ESTIMATION OF ARTIFICIAL RECHARGE

INTRODUCTION

It has been recognized that aquifers are not only sources of water but also storage reservoirs that require proper management for efficient use. With respect to management, an aquifer may be considered as a reservoir for long term storage artificially produced and as a water quality control tool because of its filtering characteristic that reclaims artificially recharged waste water. Artificial recharge may be viewed as an augmentation of the natural movement of surface water into underground formation by some method of construction, by surface spreading of water or by artificially changing natural conditions. There are two aspects associated with the assessment of artificial recharge. They are: assessment of the water actually recharged and availability of the recharged water in the zone of interest at different time. In this lecture a method has been described to predict quantity of water recharged from a spreading basin. A method has also been presented to find the temporal variation of the fraction of the recharged quantities available in a circular zone around the recharge basin.

STATEMENT OF THE PROBLEM

Schematic section and plan view of a spreading basin and the groundwater abstraction structures are shown in Figure 1. Water is recharged through the basin during a certain period of time. The groundwater is withdrawn through abstraction wells as shown in the figure. Continuous monitoring of groundwater level is done at an observation well. It is required to find the quantity of groundwater recharged through the basin and its distribution with space and time making use of the observed groundwater levels.

METHODOLOGY

The following assumptions have been made in the analysis:

i) The time parameter is discrete. The time span has been discretised by uniform time-steps.

ii) Within each time-step the recharge rate is constant but it varies from time-step to time-step.

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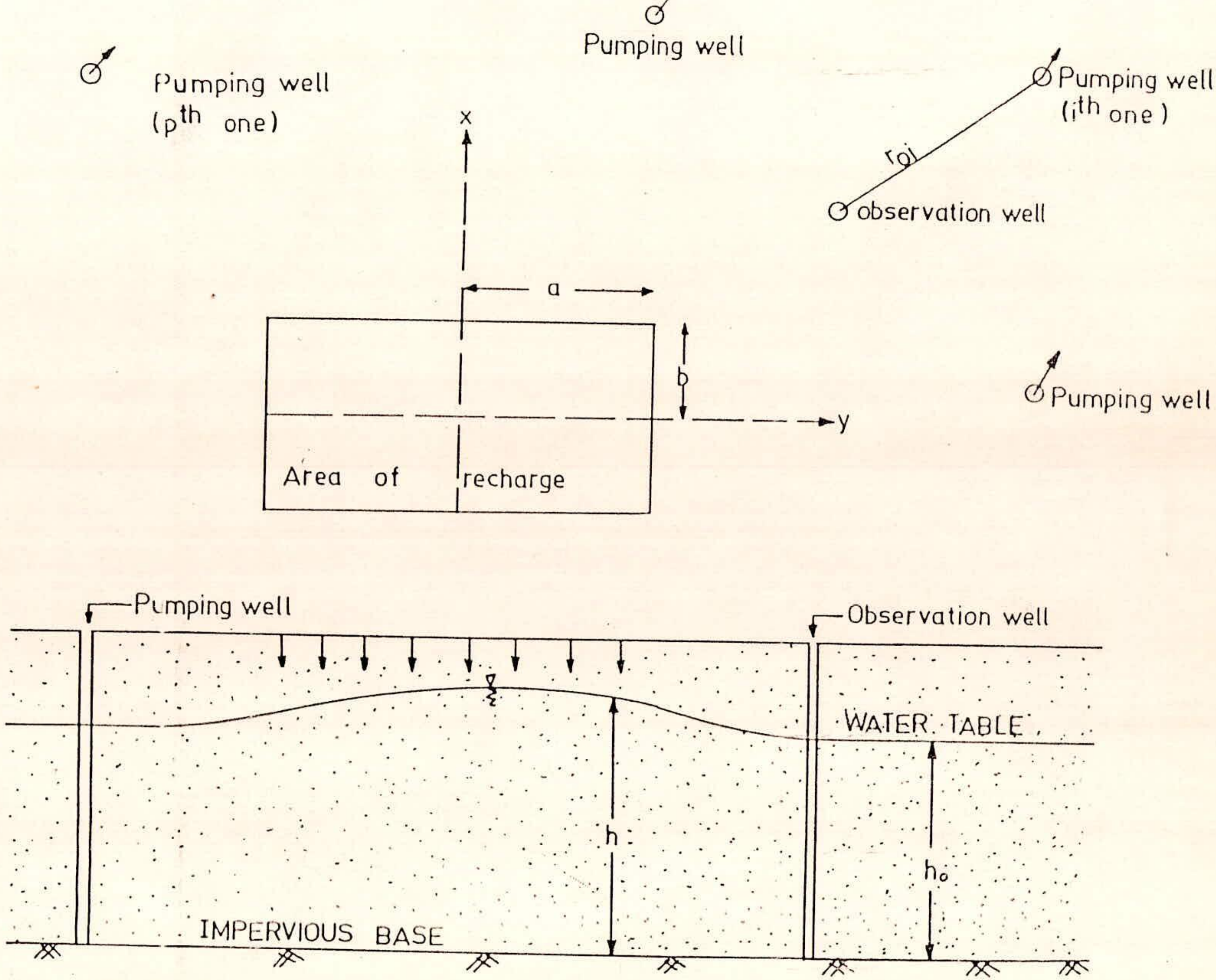

FIG. 1 SCHEMATIC PLAN AND SECTION OF A SPREADING BASIN

Hantush(1967) developed the following approximate analytical expression for the rise and fall of the water table in an infinite unconfined aquifer in response to a uniform continuous percolation from a rectangular spreading basin :

$$h^2 = h_0^2 + \frac{w\,\bar{h}\,t}{2\phi}\Big\{ F\Big[\Big(\frac{a+x}{C}\Big),\Big(\frac{b+y}{C}\Big)\Big] F\Big[\Big(\frac{a+x}{C}\Big),\Big(\frac{b-y}{C}\Big)\Big]$$

$$F\Big[\Big(\frac{a-x}{C}\Big),\Big(\frac{b+y}{C}\Big)\Big] F\Big[\Big(\frac{a-x}{C}\Big),\Big(\frac{b-y}{C}\Big)\Big]\Big\}$$

$$= h_0^2 + \Big(\frac{w\,\bar{h}\,t}{2\phi}\Big)\{ f(x, y, t) \} \qquad \ldots(1)$$

in which,

$\bar{h}$ = weighted mean of the depth of saturation during flow period

w = constant rate of percolation per unit area,

a = half of the length of the rectangular strip along the x axis

b = half of the width of rectangular strip along the y direction,

ϕ = storage coefficient of the aquifer,

k = coefficient of permeability,

t = time measured since the onset of recharge,

$F(p,q) = \int_0^1 \mathrm{erf}(p/\sqrt{z}).\mathrm{erf}(q/\sqrt{z}).dz$

$\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}}\int_0^X e^{-u^2}du$, and

$C = 2\sqrt{(k\,\bar{h}\,t/\phi)}$.

Let $\bar{h}=(h + h_0)/2$. From Hantush's solution, the rise in watertable is given by the following equation:

$$s_w(x,y,t) = \frac{w\,\bar{h}\,t}{2\phi\,(h+h_0)} f(x,y,t)$$

$$= \frac{w\,\bar{h}\,t}{4\,\phi\,[(h+h_0)/2]} f(x,y,t)$$

$$= \frac{w\,t}{4\,\phi} f(x,y,t) \qquad \ldots(2)$$

For continuous recharge at unit rate the rise in watertable is given by:

$$s_1(x,y,t) = \frac{t}{4\phi} f(x,y,t) \ldots(3)$$

Let the watertable rise due to continuous recharge at unit rate be designated as $K(x,y,t)$. If recharge takes place at unit rate during the first unit time-step and no recharge after that, the rise at the end of n^{th} unit time step, $\delta_b(x,y,n)$, is given by:

$$\delta_b(x,y,n) = K(x,y,n) - K(x,y,n-1), \text{and} \quad \ldots(4)$$

$$\delta_b(x,y,1) = K(x,y,1). \quad \ldots(5)$$

If the recharge varies with time, the rise due to recharge alone at the end of n^{th} unit time step is given by (Morel Seytoux, 1975):

$$s_r(x,y,n) = \sum_{\gamma=1}^{n} w(\gamma)\, \delta_b(x,y,n-\gamma+1) \quad \ldots(6)$$

in which $w(\gamma)$ is the recharge rate during the γ^{th} unit time step.

The drawdown at the observation well due to time variant pumping at several wells is given by (Morel -Seytoux 1975):

$$s_p(x,y,n) = \sum_{i=1}^{P} \sum_{\gamma=1}^{n} Q_i(\gamma)\delta_{wi}(n-\gamma+1) \quad \ldots(7)$$

in which,

P is the total number of wells,

$$\delta_{wi}(m) = \frac{1}{4\pi T} [E_1(\frac{r_i^2}{4\beta m}) - E_1\{\frac{r_i^2}{4\beta(m-1)}\}]$$

$\beta = T/\phi$,

T = transmissivity of the acuifer, and

ϕ = storage coefficient .

r_i = distance of the i^{th} pumping well from the observation point ,and

$E_i(X) = \int_X^{\infty} \frac{e^{-u}}{u} du$,an exponential integral.

The resultant water table rise due to recharge and pumping is given by

$$s(x,y,n) = \sum_{\gamma=1}^{n} w(\gamma)\delta_b(x,y,n-\gamma+1) - \sum_{i=1}^{P} \sum_{\gamma=1}^{n} Q_i(\gamma)\,\delta_{wi}(n-\gamma+1) \quad \ldots(8)$$

Splitting the summation into two parts and rearranging

$$w(n) = [\, s(x,y,n) - \sum_{\gamma=1}^{n-1} w(\gamma)\,\delta_b(x,y,n-\gamma+1) + \sum_{i=1}^{P} \sum_{\gamma=1}^{n} Q_i(\gamma)\,\delta_{wi}(n-\gamma+1)\,]/\delta_b(x,y,1) \quad \ldots(9)$$

Thus knowing $s(x,y,n)$, $w(n)$ can be found in succession starting from the first time-step for known withdrawal rates, $Q_i(n)$.

After knowing the quantities of water recharged by the basin at different time its spatial and temporal availability in the aquifer can be assessed as follows:

Let it be required to ascertain the amount of water available within a circular zone of radius R beneath the rectangular recharge basin at time t.

For large value of R the basin can be regarded as a point source. If recharge takes place at unit rate per unit time continuously, the drawdown at a distance R from a point source is

$$s(R,t) = \frac{1}{4\pi T} \int_{\frac{R^2}{4\beta t}}^{\infty} \frac{e^{-u}}{u}\, du \qquad \ldots(10)$$

and the gradient is given by

$$\frac{\partial s(R,t)}{\partial r} = - \frac{1}{2\pi TR} e^{-\frac{R^2}{4\beta t}} \qquad \ldots(11)$$

The quantity of water leaving the circular zone of radius R per unit time is

$$q_{RO}(t) = - 2\pi RT \frac{\partial s(R,t)}{\partial r} \qquad \ldots(12)$$

Replacing the expression of $\frac{\partial s}{\partial r}$ and simplifying, the rate at which water leaves the circular zone is found to be

$$q_{RO}(t) = e^{-\frac{R^2}{4\beta t}} \qquad \ldots(13)$$

The quantity of water which has left the zone of radius R up to time t can be expressed as :

$$Q_{RO}(t) = \int_0^t e^{-\frac{R^2}{4\beta \tau}} .d\tau$$

$$= t\, e^{-\frac{R^2}{4\beta t}} - \frac{R^2}{4\beta} E_i \left(\frac{R^2}{4\beta t}\right) \qquad \ldots(14)$$

The quantity of water which is retained within the zone of radius R up to time t is given by

$$Q_{RR}(t) = t - t\, e^{-\frac{R^2}{4\beta t}} + \frac{R^2}{4\beta} E_i \left(\frac{R^2}{4\beta t}\right) \qquad \ldots(15)$$

If recharge at unit rate per unit time takes place during the first unit time-step and no recharge thereafter, the cumulative flows, which would leave the zone of radius R up to the end of time-step n can be expressed as:

$$\delta_{CQRO}(n) = Q_{RO}(n) - Q_{RO}(n-1)$$

$$= n\, e^{-R^2/(4\beta n)} - \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta n}\right)$$

$$-(n-1)\, e^{-R^2/\{4\beta(n-1)\}} + \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta(n-1)}\right) \quad \ldots(16)$$

$$\delta_{CQRO}(1) = e^{-R^2/(4\beta)} - \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta}\right) \quad \ldots(17)$$

Similarly the cumulative quantity of water which is retained in the zone with radius R up to the end of time step n is

$$\delta_{CQRR}(n) = 1 - n\, e^{-R^2/(4\beta n)} + \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta n}\right)$$

$$+(n-1)\, e^{-R^2/\{4\beta(n-1)\}} \underline{+} \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta(n-1)}\right) \quad \ldots(18)$$

$$\delta_{CQRR}(1) = 1 - e^{-R^2/(4\beta)} + \frac{R^2}{4\beta} E_i\left(\frac{R^2}{4\beta}\right) \quad \ldots(19)$$

If recharge through the basin takes place at a rate $w(\gamma)$, the quantities of water that would be retained within zone of radius R up to n^{th} unit time-step is

$$Q_{RR}(n) = \sum_{\gamma=1}^{n} w(\gamma)\, \delta_{CQRR}(n-\gamma+1) \quad \ldots(20)$$

and the quantities which have left up to n^{th} unit time-step is:

$$Q_{RO}(n) = \sum_{\gamma=1}^{n} w(\gamma)\, \delta_{CQRO}(n-\gamma+1) \quad \ldots(21)$$

RESULTS AND DISCUSSION

Results have been presented with an aim to analyse the dissipation of recharged water in the aquifer in space and time. For this purpose, the recharge basin has been assumed to be a point source. If recharge takes place during the first unit time-step at unit rate per unit time period, the quantity of water ,that would be retained within a zone of radius 1000 m. at the end of n^{th} unit time step ,is shown in Fig.2 for T= 500 m^2 per day and for various values of ϕ .The quantities of recharged water retained are presented by the discrete kernel coefficients $\delta_{CQRR}(n)$. For

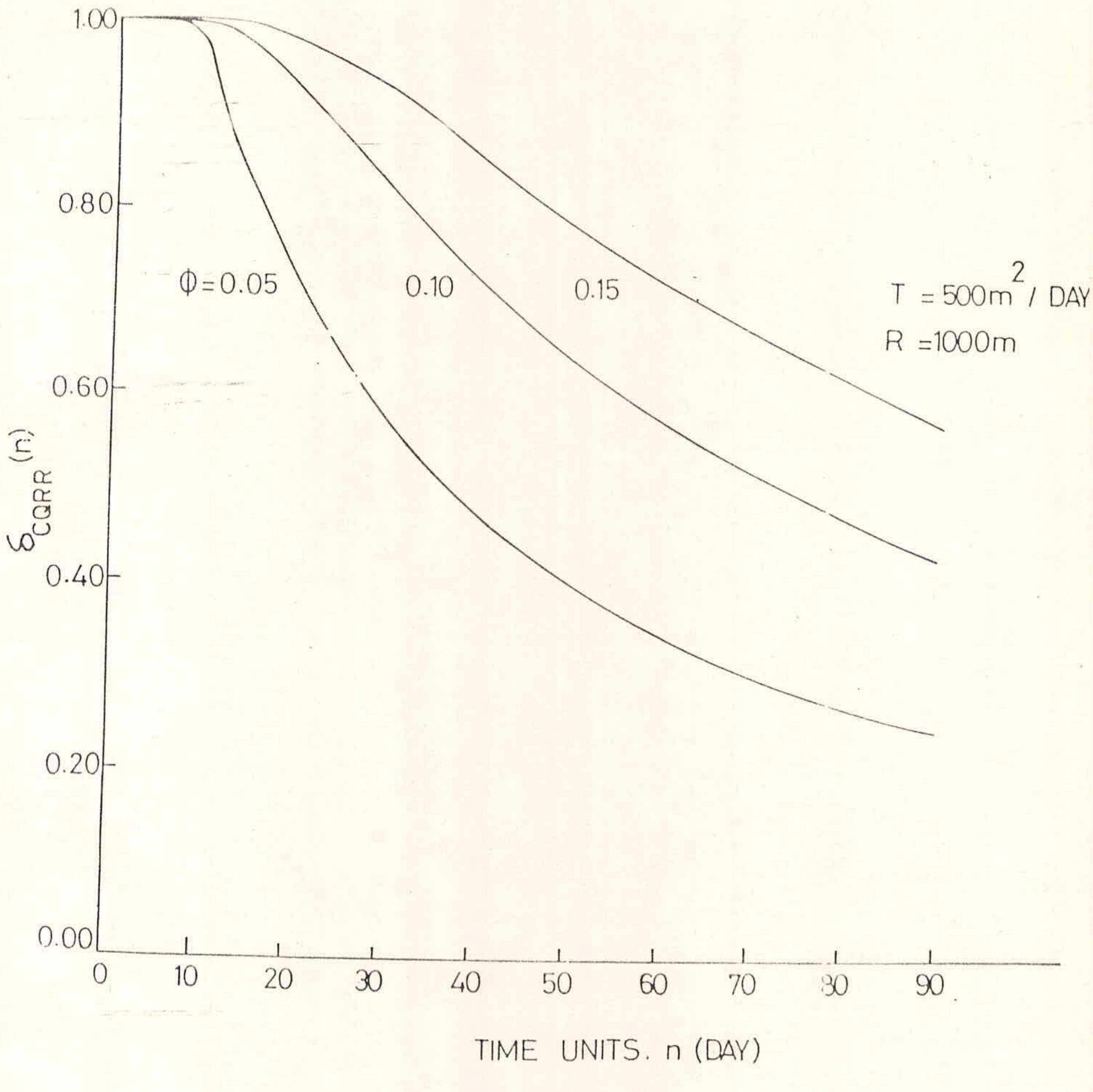

FIG. 2 VARIATION OF $\delta_{CQRR}(n)$ WITH TIME n FOR $T = 500\ m^2/day$ and $R = 1000\ m$

continuous recharge at unit rate per unit time, the cumulative quantities of flow that would leave the circular zone of radius 1000m.,have been shown in Fig.3. The discrete kernel coefficients are the properties of the linear system, using which the response for time variant recharge could be obtained. The quantities avail-able within a zone of radius R,is governed by the aquifer para - meters and the radius R .It could be seen from Fig.3 that the slope of the graph of cumulative flow retained versus time, decreases with time ,indicating that, under continuous recharge, fraction of the recharged water retained in a zone,decreases with time.For higher storage coefficient,the cumulative quantities retained up to any particular time increases. For example at the end of 50 days, for R = 1000 m, T = 500 m^2/day and ϕ = 0.05, the cumulative fraction of water retained is 0.68, where as for ϕ = 0.15, the corresponding cumulative fraction retained is 0.93. As time increases, the cumulative fraction retained decreases. For example at the end of 80 days,for ϕ = 0.05 and 0.15 the fractions retained are 0.54 and 0.83 respectively.

The dissipation of actual recharge that may occur is next considered.The size of the basin has been assumed to be 200mx 10m. A typical time variation of recharge rate per unit area for water spreading on undisturbed soil is as shown in Fig.4(1959).It has been assumed that the recharge occurs from the basin at these rates. The variation of cumulative recharge,cumulative recharge quantities that would be retained and cumulative recharge quantities that would leave a zone of radius 2000m.at different times are presented in Fig.5 for T =1000 m^2/day and for ϕ = 0.05 and 0.1. The recharge through the basin stops at n =24,i.e,at the end of 120 days.Therefore,the slope of the graph of cumulative volume of recharge versus time has become zero at n = 24 and beyond. It could be seen from Fig.5 that,for ϕ = 0.05,the quantities of water available within the zone of radius 2000 m. at the end of 73^{th} unit time (i.e.at the end of 365 days) is 1.4×10^4 m^3 of water.The total quantities that have been recharged is $1.05 \times 10^5 m^3$. Thus 13.33 percentage of the recharged water is available at the end of 365 days. It may be noted that the recharge quantities available will get modified by abstraction wells in the area.

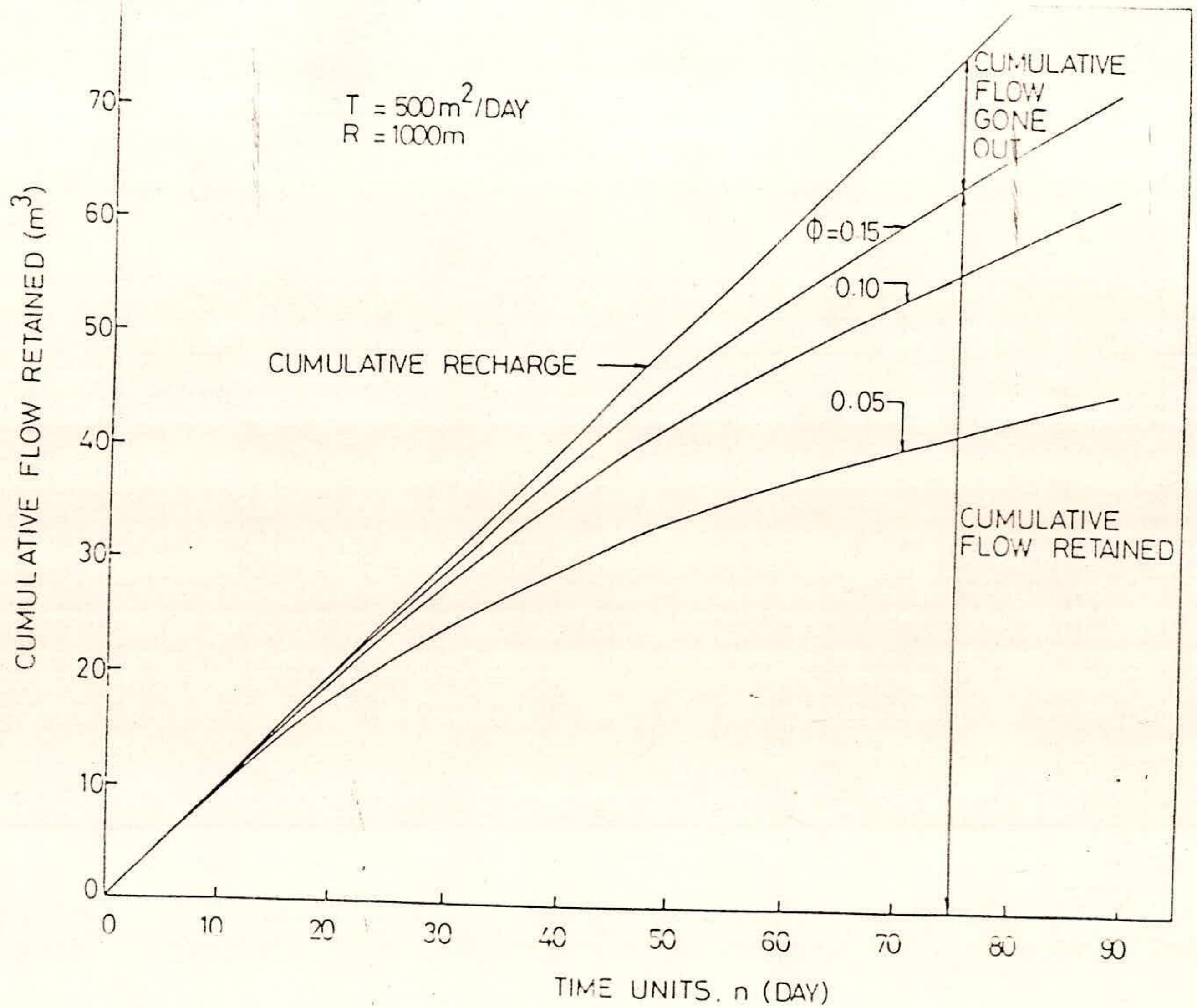

FIG. 3 VARIATION OF CUMULATIVE FLOW WITH TIME n FOR T = 500 m^2/day and R = 1000 m

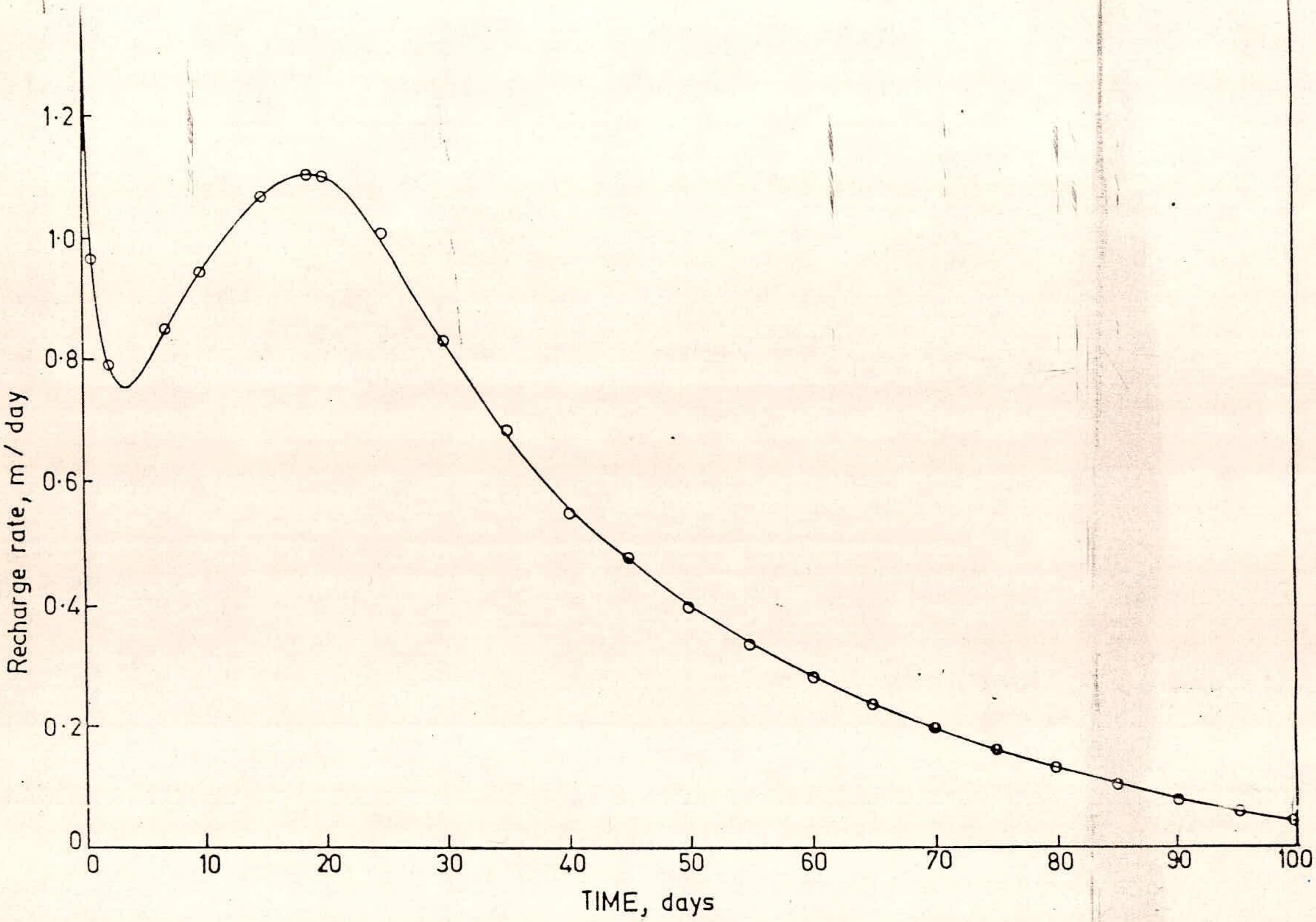

FIG. 4 TYPICAL TIME VARIATION OF RECHARGE RATE FOR WATER SPREADING ON UNDISTURBED SOIL

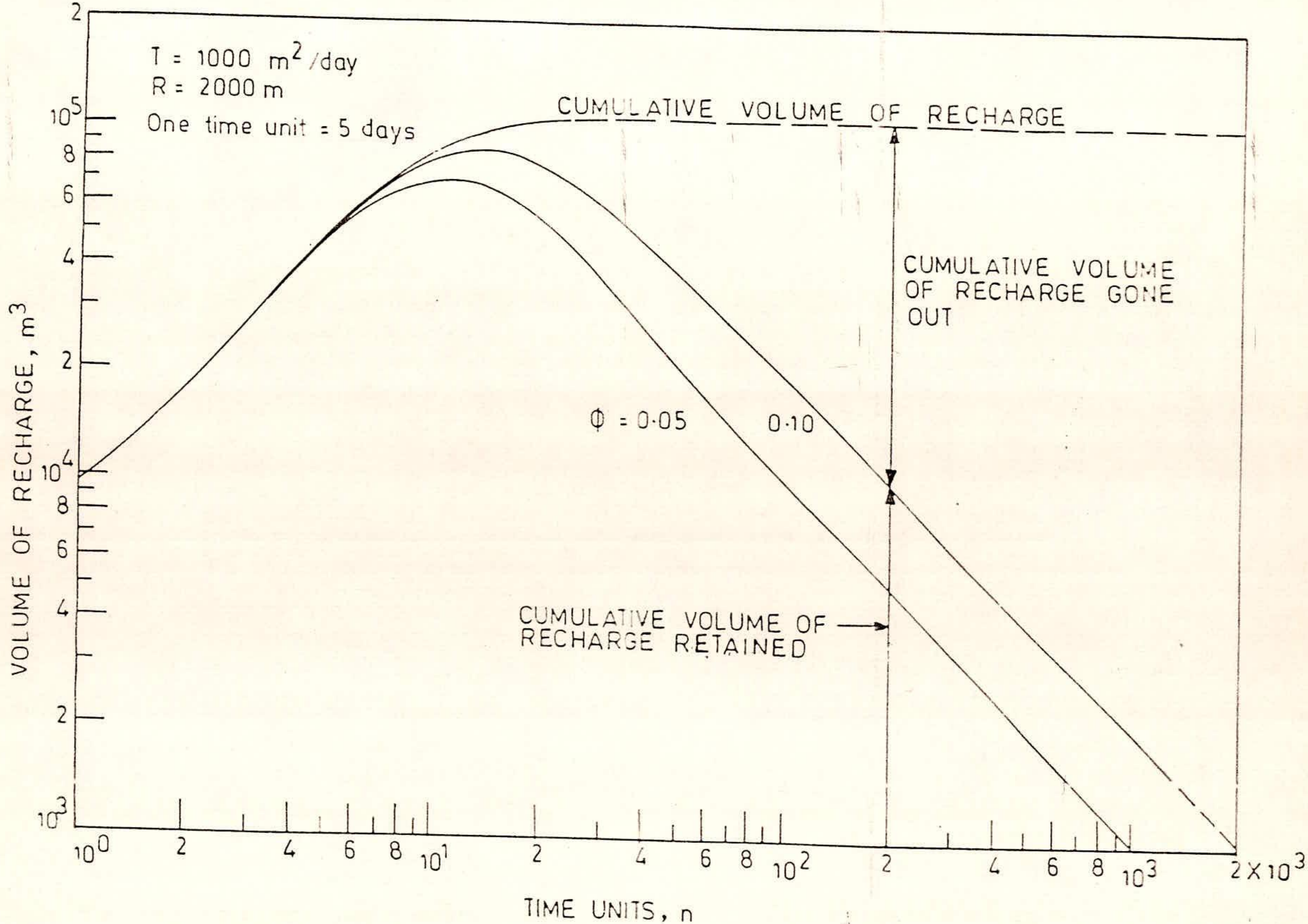

FIG. 5 VARIATION OF VOLUME OF RECHARGE WITH TIME n FOR $T = 1000\ m^2/day$ and $R = 2000$ m

CONCLUSION

A methodology which is based on discretization of time parameter, has been described to ascertain groundwater recharge affected through a rectangular recharge basin. The storativity, and transmissivity of the aquifer, dimension of the recharge basin, duration of recharge and a continuous record of water level in a observation well in the vicinity of the recharge basin, are required for the assessment.

A methodology has also been given to find the temporal variation of the fraction of the recharged quantities available in a circular zone around the recharge basin. A typical example has been given for knowing the fraction of recharged water available within a circular zone at different time .

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