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DETERMINATION OF AQUIFER RECHARGE FOR VARYING RIVER STAGES

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CONTENTS

LIST OF SYMBOLS

$B(x)$	Aquitard thickness at location x
D_o	$\frac{h_o k}{\phi}$
$H(x,t)$	Change in water table elevation at x at time t
h_o	Initial saturated thickness of aquifer
$K(x)$	Aquitard permeability
$K_Q(t)$	Flow to aquifer consequent to unit rise in river stage
$K_S(x,t)$	Rise in water table height due to unit rise in river stage
k	Coefficient of permeability
l	Width of aquifer
m	Time step
n	Time step
$q(i,n)$	Recharge rate per unit area per unit time taking place through the i^{th} strip during n^{th} unit time step
$Q(n)$	Recharge from the river to the first aquifer during time step n
S_1	Water table rise in the first aquifer
S_2	Water table rise in the second aquifer
t	Time
T	Transmissivity of an aquifer
T_1	Transmissivity of the first aquifer
T_2	Transmissivity of the second aquifer
$w(x,t)$	Recharge rate per unit area per unit time at location x and at time t

$W(i)$	Width of i^{th} strip
$W(i_o)$	Width of the river
x	Co-ordinate
Φ	Storage coefficient of an aquifer
Φ_1	Storage coefficient of the first aquifer
Φ_2	Storage coefficient of the second aquifer
α	T/Φ
α_1	T_1/Φ_1
α_2	T_2/Φ_2
$\partial_1(i,j,n)$	Rise in piezometric surface in the first aquifer at j^{th} strip at the end of n^{th} unit time step due to recharge taken place during the first unit time period through the i^{th} strip
$\partial_2(i,j,n)$	Rise in piezometric surface in the second aquifer at j^{th} strip at the end of n^{th} unit time step due to recharge taken place during the first unit time period through the i^{th} strip
Γ_r	Reach Transmissivity
τ	Time
$\sigma(n)$	River stage during time step n

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ABSTRACT

In a sedimentary groundwater basin occurrence of multiple aquifer separated by confining layers of low and negligible permeability is quite common. A stream in such basin may penetrate either partially or fully the top aquifer. During the passage of a flood the river stage changes rapidly. The rise in river stages above the aquifer water level in the vicinity of the river leads to recharge of groundwater. Consequent to rise in river stage, the upper aquifer is recharged directly through the bed and banks of the river and the lower aquifers are recharged from the top aquifer through the intervening aquitard. In the present report, the interaction of two aquifers and a river has been studied analytically. The recharge from the river to the upper aquifer and exchange of flow between the two aquifers through the aquitard have been quantified for a known fluctuation pattern in river stage. Each aquifer has been assumed to be homogeneous isotropic and of large areal extent. The time parameter has been discretised and interaction problem has been solved assuming each of the aquifers to be a linear system. The analytical solution is tractable for numerical computation. The storage coefficient of the lower aquifer controls the quantity of recharge from the upper aquifer.

1.0 INTRODUCTION

The interaction between surface water and groundwater has been examined in some detail in recent years. There are two main aspect of this process: 1) the flow of groundwater to support river flow and the flow from river to groundwater. Recharge may occur whenever the stage in a river is above that of the adjacent groundwater table, provided that the bed comprises permeable or semi-permeable material. This type of groundwater recharge may be temporary, seasonal or continuous. Also it may be a natural phenomenon or induced by man. Man can induce groundwater recharge from rivers by lowering the water table adjacent to rivers through groundwater abstraction.

In a groundwater basin it is common to identify several aquifers separated either by less permeable or impermeable layers. A river in general penetrates fully or partially the upper aquifer. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. The lower aquifer is recharged through the intervening aquitard. A single aquifer river interaction problem has been studied analytically by several investigators (Morel-Seytoux, 1975, Todd, 1955, Cooper and Rorabaugh, 1963). In the present report the interaction among a stream and two aquifers which are separated by an aquitard has been studied for varying river stages.

2.0 REVIEW

A review of aquifer recharge studies due to varying river stage has been presented here.

Cooper and Rorabaugh (1963) have studied flow into and out of the aquifer of finite length (shown in Fig.1) in response to changes in the stream stage. They solved one dimensional Boussinesq's equation under the conditions

$$H(x,0) = 0 \quad 0 \le x \le l$$

$$\frac{\partial H(l,t)}{\partial x} = 0 \quad t \ge 0$$

and

$$H(0,t) = \begin{cases} NH_o \; e^{-\partial t(1-\cos \omega t)} & 0 \le t \le \tau \\ 0 & t > \tau \end{cases}$$

where τ is the duration of the flood wave $(\omega = 2\Pi/\tau)$

$$\partial = \omega \cot \frac{\omega t_c}{2}$$

determines the asymmetry of the flood wave, t_c is the time of the flood crest, and

$$N = \frac{e^{\partial t_c}}{1 - \cos \omega t_c}$$

adjusts all curves of a given ∂ to peak at the same H_o. The solution has been carried out in two steps, one for $t \le \tau$ and another for $t \ge \tau$. For a semi-infinite aquifer, $(l = \infty)$, excited by a symmetrical flood wave $(\partial = 0)$, Cooper and Rorabough have also solved the Boussinesq equation satisfying the following initial and boundary conditions:

$$H(x,0) = 0 \qquad x \ge 0$$

$$\lim_{x \to \infty} H(x,t) = 0 \qquad t \ge 0$$

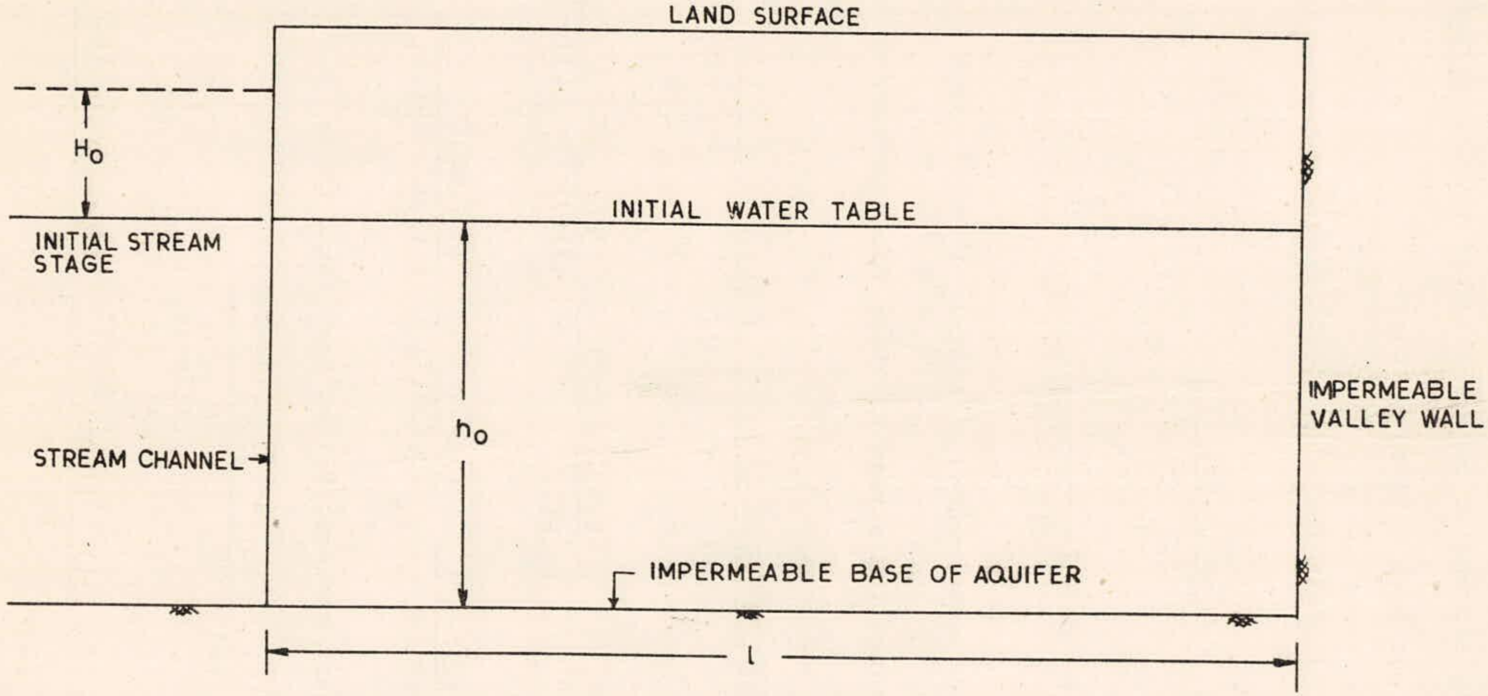

Fig.1. An aquifer of finite width bounded by a fully penetrating river

$$H(0,t) = \begin{cases} \frac{H_o}{2}(1 - \cos \omega t) & t \leq \tau \\ 0 & t > \tau \end{cases}$$

The following expression for groundwater level consequent to the passage of flood has been derived by them:

$$H_{t \leq \tau} = \frac{H_o}{2} \operatorname{erfc}\left[\frac{x}{2(D_o t)^{\frac{1}{2}}}\right] - \exp\left[-x\left(\frac{\omega}{2D_o}\right)^{\frac{1}{2}}\right]\cos\left[\omega t - x\left(\frac{\omega}{2D_o}\right)^{\frac{1}{2}}\right]$$

$$+ \frac{1}{\pi}\int_0^\infty \exp(-ut)\sin\left[x\left(\frac{u}{D_o}\right)^{\frac{1}{2}}\right]\frac{u}{u^2+\omega^2}\, du$$

and

$$H_{t>\tau} = \frac{H_o}{2}\left(\operatorname{erfc}\left[\frac{x}{2(D_o t)^{\frac{1}{2}}}\right] - \operatorname{erfc}\left\{\frac{x}{2[D_o(t-\tau)]^{\frac{1}{2}}}\right\}\right.$$

$$\left. + \frac{1}{\pi}\int_0^\infty \{\exp(-ut) - \exp[-u(t-\tau)]\}\sin\left[x\left(\frac{u}{D_o}\right)^{\frac{1}{2}}\right]\frac{u}{u^2+\omega^2}du\right)$$

The groundwater flow into the stream has been found to vary according to

$$Q_{t \leq \tau} = \frac{H_o}{2}(\omega\phi^2 D_o)^{\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left\{\frac{1}{(2\omega t)^{\frac{1}{2}}}\left[-\frac{(2\omega t)^2}{3} + \frac{(2\omega t)^4}{(3)(5)(7)} - \frac{(2\omega t)^6}{(3)(5)(7)(9)\ (11)} + \ldots\right]\right.$$

and

$$Q_{t>\tau} = \frac{H_o}{2}(\omega\,\phi^2 D_o)^{\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left\{\frac{1}{(2\omega t)^{\frac{1}{2}}}\left[-\frac{(2\omega t)^2}{3} + \frac{(2\omega t)^4}{(3)(5)(7)} - \frac{(2\omega t)^6}{(3)(5)(7)(9)(11)} + \ldots\right]\right.$$

$$\left. - \frac{1}{(2\omega t-4\pi)^{\frac{1}{2}}}\left[-\frac{(2\omega t-4\pi)^2}{3} + \frac{(2\omega t-4\pi)^4}{(3)(5)(7)} - \frac{(2\omega t-4\pi)^6}{(3)(5)(7)(9)(11)} + \ldots\right]\right\}$$

The expression for bank storage, $\forall$, has been found to be

$$\forall_{t \leq \tau} = \frac{H_o}{2}\left(\frac{\phi^2 D_o}{\omega}\right)^{\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2\omega t)^{(4n+1)/2}}{\prod_{m=1}^{2n}(2m+1)}$$

and

$$\forall_{t>\tau} = \frac{H_o}{2}\left(\frac{\phi^2 D_o}{\omega}\right)^{\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}[(2\omega t)^{(4n+1)/2} - (2\omega t-4\pi)^{(4n+1)/2}]}{\prod_{m=1}^{2n}(2m+1)}$$

For a step rise, σ, in a fully penetrating river the rise in water table height at a distance x from the bank and at time t after the onset of rise is given by:

$$\partial(x,t) = \sigma\,[1-\mathrm{Erf}(\frac{x}{\sqrt{4\alpha t}})] \qquad \alpha=T/\phi$$

Let, the change in water table height due to unit change in river stage be designated by $K_S(x,t)$ i.e.

$$K_S(x,t) = 1-\mathrm{Erf}(x/\sqrt{4\alpha t})$$

The flow from the river to the aquifer through the river banks consequent to a step rise in river stage varies according to the law

$$Q(t) = \frac{2T}{\sqrt{\pi\alpha t}}\,\sigma$$

Let the flow rate consequent to unit change in river stage be designated by $K_Q(t)$ i.e.

$$K_Q(t) = \frac{2T}{\sqrt{\pi\alpha t}}$$

For varying river stages the flow rate could be determined using convolution technique. Discretising the time parameter and making use of convolution technique Morel Seytoux (1975) has derived the expressions for flow rate and water table height as described below:

The flow rate for varying river stage is

$$Q(t) = \int_0^t K_Q(t-\tau)\,(\frac{d\sigma}{d\tau})\,d\tau$$

Discretising the time span by uniform time steps and assuming that $\frac{d\sigma}{d\tau}$ is constant within a time step but $\frac{d\sigma}{d\tau}$ varies from step to step, the flow rate during time step n has been found to be

$$Q(n) = \sum_{\gamma=1}^{n} [\sigma(\gamma)-\sigma(\gamma-1)]\,\partial(n-\gamma+1)$$

in which $\partial(m) = \int_{o}^{1} K_Q(m-\tau)d\tau$

$$= \int_{o}^{1} \frac{2T}{\sqrt{\pi\alpha(m-\tau)}}\, d\tau$$

$$= 4\sqrt{(\frac{\phi T}{\pi})}\,[\sqrt{m}-\sqrt{(m-1)}\,]$$

A digital model of multiaquifer system has been developed by Bredehoeft and Pinder (1970) assuming horizontal flow in the aquifers and vertical flow through the confining layers which separate the aquifers. These assumptions have reduced the mathematical problem to one of solving coupled two dimensional equation for each aquifer in the system. An iterative, alternating-direction-implicit scheme has been used to solve the system of simultaneous, finite difference equations which describe the response of the aquifer system to applied stresses. The quasi three-dimensional model has been developed to simulate a groundwater system having any number of aquifers. The aquifers may have confined or unconfined hydraulic conditions. The aquifers are assumed to be horizontal, nonhomogeneous and isotropic. The confining layers separating the aquifers are assumed to permit one-dimensional vertical flow with or without storage in the confining layers. The flow model assumed for a specific basin is shown in Fig.2

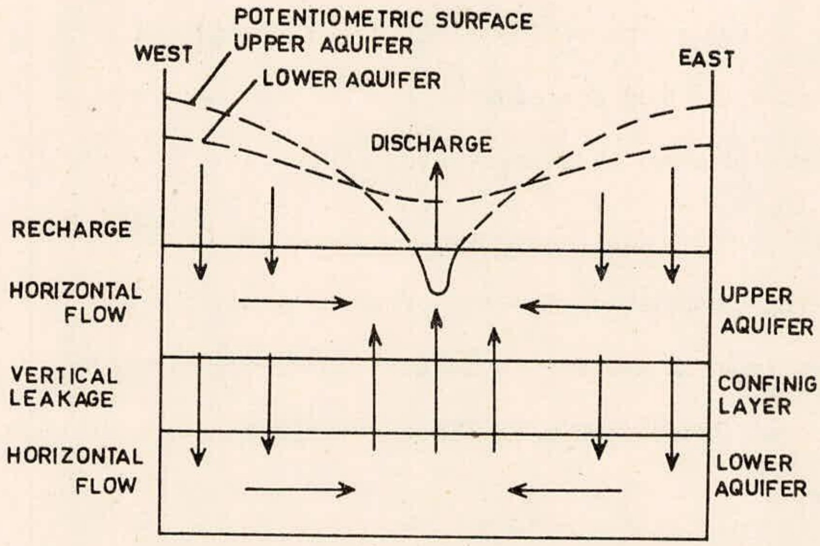

Fig.2. Flow model of the aquifer system for a basin

PROBLEM DEFINITION AND METHODOLOGY

3.1 Statement of the Problem

A schematic section of a partially penetrating river in a two layered multiaquifer system is shown in Fig.3 . The two aquifers are separated by an aquitard. Each aquifer is homogeneous, isotropic and infinte in areal extent. The aquitard's thickness and vertical permeability vary with space. The river and the two aquifers are initially at rest condition. Due to passage of a flood, the river stages changes with time. The changes are identical over a long reach of the river. It is required to find the recharge from the river to the top aquifer and the exchange of flow between the two aquifers through the intervening aquitard.

3.2 Methodology

The following assumptions are made for the analysis:

1) The flow in each aquifer is in horizontal direction and one dimensional Boussinesq's equation governs the flow in each aquifer.

2) The flow in the aquitard is vertical and there is no release of water from the aquitard's storage.

3) The variation in aquitard's thickness over a large length parallel to the river is negligible. However, the variation in aquitard's thickness along a line normal to the river axis has been taken care of.

4) The aquifers and the aquitard are divided into identical strips with varying width.

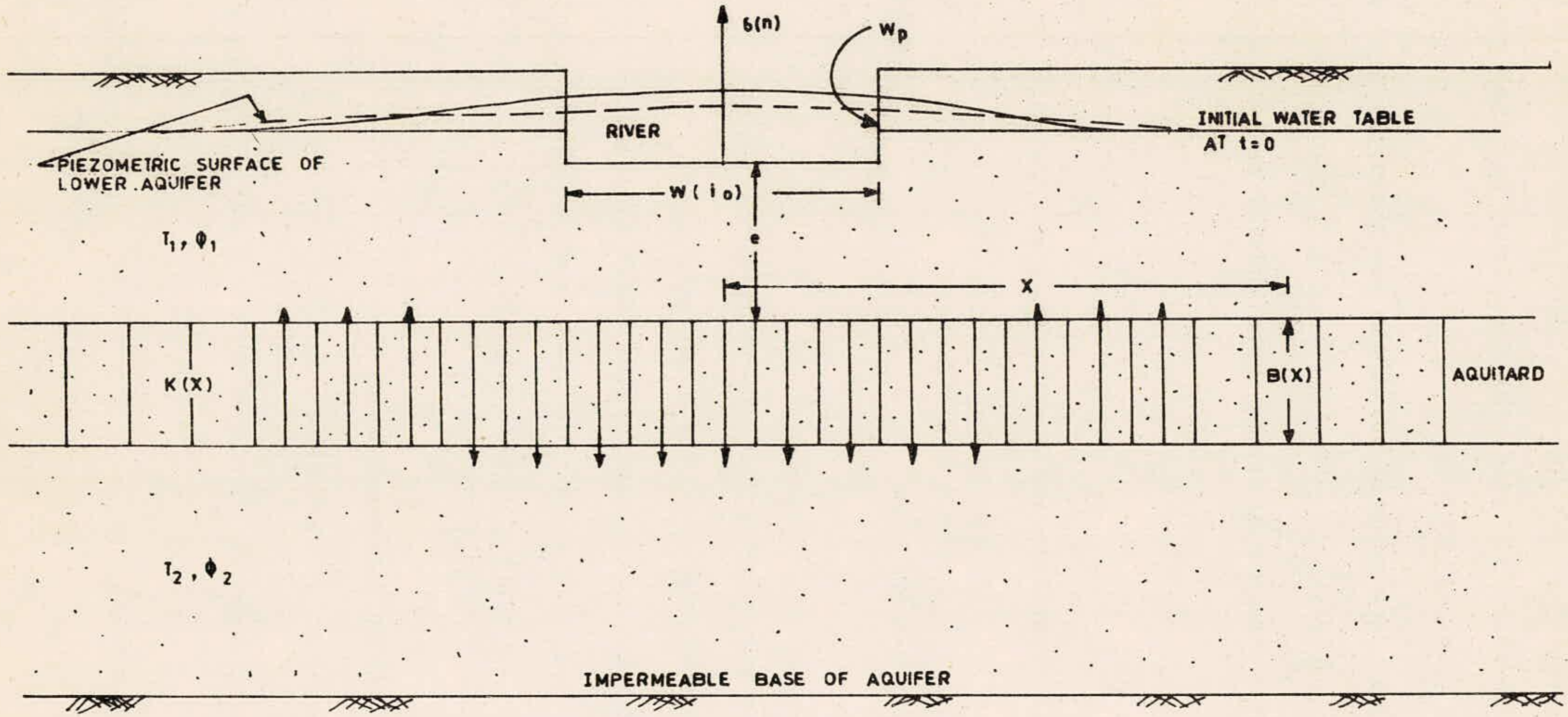

Fig.3. A partially penetrating river and two aquifers seperated by an aquitard

5) At large distance from the river the difference in piezometric surfaces is negligible and therefore the exchange of flow between the aquifers at large distances from the river has been assumed to be negligible.

6) The exchange of flow between the river and the upper aquifer is linearly proportional to the difference in the potentials at the river boundary and in the upper aquifer below the river bed.

ANALYSIS

The differential equation which governs the flow in the first aquifer is

$$T_1 \frac{\partial^2 S_1}{\partial x^2} = \phi \frac{\partial S_1}{\partial t} - w(x,t) \qquad \ldots(1)$$

in which

S_1 = the water table rise in the first aquifer,

T_1 = transmissivity of the first aquifer,

ϕ_1 = storage coefficient of the first aquifer,

$w(x,t)$ = recharge rate per unit area which is equal to

$$K(x) \frac{S_1(x,t) - S_2(x,t)}{B(x)}$$

$K(x)$ = aquitard's permeability, and

$B(x)$ = aquitard's thickness.

The differential equation that governs the flow in the lower aquifer is

$$T_2 \frac{\partial^2 S_2}{\partial x^2} = \phi_2 \frac{\partial S_2}{\partial t} + w(x,t) \qquad \ldots(2)$$

in which

T_2 = transmissivity of the lower aquifer

ϕ_2 = storage coefficient of the lower aquifer, and

S_2 = rise in piezometric surface in the lower aquifer.

Solutions to equations (1) and (2) are required to satisfy the following initial and boundary conditions:

If the aquifers and the river were initially at rest, the initial condition to be satisfied is:

$$S_1(x,0) = 0 \text{ and } S_2(x,0) = 0 .$$

The boundary conditions to be satisfied are:

$$S_1(\pm\infty,t) = 0 \text{ and } S_2(\pm\infty,t) = 0$$

At the river and upper aquifer interface recharge from the river to the upper aquifer takes place in a manner similar to that from an overlying bed source to an underlying aquifer through an intervening aquitard. The river resistance and the aquitard resistance are analogous. If the river fully penetrates the upper aquifer, then $S_1(r,n)=\sigma(n)$ i.e. the rise in water table height at the river node in the upper aquifer is equal to rise in river stage, $\sigma(n)$. For a partially penetrating river $S_1(r,n)\neq\sigma(n)$ and $S_1(r,n)$ is to be determined as a part of the solution. The unknown recharge which has been assumed to be linearly proportional to the potential difference, $[\sigma(n)-S_1(r,n)]$, is to be incorporated at the river node.

The solution to the problem has been obtained following the principle of superposition.

The upper and the lower aquifer have been divided identically into a number of strips of varying width as shown in Fig.3. Had the recharge taken place at unit rate per unit area through the i^{th} strip alone, the rise in piezometric surface in the lower aquifer would have been as given below:

$$S_2(x,t) = F(x,T_2,\ \phi_2,W(i),t) - \frac{[x^2+0.25W^2(i)]}{2T_2}$$

$$\text{for } |x| < \frac{W(i)}{2}$$

$$= F(x,T_2,\phi_2,W(i),t) - \frac{\sqrt{x^2}\,W(i)}{2T_2}$$

$$\text{for } |x| > \frac{W(i)}{2} \qquad \ldots(3)$$

in which

$$F(x,T_2,\phi_2,W(i),t) = \frac{t}{2\phi_2}[\mathrm{erf}\{\frac{x+0.5W(i)}{\sqrt{4\alpha_2 t}}\} - \mathrm{erf}\{\frac{x-0.5W(i)}{\sqrt{4\alpha_2 t}}\}]$$

$$+ \frac{1}{4T_2}[\{x+0.5W(i)\}^2\ \mathrm{erf}\{\frac{x+0.5W(i)}{\sqrt{4\alpha_2 t}}\}$$

$$- \{x-0.5W(i)\}^2\ \mathrm{erf}\{\frac{x-0.5W(i)}{\sqrt{4\alpha_2 t}}\}]$$

$$+ \frac{\sqrt{\alpha_2 t}}{2T_2\sqrt{\pi}}[\{x+0.5W(i)\}\ \exp\{-\frac{(x+0.5W(i))^2}{4\alpha_2 t}\}$$

$$- \{x-0.5W(i)\}\ \exp\{-\frac{(x-0.5W(i))^2}{4\alpha_2 t}\}] \qquad \ldots(4)$$

α_2 = T_2/ϕ_2

$W(i)$ = width of the i^{th} strip

x = distance measured from the centre of the i^{th} strip to the point of observation

Had the recharge taken place for the first unit time period through the i^{th} strip alone, the rise in piezometric surface at the end of n^{th} unit time step would have been as stated below:

$$S_2(x,1) = F(x,T_2,\phi_2,W(i),1) - \frac{\sqrt{x^2}W(i)}{2T}$$

$$\text{for } |x| > \frac{W(i)}{2}$$

$$= F(x,T_2,\phi_2,W(i),1) - \frac{1}{2T_2}[x^2+0.25W^2(i)]$$

$$\text{for } |x| < \frac{W(i)}{2} \qquad \ldots(5)$$

$$S_2(x,n) = F(x,T_2,\phi_2,W(i),n) - F(x,T_2,\phi_2,W(i),n-1)$$

$$\text{for } n > 2 \text{ and for all } x \qquad \ldots(6)$$

Let the rise in piezometric surface at the j^{th} strip at the end of n^{th} unit time step due to recharge taken place during the first unit time period through the i^{th} strip be designated as $\partial_2(i,j,n)$. Hence,

$$\partial_2(i,j,n) = F(|x(i)-x(j)|,\ T_2,\phi_2,W(i),n)$$

$$-F(|x(i)-x(j)|,\ T_2,\phi_2,W(i),n-1)$$

$$\text{for } n>2 \qquad \ldots(7)$$

$$\partial_2(i,j,1) = F(|x(i)-x(j)|,\ T_2,\phi_2,W(i),1)$$

$$- \frac{\sqrt{(x(i)-x(j))^2}}{2T_2} W(i) \qquad \ldots(8)$$

$$\partial_2(i,i,1) = F(0,T_2,\phi_2,W(i),1) - \frac{1}{2T_2}(0.25W^2(i)) \qquad \ldots(9)$$

Dividing the time span into discrete time steps, and assuming that, the recharge per unit area is constant within each time step but varies from step to step, the rise in piezometric surface under j^{th} strip due to time variant recharge through the i^{th} strip alone can be written as

$$S_2(j,n) = \sum_{\gamma=1}^{n} q(i,\gamma)\,\partial_2(i,j,n-\gamma+1) \qquad \text{...(10)}$$

in which $q(i,\gamma)$ is the recharge rate per unit area per unit time which is taking place through the i^{th} strip during time step γ. When recharge takes place through all the strips, the resultant rise in piezometric surface can be written as

$$S_2(j,n) = \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,j,n-\gamma+1) \qquad \text{...(11)}$$

$q(\rho,\gamma)$ are unknown priori. The procedure for determining $q(\rho,\gamma)$, $\gamma=1,2,n$ and $\rho=1,2,\ldots R$ is described below. The recharge which takes place from the upper aquifer to the lower aquifer through i^{th} strip of the aquitard can be expressed as:

$$q(i,n) = \frac{K(i)}{B(i)}[S_1(i,n)-S_2(i,n)] \qquad \text{...(12)}$$

Let the i_o^{th} strip comprise the bottom width of the river. The rise in water table height in the upper aquifer, $S_1(i,n)$ is given by

$$S_1(i,n) = \sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)}\,\partial_1(i_0,i,n-\gamma+1) - \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_1(\rho,i,n-\gamma+1) \qquad \text{...(13)}$$

in which

$Q(\gamma)$ = river recharge,

$W(i_0)$ = width of river,

$$\partial_1(\rho,i,n) = F(\,|x(\rho)-x(i)|,\ T_1,\phi_1,W(i),n) - F(|x(\rho)-x(i)|,\ T_1,\phi_1,W(i),n-1) \quad \text{for } n>2, \qquad \text{...(14)}$$

$$\partial_1(\rho,i,1) = F(|x(\rho)-x(i)|,T_1,\phi_1,W(i),1) - \frac{\sqrt{(x(\rho)-x(i))^2}W(i)}{2T_1}, \text{ and} \qquad \ldots(15)$$

$$\partial_1(\rho,\rho,1) = F(o,T_1,\phi_1,W(\rho),1) - \frac{1}{2T_1}(0.25W^2(\rho)) \qquad \ldots(16)$$

The first summation in right side of equation (13) represents the rise in water table height due to river recharge taken place upto time step n. The 2nd summation is decline in water table height due to recharge from all strips taken place upto time step n.

Similarly the rise in piezometric surface in the lower aquifer is given by

$$S_2(i,n) = \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,i,n-\gamma+1) \qquad \ldots(17)$$

Substituting $S_1(i,n)$ and $S_2(i,n)$ in equation (12),

$$q(i,n) = \frac{K(i)}{B(i)}\Big[\sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)}\,\partial_1(i_0,i,n-\gamma+1) - \sum_{\rho=1}^{R}\sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_1(\rho,i,n-\gamma+1) - \sum_{\rho=1}^{R}\sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,i,n-\gamma+1)\Big] \qquad \ldots(18)$$

Splitting the temporal summation into two parts and rearranging

$$q(i,n)\frac{B(i)}{K(i)} + \sum_{\rho=1}^{R} q(\rho,n)[\partial_1(\rho,i,1)+\partial_2(\rho,i,1)] - \frac{Q(n)}{W(i_0)}\,\partial_1(i_0,i,1)$$

$$= \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)}\,\partial_1(i_0,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\, \partial_1(\rho,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\, \partial_2(\rho,i,n-\gamma+1) \qquad \ldots(19)$$

or

$$q(i,n)\left[\frac{B(i)}{K(i)} + \partial_1(i,i,1) + \partial_2(i,i,1)\right]$$

$$+ \sum_{\substack{\rho=1 \\ \rho\neq i}}^{R} q(\rho,n)\left[\partial_1(\rho,i,1) + \partial_2(\rho,i,1)\right]$$

$$- \frac{Q(n)}{W(i_0)}\, \partial_1(i_0,i,1)$$

$$= \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)}\, \partial_1(i_0,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\left[\partial_1(\rho,i,n-\gamma+1) + \partial_2(\rho,i,n-\gamma+1)\right] \ldots(20)$$

'R' number of equations could be written for 'R' number of recharge strips. However, the unknowns at any time 'n' are the 'R' number of unknown $q_i(n)$ and the river recharge $Q(n)$. One more equation could be written as:

$$Q(n) = \Gamma_r[\sigma(n) - S_1(i_0,n)] \qquad \ldots(21)$$

in which

Γ_r = reach transmissivity,

$\sigma(n)$ = river stage during time step n, and

$S_1(i_0,n)$ = rise in piezometric surface under the river bed or in the vicinity in case of a fully penetrating river during time step n.

Substituting for $S_1(i_0,n)$

$$\frac{Q(n)}{\Gamma_r} = \sigma(n) - \left[\sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1) - \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_1(\rho,i_0,n-\gamma+1) \right] \qquad \ldots(22)$$

Splitting the temporal summation into two parts and rearranging

$$Q(n)\left[\frac{1}{\Gamma_r} + \frac{\partial_1(i_0,i_0,1)}{W(i_0)}\right] - \sum_{\rho=1}^{R} q(\rho,n)\,\partial_1(\rho,i_0,1) = \sigma(n) - \left[\sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1) - \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\,\partial_1(\rho,i_0,n-\gamma+1) \right]$$

Let R be equal to $2i_0-1$.

The set of (R+1) equations represented by equation (19) and (22) can be written in the following matrix form

$$[A].[B] = [C] \qquad \ldots(23)$$

in which

$$[C] = \begin{bmatrix} \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,1,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\{\partial_1(\rho,1,n-\gamma+1)+\partial_2(\rho,1,n-\gamma+1)\} \\ \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,2,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\{\partial_1(\rho,2,n-\gamma+1)+\partial_2(\rho,2,n-\gamma+1)\} \\ \sum_{\gamma=1}^{n-1} Q(\gamma)\,\partial_1(i_0,2i_0-1,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\,\partial_1(\rho,2i_0-1,n-\gamma+1)+\partial_2(\rho,2i_0-1,n-\gamma+1) \\ \sigma(n) - \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1) + \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\partial_1(\rho,i_0,n-\gamma+1) \end{bmatrix}$$

Hence $[B] = [A]^{-1}[C]$

For the first time period

$[C] = [0,0,\ldots\ldots\ldots\ldots,\sigma(1)]^T$

Thus $q(i,n)$, $i=1,2,\ldots\ldots 2i_0-1$ and $Q(n)$ can be solved in succession starting from time step 1.

$$[A] = \begin{bmatrix} \frac{B(1)}{K(1)}+\partial_1(1,1,1)+\partial_2(1,1,1) & \partial_1(2,1,1)+\partial_2(2,1,1) & \cdots & \partial_1(2i_0-1,1,1)+\partial_2(2i_0-1,1,) & -\frac{\partial_1(i_0,1,1)}{W(i_0)} \\ \partial_1(1,2,1)+\partial_2(1,2,1) & \frac{B(2)}{K(2)}+\partial_1(2,2,1)+\partial_2(2,2,1) & \cdots & \partial_1(2i_0-1,2,1)+\partial_2(2i_0-1,2,1) & -\frac{\partial_1(i_0,2,1)}{W(i_0)} \\ \vdots & \vdots & & & \\ \partial_1(1,i_0,1)+\partial_2(i,i_0,1) & \partial_1(2,i_0,1)+\partial_2(2,i_0,1) & \cdots & \partial_1(2i_0-1,i_0,1)+\partial_2(2i_0-1,i_0,1) & -\frac{\partial_1(i_0,i_0,1)}{W(i_0)} \\ \vdots & \vdots & & & \\ \partial_1(1,2i_0-1,1)+\partial_2(1,2i_0-1,1) & \partial_1(2,2i_0-1,1)+\partial_2(2,2i_0-1,1) & \cdot & \frac{B(2i_0-1)}{K(2i_0-1)}+\partial_1(2i_0-1,2i_0-1,1) +\partial_2(2i_0-1,2i_0-1,1) & -\frac{\partial_1(i_0,2i_0-1,1)}{W(i_0)} \\ -\partial_1(1,i_0,1) & -\partial_1(2,i_0,1) & \cdots & -\partial_1(2i_0-1,1_0,1) & \frac{1}{\Gamma_r}+\frac{\partial_1(i_0,i_0,1)}{W(i_0)} \end{bmatrix}$$

$$[B] = [q(1,n),\ q(2,n)\ldots\ldots\ldots q(i_0,n),\ldots\ldots\ldots q(2i_0-1,n),\ Q(n)]^T$$

4.0 RESULTS AND DISCUSSIONS

Assuming that a finite number of strips on both sides of the river takes part in the stream aquifer interaction, the response function coefficients are determined for known set of aquifer parameters T_1 ϕ_1, T_2 and ϕ_2 and for an assumed integer value of i_0 making use of eqns. (7),(8),(9) and (14),(15),(16). The appropriate width of the strips and their number, $(2i_0-1)$, could be ascertained only after assessing the magnitude of recharge at the farthest strip from the river occurring towards the end of excitation. With known $B(i)$,$K(i)$,$\partial_1(i,j,l)$, and $\partial_2(i,j,l)$, the element of matrix [A] are calculated and inverse of the matrix [A] is found. For known value of the river stage during the first time step the recharge rates at each recharging strip and the recharge from the river to the upper aquifer are determined for the first time step. Evaluating the element of the matrix [C] in succession, the recharge occurring through each of the $(2i_0-1)$ strips and the recharge from the river to the upper aquifer are determined in succession for other time step starting from time step 1.

The variations of $Q(n)$, and $W(i_0)q(i_0,n)$ with time presented in Fig.4 pertain to a unit step rise in the river stage. The term $W(i_0)q(i_0,n)$ represents the recharge taking place from the upper aquifer to the lower aquifer through a strip whose width is equal to that of the river. The aquitard has been assumed to be homogeneous. It could be seen that the recharge from the river to the aquifer decreases with time in case of a step rise in river stage. The aquitard resistance controls the recharge from the upper aquifer to the lower one. When the resistance increases by 10 times, from 100 day to 1000 day, the recharge rate during the 24th day

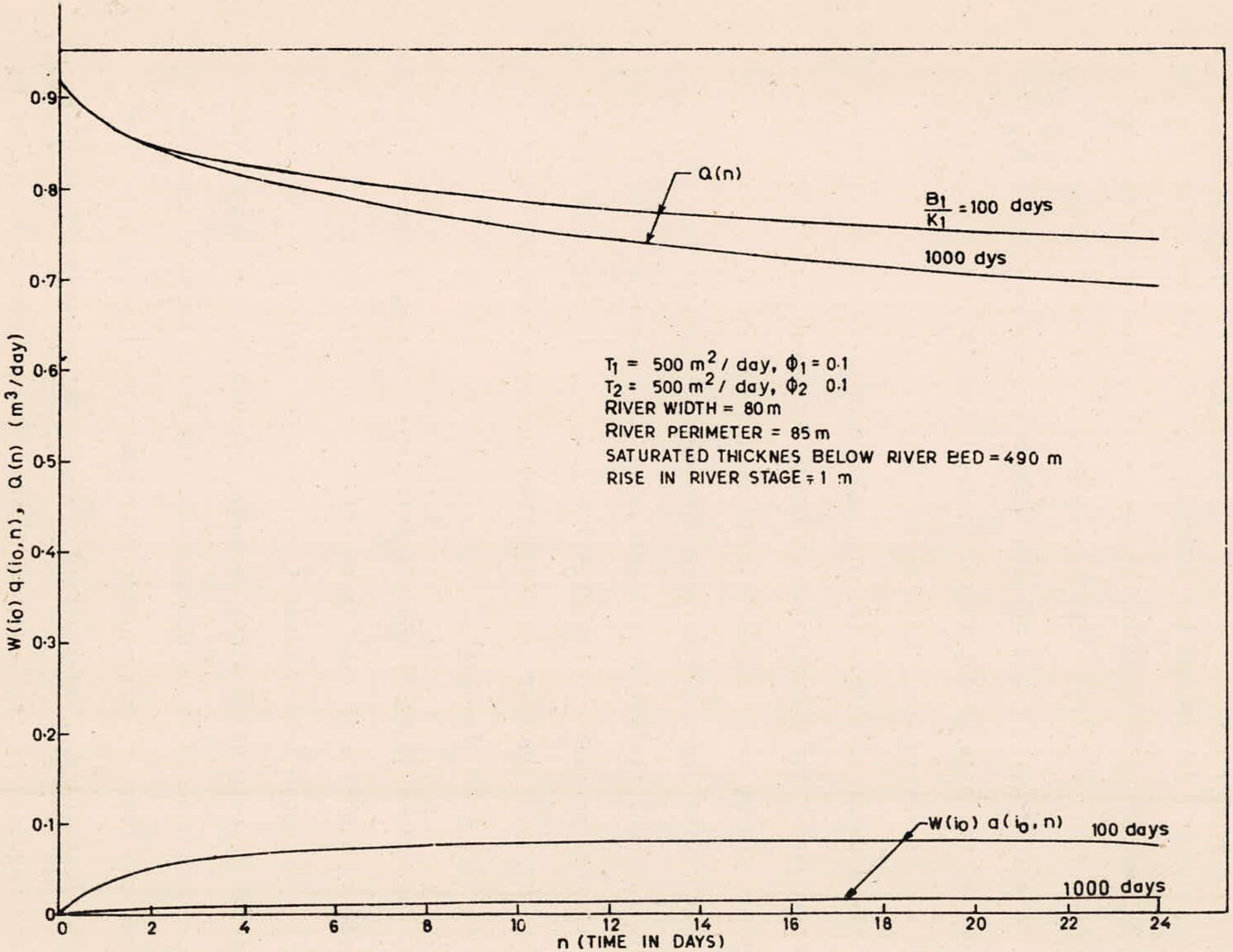

Fig.4. Variation of recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer through part of aquitard below river bed with time due to unit step rise in river stage

reduces by 77%. The increase in aquitard resistance also leads to a decrease in the river recharge. For assumed aquifer parameters, if the resistance is increased from 100 day to 1000 day the river recharge during the 24th day is reduced by 7.2%. The variations of recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer through the aquitard below the river bed, with time due to varying river stages are shown in Fig.5. The upper and lower aquifers are assumed to be identical having transmissivity of 500 m^2/day, and storage coefficient of 0.1. The aquitard resistance has been assumed to be 100 day. Results have been presented for two values of river width. As seen from figure reduction in river width from 200m to 80m does not reduce the river recharge appreciably. The cumulative recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer for the multiaquifer system comprising identical aquifer are shown in Fig.6. It could be seen that for the river with 200m width, at the end of 24 days, 45% of the river recharge has entered into the lower aquifer through the aquitard having a resistance of 100 day.

The river recharge to aquifer and the recharge from the upper aquifer to the lower one are governed by the aquifer parameters. The results for comulative recharge presented in Figs.7 and 8 are for ϕ_2= 0.1 and 0.001 respectively. It is seen that all other parameters remaining same if ϕ_2 decreases, the cumulative recharge to the second aquifer decreases. If ϕ_2 decreases from 0.1 to 0.001, the cumulative recharge at the end of 24 days decreases by 55%. In reality the aquitard thickness and conductivity vary from place to place. The recharge that would occur through a window in the aquitard has been presented in Fig.9. The aquitard resistance is assumed to 1000 day except for the part under the river bed. The aquitard under

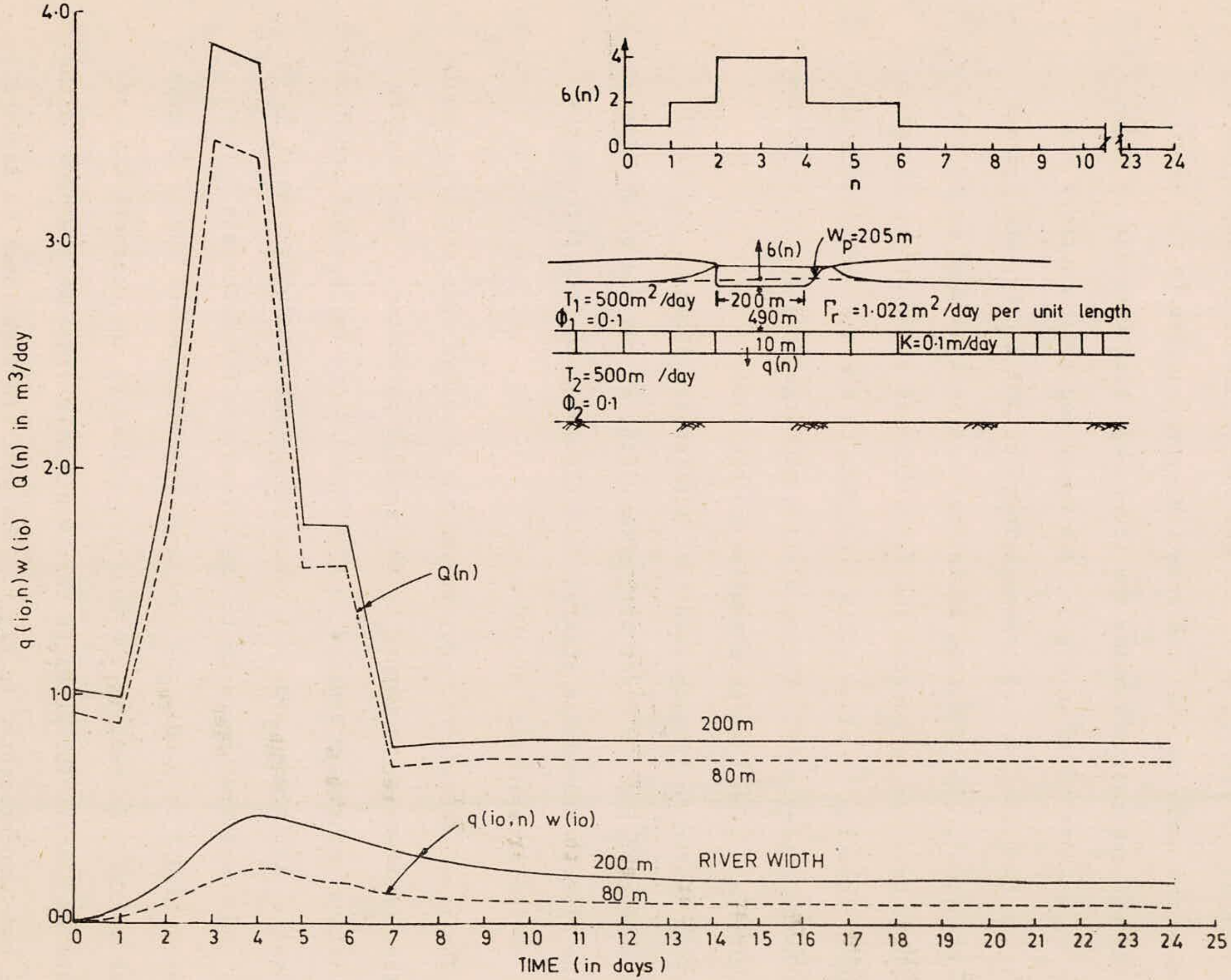

Fig.5. Variation of recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer through part of aquitard below river bed with time in response to a typical varying river stages

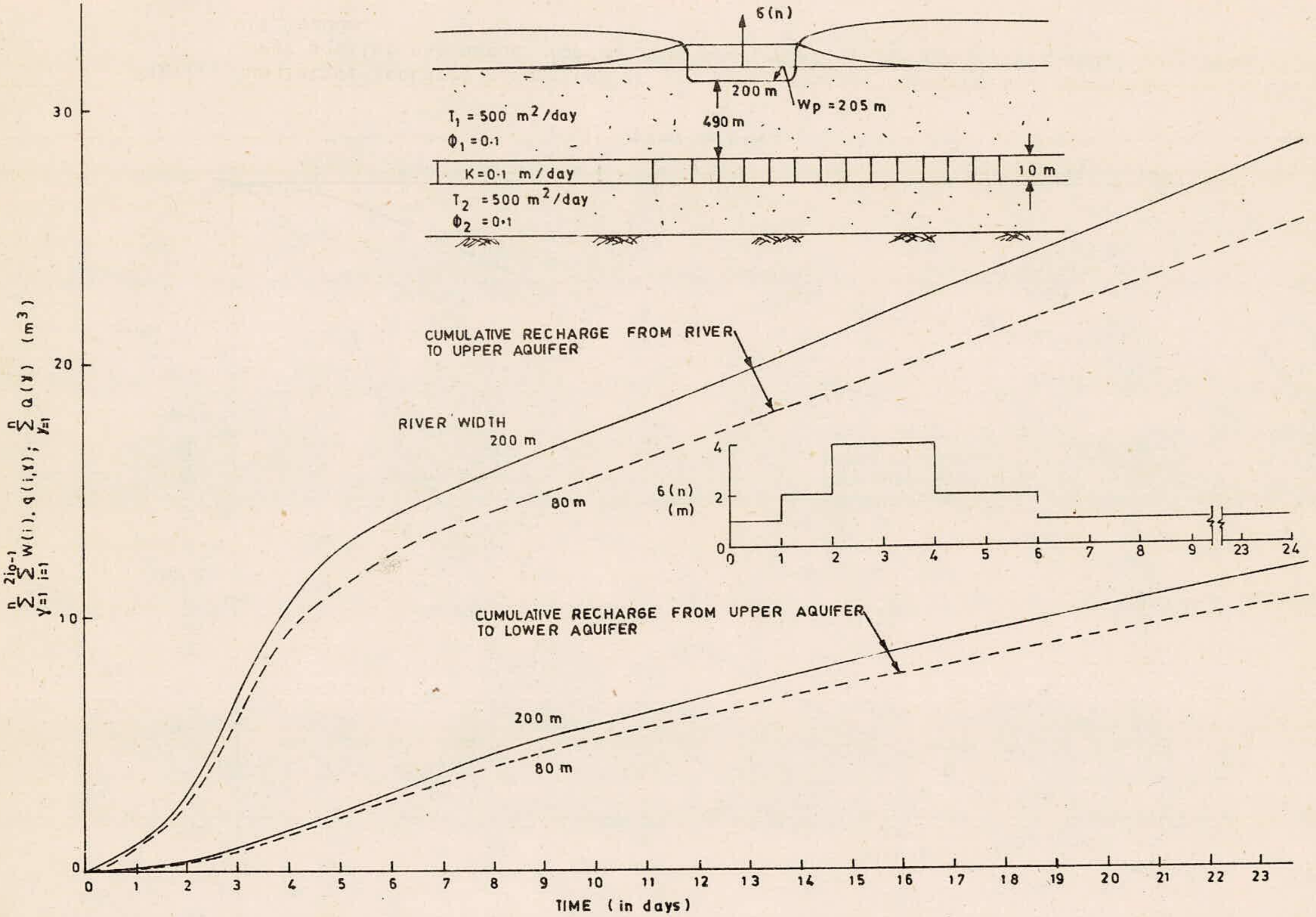

Fig.6. Cumulative recharge from river to the upper aquifer and from the upper aquifer to the lower aquifer evaluated for T_1=500m^2/day, ϕ_1=0.1, T_2=500m^2/day, ϕ_2=0.1, aquifard resistance=100 day in response to a typical varying river stages

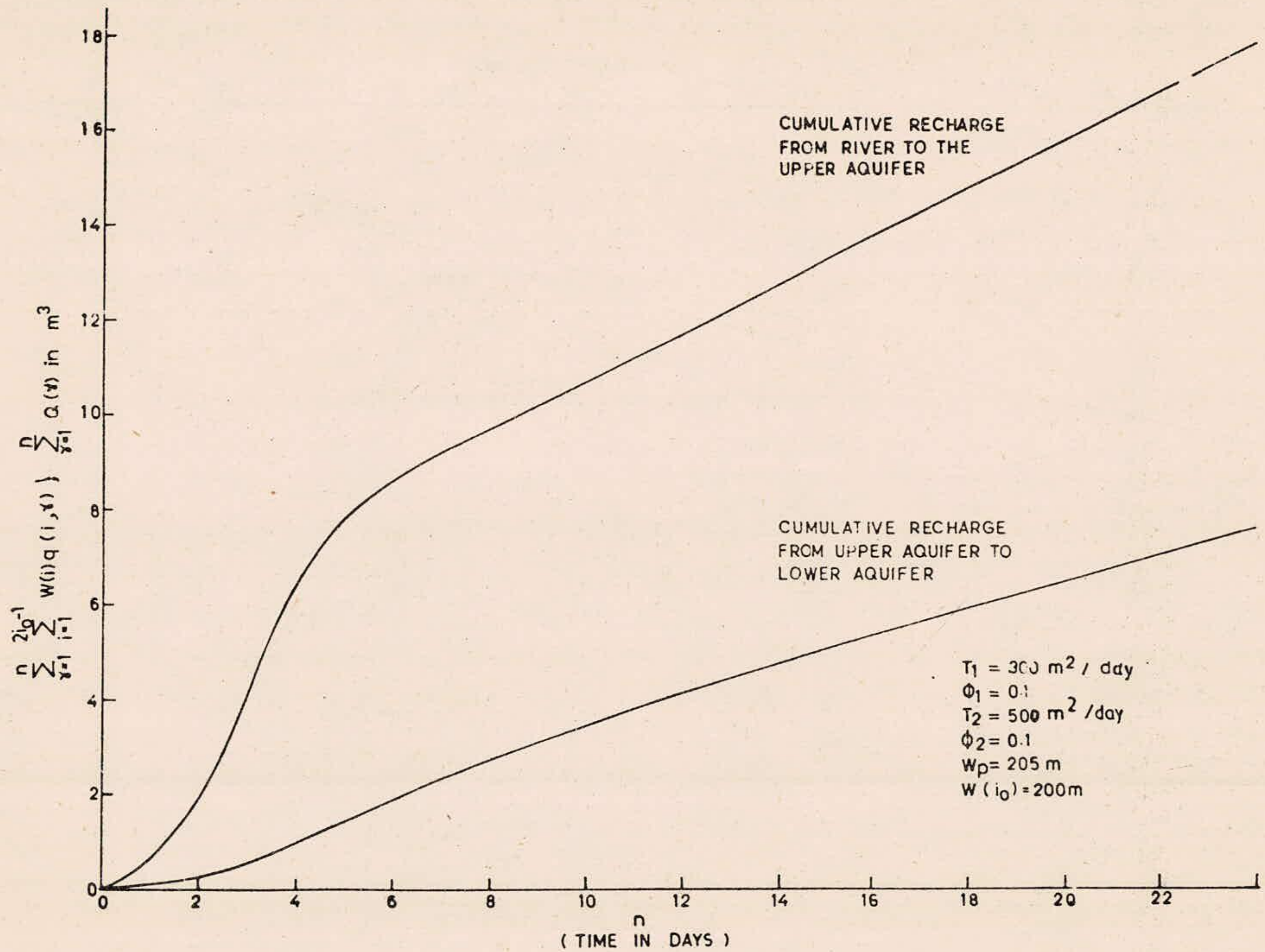

Fig.7. Cumulative recharge from river to the upper aquifer and from the upper aquifer to the lower aquifer evaluated for T_1=300m^2/day, ϕ_1=0.1, T_2=500m^2/day, ϕ_2=0.1, w_p=205m, $W(i_0)$=200m

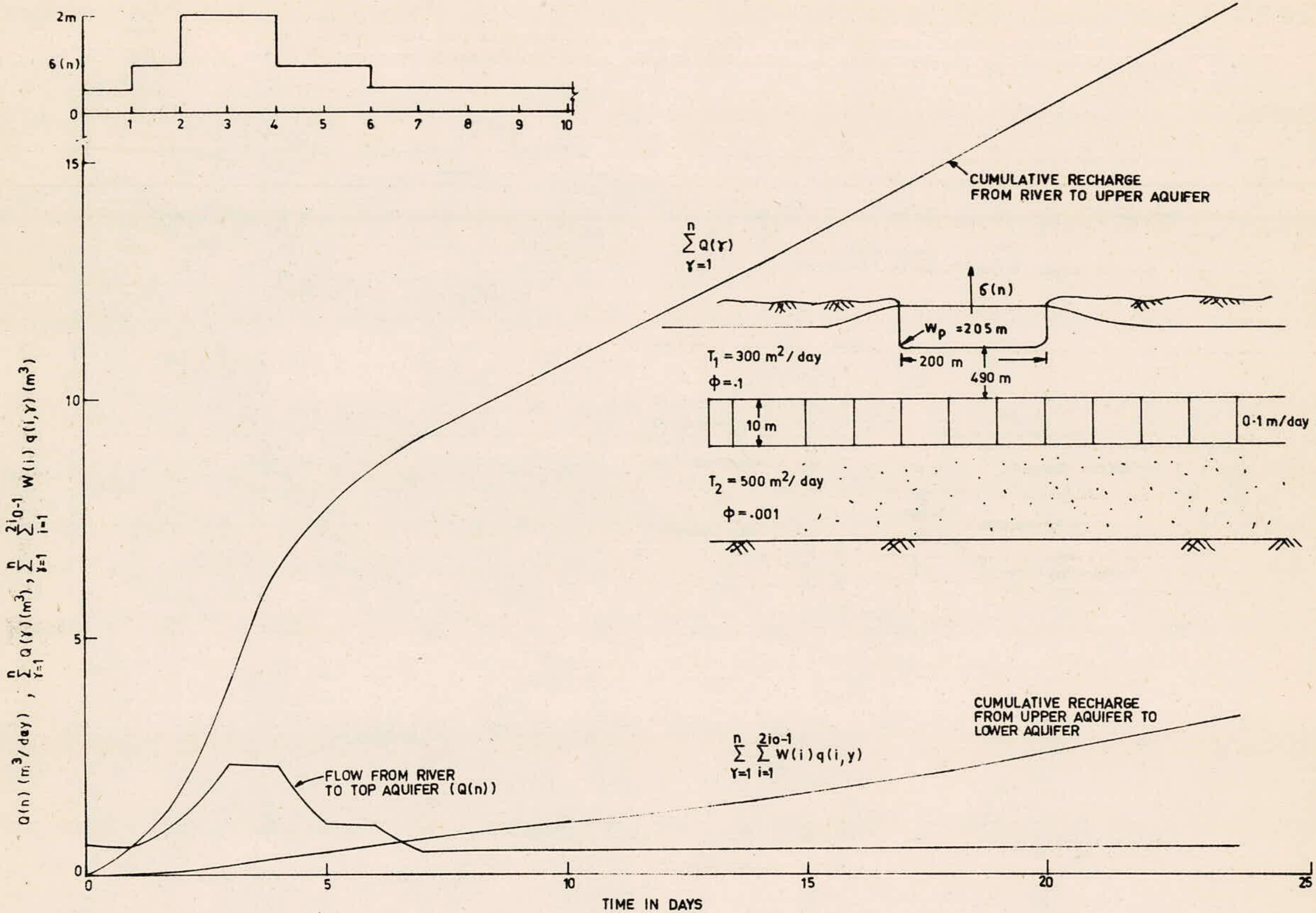

Fig.8. Cumulative recharge from river to the upper aquifer and from the upper aquifer to the lower aquifer evaluated for T_1=300m²/day, ϕ_1=0.1, T_2=500m²/day, ϕ_2=0.001, w_p=205m, $W(i_0)$=200m

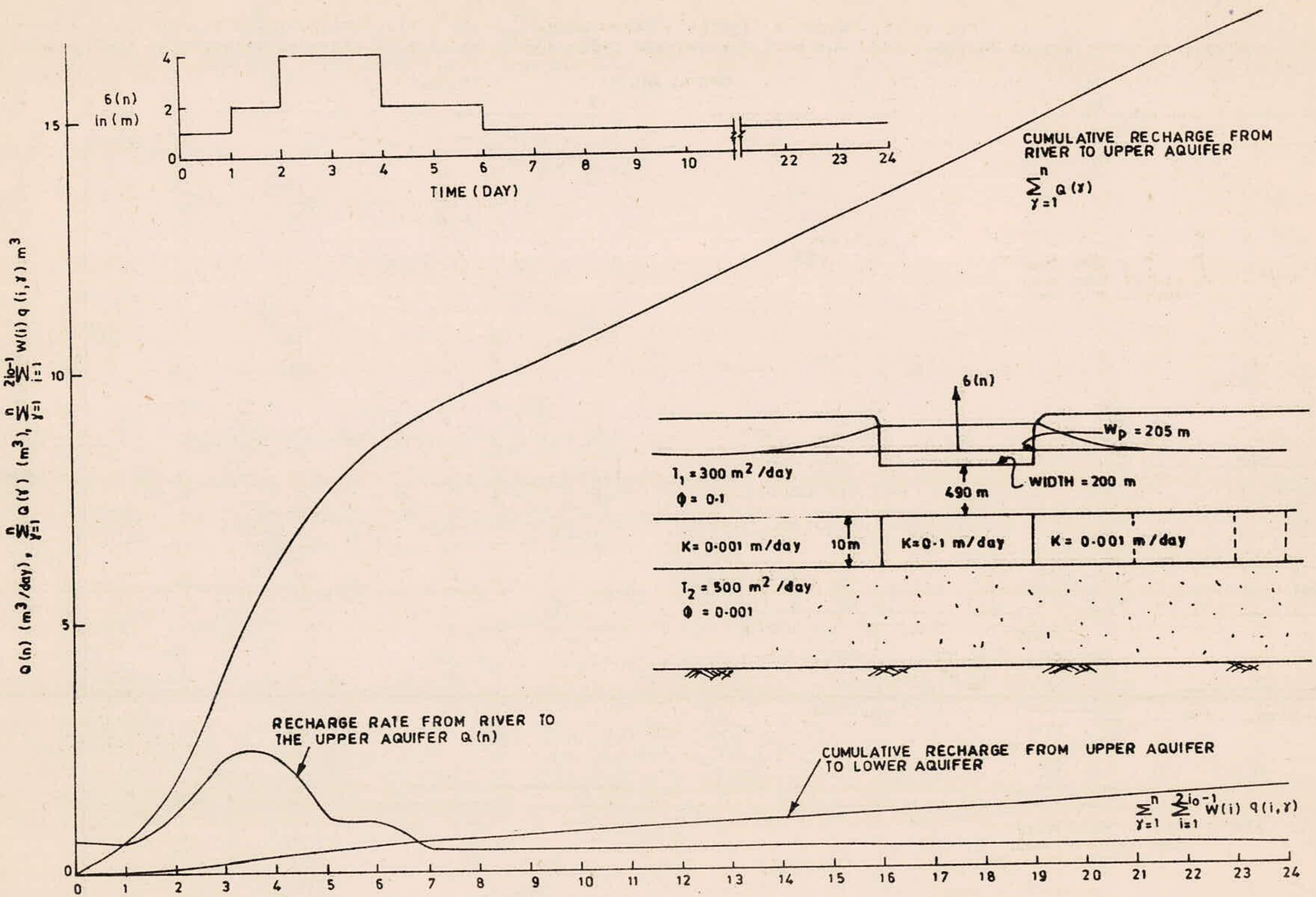

Fig.9. Cumulative recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer through an window below river bed evaluated for T_1=300m^2/day, ϕ_1=0.1, T_2=500m^2/day, ϕ_2=0.001, w_p=205m and $W(i_0)$=200m

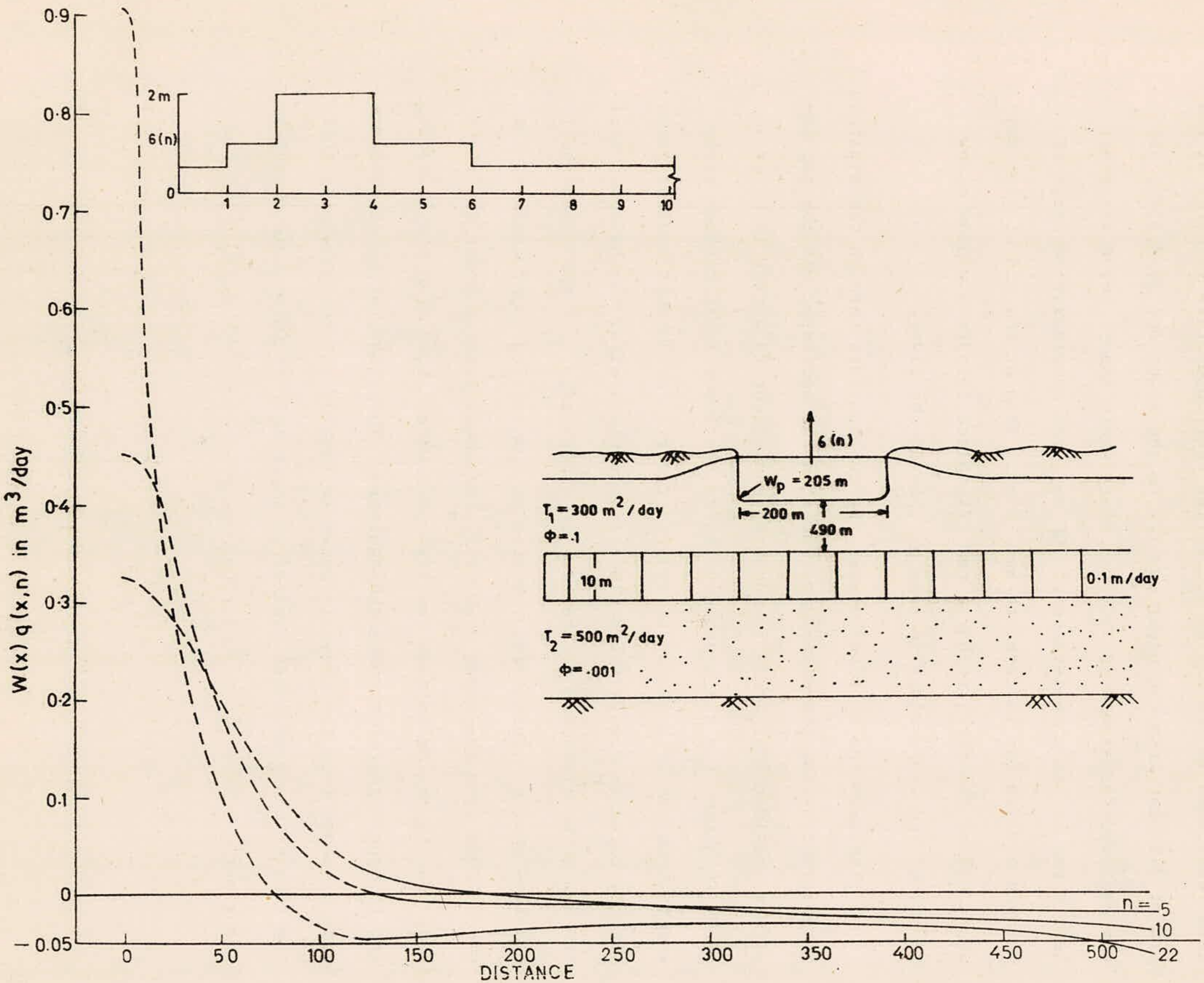

Fig.10. Distribution of recharge from upper aquifer to lower aquifer evaluated for T_1=300m^2/day, ϕ_1=0.1, T_2=500m^2/day, ϕ_2=0.001, river width=200m, w_p=205m

the river bed is assumed to have a resistance of 100 day. If the aquitard resistance is 100 day at all strips, the cumulative recharge at the end of 24th day from the upper aquifer to the lower one is $3.35 m^3$. If the aquitard resistance is 1000 day every where except below the river bed, where it has a resistance of 100 day, the cumulative recharge at the end of 24th day is found to be $1.55 m^3$. Thus at the end of 24th day about 43% of the total recharge can take place through a window below the river bed whose width is same as that of the river.

The exchange of flow between the two aquifers through the aquitard is governed by the piezometric surfaces in the aquifers. If the two aquifers are identical it is found that, consequent to a rise in river stage recharge from the upper aquifer to the lower aquifer takes place which decreases with distance from the river. For unequal aquifer parameters, recharge always takes place from upper aquifer to the lower aquifer below the river bed due to rise in river stage. If the lower aquifer has less storage coefficient compared to that of the upper aquifer, the water flows from the upper aquifer to the lower aquifer under the river bed. But in region outside the river bed water flows from the lower aquifer to upper aquifer through the aquitard. The distribution of recharge with distance from the river is shown in Fig.10. It could be seen that the flow enters from lower aquifer to upper aquifer in regions away from the river.

5.0 CONCLUSION

A mathematical model has been developed to study recharge from a river to a multiaquifer system for varying river stages. The analytical solution is tractable for numerical calculation. The solution has been obtained by discretising the time parameter and using unit response function coefficients.

It is found that the storage coefficient of the lower aquifer controls the recharge from the upper aquifer, besides the aquitard resistance. If two identical layers are separated by an aquitard with resistance of 100 day 45% of the river recharge enters to the lower aquifer. A decrease in river width from 200m to 80m does not change the recharge rate appreciably.

A window in the aquitard located under the river with width equal to that of the river can cause 43% of the recharge that would take place through a window of very large width.

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