ANALYSIS OF AQUIFER TEST FOR DUG WELLS/DUG CUM BORE WELLS

INTRODUCTION

Large-diameter wells are mostly used for ground-water abstraction from a shallow unconfined aquifer of low transmissivity. A typical large diameter dug well in a hard rock region has a diameter that may vary from 3 to 8 meters (Limaye,1984). The large-diameter wells have the characteristics that during the early abstraction phase, most of the water is pumped from the well storage .In many cases more water is drawn from the aquifer during the recovery phase than during the pumping phase (Rushton and Singh, 1987). Large-diameter wells are widely used because of their low cost and simplicity of construction.

If necessary, an aquifer test can be conducted in a large-diameter well. In such a test the aquifer response may be recorded either in the large-diameter well itself or at a nearby observation well of negligible diameter. A large-diameter well can also serve as an observation well if an aquifer test is conducted in a production well of negligible diameter. Barker (1984) has shown that, if a pumping test is conducted in a production well of negligible diameter, the drawdown in a large-diameter observation well is identical to the drawdown in an observation well if roles of the wells are reversed. The storage associated with a large-diameter production or observation well modifies and causes delay in the aquifer response. Therefore, storage effect should be duly considered while solving a direct or an inverse problem.

Analysis of unsteady flow to a large-diameter well during pumping has been done by several investigators. Foremost among the solutions is that of Papadopulos and Cooper(1967). According to them the well storage dominates the time drawdown curve up to a time 't' given by $t=25\ r_c^{2}/T$,in which r_c is radius of the well casing and T is transmissivity of the aquifer. Using the solution of Papadopulos and Cooper, the aquifer response can be estimated

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at the large-diameter production well and at other observation wells which have negligible storage. An approximate discrete kernel method for analysing unsteady flow to a large diameter well has been presented by Patel and Mishra(1983). The method is approximate as the discrete kernel coefficients are generated making use of the Theis' solution, which is based on the assumption that the well is infinitesimally narrow. Exact solution to the problem of unsteady flow to a well of finite radius for uniform withdrawal from aquifer storage is yet to be found. An asymptotic solution has been obtained by Hantush (1964) according to which the Theis' formula is valid for any value of the well screen radius, r_w, at a nondimensional time parameter, $4Tt/(\phi r_w^2)$, larger than 120. This limitation should be considered while analysing a large-diameter well problem by the discrete kernel approach. A solution to the problem of unsteady flow to a large-diameter well by the discrete kernel approach has been compared with the exact solution given by Papadopulos and Cooper(1967) and the approximate discrete kernel method is found to compare well with the exact solution (Patel and Mishra, 1983). In the present lecture the application of discrete kernel has been discussed for analysing unsteady flow to a large diameter well during recovery phase(Mishra, and Chachadi, 1985).

STATEMENT OF THE PROBLEM

A schematic cross section of a large-diameter well in a homogeneous, isotropic, confined aquifer of infinite areal extent is shown in Fig.(1). It is assumed that the aquifer prior to

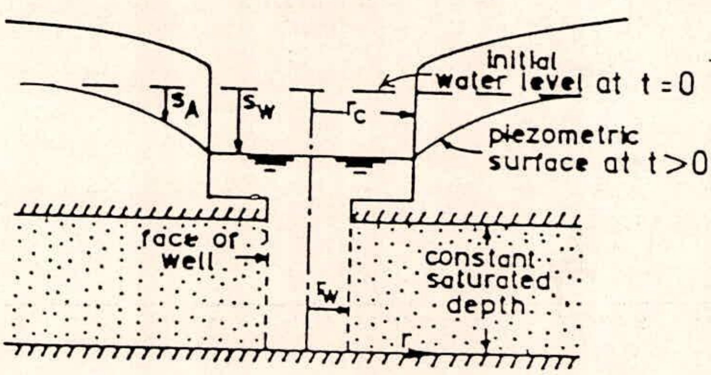


FIG. 1 - Schematic cross section of a large-diameter well

pumping was at rest condition. The radius of the well screen is r_w, and that of the well casing r_c. Pumping is carried out at a uniform rate up to time t_p. The problem is to determine the drawdown in piezometric surface at the well face and at any distance, r, from the center of the well during the recovery period , contribution of well storage to pumping, and recovery of well storage.

ANALYSIS

The following assumptions have been made in the analysis:

a) The radius of the production well screen is small.

b) The time parameter is discrete. Within each time step the abstraction rate from the well storage and that derived from the aquifer storage are separate constants.

c) At any time the drawdown in the piezometric surface in the aquifer at the well face is equal to the drawdown in the water level in the well.

The basic differential equation for an axially-symmetric radial unsteady ground water flow in a homogeneous, isotropic, confined aquifer of uniform thickness is given by

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad ...(1)$$

where s = drawdown; r = distance measured from the centre of the well; t = time; ϕ = storage coefficient; and T = transmissivity of the aquifer.The required solution to the differential eq.(1) needs to satisfy the following boundary condition to account for well storage:

$$2\pi r_w T \frac{\partial s}{\partial r}\Big|_{r=r_w} - \pi r_c^2 \frac{\partial s_w(t)}{\partial t} = -Q_p(t) \qquad ...(2)$$

in which $s_w(t)$ is the drawdown in the well ,and $Q_p(t)$ is the pumping rate. The boundary condition to be satisfied at $r=\infty$ is :

$$s(\infty,t) = 0 \qquad ...(3)$$

For the assumption made in the analysis another boundary condition required to be satisfied is:

$$s(r_w,t)=s_w(t) \qquad ...(4)$$

Besides the above mentioned boundary conditions the solution has to satisfy the initial condition

$$s(r,0)=0 \qquad \ldots(5)$$

For the initial condition $s(r,0) = 0$,and the boundary condition $s(\infty,t)=0$, solution to the above differential equation, when a unit impulse quantity of water is withdrawn from the aquifer storage through a well with negligible radius, is given by (Muskat,1937,Carslaw and Jaeger, 1959)

$$s(r,t) = \frac{e^{-\phi r^2/(4Tt)}}{4\pi Tt} \qquad \ldots(6)$$

The response of the aquifer to a unit impulse excitation has been defined as unit impulse kernel (Morel-Seytoux, 1975). Designating the unit impulse kernel for drawdown as $k(t)$, the drawdown caused by variable abstraction can be found using the expression (Muskat, 1937,Carslaw and Jaeger,1959)

$$s(r,t) = \int_0^t Q_A(\tau)\, k(t-\tau)d\tau \qquad \ldots(7)$$

where $Q_A(\tau)$ is the variable abstraction rate from the aquifer storage at time τ.

Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step but varies from step to step, drawdown at the end of n^{th} time step can be written as (Morel-Seytoux, 1975)

$$s(r,n) = \sum_{\gamma=1}^{n} \delta_r(n-\gamma+1)Q_A(\gamma) \qquad \ldots(8)$$

in which the discrete kernel coefficients $\delta_r(M)$ is given by

$$\delta_r(M) = \int_0^1 k(M-\tau)d\tau = \frac{1}{4\pi T}\left[E_1\left(\frac{\phi r^2}{4M}\right)-E_1\left(\frac{\phi r^2}{4T(M-1)}\right)\right] \qquad \ldots(9)$$

where $E_1(X)$ is an exponential integral defined as $E_1(X) = \int_X^\infty \frac{e^{-y}}{y}\,dy$. The Theis'well function for a nonleaky aquifer is equal to this exponential integral. The discrete kernel coefficient $\delta_r(M)$ is the response of a linear system at the end of M^{th} unit time step to a unit pulse excitation given to the system during the first unit time step. In the coefficient $\delta_r(M)$, 'M' is an index and it has no dimension, but the term 'M', which appears in the exponential integral $E_1\{\phi r^2/(4TM)\}$,is an integer having the dimension of time. For computing the dimensionless term $\phi r^2/(4TM)$, a transmissivity value corresponding to a unit time step size is to be used. In both the terms, $\delta_r(M)$ and $\phi r^2/(4TM)$, values of M are numerically equal.

Let the total time of pumping, t_p, be discretised to 'm' units of equal time steps. The quantity of water, $Q_p(n)$, pumped during any time step 'n' can be written as:

$$Q_A(n) + Q_W(n) = Q_p(n) \qquad \ldots(10)$$

in which

$Q_A(n)$ = water withdrawn from the aquifer storage through the production well during the n^{th} time step ; and

$Q_W(n)$ = water withdrawn from the production well storage during the n^{th} time step .

For $n > m$, $Q_p(n)=0$. Otherwise $Q_p(n)$ is equal to the rate of pumping during the n^{th} time step. Equation (10) is the alternate statement of boundary condition stated in equation(2).

The drawdown, $s_w(n)$, in the water level at the production well at the end of n^{th} time step due to abstraction from the production well storage is given by

$$s_w(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(11)$$

where $Q_W(\gamma)$ represents rate of withdrawal from the production well storage or the replenishment during time step γ. $Q_W(\gamma)$ values are unknown a priori. A negative value of $Q_W(\gamma)$ means that there is a replenishment of the well storage which occurs during the recovery period.

The drawdown in the piezometric surface in the aquifer at the production well face at the end of n^{th} time step due to abstraction from the aquifer is given by

$$s(r_w,n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{r_w}(n-\gamma+1) \qquad \ldots(12)$$

Because $s(r_w,n) = s_w(n)$, therefore,

$$\sum_{\gamma=1}^{n} Q_A(\gamma)\ \delta_{r_w}(n-\gamma+1) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(13)$$

Rearranging,

$$Q_A(n)\,\delta_{r_w}(1) - \frac{1}{\pi r_c^2} Q_W(n)$$

$$= \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\,\delta_{r_w}(n-\gamma+1) \qquad \text{...(14)}$$

Eqs. 10, and 14 can be expressed in the following matrix form:

$$\begin{bmatrix} 1 & , & 1 \\ \delta_{r_w}(1) & , & -\dfrac{1}{\pi r_c^2} \end{bmatrix} \cdot \begin{bmatrix} Q_A(n) \\ Q_W(n) \end{bmatrix} =$$

$$\begin{bmatrix} Q_P(n) \\ \dfrac{1}{\pi r_c^2} \displaystyle\sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\,\delta_{r_w}(n-\gamma+1) \end{bmatrix} \qquad \text{...(15)}$$

In particular, for the first time step ,the right-hand side vector is [$Q_p(1)$, 0] .$Q_A(n)$,and $Q_W(n)$,can be solved in succession starting from the first time step . Once $Q_A(n)$, $Q_W(n)$, are known, the drawdown at any point in the aquifer at a distance 'r' from the production well can be known using the relation

$$s(r,n) = \sum_{\gamma=1}^{n} \delta_{r}(n-\gamma+1)Q_A(\gamma) \qquad \text{...(16)}$$

in which r is the distance between the point under consideration and the large-diameter well.

RESULTS AND DISCUSSION

The discrete kernel coefficients $\delta_{r_w}(M)$ have been generated for known values of transmissivity, storage coefficient,and radius of the production well screen . The exponential integrals have been evaluated making use of the polynomial and rational approximations given by Gautschi and Cahill(1964). The computational efficiency of these approximations has been brought out by Huntoon (1980). Using a matrix inversion technique, eq.(15)

is solved for $Q_A(n)$, and $Q_W(n)$, in succession starting from the first time step for known values of $Q_p(n)$ and r_c. The drawdowns in water level in the production well and in the observation well are obtained using eqs.(11) and (16) respectively. The drawdowns have been computed for a constant pumping rate, Q, and have been presented here.

The sensitivity analysis of drawdown to size and number of time steps has already been presented by Mishra and Chachadi (1985). A time step size of t/10 should be adopted to compute the response at time t.

The variations of $s_w(t)/[Q/(4\pi T)]$ with $4Tt/(\phi r_w^2)$ at the production well for m=2,5,10,25,50,100,250,and 500 are shown in Figs.[(2a)] through [(2g)] for different values of α where $\alpha = \phi(r_w/r_c)^2$ The parameter α quantifies the storage of the production well. $s_w(t)$ is the drawdown at the well face at time t and $s_w(t)/[Q/(4\pi T)]$ can be regarded as the well function for a large-diameter well. The type curves in Figures [2(a)] through [2(g)] contain the response of an aquifer during the abstraction as well as recovery phase. Each of the recovery curves is characterized by a nondimensional time factor $4Tt_p/(\phi r_w^2)$, at which it deflects from the time-drawdown curve of the abstraction phase. The nondimensional time factor, $4Tt_p/(\phi r_w^2)$, can be used to check the accuracy of the aquifer parameters determined by curve matching.

The variations of $s_r(t)/[Q/4\pi T)]$ with $4Tt/(\phi r^2)$ are shown in Figures [3(a)] through [3(g)] for an observation point located at a distance of $10r_w$ from the center of the well for $r/r_w = 10$ and for different values of α. The results have been given for various durations of pumping. The nondimensional time factor $4Tt_p/(\phi r^2)$ at which the pumping discontinued, have been indicated in the figures. It can be seen from the figures that water level continues to fall at the observation point after the abstraction ceased. Such phenomenon occurs due to the fact that the aquifer continues to supply water to refill the well even after pumping is discontinued.

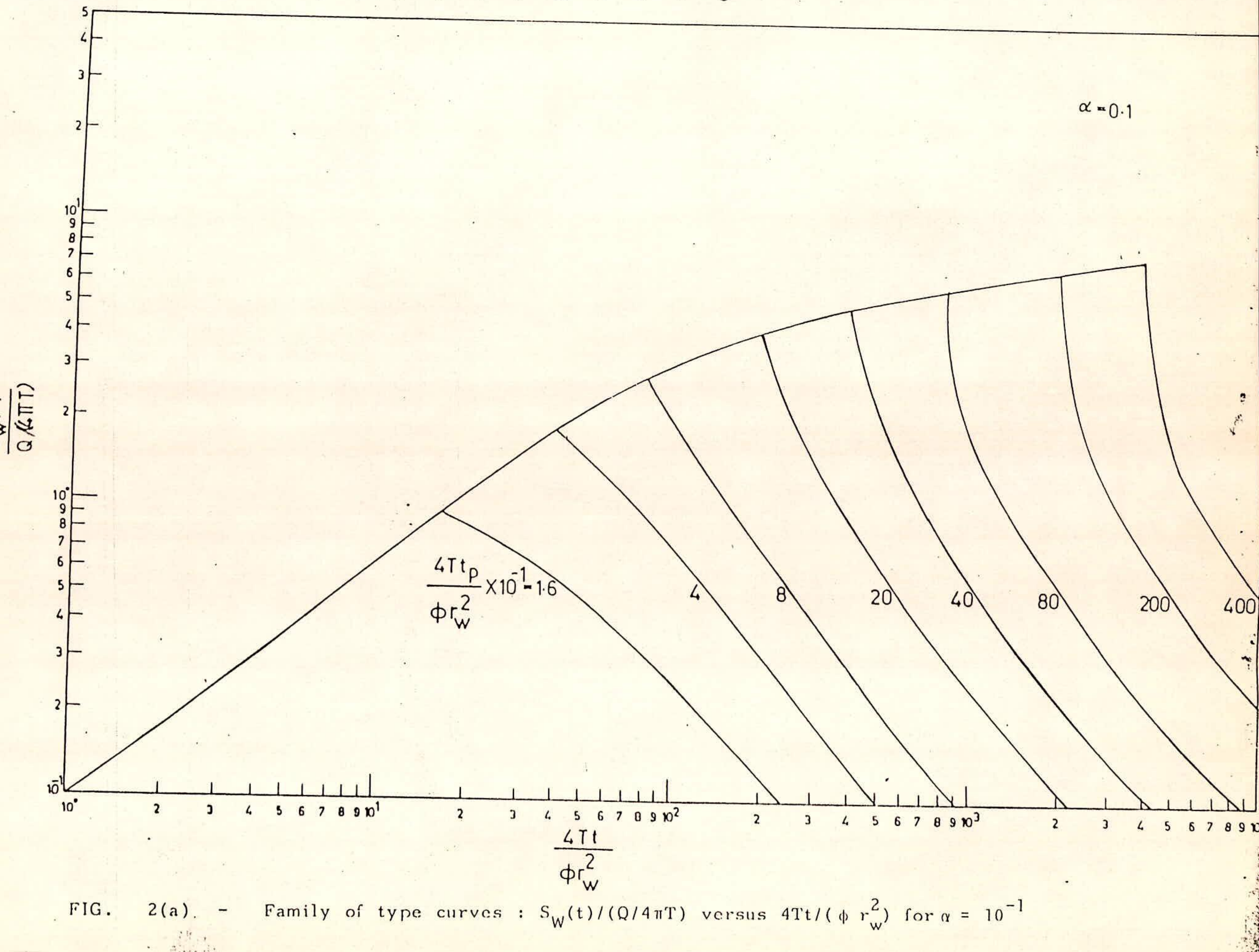


FIG. 2(a). - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-1}$

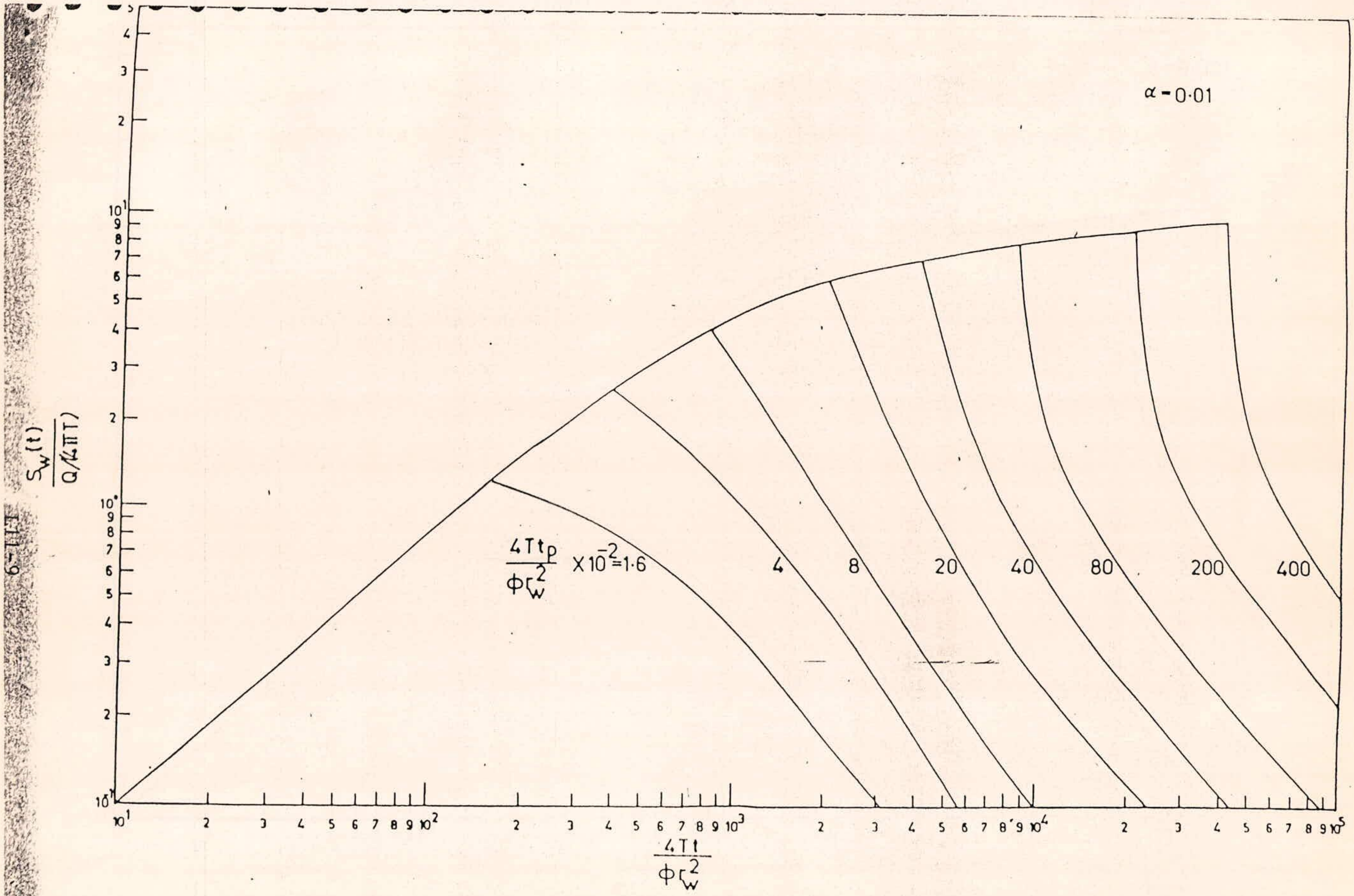


FIG. 2(b) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-2}$.

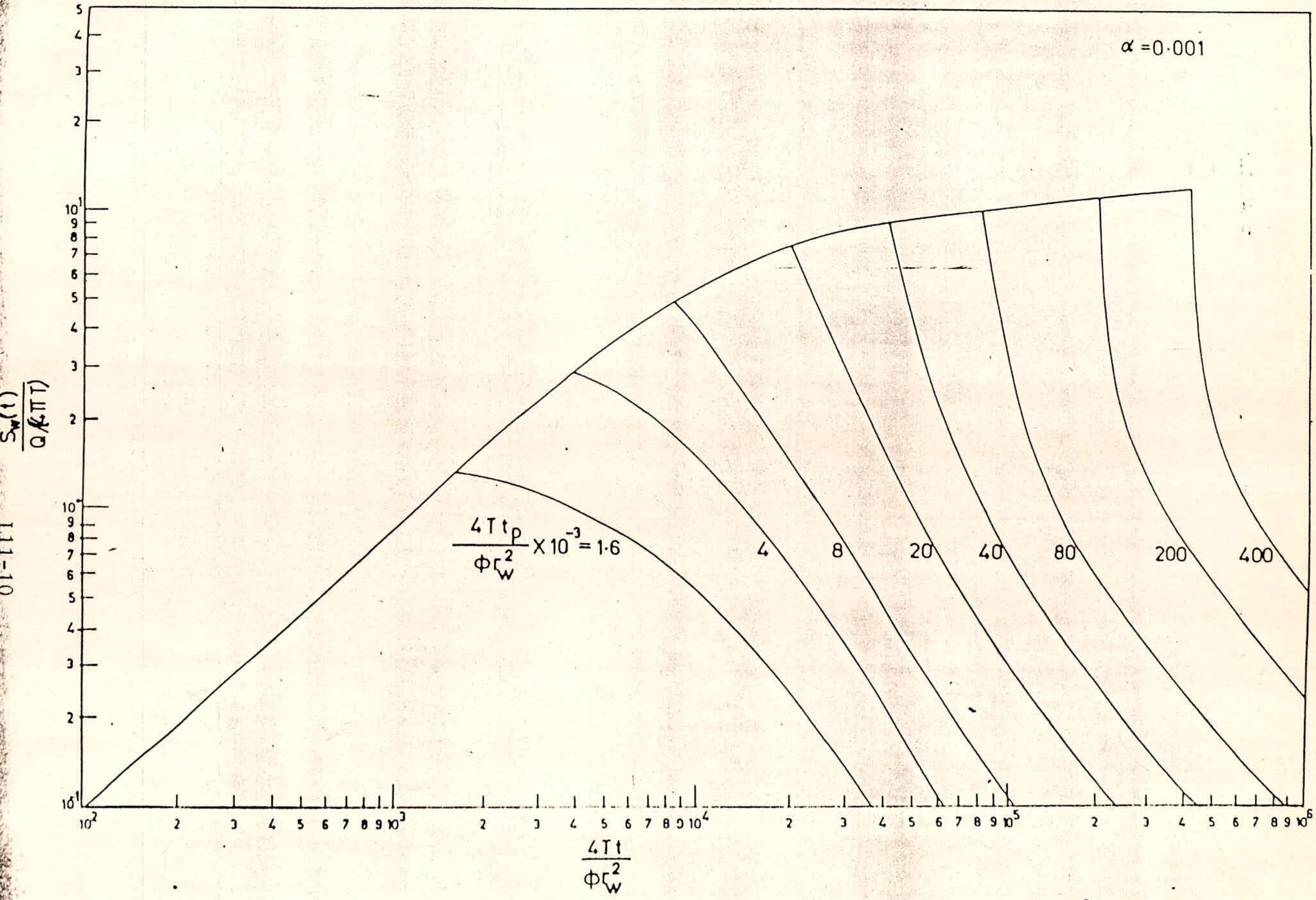


FIG. 2(c) - Family of type curves : $S_w(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-3}$.

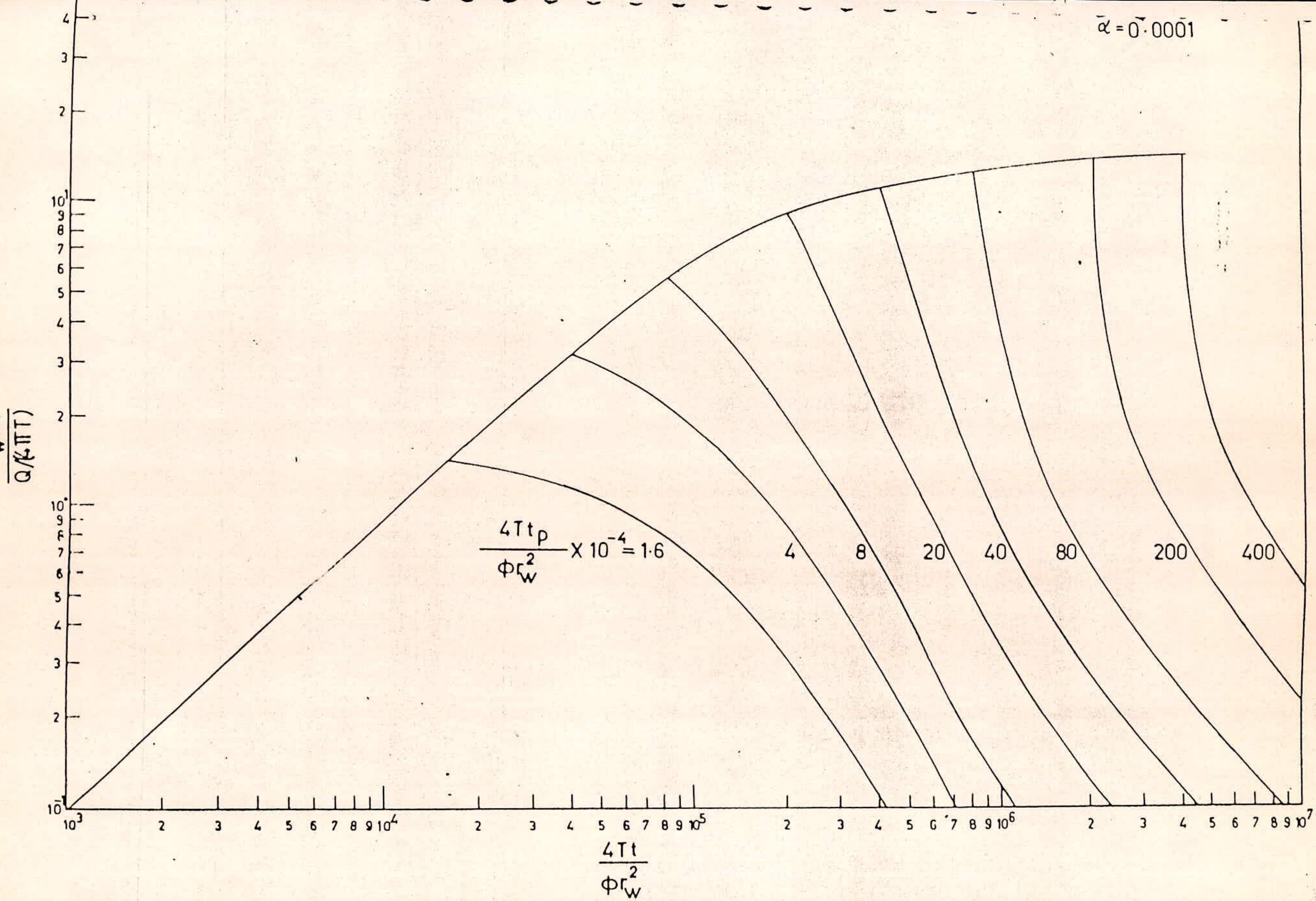


FIG. 2(d) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-4}$.

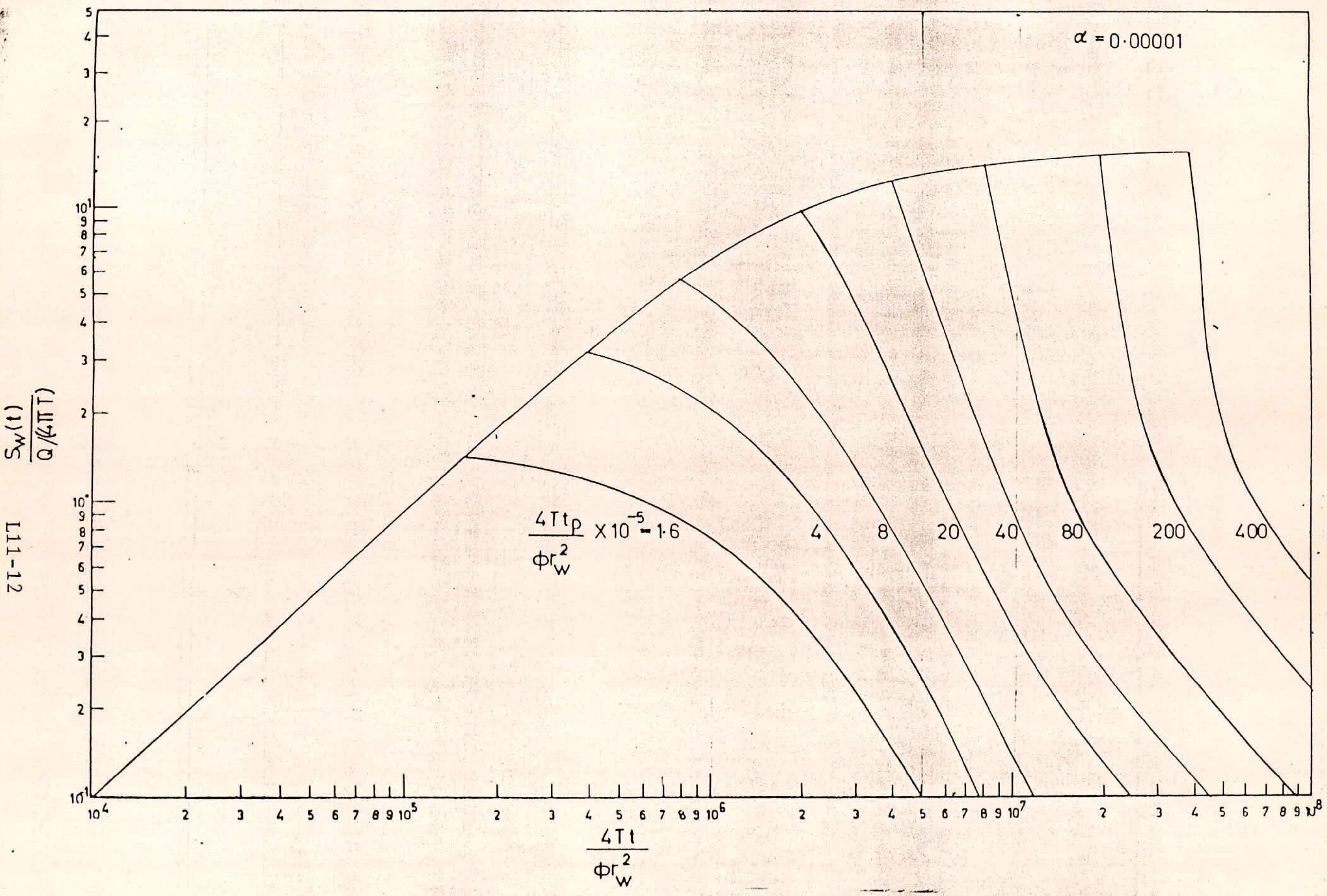


FIG. 2(e) - Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi r_w^2)$ for $\alpha = 10^{-5}$.

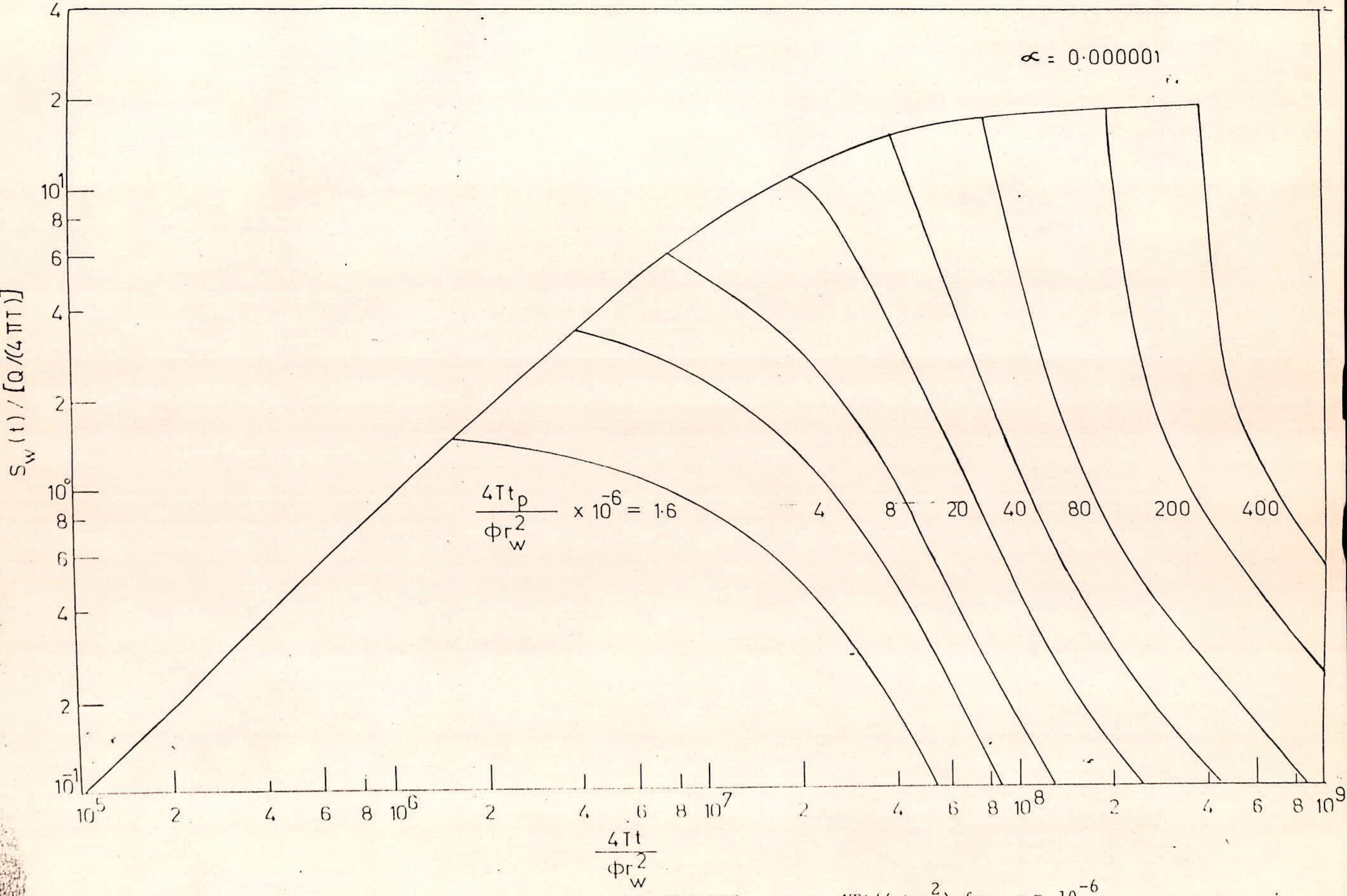


FIG. 2(f) – Family of type curves : $S_W(t)/(Q/4\pi T)$ versus $4Tt/(\phi\, r_w^2)$ for $\alpha = 10^{-6}$.

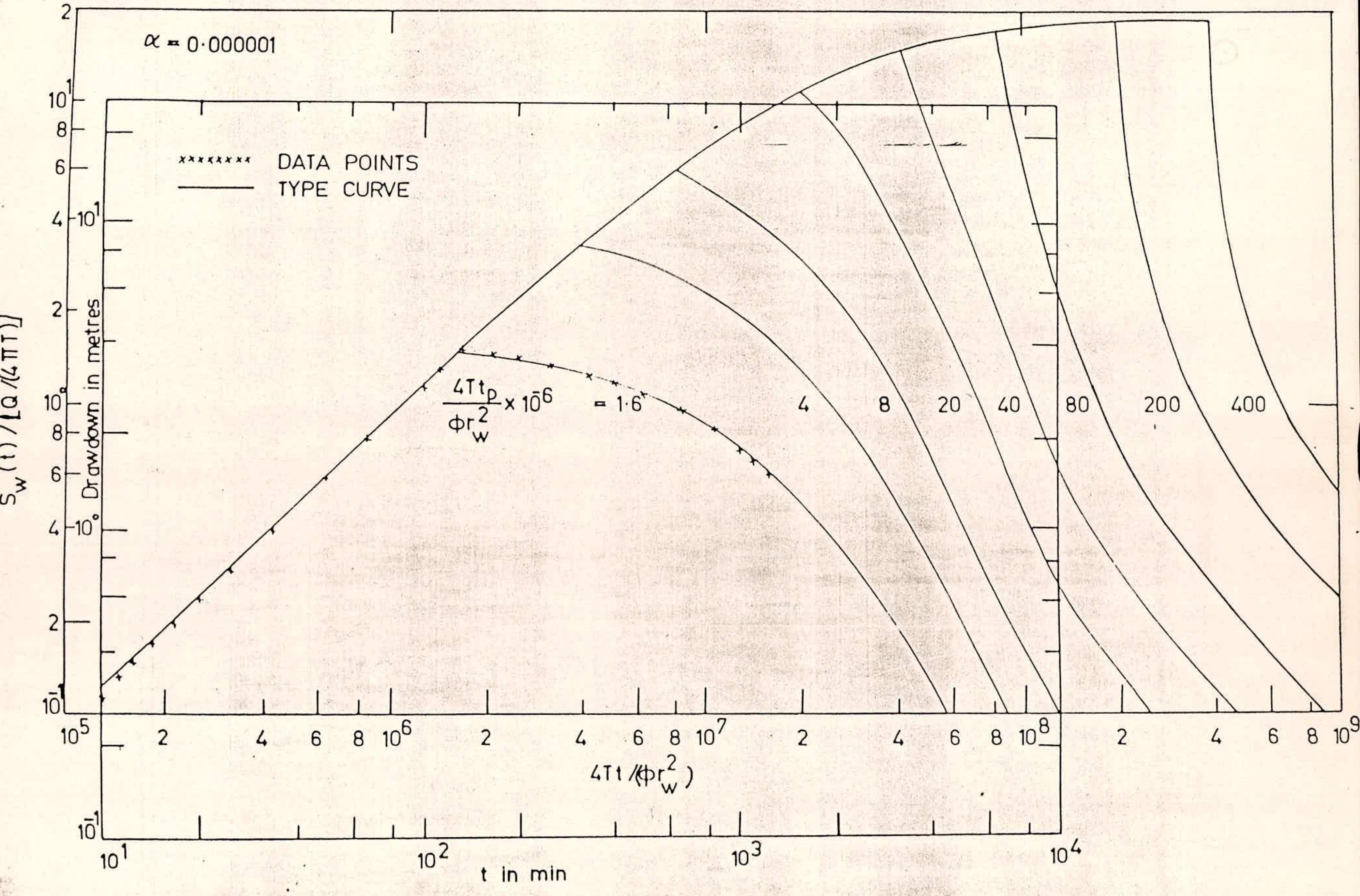


FIG. 2(g) - Matching of pumping test data from large-diameter abstraction well with the type curve

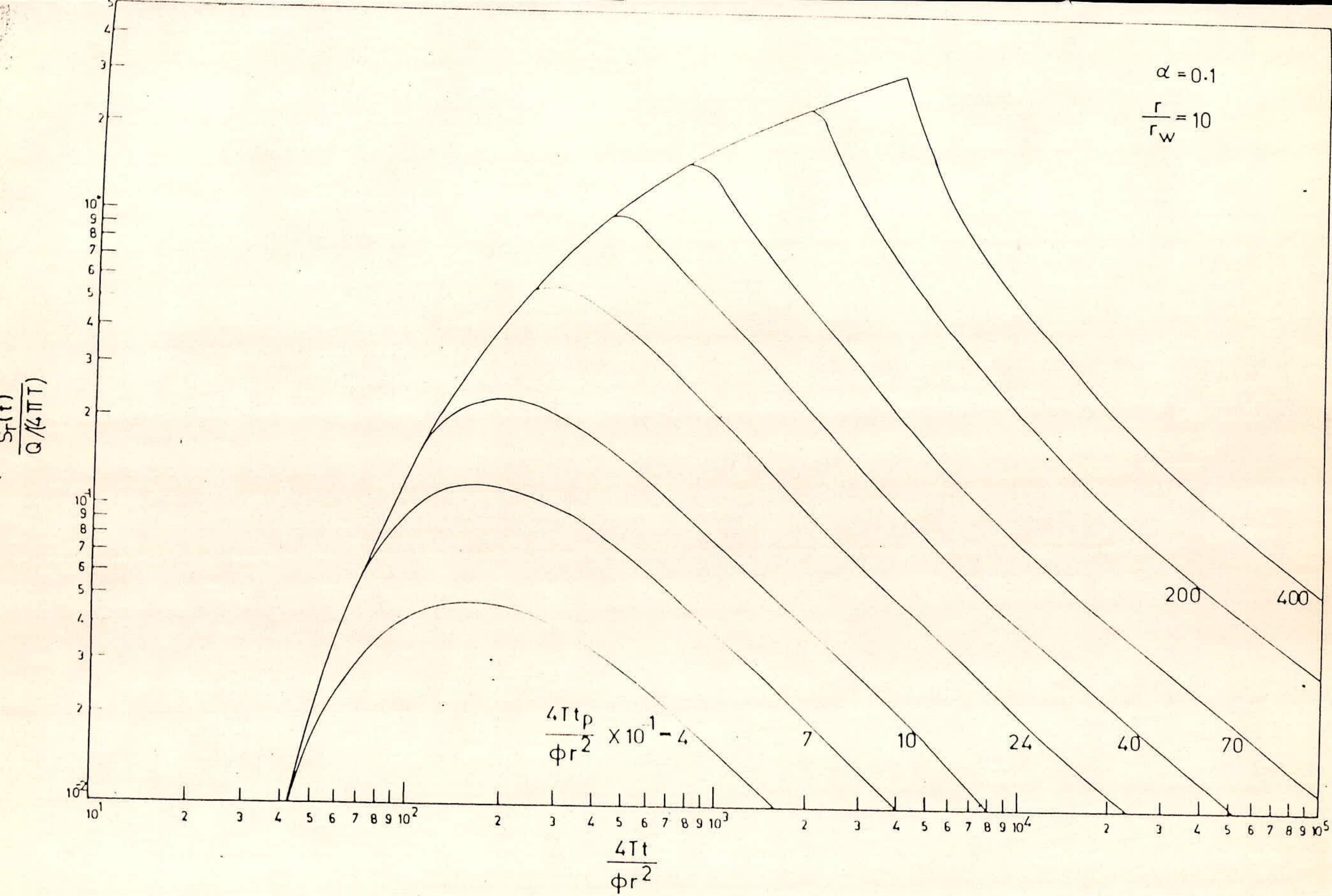
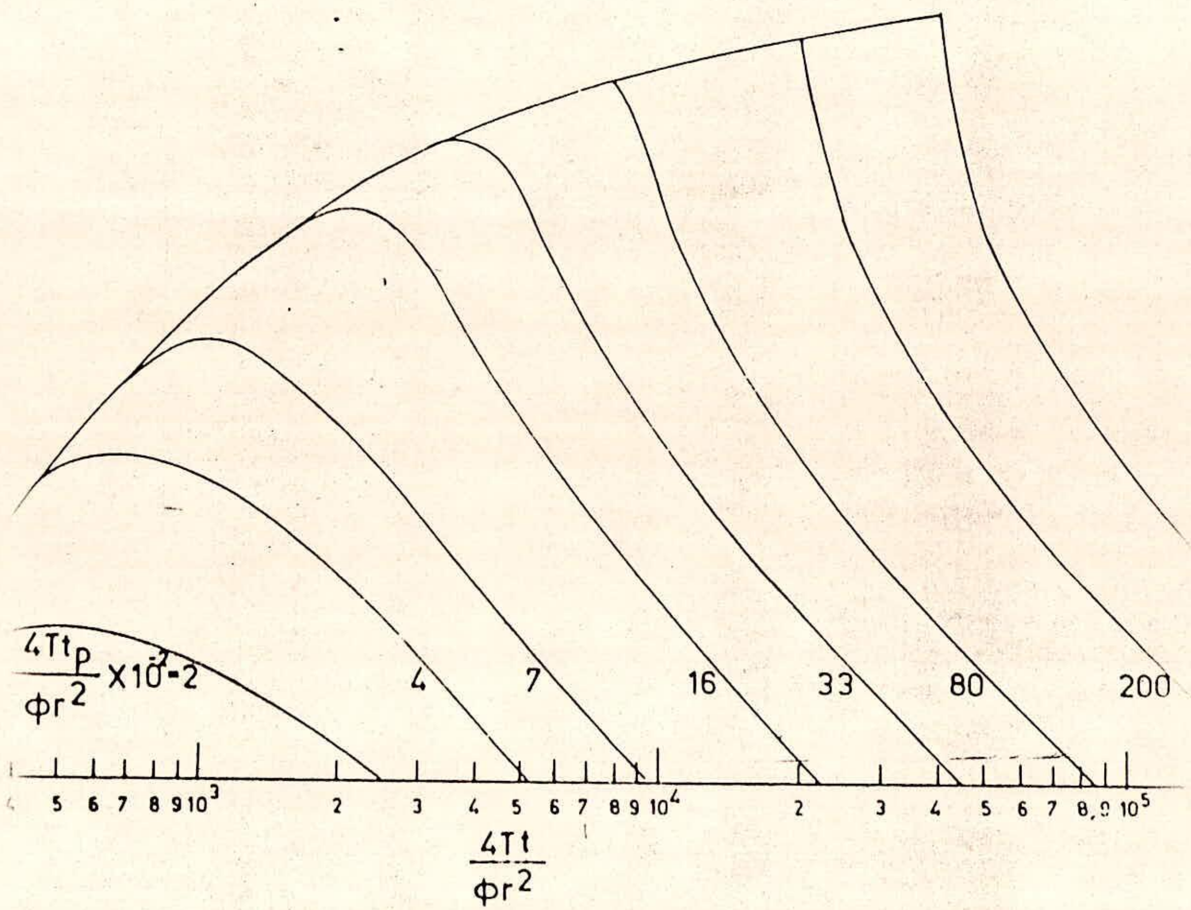


FIG. 3(a) – Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$ and $\alpha = 10^{-1}$.



– Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and α

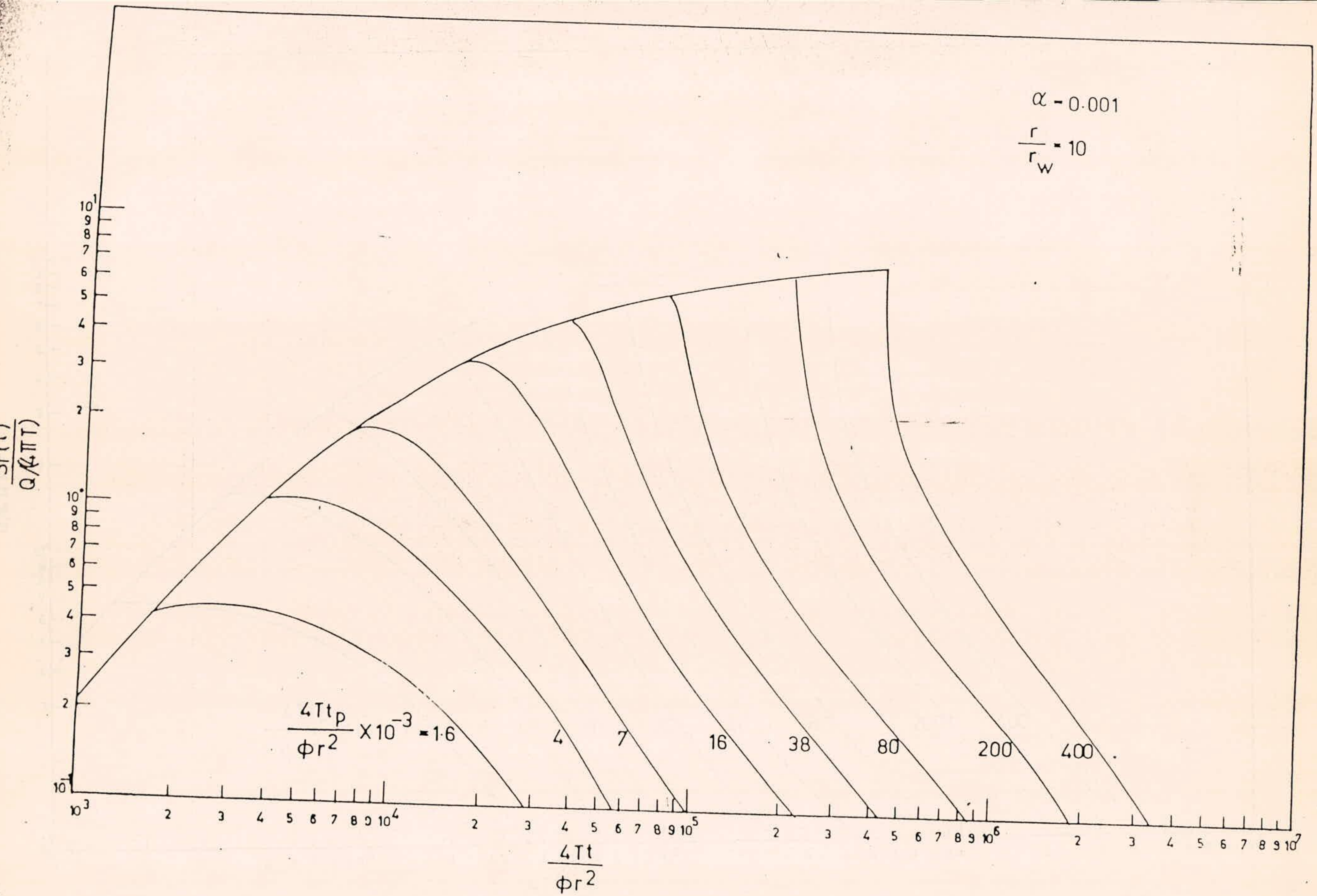


FIG. 3(c) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and $\alpha = 10^{-3}$.

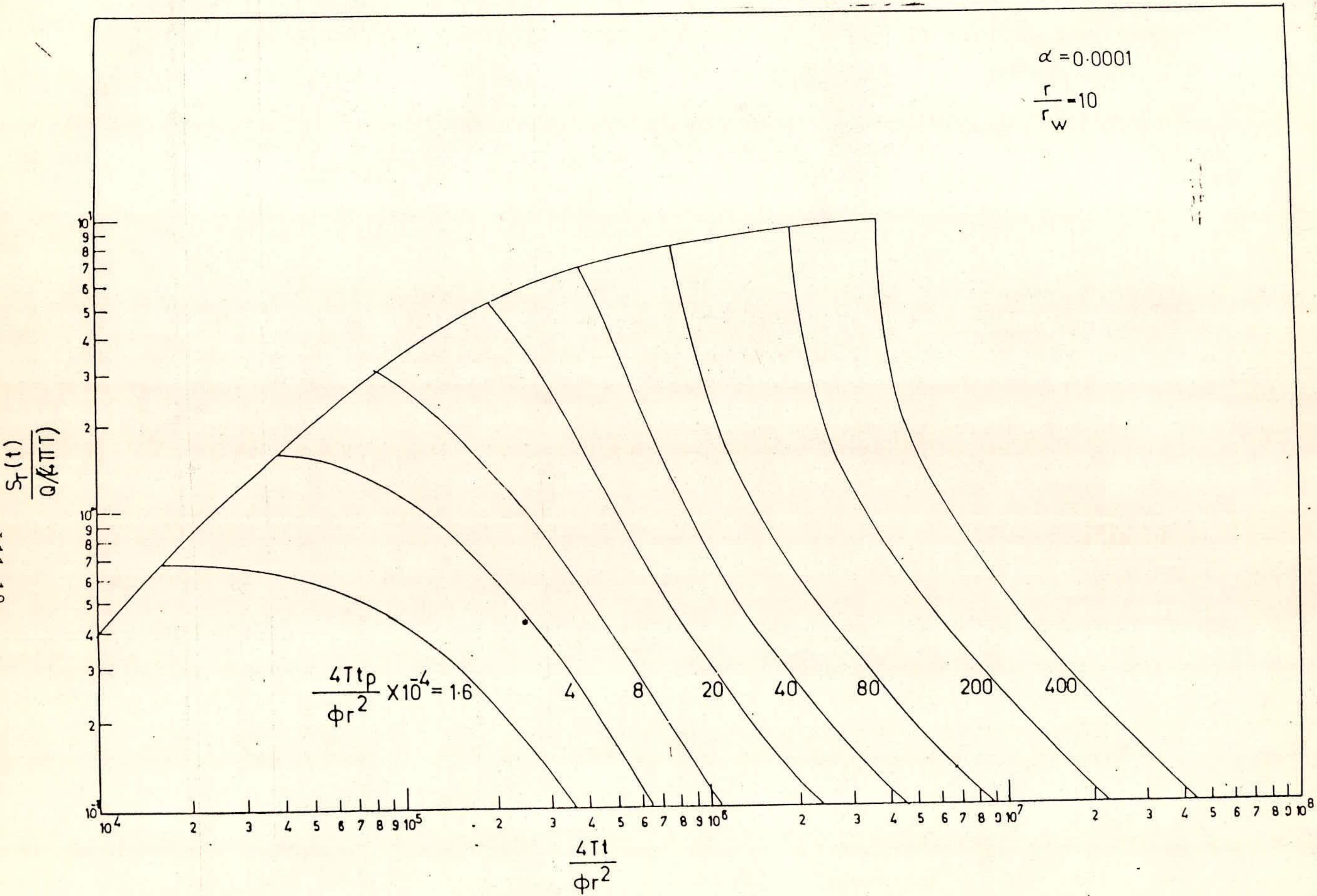


FIG. 3(d) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and $\alpha = 10^{-4}$.

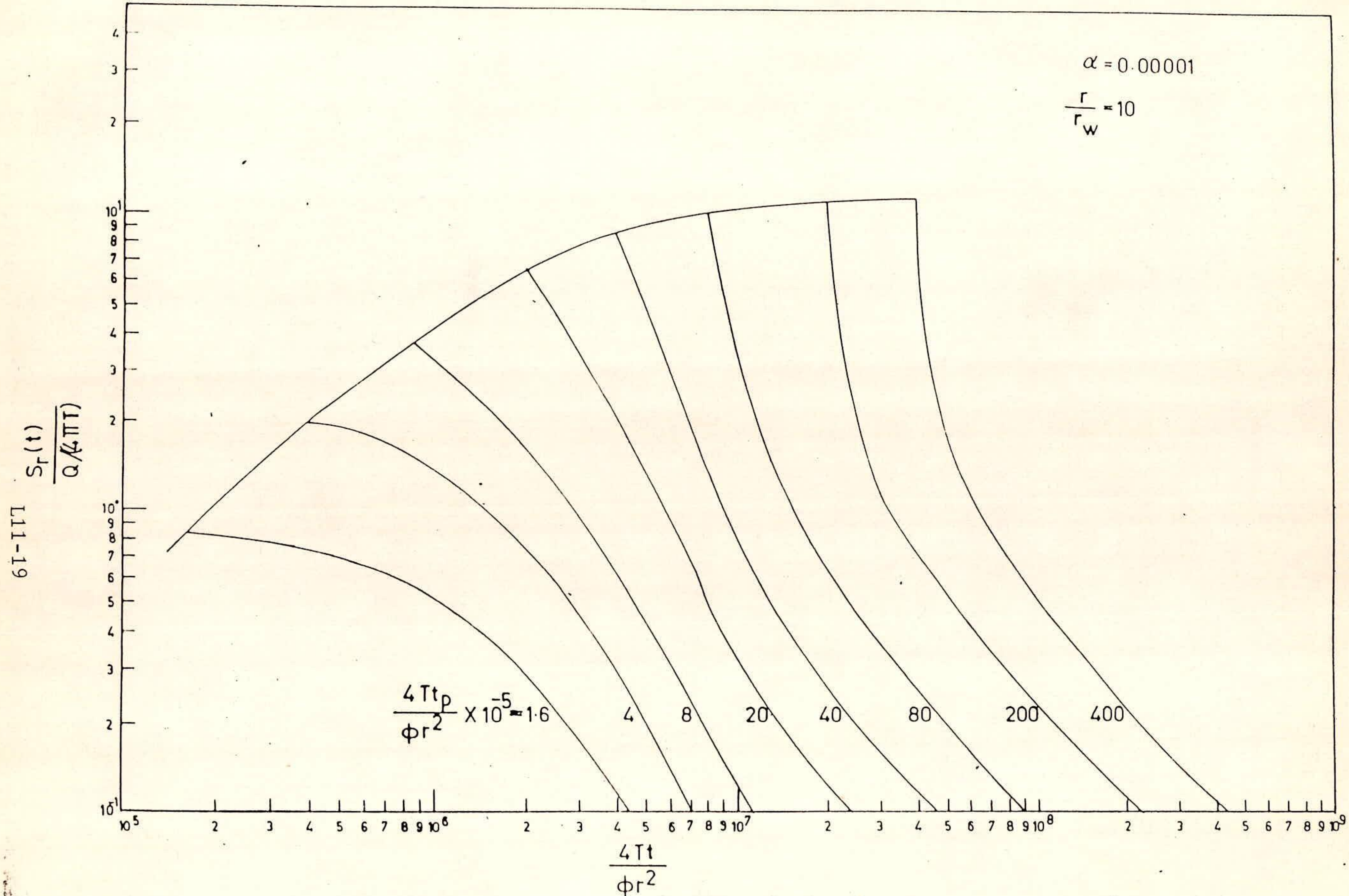


FIG. 3(e) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for $r/r_w = 10$, and $\alpha = 10^{-5}$.

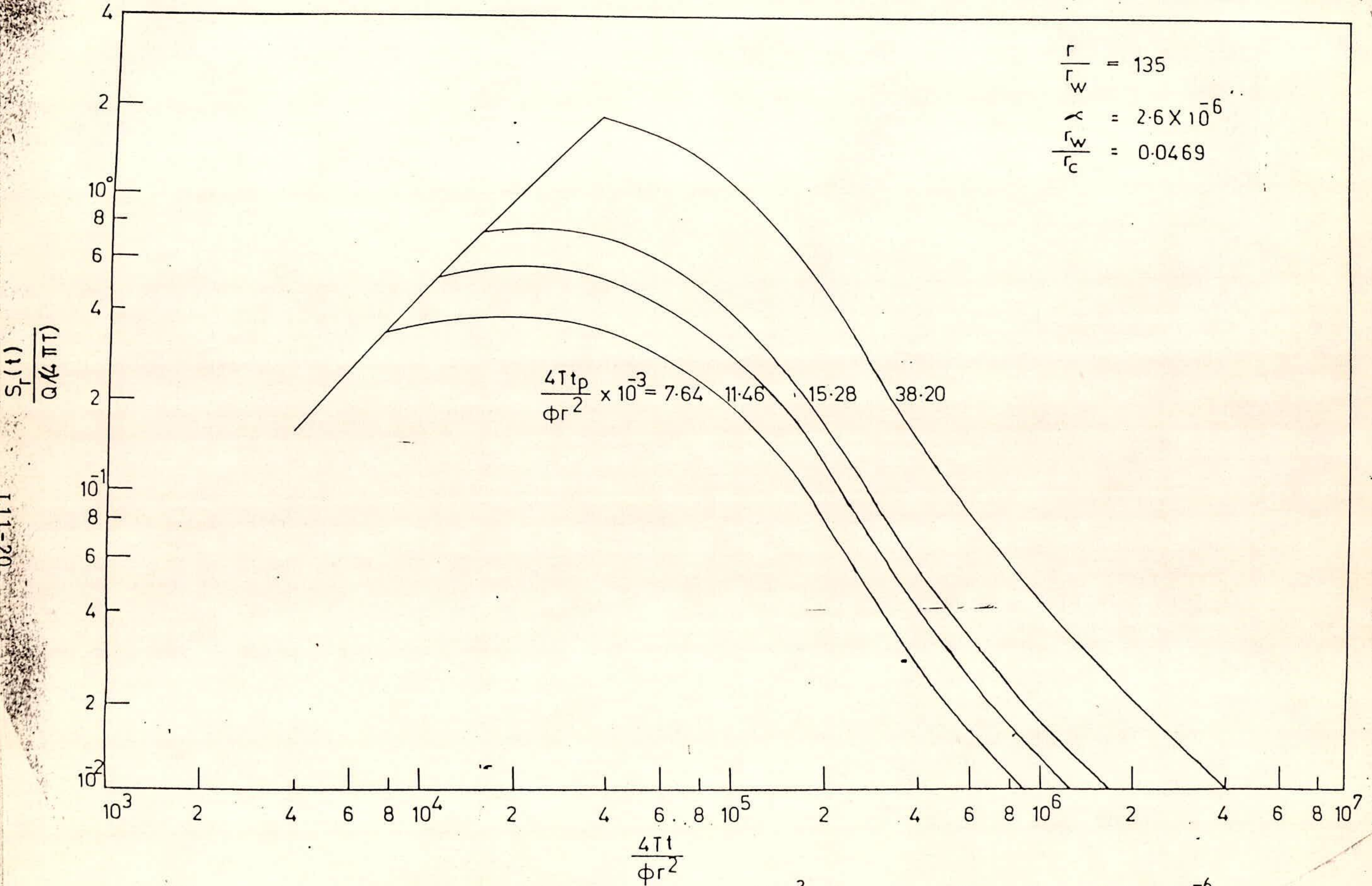


FIG. 3(f) - Variation of $S_r(t)/(Q/4\pi T)$ with $4Tt/(\phi r^2)$ for r/r_w = 135 and $\alpha = 2.6 \times 10^{-6}$.

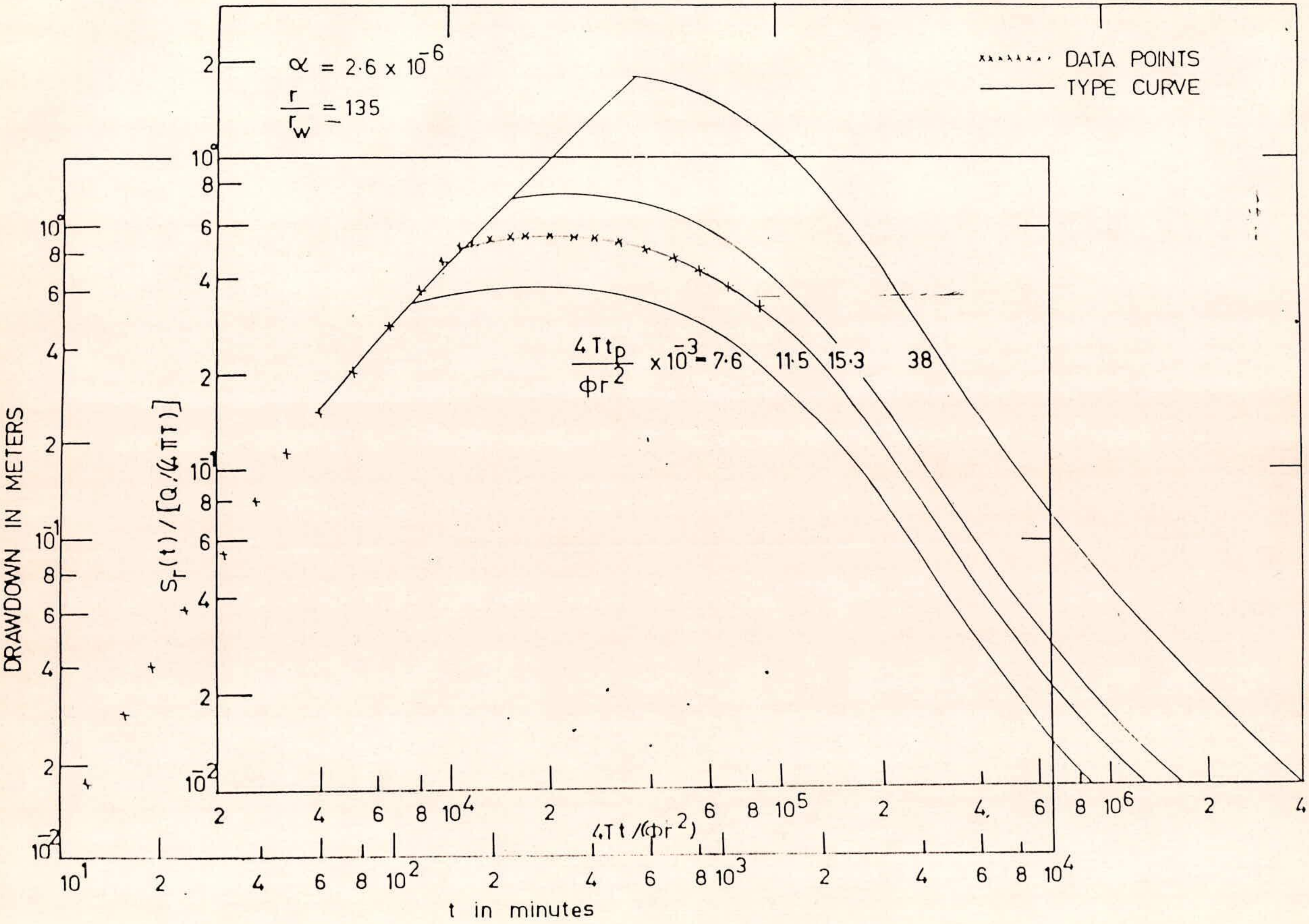


FIG. 3(g) - Matching of pumping test data from an observation well located near a large-diameter abstraction well with the type curve

The family of type curves presented in Figures [2(a)] through [2(g)] and [3(a)] through [3(g)] provide an accurate means of determining parameters of a confined aquifer. Rushton and Holt (1981) have estimated aquifer parameters for a large-diameter well using numerical technique. They have used the drawdown data of abstraction phase, and the recovery phase at the well point and at an observation point in the vicinity of the well. These data have been used for estimating aquifer parameters by curve matching techniques with the help of the type curves presented herein. The time-drawdown curve at the well face matches with the type curve corresponding to α = .000001 and $4Tt_p/(\phi r_w^2) = 1.6\times10^6$ which has been presented in Figure [2(g)]. The duration of pumping, t_p, obtained through matching is 136.9 minutes. The true pumping period reported by Rushton and Holt is 135 minutes. A proper matching ensures an agreement between true duration of pumping and the duration estimated through curve matching. Sufficient recovery data are necessary to have a unique match. The time-drawdown curve at the observation point matches closely with the type curve corresponding to $\alpha = 2.6\times10^{-6}$ and $4Tt_p/(\phi r^2) = 11.5\times10^3$. The matching has been shown in Figure [3(g)]. Table (1) shows the values estimated by Rushton and Holt and the values evaluated with the help of the type curves. The drawdowns at the well face and at the observation point calculated by discrete kernel approach using the estimated aquifer parameters are shown in Figures (4) and (5) respectively. The observed drawdown also have been plotted in these figures. But for the last part of the time-drawdown curve during recovery, the observed and calculated drawdown fairly match.

Table 1 Comparison of Aquifer Parameters Obtained by Numerical Method and Discrete Kernel Approach

Method	Data from	T(m^2/day)	ϕ
Numerical method (Rushton and Holt)	-	24 to 29	0.0006 to 0.001

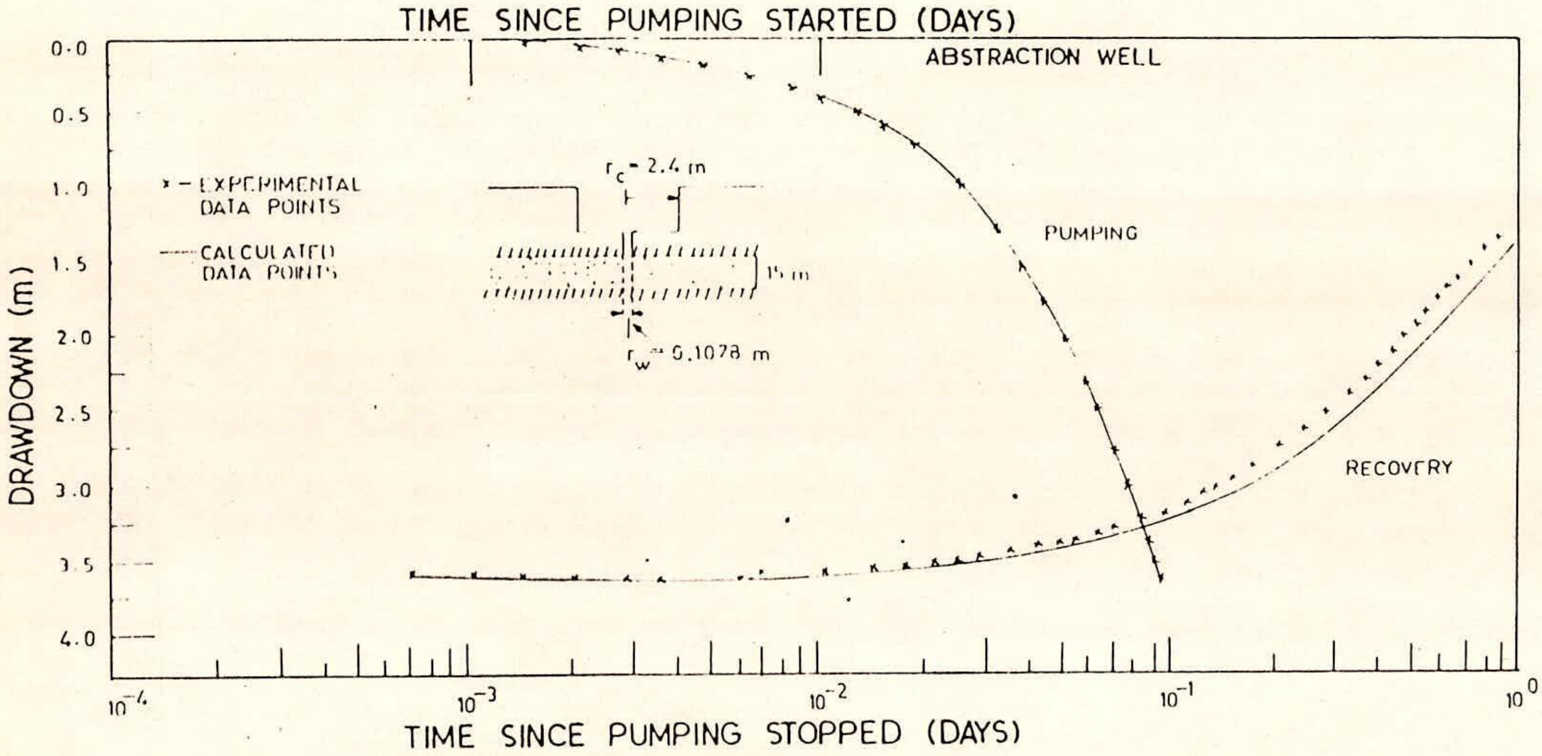


FIG. 4 - Comparison of observed and computed drawdowns at the, abstraction well face

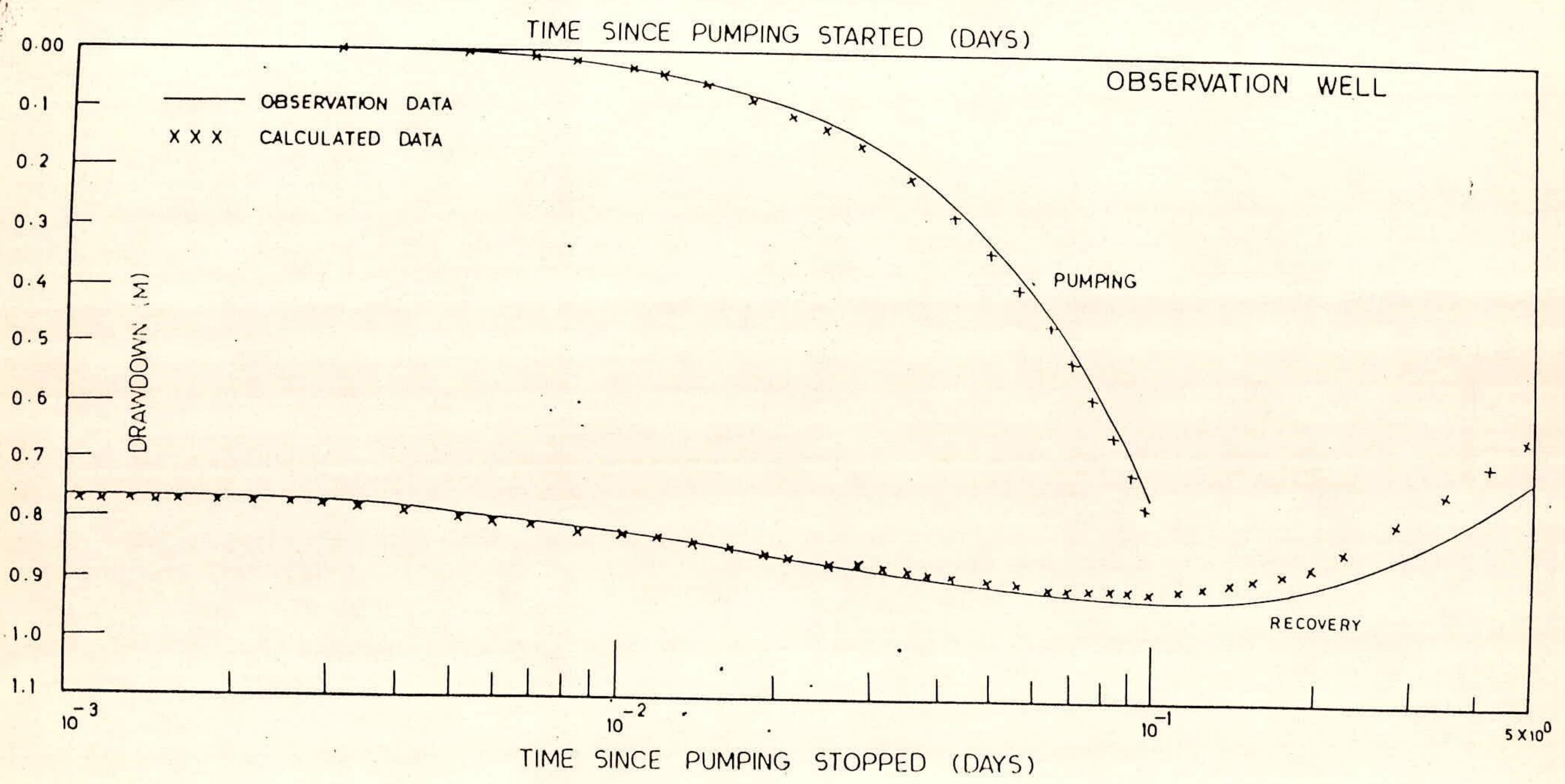


FIG. 5 - Comparison of observed and computed drawdowns at the observation well

Discrete kernel approach	Discharging well	22	0.00045
	Piezometric	30	0.0012

The quantity of water withdrawn from well storage during pumping and the replenishment that occurs during recovery are presented in Figures [6(a)] and [6(b)] for a large-diameter well with r_w/r_c ratio equal to 0.633 and 0.8. The pumping has been discontinued at the end of the 100th time step. The results have been given for two sets of aquifer parameters in which only the value of storage coefficient differs. For example α = 0.1 refers to ϕ = 0.25, and α =0.000001 corresponds to ϕ = 0.0000025. The value of transmissivity has been assumed to be 0.5 m^2 per unit time period. It can be seen from the figure that more water is withdrawn from the storage of that well which has been constructed in the aquifer having a lower storage coefficient.

The variation of $[\sum_{\gamma=m+1}^{n} -(Q_W(\gamma)]/[\sum_{\gamma=1}^{m}(Q_W(\gamma)]$ with $4Tt/(\phi r_w^2)$, is shown in Fig. (7) for different values of ϕ and m. The time t is measured since cessation of pumping. $\sum_{\gamma=1}^{m} Q_W(\gamma)$ represents the quantity of water withdrawn from well storage during pumping. $-\sum_{\gamma=m+1}^{n} Q_W(\gamma)$ represents the quantity of water recouped up to time-step n. It can be seen from the figure (7) that the time of 90 percent recovery of a well storage is nearly same for different durations of pumping. Smaller the value of storage coefficient longer will be the duration for 90 percent recovery. For example from figure (7), for t_p = 6 hours, T = 150 m^2/day, ϕ = 0.01 the value of t for 90 percent recovery of well storage is 8.8×10^{-3} hours. For ϕ = 0.001, for corresponding value is 11.21×10^{-3} hours.

CONCLUSIONS

Based on the study the following conclusions are drawn:

(i) Computation of drawdown during the early stages of pumping and during recovery is sensitive to the time step size.

(ii) Accuracy in the computation of drawdown for any time step size improves with increase in the number of time steps used for computation.

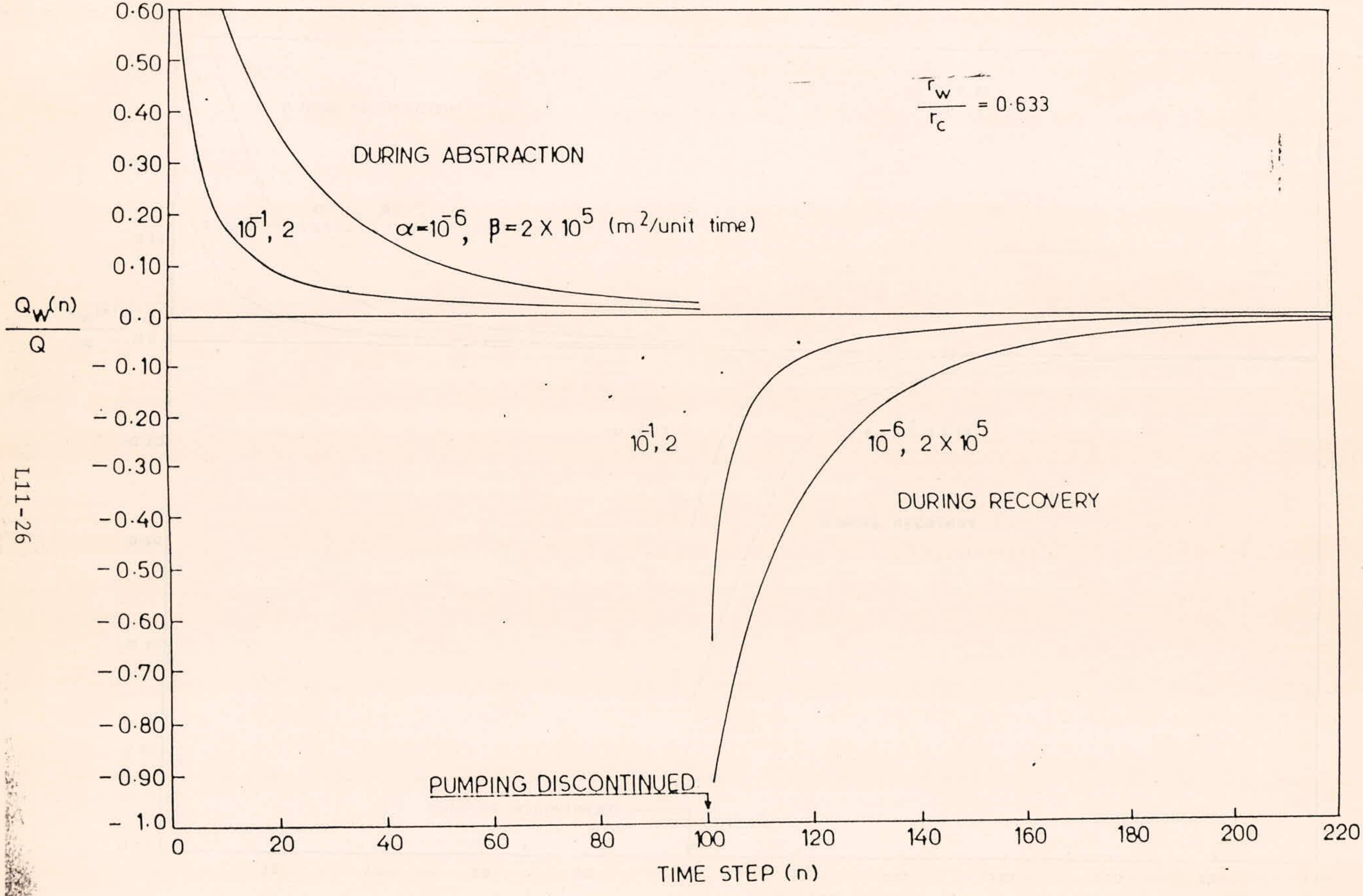


FIG. 6(a) - Variation of contributions from well storage to pumping during abstraction and replenishment during recovery for $r_w/r_c = 0.633$.

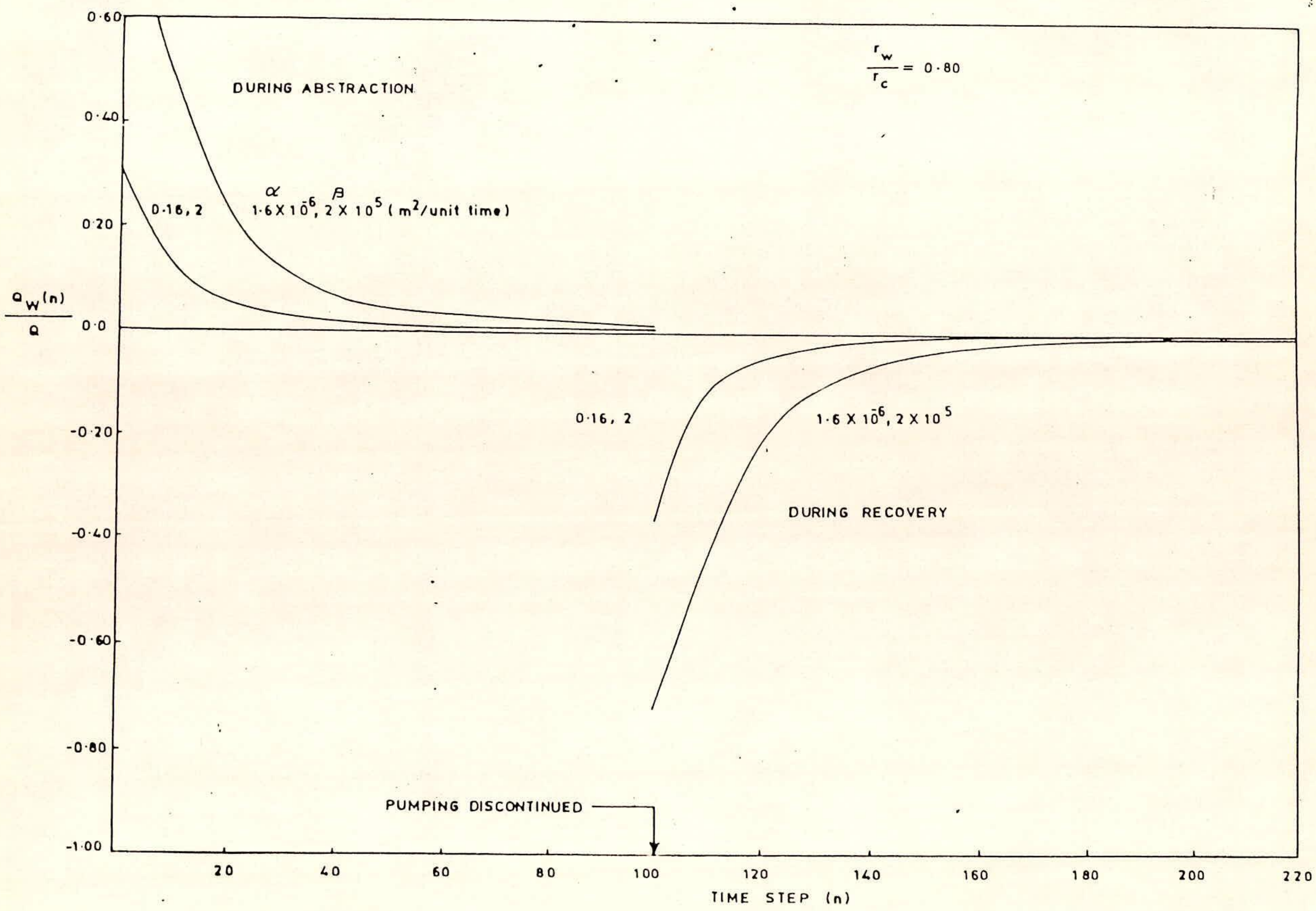


FIG. 6(b) - Variation of contribution from well storage to pumping during abstraction and replenishment during recovery for $r_w/r_c = 0.8$.

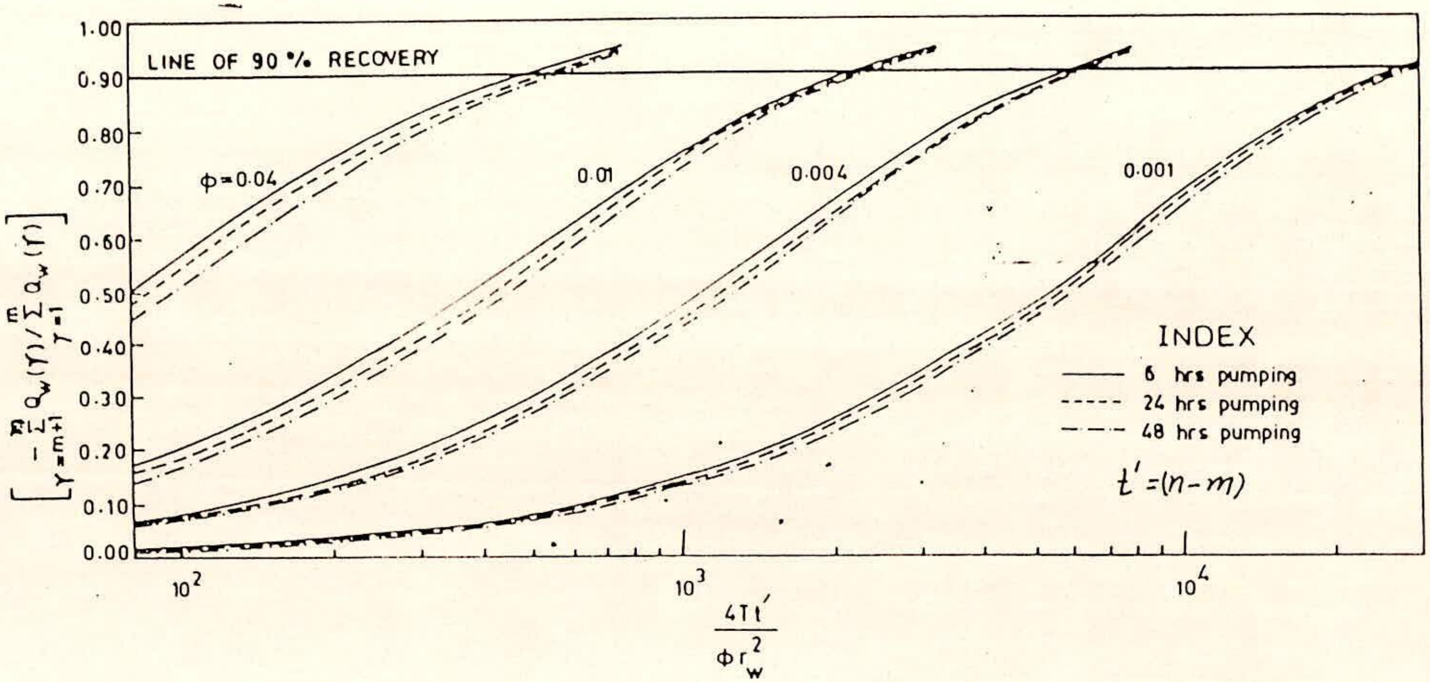


FIG. 7 - Rate of replenishment of well storage with time for ϕ = 0.04. 0.01, 0.004, 0.001 and t_p = 6, 24 and 48 hours.

(iii) Rate of contribution of well storage to pumping and rate of replenishment during recovery are higher for aquifers with lower storage coefficient.

(iv) Calculation of drawdown during recovery using Theis recovery formula is not valid for a large-diameter well.

(v) The type curves which incorporates the response of the aquifer during recovery can provide an accurate means of determining aquifer parameters.

(vi) The duration of pumping, t_p, computed from the non-dimensional time factor $4Tt_p/(\phi r_w^2)$ through type curve matching and its comparison with actual duration of pumping recorded in the aquifer test helps in selecting appropriate type curve for matching.

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