MODELLING SOIL MOISTURE PROCESSES

C.P.Kumar*

1.0 Introduction

The vertical movement of soil moisture in the liquid phase between the surface and the water table can be subdivided into the following three categories according to predominant forces involved.

(i) Infiltration and exfiltration:

Alternate wetting and drying of soil surface during consecutive storm and interstorm periods will cause a penetration of the medium by an unsteady wave like diffusion of liquid soil moisture into the soil during wet surface (storm) periods under the complementary effects of capillarity and gravity and out of the soil during dry surface (interstorm) periods when capillarity opposes gravity. With increasing depth of penetration, diffusion reduces the soil moisture gradients and thus reduces the effect of capillarity until moisture movement becomes dominated by gravity. The depth at which surface induced capillary forces becomes negligible determines the penetration depth of the surface process and is used to define the thickness of the zone of soil moisture. The presence of transpiring vegetation adds another mechanism for moisture extraction distributed over a depth which is related to root structure.

(ii) Percolation:

Liquid soil moisture moves out of the bottom of the zone of soil moisture and percolates downward under the domination of gravity forces until it encounters the increasing soil moisture gradients lying above the water table. At some depth upward capillary forces will be prominent defining the bottom of this intermediate zone.

---

* Scientist, National Institute of Hydrology, Roorkee, U.P., India.

(iii) Capillary rise:

Between the water table and the intermediate zone there is a capillary fringe in which gravity and capillarity again jointly govern the liquid soil moisture movement.

2.0 The Mathematical Model

The continuous variation of soil moisture with time and depth in homogeneous bare soil can be known by solving Richard's equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{\partial K(\theta)}{\partial z} \qquad \ldots(1)$$

satisfying the initial condition, i.e. soil moisture variation with depth at a given time, and the boundary conditions those prevail at the soil surface and at the lower boundary of the zone of aeration. In above equation θ is the effective volumetric moisture content, which is equal to the volume of active soil moisture divided by the total volume, t is the time, $K(\theta)$ is the effective hydraulic conductivity. $D(\theta)$ is the diffusivity defined as $D(\theta) = K(\theta)\frac{\partial \psi(\theta)}{\partial \theta}$, where $\psi(\theta)$ is the soil matrix potential. z represents the vertical space co-ordinate and z is positive downward.

For vegetated surface the internal extraction of soil moisture by the plant roots needs to be incorporated. The local extraction rate will be a function of the plant species through root structure, and effective leaf area. It will be also a function of the climate through the potential rate of evaporation and will be sensitive to the soil moisture content. The root extraction is considered by including an appropriate sink term in Richard's equation and the final equation is expressed as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{\partial K(\theta)}{\partial z} - g_r(z,\theta) \qquad \ldots(2)$$

Solution to the equation is governed by general initial $\theta_o(z)$ and boundary θ_1 and θ_2 conditions such as shown in fig.1 .Since no exact analytical solution has yet been found, therefore, a numerical technique has to be adopted for solving the equation.

The soil water diffusivity term $D(\theta)$ is infinite for $\theta = \theta_s$ and $\theta = \theta_r$, in which, θ_s = saturated moisture content, θ_r = residual moisture content,. It is therefore preferable to solve Richards' equation in terms of soil water pressure h instead of volumetric water content θ. Richards equation in terms of soil water pressure is given by :

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\frac{\partial h}{\partial z}\right] - \frac{\partial K(h)}{\partial z} \qquad \ldots(3)$$

in which C(h) is the specific water capacity defined as:

$$C(h) = d\theta/dh \qquad \ldots(4)$$

The value of C(h) is zero for $\theta = \theta_s$ and $\theta = \theta_r$.

3.0 Model Parameters

For a proper description of the unsaturated flow, a correct description of the two hydraulic functions, $K(\theta)$ and $\psi(\theta)$, is important. The hydraulic conductivity, $K(\theta)$, decreases strongly as the moisture content, θ, decreases from saturation. The experimental procedure for measuring $K(\theta)$ at different moisture contents is rather difficult and not very reliable. Alternatively procedures have been suggested to derive the $K(\theta)$ function from more easily measurable characterizing properties of the soil. In many studies, the hydraulic conductivity of the unsaturated soil is defined as product of a non-linear function of the effective saturation, and hydraulic conductivity at saturation. The relation is given by $K(\theta) = K_{sat}S_e^n$ in which S_e= effective saturation equal to $(\theta-\theta_r)/(\theta_s-\theta_r)$, K_{sat} = hydraulic conductivity at saturation. The value of n is found to be 3.5 for coarse textured soils. n will vary with soil type. In literature established empirical correlation between n and soil characteristic is available .

The relationship between the suction head $\psi(\theta)$ and moisture content, θ, which is usually termed as the water retention curve or the soil moisture characteristics, is basically determined by the textural and the structural composition of the soil. Also the organic matter content may have an influence on the relationship. A characteristic feature of the water retention curve is that ψ decreases fairly rapidly with moisture content. Hysteresis effects may appear, and, instead of being a single valued relationship, the $\psi-\theta$ relation consists of a family of curves. The actual curve will have to be determined from the history of wetting and drying.

Based on experimental findings the following relationship for diffusivity has been suggested by Miller and Bresler (1977)

$$D(S_e)=(\alpha\, m^2)\exp(\beta S_e) \quad \ldots(5)$$

in which α and β appear to be universal constants both dimensionless, with suggested values: $\alpha=0.001$ and $\beta=8$. The third parameter ,m, is a unique constant for each soil .Its value can be estimated from observations of the visual wetting front by infiltration in an air dry soil:

$$m = X_f / \sqrt{t} \quad \ldots(6)$$

where X_f is the distance of the wetting front at the time t.

4.0 Numerical Schemes

Haverkamp et.al (1977) have compared six different schemes in terms of execution time,and accuracy. According to Haverkamp et.al all the following schemes yield good agreement with water content profiles measured experimentally at various times.

Scheme 1 : Explicit scheme (Eq. 3)

$$h_i^{j+1} = h_i^j + \frac{\Delta t}{C_i^j\,\Delta z}\left[K_{i+1/2}^j \left(\frac{h_{i+1}^j - h_i^j}{\Delta z} - 1\right) - K_{i-1/2}^j \left(\frac{h_i^j - h_{i-1}^j}{\Delta z} - 1\right)\right] \quad \ldots(7)$$

Scheme 2 :Implicit scheme with explicit linearization(Eq.3)

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta z}\left[K_{i+1/2}^j \left(\frac{h_{i+1}^{j+1} - h_i^{j+1}}{\Delta z} - 1\right) - K_{i-1/2}^j \left(\frac{h_i^{j+1} - h_{i-1}^{j+1}}{\Delta z} - 1\right)\right] \quad \ldots(8)$$

Scheme 3:Implicit scheme with implicit linearization(Eq.3)
(prediction-correction)

Prediction(estimation of C_i^j and K_i^j)

$$\frac{2C_i^j}{K_i^j}\,\frac{h_i^{j+1/2} - h_i^j}{\Delta t} = \Delta_z^2 h_i^{j+1/2} + \frac{1}{K_i^j}\,\Delta_z K_i^j\,(\Delta_z h_i^j - 1) \quad \ldots(9)$$

Correction(estimation of h_i^j)

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}} \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{2} \Delta_z^2 (h_i^{j+1} + h_i^j) + \frac{1}{K_i^{j+1/2}} \Delta_z K_i^{j+1/2} (\Delta_z h_i^{j+1/2} - 1) \quad ...(10)$$

where,

$$\Delta_z^2 h_i^j = \frac{h_{i+1}^j - 2h_i^j + h_{i-1}^j}{(\Delta z)^2} \;;$$

$$\Delta_z h_i^j = \frac{h_{i+1}^j - h_{i-1}^j}{2\Delta z} \;; \text{ and}$$

$$\Delta_z K_i^j = \frac{K_{i+1}^j - K_{i-1}^j}{2\Delta z}$$

By applying the boundary condition i.e. $h(Ø,t)$ during storm and interstorm periods, the soil moisture distribution with depth can be ascertained at desired time.

5.0 Boundary conditions:

Appropriate boundary conditions are to be applied i)during storm till ponding time, ii)during storm after ponding ,and iii)during inter storm period. For variable rainfall pattern,the expression for ponding time is (Morel-Seytoux,1982):

$$t_p = t_{J-1} + \frac{1}{R(J)} \left[\frac{(\tilde{\theta} - \theta_i) H_f}{R^*(J) - 1} - \sum_{\gamma=1}^{J-1} R(\gamma)\ (t_\gamma - t_{\gamma-1}) \right] \quad ...(11)$$

where $R(J)$ is the rainfall at J^{th} time and $R^*(J) = R(J)/\tilde{K}$, $\tilde{K}$ is the hydraulic conductivity at natural saturation, and H_f=average capillary drive which can be determined by method suggested by Bouwer and it is given by

$$H_f = \int_Ø^{h_{ci}} k_{rw}(\theta)\ dh_c \quad ...(12)$$

where $k_{rw}(\theta) = K(\theta) / \tilde{K}$, h_c= capillary pressure head, h_{ci} = capillary pressure head corresponding to the initial soil moisture θ_i ,prevailing at the onset of storm.

The finite difference equations(7), (8),(9) and (10) at the node next to the upper boundary node take the following forms respectively to satisfy the Neumann type boundary condition that prevails till ponding time :

$$h_2^{j+1} = h_2^j + \frac{\Delta t}{C_2^j \Delta z}\left[K^j_{2+1/2} \left(\frac{h^j_3 - h^j_2}{\Delta z} - 1\right) + R(j)\right] \qquad \ldots(13)$$

$$C_2^j \frac{h_2^{j+1} - h^j_2}{\Delta t} = \frac{1}{\Delta z}\left[K^j_{2+1/2}\left(\frac{h_3^{j+1} - h_2^{j+1}}{\Delta z} - 1\right) + R(j+1)\right] \qquad \ldots(14)$$

Prediction (estimation of C_2^j and K_2^j)

$$\frac{2C_2^j}{K_2^j} \frac{h_2^{j+1/2} - h^j_2}{\Delta t} = \frac{1}{(\Delta z)^2}\left[(h_3^{j+1/2} - h_2^{j+1/2}) + R(j+1)\frac{\Delta z}{K^j_{2-1/2}}\right]$$

$$+ \frac{1}{2\Delta z}\left[(h_3^j - h_2^j) - R(j)\frac{\Delta z}{K^j_{2-1/2}}\right] \qquad \ldots(15)$$

Correction(estimation of h_2^j)

$$\frac{C_2^{j+1/2}}{K_2^{j+1/2}} \frac{h_2^{j+1} - h^j_2}{\Delta t} = \frac{1}{2(\Delta z)^2}\left[R(j+1)\frac{\Delta z}{K^{j+1/2}_{2-1/2}} - h_2^{j+1} + h_3^{j+1}\right.$$

$$\left. + R(j)\frac{\Delta z}{K^j_{2-1/2}} - h_2^j + h_3^j\right]$$

$$+ \frac{1}{K_2^{j+1/2}} \Delta_z K_2^{j+1/2}\left[\frac{h_3^{j+1/2} - h_2^{j+1/2}}{2\Delta z} - \frac{R(j+1)}{2K^{j+1/2}_{2-1/2}} - 1\right] \qquad \ldots(16)$$

After ponding the upper boundary condition is satisfied by assigning h_1^j equal to the depth of ponded water on the ground surface.

During inter storm period the upper boundary condition is given by the relation (Hillel,1977):

$$h_1^j = \frac{R\ T(j)}{M\ g} \log_e f(j) \qquad \ldots(17)$$

where R is the universal gas constant (ergs $mole^{-1}$ degree K^{-1}), T is the temperature ^oK, g is the acceleration due to gravity (cm per sec per sec), M is the molecular weight of water (gm per mole) and f is the relative humidity of air.

-xxx-

References

1. Eagleson, P.S. (1978),"Climate, Soil and Vegetation", Water Resources Research, Vol. 14, No. 5.
2. Haverkamp, R., M. Vauclin, J. Touma, P.J. Wierenga, and G. Vachaud (1977), "A Comparison of Numerical Simulation Models for One-Dimensional Infiltration", Soil Sci.Soc. Am. J., Vol.41, 1977, pp. 285-294.
3. Hillel, D.(1977), "Computer Simulation of Soil-Water Dynamics: A Compendium of Recent Work", International Development Research Centre. Ottawa, 214 pp.
4. Miller, R. D. and E. Bresler (1977),"A Quick Method for Estimating Soil Water Diffusivity Functions", Soil Sci. Soc. American J., Vol. 41, pp. 1020-1022.
5. Morel-Seytoux,H.J.,(1982).Physical Hydrology:Ensemble of Lecture Notes and Class Handouts Developed since 1977, HYDROPOWER Programme,Colorado State University,Fort Collins.