UNSTEADY FLOW TO A MULTI-AQUIFER ARTESIAN WELL

Introduction

In a sedimentary groundwater basin, the occurrence of multiple aquifers separated by confining layers of low and negligible permeability is quite common. A water well in such a basin may have to be constructed tapping more than one aquifer in order to provide the requisite yield. If the aquifers are separated by confining layers of negligible permeability (aquicludes) interaction among the aquifers tapped by the well takes place only through the well screens. Wells, which are open to two or more water bearing strata, which have different hydraulic properties and which are not closely connected except by the well itself, are referred to as multi-aquifer wells (Papadopulos, 1966). A solution for unsteady flow to a well, tapping two confined aquifers having different potentiometric surfaces prior to well construction was obtained by Papadopulos. The Laplace transform technique has been used to obtain an exact expression for head distribution but the solution is intractable for numerical calculation. Subsequently, asymptotic solutions for both head and discharge distribution, amenable to computation which yield results accurate enough for practical application, have been derived by Papadopulos (1966); however, no numerical results were presented. Using the integral transform technique, unsteady flow to a multi-aquifer well, which is open to two aquifers, has also been analysed by Khader & Veerankutty (1975), who have presented numerical results for the contributions of individual aquifers to the total discharge of a well. Using Laplace transform technique, solutions for unsteady flow to a multi-aquifer well which is open to several aquifers (more than two) have been obtained by Wikramaratna (1984).

Wikramaratna in his paper assumed all the aquifers to have equal potentiometric surfaces prior to pumping and has presented results for a two aquifer system. A solution of unsteady flow to a multi-aquifer well which taps two aquifers, by a discrete kernel approach, has been given by Mishra et al. (1985). In their analysis, it has been assumed that the initial potentiometric surfaces prior to pumping are the same in all the aquifers. In the present lecture, unsteady flow to a well which is open to several aquifers where different potentiometric surfaces prevail prior to well construction, has been analysed.

Statement of the Problem

A schematic cross section of a well tapping a number of confined aquifers which are separated by aquicludes is shown in Fig.1. Each of the aquifers is homogeneous, isotropic, and infinite in areal extent. The potentiometric surfaces in the aquifers are different from each other, and prior to the well construction all the aquifers were at rest. After construction the well remains unpumped for a period t_o during which exchange of flow occurs among the aquifers through the well screens owing to the difference in initial heads. The multi-aquifer well is pumped subsequently at a constant rate for a period t_p. It is required to find the following:

(a) the exchange of flows that takes place through the well screen among the aquifers prior to pumping due to the difference in piezometric surfaces,

(b) the contributions of each of the aquifers to the discharge of the well during pumping,

(c) the exchange of flows that takes place among the aquifers after stoppage of pumping, and

(d) drawdowns in the piezometric surfaces at various times after well construction, during and after pumping.

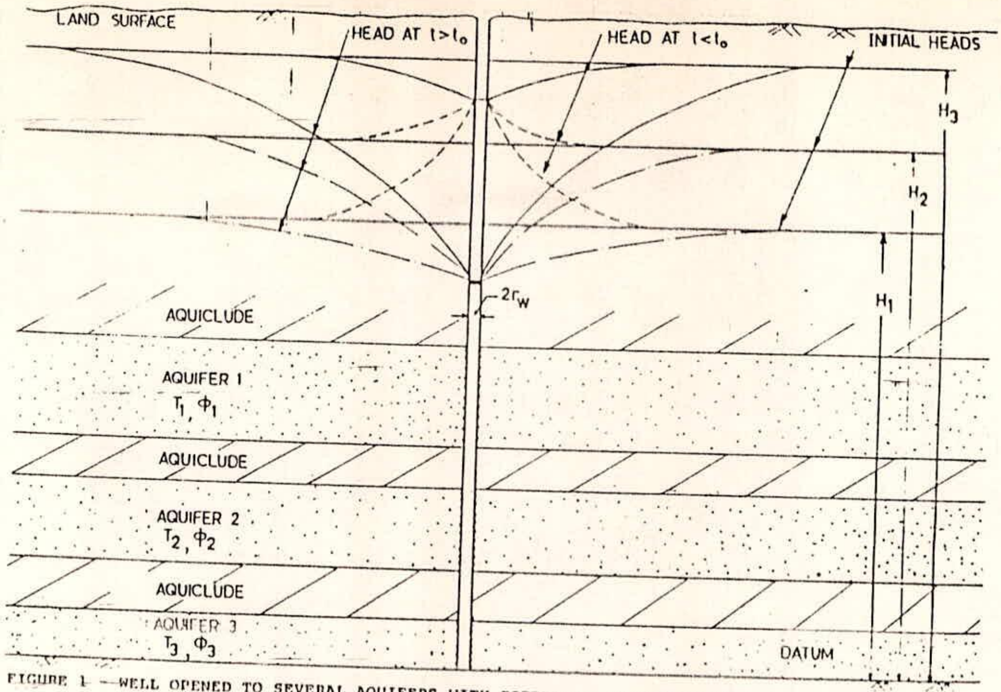

FIGURE 1 — WELL OPENED TO SEVERAL AQUIFERS WITH DIFFERENT POTENTIOMETRIC SURFACE SEPARATED BY AQUICLUDES

Analysis

The following assumptions have been made in the analysis:

(a) at any time, the drawdowns in all the aquifers at the well face are equal,

(b) the time parameter is discrete; within each time step, the abstraction rates of water derived from each of the aquifers and from well storage are separate constants.

The set of differential equations which describes the axially symmetric, radial, unsteady flow in the aquifers is given by

$$\frac{\partial^2 h_i}{\partial r^2} + \frac{1}{r}\frac{\partial h_i}{\partial r} = \frac{\phi_i}{T_i}\frac{\partial h_i}{\partial t} \; ; \; i = 1,2,\ldots m \; ; \; r > r_w \qquad \ldots(1)$$

in which r_w = radius of the well screen ; M = total number of aquifers tapped by the well; h_i = head at a distance r from the well at time t in the i^{th} aquifer; T_i = transmissivity of the i^{th} aquifer, and ϕ_i = storage coefficient of the i^{th} aquifer.

Solution to equation (1) is to be found for the initial conditions

$$h_i(r,0) = H_i \quad ; \; i = 1,2,\ldots,M \qquad \ldots (2)$$

in which H_i is the initial head in the i^{th} aquifer prior to well construction.

The boundary conditions to be satisfied are:

$$h_i(\infty, t) = H_i \qquad \ldots(3)$$

$$h_1(r_w, t) = h_2(r_w, t) = \ldots = h_M(r_w, t) = h_w(t) \qquad \ldots(4)$$

$$\sum_{i=1}^{M} 2\pi r_w T_i \left.\frac{\partial h_i}{\partial r}\right|_{r=r_w} - \pi r_w^2 \frac{\partial h_w(t)}{\partial t} = Q_p(t) \qquad \ldots(5)$$

In equation (5), $h_w(t)$ represents the head in the well and $Q_p(t)$ the pumping rate.

Let the duration t_o and t_p be discretized to m and n units of equal time-steps respectively. Let the pumping rate $Q_p(t)$ be equal to Q. Let $Q_i(I)$ and $Q_w(I)$ be the contributions of the i^{th} aquifer and well storage respectively during the I^{th} unit time period. Using the second assumption, the boundary condition prescribed by equation (5) can be rewritten as:

$$Q_1(I) + Q_2(I) + Q_3(I) + \ldots + Q_M(I) + Q_W(I) = Q_P(I) \qquad \ldots(6)$$

$$\begin{aligned} Q_P(I) &= 0 \text{ for } I \leq m \\ &= Q \text{ for } m < I \leq (m+n) \\ &= 0 \text{ for } I > (m+n) \end{aligned}$$

Had the aquifers been tapped separately, for the initial condition $h_i(r,0) = H_i$, and the boundary condition $h_i(\infty, t) = H_i$, solution to differential equation (1), if unit quantity of water is withdrawn from the i^{th} aquifer at time t = 0, is (Carslaw and Jaeger, 1959)

$$H_i - h_i(r,t) = \frac{1}{4\pi T_i t} \exp\left[-\frac{r^2}{4\beta_i t}\right] \quad ; \; \beta_i = \frac{T_i}{\phi_i} \qquad \ldots(7)$$

Defining a unit impulse kernel

$$k_i(t) = \frac{1}{4\pi T_i t} \exp\left[-\frac{r^2}{4\beta_i t}\right] \qquad \ldots(8)$$

and designating $H_i - h_i(r,t) = s_i(r,t)$, which is the drawdown in the piezometric surface in the i^{th} aquifer, drawdown for variable withdrawal from the i^{th} aquifer can be written in the form:

$$s_i(r,t) = {}_0\int^t Q_i(\tau)k_i(t-\tau)d\tau \qquad \ldots(9)$$

$Q_i(\tau)$ being the withdrawal rate from the i^{th} aquifer at time τ. Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step but varies from step to step, the drawdown at the end of time step I in the i^{th} aquifer at a distance r from the well can be written as (Morel-Seytoux, 1975):

$$s_i(r,I) = \Sigma^I_{\gamma=1}\ \partial_{r,i}(I-\gamma+1)Q_i(\gamma) \qquad \ldots(10)$$

in which the discrete kernel coefficient $\partial_{r,i}(I)$ is defined as

$$\partial_{r,i}(I) = \int_0^1 k_i(I-\tau)d\tau$$
$$= \frac{1}{4\pi T_i}\left[E_1\left\{\frac{r^2}{4\beta_i I}\right\}-E_1\left\{\frac{r^2}{4\beta_i(I-1)}\right\}\right] \qquad \ldots(11)$$

The exponential integral $E_1(x)$ appearing in equation (11) is defined as (Abramowitz and Stegun ,1970):

$$E_1(x)={}_x\int^\infty \exp(-u)u^{-1}du.$$

If M aquifers are tapped by a single well, at any time after the well construction and before the commencement of pumping, the i^{th} aquifer will either loose or gain water depending on whether the composite hydraulic head at the well at that time is less than the initial hydraulic head in the i^{th} aquifer or more. During pumping there will be contribution from each of the aquifers through the respective well screen.

If $Q_i(\gamma)$, $\gamma = 1,2,\ldots,I$, are the contributions by i^{th} aquifer, drawdown at the well face in the i^{th} aquifer at the end of time step I is given by

$$s_i(r_w,I) = \Sigma^I_{\gamma=1}\ Q_i(\gamma)\ \partial_{rwi}(I-\gamma+1) \qquad \ldots(12)$$

in which the discrete kernel coefficient $\partial_{rwi}(i)$ is defined as:

$$\partial_{rwi}(I) = \frac{1}{4\pi T_i}\left[E_1\left\{\frac{r_w^2}{4\beta_i I}\right\}-E_i\left\{\frac{r_w^2}{4\beta_i(I-1)}\right\}\right] \qquad \ldots(13)$$

Thus the head at the well face in the i^{th} aquifer at the end of time step I can be expressed by the relation

$$h_i(r_w, I) = H_i - \Sigma_{\gamma=1}^{I} Q_i(\gamma)\, \partial_{rwi}(I-\gamma+1) \qquad \ldots(14)$$

Since the heads at the well face at the end of any time step I in all the aquifers are equal, therefore,

$$H_1 - \Sigma_{\gamma=1}^{I} Q_1(\gamma)\, \partial_{rw1}(I-\gamma+1) = H_2 - \Sigma_{\gamma=1}^{I} Q_2(\gamma)\partial_{rw2}(I-\gamma+1)$$

$$= H_3 - \Sigma_{\gamma=1}^{I} Q_3(\gamma)\, \partial_{rw3}(I-\gamma+1) = H_M - \Sigma_{\gamma=1}^{I} Q_M(\gamma)\partial_{rwM}(I-\gamma+1) \qquad \ldots(15)$$

The above set of equations can be written as:

$$-Q_i(I)\partial_{rw1}(1) + Q_2(I)\partial_{rw2}(1) = H_2 - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_2(\gamma)\partial_{rw2}(I-\gamma+1) \qquad \ldots(16)$$

$$-Q_1(I)\partial_{rw1}(1) + Q_3(I)\partial_{rw3}(1) = H_3 - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_3(\gamma)\partial_{rw3}(I-\gamma+1) \qquad \ldots(17)$$

$$-Q_1(I)\partial_{rw1}(1) + Q_M(I)\partial_{rwM}(1) = H_M - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_M(\gamma)\partial_{rwM}(I-\gamma+1) \qquad \ldots(18)$$

Let H_{max} be the maximum value of H_i. Head in the well in consequence to abstraction from well storage is given by

$$h_w(I) = H_{max} - \Sigma_{\gamma=1}^{I} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad \ldots(19)$$

$h_w(I)$ is equal to $h_i(r, I)$ for all values of i and I at $r = r_w$. Using this relation one more equation can be written as:

$$H_1 - \Sigma_{\gamma=1}^{I} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) = H_{max} - \Sigma_{\gamma=1}^{I} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad \ldots(20)$$

Rearranging

$$-Q_1(I)\vartheta_{rw1}(1) + \frac{Q_w(I)}{\pi r_w^2} = H_{max} - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\vartheta_{rw1}(I-\gamma+1)$$

$$-\Sigma_{\gamma=1}^{I} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad ...(21)$$

In matrix notation, equations (6), (16), (17), (18) and (21) can be written as

$$[A]\ [B] = [C] \qquad ...(22)$$

in which

$$[A]= \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -\vartheta_{rw1}(1) & \vartheta_{rw2}(1) & 0 & \cdots & 0 & 0 \\ -\vartheta_{rw1}(1) & 0 & \vartheta_{rw1}(1) & \cdots & 0 & 0 \\ \vdots & & & & & \\ -\vartheta_{rw1}(1) & 0 & 0 & \cdots & \vartheta_{rwM}(1) & 0 \\ -\vartheta_{rw1}(1) & 0 & 0 & \cdots & 0 & 1/(r_w^2\pi) \end{bmatrix}$$

$$[B]= [Q_1(I), Q_2(I), Q_3(I), \ldots, Q_M(I), Q_w(I)]'$$

$$[C]= \begin{bmatrix} Q_p(I) \\ H_2 - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\vartheta_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_2(\gamma)\vartheta_{rw2}(I-\gamma+1) \\ H_3 - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\vartheta_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_3(\gamma)\vartheta_{rw3}(I-\gamma+1) \\ H_M - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\vartheta_{rw1}(I-\gamma+1) - \Sigma_{\gamma=1}^{I-1} Q_M(\gamma)\vartheta_{rwM}(I-\gamma+1) \\ H_{max} - H_1 + \Sigma_{\gamma=1}^{I-1} Q_1(\gamma)\vartheta_{rw1}(I-\gamma+1) - 1/(r_w^2\pi)\Sigma_{\gamma=1}^{I-1} Q_w(\gamma) \end{bmatrix}$$

In particular, for time step 1

$$[C] = [0,\ H_2-H_1,\ H_3-H_1,\ \ldots H_M-H_1,\ H_{max}-H_1]'$$

$Q_1(I)$, $Q_2(I)$, $Q_3(I)$, $Q_M(I)$ and $Q_w(I)$ can be solved in succession starting from time step 1 using the relation

$$[B] = [A]^{-1} [C] \qquad \ldots(23)$$

Knowing Q (I) values, the drawdown in the i^{th} aquifer can be calculated using equation (10).

Results and Discussion

Though the analysis has been done for a multi-aquifer well which is open to any number of aquifers, results have been presented for a well which is open to three confined aquifers only. The duration starting from completion of the well to commencement of pumping and the pumping period are discretized to m and n units with equal time steps. For known values of T_1, ϕ_1, T_2, ϕ_2, T_3, ϕ_3, and r_w, the discrete kernel coefficients, $\partial_{rwi}(I)$, are generated making use of equation (13) for different integer values of I. For Known values of m, n, H_i and $Q_P(I)$, the values of $Q_i(I)$ and $Q_w(I)$ have been found in succession, starting from step 1.

The variations of $Q_1(t)/[\bar{T}(H_{max}-H_1)]$ and $Q_2(t)/[\bar{T}(H_{max}-H_2)]$ with $4\bar{T}t/(\bar{\phi}r_w^2)$ for $Q_P(t)=0$ are shown in Fig.2 for $(H_{max}-H_1)/(H_{max}-H_2)=1$, $T_1:T_2:T_3 = 3:2:1$. $\phi_1:\phi_2:\phi_3 := 3:2:1$. $\bar{T}$ and $\bar{\phi}$ are the arithmetic mean values of of transmissivities and storage coefficients respectively. These results pertain to the case where (i) all the aquifers have equal hydraulic diffusivity, and (ii) the initial hydraulic heads in the first and in the second aquifer are the same and less than that of the third aquifer. It is found from Fig.2 that when the aquifers have equal hydraulic diffusivity values and have same potentiometric surfaces, the flow quantities received by them from the third aquifer are in proportion to their respective transmissivity values. In other words if aquifer 1 and aquifer 2 have equal hydraulic diffusivity and have same initial potentiometric surface and if they are receiving water from the third aquifer in which the potentiometric surface is at a higher level, then at all times $Q_1(t)/Q_2(t)$ values are equal to T_1/T_2.

The response of a multi-aquifer system to pumping can conveniently be decomposed to the following two parts:

Part 1: Response due to difference in potentiometric surfaces as existing in the field but $Q_p(t) = 0$.

Part 2: Response due to pumping when all the initial hydraulic heads are equal to the lowest initial hydraulic head i.e.:

$$Q_p(t) = \begin{cases} 0 & \text{for } 0 < t \leq t_o \\ Q & \text{for } t_o < t \leq (t_o + t_p) \\ 0 & \text{for } t > (t_o + t_p) \end{cases}$$

The response of an aquifer corresponding to part 1 and 2 when added would give its response to the pumping for the case when the potentiometric surfaces are at different levels. This can be seen from the results presented in Tables 1,2 ,and 3.

Conclusion

A procedure that uses a unit response function has been described to analyse unsteady flow to a well opened to several aquifers, which are separated by aquicludes and in which the potentiometric surfaces are at different levels. The solution is tractable for numerical calculation. The contributions of each aquifer and composite hydraulic head at the well point have been evaluated when a well is open to three confined aquifers. Aquifers having the same initial hydraulic head and equal hydraulic diffusivity, receive water from the aquifer of higher potentiometric surface in proportion to their respective transmissivity values.

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References

Abramowitz, M & Stegun, I.A. (1970) Handbook of Mathematical Functions Dover, New York.

Carslaw, H.S. & Jaeger, J.C. (1959) Conduction of Heat in Solids. Oxford University Press, New York.

Khader, A. & Veerankutty, M.K. (1975) Transient well flow in an unconfined-confined aquifer system. J. Hydrol. 26, 123-140.

Mishra, G.C., Nautiyal, M.D. & Chandra, S. (1985) Unsteady flow to a well tapping two aquifers separated by an aquiclude. J.Hydrol. 19, 357-369.

Morel-Seytoux, H.J. (1975) Optimal legal conjunctive operation of surface and ground water. Proc. Second World Congress, Int. Water Resour. Assoc. (New Delhi) 4, 119-129.

Papadopulos, I.S. (1966) Non-steady flow to multi aquifer well. J. Geophys. ERes. 71, 4791-4797.

Wikramaratna, R.S. (1984) An analytical solution for the effects of abstraction from a multiple-layered confined aquifer with no cross flow. Wat. Resour. Res. 20(8), 1067-1074.

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Table 1 Contributions of each aquifer and head at the well point evaluated for $T_1 = 500\ m^2\ day^{-1}$, $T_2 = 400\ m^2\ day^{-1}$, $T_3 = 300\ m^2 day^{-1}$, $\phi_1 = 0.003$, $\phi_2 = 0.002$, $\phi_3 = 0.0001$, $r_w = 0.1m$, $H_1 = 200.0$ m, $H_2 = 201.0$ m, $H_3 = 202.00$ m, and $Q_P(t) = 0$

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$h_i(r_w,t)$ (m)
1	-283.359	59.668	223.691	200.7897
2	-272.993	56.987	216.005	200.7912
3	-267.301	55.533	211.768	200.7920
5	-260.480	53.805	206.675	200.7930
10	-251.782	51.622	200.159	200.7943
11	-250.634	51.338	199.295	200.7945
12	-249.593	51.079	198.514	200.7946
14	-247.771	50.627	197.144	200.7949
16	-246.216	50.244	195.971	200.7951
18	-244.859	49.909	194.950	200.7953
20	-243.658	49.612	194.046	200.7955
21	-243.107	49.476	193.631	200.7956
25	-241.156	48.996	192.160	200.7959
35	-237.479	48.097	189.382	200.7964
40	-236.051	47.749	188.302	200.7966

Table 2 Contributions of each aquifer and head at the well point evaluated for $T_1 = 500\ m^2\ day^{-1}$, $T_2 = 400\ m^2\ day^{-1}$, $T_3 = 300\ m^2\ day^{-1}$, $\phi_1 = 0.003$, $\phi_2 = 0.002$, $\phi_3 = 0.0001$, $r_w = 0.1$ m, $H_1=H_2=H_3=200.0$ m, $t_o = 10$ days, $t_p = 10$ days, $Q = 1000\ m^3\ day^{-1}$

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$h_i(r_w,t)$ (m)
10	0	0	0	200
11	433.511	343.237	223.250	198.7966
12	432.923	342.907	224.170	198.7505
14	432.377	342.601	225.021	198.7044
16	432.078	342.429	225.493	198.6774
18	431.870	342.315	225.815	198.6582
20	431.712	342.226	226.062	198.6434
21	-1.86490	-1.04855	2.91547	199.8404
25	- .77452	- .43699	1.21180	199.9269
35	- .33449	- .18934	.52395	199.9660
40	- .25906	- .14970	.40880	199.9730

Table 3 Contributions of each aquifer and head at the well point edvaluated for $T_1 = 500\ m^2\ day^{-1}$, $T_2 = 400\ m^2\ day^{-1}$, $T_3 = 300\ m^2\ day^{-1}$, $\phi_1 = 0.003$, $\phi_2 = 0.002$, $\phi_3 = 0.0001$, $r_w = 0.1$ m, $H_1 = 200.0$ m, $H_2 = 201.0$ m, $H_3 = 202.00$ m, and $Q_P(t) = 1000\ m^3\ day^{-1}$, $t_o=10$ days, $t_P=10$ days

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$h_i(r_w,t)$ (m)
1	-283.359	59.668	223.691	200.7897
2	-272.993	56.987	216.005	200.7912
3	-267.301	55.533	211.768	200.7920
5	-260.480	53.805	206.675	200.7930
10	-251.782	51.622	200.159	200.7943
11	182.879	394.576	422.542	199.5911
12	183.332	393.988	422.678	199.5451
14	184.608	393.229	422.163	199.4993
16	185.862	392.674	421.464	199.4725
18	187.012	392.221	420.767	199.4536
20	188.056	391.839	420.105	199.4389
21	-244.971	48.429	196.544	200.6360
25	-241.929	48.560	193.369	200.7228
40	-236.308	47.600	188.708	200.7697

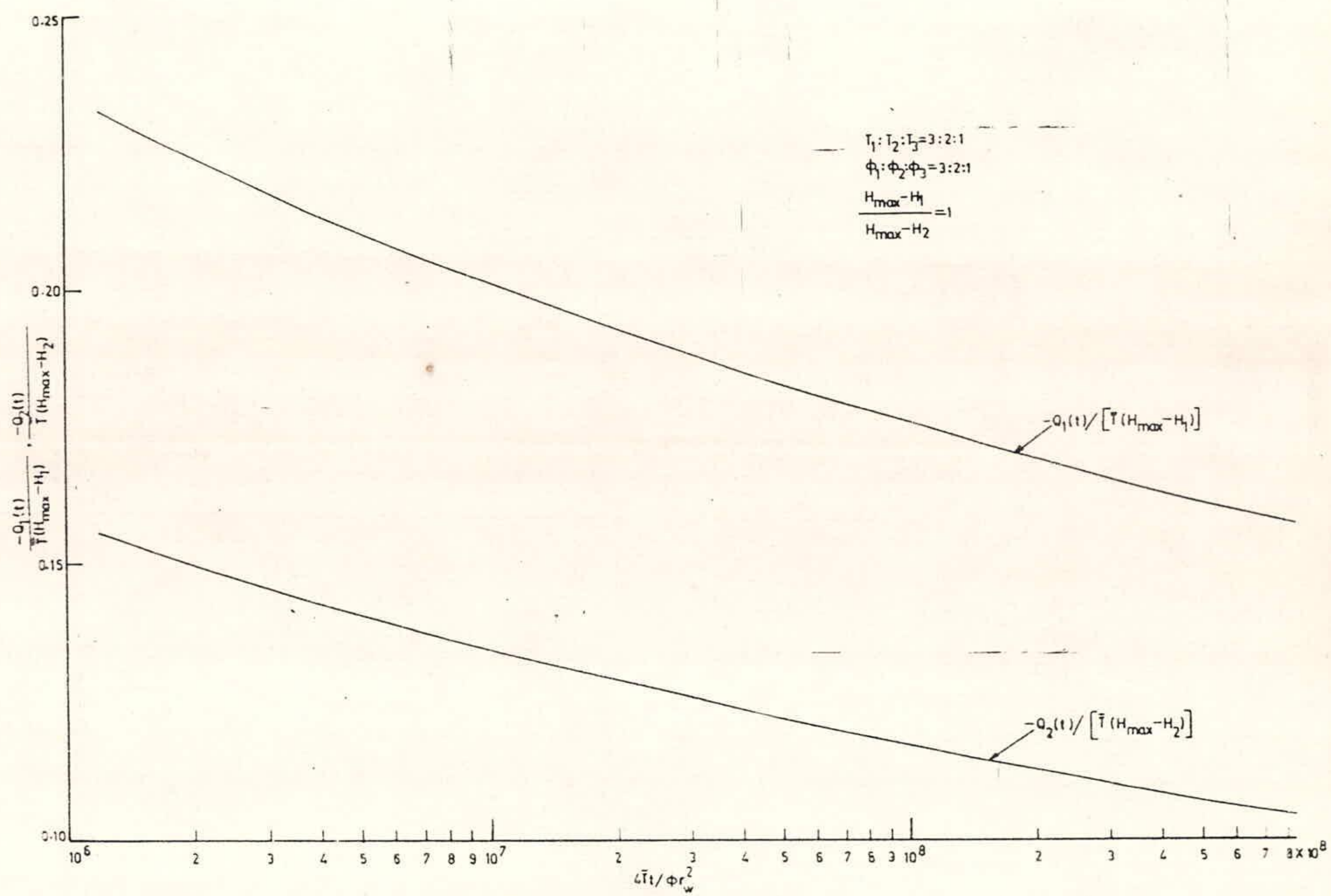

FIGURE 2 - VARIATIONS OF $-Q_1(t)/\{\bar{T}(H_{max}-H_1)\}$ AND $-Q_2(t)/\{\bar{T}(H_{max}-H_2)\}$ WITH $4\bar{T}t/(\bar{\phi}r_w^2)$ WHEN $T_1:T_2:T_3=3:2:1, \phi_1:\phi_2:\phi_3=3:2:1$ AND $(H_{max}-H_1)/(H_{max}-H_2) = 1$