leq S^Rmax \qquad \ldots(16b)$$

$$qr \leq (\text{inflow})\ r \qquad \ldots(16c)$$

where:

qr	= amount of inflow (MCM) which is controlled and/or developed in a period.
(inflow) r	= known inflow (MCM) into Rajghat reservoir.
crq	= unit cost (Rs/MCM) of controlling inflow.
crs	= unit cost (Rs/MCM) of storing water.
negative S^R	= means target flows (MCM) are not met in those periods.

5.0 SUB-SYSTEM II - MATATILA RESERVOIR

$$Kn\ (Sm) = \text{Opt}\ \{(cmp*xmp)+(cmi*xmi)+pmp(Xmp-xmp) + xmp,xmi,pmi\ (Xmi-xmi-xmp) + Kn\text{-}l\ (S'm)\ ((l+r)/12)^{-1}\ )\} \qquad \ldots(17)$$

subject to:

$$xmp \leq Xmp \qquad \ldots(18a)$$

$$xmp + xmi \leq Xmi \qquad \ldots(18b)$$

$$Sm+im+(xrc+xrs)-xmp-xmi-xms-em= S'm \qquad \ldots(18c)$$

$$Sm \geq Smmin \qquad \ldots(18d)$$

$$S'm \leq Smmax \qquad \text{...(18e)}$$

where:

Sm = the state of Matatila reservoir when there are n stages to the end of the plan.

xmp = controlled releases (MCM) through dam powerhouse at Matatila.

Xmp = target releases (MCM) through dam powerhouse at Matatila according to generation plan

xmi = non-power releases (MCM) from Matatila for irrigation in downstream area.

Xmi = target releases (MCM) from Matatila and Rajghat for downstream use.

xms = spillover (MCM) Matatila dam.

cmp = unit operation cost (Rs/MCM) of release through power plant at Matatila.

pmp = unit penalty cost (Rs/MCM) of deficit release through power plant at Matatila.

cmi = unit operation cost (Rs/MCM) of non-power release from Matatila for irrigation in downstream area.

pmi = unit penalty cost (Rs/MCM) of deficit release from Matatila for use for downstream area.

xrc = controlled release (MCM) from Rajghat to Matatila reservoirs.

xrs = spillover (MCM) from Rajghat dam.

im = interim catchment flow (MCM) to Matatila reservoir.

em = evaporation loss (MCM) from Matatila reservoir.

r = interest rate (%).

Smmin = dead storage (MCM) in Matatila reservoir.

Smmax = maximum storage (MCM) in Matatila reservoir.

5.1 Simplified inventory control model for Matatila Reservoir operation

$$Kn\,(Sm) = Opt\,\{(cmq*qm)+(cms*Sm)+Hn\text{-}1(S'm)\}qm \qquad \text{...(19)}$$

subject to:

$$Sm + qm - (Qm+em) = S'm \qquad \text{...(20a)}$$

$$Sm \leq Smmax \qquad \text{...(20b)}$$

$$qm \leq (inflow)m \qquad \text{...(20c)}$$

where:

qm	=	amount of inflow (MCM) which is released in a controlled manner to meet power generation and irrigation demand in downstream area.
(inflow)m	=	interim catchment runoff (MCM) between Rajghat and Matatila (im), and release from Rajghat.
	=	im + (inflowr - qr)
(inflow)r	=	inflow at Rajghat dam
qr	=	amount of inflow at Rajghat which is controlled and/or developed in a period

6.0 REAL TIME OPERATION FOR FLOOD CONTROL

The operating decisions are releases from Rajghat and Matatila reservoirs so that constraints on river channel flow, reservoir storage and other operating constraints (irrigation and power generation requirements) are satisfied. Daily flow forecasts for a finite number of future time periods (30 days) and remaining monsoon season (i.e. up to end of September) forecasts are important inputs. Although daily releases are determined for a finite number of future time periods but implemented for one immediately next period after which next set of observations become available to update information on the state of system and then entire process is repeated.

Daily Flow Forecast: Many forecasting models are available. As an example model is given below:

$$FF\ (K) = AF(K):\ (Z(K) + 1) \qquad \ldots(21)$$

where:

FF(K)	=	Forecasted flow on day K,
AF(K)	=	Actual flow on day K (from historical record),
K	=	n indicates nth day in future,
Z(1)	=	0: 0, Z(2) = a unit normal random variate: and for K 3,
Z(K)	=	RH01.Z (K-1) + RH02.Z (K-2) + (normal random variate),
RH01	=	0.9945, RH02 = 0.006 determined by comparing forecasted.

6.1 Remaining Season Forecast (RSF)

This is obtained by applying Thomas Fiering model or any other model based on Markovian process.

System Variables and Parameters:

Actual vacant storage space: It depends on storage capacity and present storage.
Travel time between two reservoirs: It depends on size of the release.
Rule curves for the reservoirs: Required vacant storage space as a function of forecast of remaining season runoff
Release pattern
Programming Decisions: Based on forecast of remaining season runoff, required vacant flood control space is computed from rule curves and compared with actual vacant flood control space.

If vacant space is deficient, releases are raised above inflow limited to channel capacity. Amount of release depends on period of remaining season. Flood control regulations permit releases greater than channel capacity under certain conditions. No dam is allowed to overflow.

If vacant space is sufficient, then water balance is applied to Rajghat reservoir to check overflow of dam on the basis of today's storage in Rajghat, forecast of daily flows for next 30 days and release pattern (to meet target demands). If Rajghat dam is likely to overflow in next 30 days, then releases are raised above normal releases. Conservation storage requirement and channel capacity are the constraints on release pattern.

The 30 day release sequence and interim catchment flow forecast are used to decide inflows into Matatila reservoir. Decision process to check overflow of Matatila is similar to that for Rajghat.

Release from Rajghat does not create vacant space in the total system as it will occupy storage in Matatila. Therefore flood control regulations should specify vacant storage space in Matatila, say two third of vacant space in the system.

7.0 APPLICATION EXAMPLE

Table 1 shows conventional reservoir operation study of Matatila reservoir for 75 percent dependable year. Instead of carrying out study for a single year, it can be carried out for a number of years and reliability parameters can be estimated. Long term simulation study for joint operation of the two reservoirs can be carried out in a similar manner as shown in Table 1.

Table 2 shows the comparison of operation table between project report and DP study for both the Rajghat and Matatila reservoirs. Results revealed that the optimal annual inflow that can be controlled and/or developed in the reservoir will be 18000 MCM for Rajghat and 45000 MCM for Matatila. The optimal inflow is sufficient to satisfy water demand including irrigation, power requirement and evaporation losses. The balance storage in each reservoir will be reduced. Thus, an additional space for flood control will be available.

Operation curve of optimum controlled inflows and storage for Matatila reservoir is shown in Fig. 2. The optimal storage and inflow as per DP study is lower as compared to the project report.

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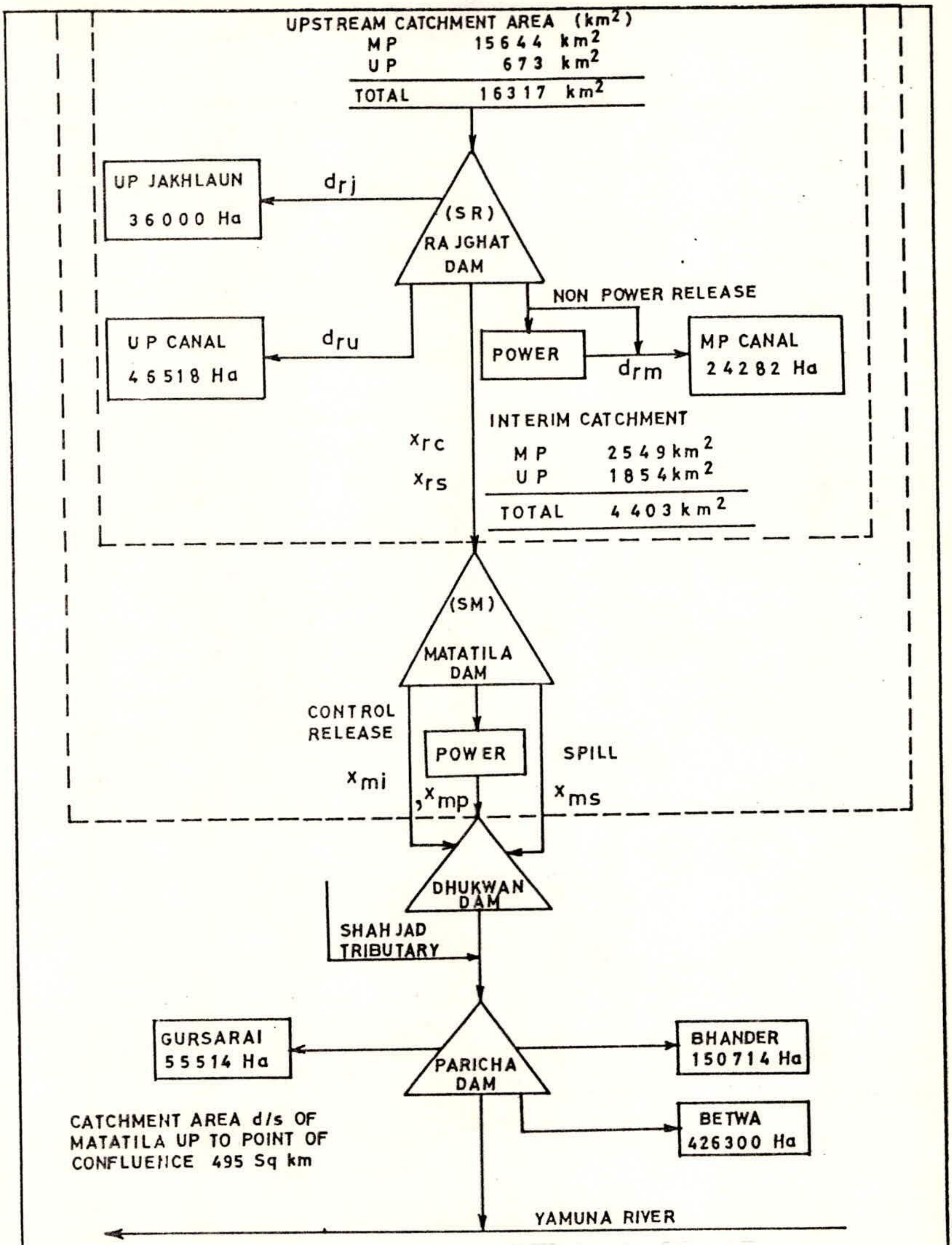

Fig. 1 Schematic Diagram of Lower Betwa System

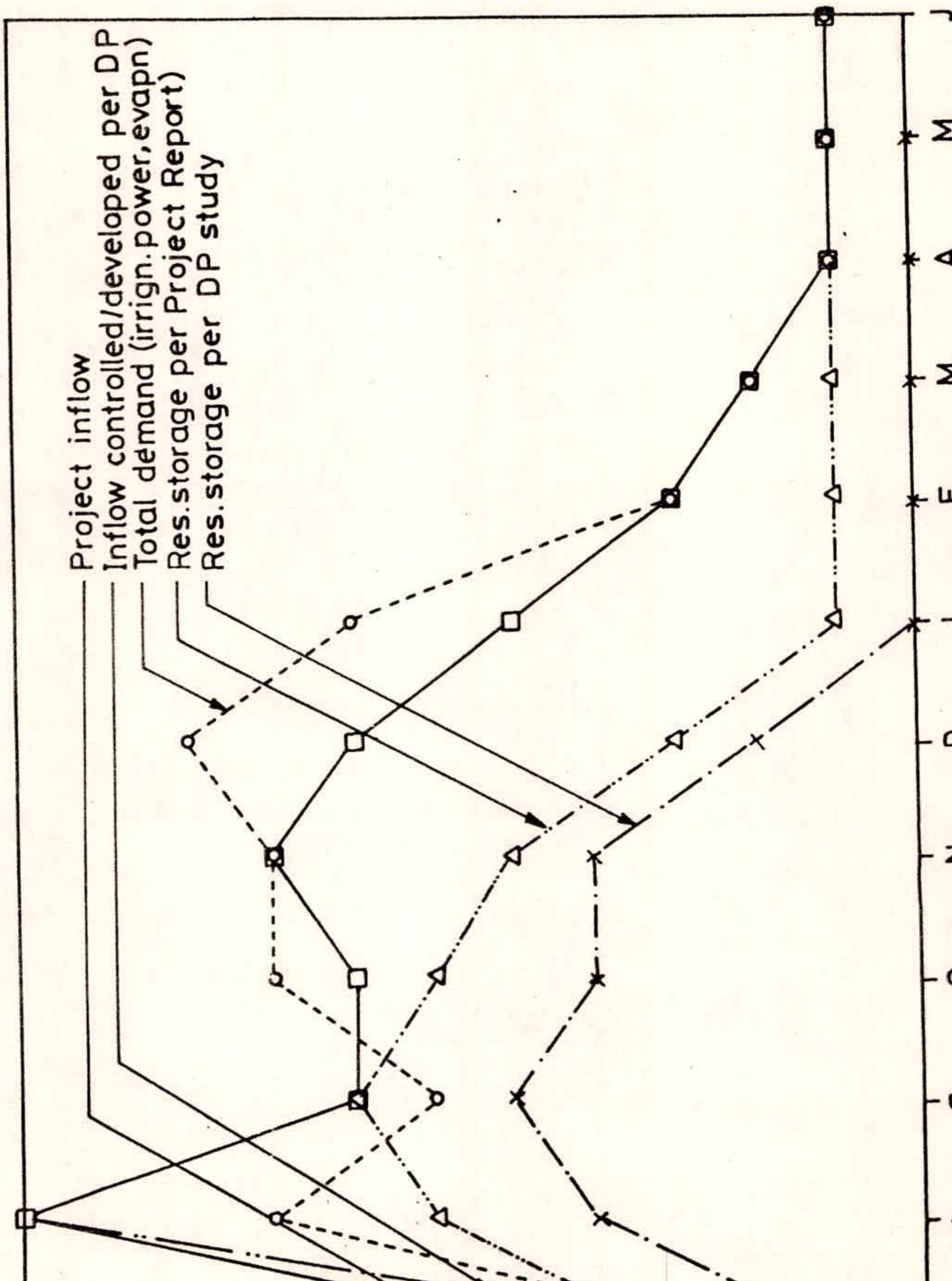

Fig. 2 Operation Curve for Matatila Reservoir
(Project report vs DP study)

Table 1a: Operation Table for Matatila Reservoir for 75% Dependable Year
(All quantities from column 2 to 10 are in MCM)

FRL	=	308.50 m
DSL	=	295.73 m
Live Storage	=	833.00 MCM
Dead Storage	=	50.00 MCM
Silt Storage	=	29.00 MCM

MONSOON

Month	INFLOW			OUTFLOW						BALANCE STORAGE [COL 4-10]	STORAGE ELEVATION IN METER
	NET INFLOW	CARRY OVER [VIDE COL 10]	TOTAL INFLOW [COL 2+3]	IRRIGATION				EVAP. LOSSES	TOTAL OUTFLOW [COL 8+9]		
				UP		MP	TOTAL [COL 5+6+7]				
(M)	(IN)	(SM)	(qm)	GURUSARAI (xmg)	BETWA (xmb)	BHANDER (xmh)	(xmq)	(em)	(Qm)	(SM)	(EL)
1	2	3	4	5	6	7	8	9	10	11	12
Jul	383	59	442	20	96	95	211	10	221	221	301.64
Aug	1103	221	1324	55	375	344	774	14	788	536	306.04
Sep	680	536	1216	31	320	236	587	10	597	619	307.66
Oct 1-15	342	619	961	26	124	190	340	7	347	614	307.56
Sub total	2508	-	-	132	915	865	1912	41	-	-	-

Notes:

1. 75% dependable year means 46 years of record (1925-71) multiply by 3/4 = 35.5, say 35,
   46-35 = 11, hence 12th year of record (arranged in ascending order) will be the 75% dependable year.
2. Carry over storage of 59 MCM (Col 4) is the balance of the previous year (1935) from long term operation table.
3. MP = State of Madhya Pradesh.
4. UP = State of Uttar Pradesh.

Table 1b: Operation Table for Matatila Reservoir for 75% Dependable Year
(All quantities from column 2 to 10 are in MCM)

NON-MONSOON

Month	INFLOW			OUTFLOW						BALANCE STORAGE	STORAGE ELEVATION IN METER
	NET INFLOW	CARRY OVER [VIDE COL 10]	TOTAL INFLOW [COL 2+3]	IRRIGATION				EVAP. LOSSES	TOTAL OUTFLOW [COL 8+9]		
				UP		MP	TOTAL [COL 5+6+7]				
(M)	(IN)	(SM)	(qm)	GURSARAI (xmg)	BETWA (xmb)	BHANDER (xmh)	(xmq)	(em)	(Qm)	(SM)	(EL)
1	2	3	4	5	6	7	8	9	10	11	12
Oct 16-31	342	614	956	26	154	220	400	6	406	550	306.27
Nov	785	550	1335	55	433	350	838	11	849	486	305.58
Dec	674	486	1160	50	445	348	843	8	851	309	303.22
Jan	516	309	825	52	348	283	683	8	691	134	299.71
Feb	288	134	422	41	83	202	326	7	333	89	298.26
Mar	216	89	305	30	72	127	229	7	236	69	297.49
Apr	90	69	159	0	40	45	85	11	96	63	297.19
May	83	63	146	0	39	39	78	9	87	59	296.89
Jun	82	59	141	9	55	0	64	10	74	67	297.28
sub total	3076	-	-	263	1669	1614	3546	77	-	-	-
ANNUAL	5584	-	-	395	2584	2479	5458	118	-	-	-

Table 2: Operation Table for Rajghat and Matatila Reservoirs
(Pre-Project Report Vs DP Study)

Month	Net Inflow (100 MCM)				Carryover (vide Col 16-19) in 100 MCM				Total Inflow (100 MCM)				Total outflow Demand (100 MCM)		Balance Storage (100 MCM)			
	Pro. Rep.		DP Study		Pro. Rep.		DP Study		Pro. Rep.		DP Study				Pro. Rep.		DP Study	
	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT	RAJ	MAT
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Jul	4	4	0	2	6	1	6	1	18	5	6	3	8	2	18	3	6	3
Aug	8	11	5	11	8	3	6	1	16	14	11	12	2	8	14	6	9	4
Sep	6	7	6	2	12	6	8	4	18	13	15	11	2	6	16	7	13	5
Oct	2	7	2	2	14	7	11	5	16	14	15	12	2	8	14	6	13	4
Nov	1	9	1	8	13	5	13	4	14	13	14	12	3	8	11	5	11	4
Dec	1	7	1	7	11	5	11	4	12	12	12	11	4	9	8	3	8	2
Jan	1	5	1	5	8	3	8	2	9	9	8	7	4	2	5	1	5	0
Feb	1	3	1	3	5	1	5	0	6	4	6	3	4	3	2	1	2	0
Mar	1	2	1	2	2	1	2	0	3	3	3	2	3	2	0	1	0	0
Apr	1	1	0	1	1	1	1	0	2	2	1	1	0	1	2	1	1	0
May	1	1	0	1	1	1	0	2	2	2	0	1	0	1	2	1	0	0
Jun	1	1	0	1	2	1	0	2	3	2	0	1	0	1	3	1	0	0
Annual	28	57	18	45	83	35	71	25	119	93	91	76	24	56	95	36	68	22

Notes:

1. Operation table project data based on 75% dependable year.
2. Operation table for DP study derived from QSB computer application package.
3. Total inflow = net inflow + carry over storage.
4. Total outflow includes irrigation, power requirements and evaporation losses.
5. Balance storage = Total inflow - Total outflow
6. Net inflow as per DP study corresponds to optimum inflow to be controlled and/or developed.