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Stochastic Models for Reservoir Planning and Operation

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Abstract : *An overview of Stochastic optimization models used in reservoir planning and operation is presented. Various models are discussed which consider the stochastic nature of system parameters involved in the determination of optimum operating rules and reservoir capacities. Stochastic optimization methodologies are discussed in considerable detail applicable to real time reservoir operation.*

INTRODUCTION

Reservoir models framed in stochastic environment are of great importance, since they incorporate the uncertainty of system parameters into the decision or design process. The stochasticity is induced by the hydrological variables (e.g., streamflows, rainfall, etc.) which form the inputs to the reservoir system. Deterministic models on the other hand ignore the stochasticity of system (stochastic nature of inputs, demand, prices etc.), thus having limited applications. The stochastic optimization techniques use the statistical aspects of the input variables (streamflows) to arrive at the optimum operating policy for the reservoir under consideration.

This paper presents an overview of stochastic optimization models under two main classifications namely explicit and implicit stochastic optimisation models. Explicit stochastic optimization (ESO) approach uses the expected value of objective function along with statistical properties of inputs to arrive at optimum decision, whereas implicit stochastic optimization (ISO) utilizes simulation, deterministic optimisation and multivariate analysis. The important aspects of these models are dealt in considerable detail, wherever possible specific examples are provided. The ESO models discussed include Chance constrained Linear Programming (CCLP) models, Stochastic Dynamic programming (SDP) models, Chance Constrained Dynamic programming (CCDP) models and Stochastic Linear programming (SLP) models. The fig. 1. gives the classification of different models under stochastic environment.

Explicit Stochastic optimization (ESO)

In the ESO, the probabilistic nature of the hydrologic variables is explicitly included in the optimisation model itself. The output from the model would therefore give a steady state or long term operating policy. The SDP, CCLP, SLP and CCDP are some commonly used ESO techniques. The ESO technique was first proposed by Manne (1960), which was used by many researchers (Gablinger and Loucks, 1970; Loucks and Falkson, 1970)

STOCHASTIC DYNAMIC PROGRAMMING MODELS

The Stochastic Dynamic Programming (SDP) is extensively used to derive the long term steady state operating policies for reservoir systems (Butcher, 1971). The random input is usually assumed to form a single step Markov process and the objective is to maximize (or minimize) the expected system performance measure. A deterministic DP can be easily

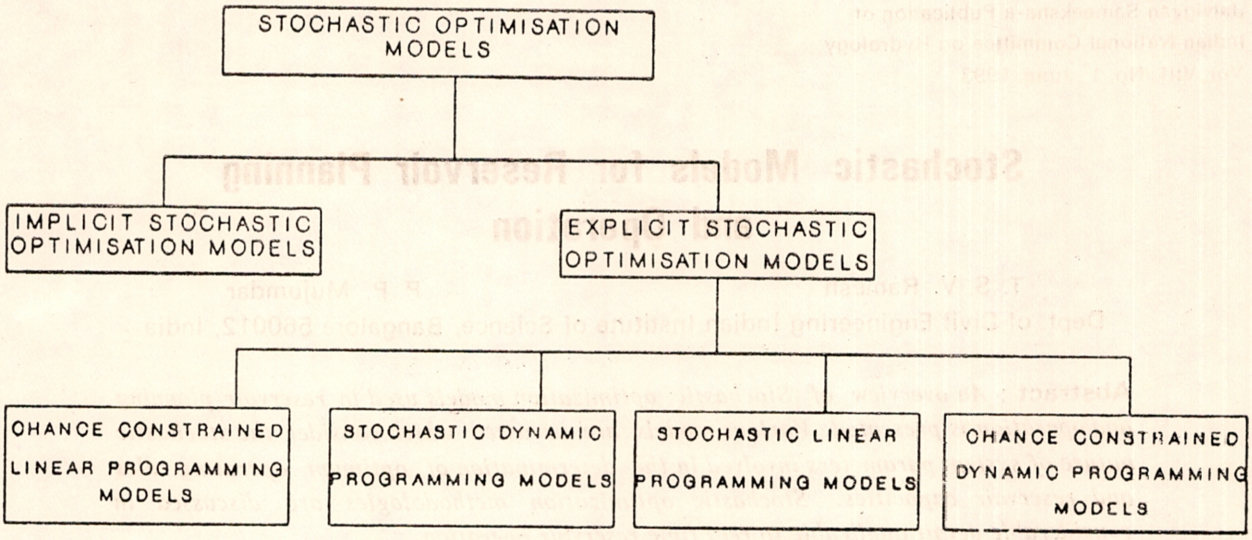

Fig. 1. STOCHASTIC MODELS FOR RESERVOIR OPERATION

generalized to Stochastic DP using recursive relationship and transition probabilites. A simple recursive relationship in the deterministic environment may be written as

$$F_t(S_i) = \underset{L}{\mathrm{MAX}}\,[B(S_i, L) + F_{t+1}(S_j)] \quad (1)$$

The equation 1 represents a recursive relationship of a backward moving DP algorithm. The system can be in any state say S_i for all i (i.e. i=1...n). S_i defines the discrete state of the system (i=1, 2,.. n). 'B' is the benefit function dependent upon current state 'i' and decision 'L'. In deterministic DP, $F_{t+1}(S_j)$ is known with certainty as starting from a state S_i in period 't', the transformation of system to state S_j in period 't+1' is a non random process. In SDP, however one or more inputs are random and therefore the state transformation is governed by the probability distribution of the random variables.

The basic recursive relationship of stochastic DP using Markov process may then be written as follows

$$F_t(S_i) = \mathrm{MAX}\,[B(S_i, L) + \sum_{j=1}^{n} P^t_{ij} * F_{t+1}(S_j)] \quad (2)$$

Where P^t_{ij} represents the probability (called transition probability) that the system is in state 'j' in period 't+1', given that it is in state 'i' in period 't'. In simple terms the recursive relationship defines the maximum expected value of future performance of the system, starting from the current period 't'.

A simple (reservoir operating policy) SDP model can be used to find out optimal reservoir operating policy. For a single reservoir whose capacity, release targets and storage targets are fixed, the main aim is to maximize system performance. System performance is a function of release, storage and inflows.

Discretisation of possible random inflows and initial (possible) storage volumes to a set of predetermined intervals forms the first step in SDP. The intervals for inflow are a range of discrete values represented by index 'i' and the storage volumes by 'k'. Then Q_{it} and S_{kt} represent the inflow and storage belonging to a particular interval 'i' and 'k' respectively in time period 't'. Along the same lines the inflows and storage volumes are discretised into intervals 'j' and 'L' for the time period 't+1' (next period). Then the release possible for all combinations of intervals is determined by continuity equation

$$R_{kit} = S_{kt} + Q_{it} - E_{klt} - S_{l,t+1}$$

$$\forall\ k, i, l, t \quad (3)$$

where E_{klt} represents possible evaporation and seepage losses

The system performance measure B_{kilt}, is in general a function of release R_{kilt} and storage S_{kt}. Fig. 2 depicts the relationship between the stage 'n' and the within year period 't', in the SDP solutions. The recursive relationship is solved iteratively till a steady state solution is acheived. A simple problem with two possible intervals for inflows and storage volumes considered for two time periods is solved in detail (Loucks et al., 1981).

n=8	n=7	n=6	n=5	n=4	n=3	n=2	n=1
t=1	t=2	t=1	t=2	t=1	t=2	t=1	t=2

n : stages, t = time periods

Fig. 2. Backward moving Stochastic dynamic programming process

SDP models are very useful in arriving at long term reservoir operating policies. The technique is well suited for sequential decision processes and can easily handle nonlinear objective functions with constraints also, It however suffers from a serious limitation due to the "curse of dimensionality", since the random variables add to state space, The SDP formulation is entirely problem dependent, hence no general algorithms or packages are available to solve reservoir related problems. It has been proved that SDP solution is computationally difficult for multireservoir problems (Schwieg and Cole, 1968).

It is known from the past experience (Karmouz and Houck, 1987) that SDP cannot perform well for large reservoir systems, for this reason it is almost never used for systems with more than two or three reservoirs. Discretisation is very important part of SDP which requires serious attention while formulation.

CHANCE CONSTRAINED LINEAR PROGRAMMING MODELS

Chance constrained linear programming models are basically explicit stochastic optimization models where risk is made explicit in the reservoir design / operation by the usage of chance constraints. The models determine the reservoir operating rules or the design, performing at the levels of certainty fixed by the designer or decision maker. The level of certainty is the "reliability level". Chance constrained models are effectively used as preliminary models in arriving at cost effective reservoir designs and optimal operating policies.

A simple chance constraint may be written as

$$Pr\ (R_t \geqslant D_t) \geqslant \alpha$$

R_t and D_t represent the release and demand at any time respectively, then the constraint specifies that the release made is greater than the demand for at least α*100% of the time. The chance constraints of this nature cannot be included in linear programming formulations straightaway, hence they need to be converted to deterministic equivalents for use in LP solution.

The transformation of chance constraints to their deterministic equivalents is possible by linear decision rule proposed by Revelle et al. (1969). Linear decision rule is a very simple and useful tool in converting chance constrained problems (in reservoir design) into simple LP formulations/problems. The simplest form of linear decision rule is as follows.

$R = S - b$ where R is the release made

S : storage at the end of the previous period

b : deterministic decision parameter (operating policy parameter)

In stochastic framework, where the inputs (streamflows) to the reservoir system are considered random variables unknown in advance, the linear decision rule facilitates the use of chance constraints through their deterministic equivalents in a LP framework.

A reservoir problem is presented here for complete understanding of chance constraints and their significance. A reservoir problem is taken up to find optimal operating policy with the requirements satisfied at predetermined reliability level while aiming at minimum reservoir capacity. A simple chance constraint chosen for the problem imposes on the model the condition that the release made is greater than the demand with a reliability level (i.e. $Pr(R_t \geqslant D_t) \geqslant \alpha$). The model is then solved for different reliability levels.

The model is written as

MINIMIZE K;

Subject to :

$$Pr(R_t \geqslant D_t) \geqslant \alpha; \ \forall t \qquad (4)$$

$$P_r(S_t \leqslant K) \geqslant P \approx 1 \quad \forall t \qquad (5)$$

K : Reservoir capacity

R_t : Release made in time period t

D_t : Demand to be met in time period t.

α : Reliability level

S_t : Storage

Conversion of chance constraints to deterministic equivalents :

The linear decision rule is given by the equation below

$$R_t = S_t + Q_t - b_t \qquad (6)$$

b_t : deterministic (operating policy) parameter

Q_t : inflow occurring in that period.

Then the continuity equation for the reservoir system is as follows

$$S_{t+1} = S_t + Q_t - R_t \qquad (7)$$

Using linear decision rule (eq. 6) the equation can be written as

$$S_{t+1} = b_t \text{ or } S_t = b_{t-1} \qquad (8)$$

Using equations (6) and (7) the chance constraint (4) can be changed into deterministic equivalent as follows

$$Pr(Q_t \leqslant D_t + b_t - b_{t-1}) \leqslant (1-\alpha) \qquad (9)$$

Here Q_t is streamflow value in the period 't' represented as random variable having normal probability distribution. Then the equation (9) represents the cumulative distribution function (CDF) of Q_t. Taking inverse of CDF we obtain

$$(Q_t \leqslant D_t + b_t - b_{t-1}) \leqslant F^{-1} Q_t (1-\alpha) \qquad (10)$$

The streamflows are assumed to be normally distributed. The chance constraint (5) can also be transformed to deterministic equivalent by use of LDR (linear decision rule)

The complete model with deterministic LP formulation is follows

Min K;

S.T.

$$D_t - b_t - b_{t-1} - \mu_t \leqslant \sigma_t {}^* F^{-1} Q (1-\alpha); \ \forall t$$

$$b_{t-1} - K \leqslant 0; \qquad \forall t$$

μ_t : mean monthly inflow value.

σ_t : standard deviation value.

In the present case a LINGO package running on DOS environment is used to solve the model for different reliability levels for known demand. For a particular reliability, the demand levels are increased to examine the maximum demand that can be satisfied with an associated (minimum) reservoir capacity. The complete results for several runs of the model with different reliability levels is presented in the table 1. For each reliability level, the inflow pattern imposes a constraint on the value of demand that can be satisfied beyond which the solution becomes infeasible. For a constant demand, the reservoir capacity increases as the reliability level increases, This is indicated by fig. 3.

The model presented is a simple model, but extensions are possible to include many constraints (constraints related to free board, flood control and various others). The model can also be modified for multireservoir planning problems.

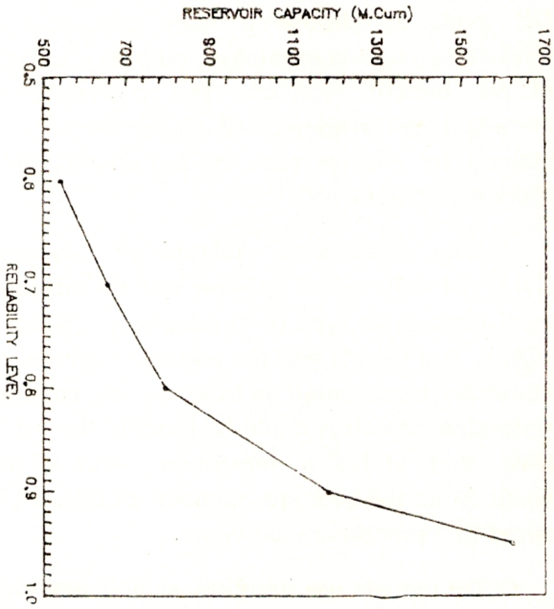

Fig. 3. Variation of Reservoir Capacity with Reliability (for constant demand)

Table 1

Reliability Level (α)	Maximum Demand that can be met (M.cum)	Reservoir Capacity (M. cum)
0.6	832	5137
0.7	707	4499
0.8	554	3644
0.9	354	2560
0.95	182	1623

Chance constrained linear programming (CCLP) models can be used as preliminary screening models before an optimal decision policy is chosen from a class of possible decision procedures. CCLP models can give a precise representation of relationship between operating policy and reservoir capacity. CCLP models have been applied to variety of reservoir management problems (Revelle and Kirby, 1970; Joeres et al., 1971; Nayak and Arora, 1971; Eastman and Revelle, 1973; Nayak end Arora, 1974).

The linear decision rule which forms the crux of the CCLP models has never been accepted as the best rule. It has always been questioned for the usage af deterministic parameter in the stochastic environment. Loucks and Dorfman (1975) evaluated different linear decision rules applicable to reservoir problems. Many researchers believe that CCLP models result in conservative dasigns and does not yield optimum results (Loucks, 1970). A complete analysis of CCLP model was provided by Sniedovich (1980). Still today CCLP models remain as basic stochastic optimisation models used for preliminary designs, even after major chenges in their formulation and refinement (in linear decision rule).

STOCHASTIC LINEAR PROGRAMMING MODELS

SLP models use linear programming techniques to arrive at optimum operating policy decisions (Loucks et al. 1981). In these type of models, the inflows are assumed to follow a single step Markov chain. Using SDP one can obtain steady state operating policy, from where steady state probabilities of release and storage can be derived. But stochastic linear programming is a reverse process where steady state probabilities of release and storage are solved for and the resulting information is interpreted to transform them (steady state probabilities) into releases and storages.

The same type of problem as discussed in the SDP model can be used to apply the SLP methodology to solve for optimum operating policy. An LP model can be proposed to solve for steady state probabilities (PS_{kt} and PP_{kilt}) of all storage volumes S_{kt} and releases R_{kit}. The steady state probabilities obtained are optimum steady state probabilities of releases and storages. Then the actual values of storages and releases associated with optimum operating policy (which minimizes expected system performance) can be calculated. The expected system performance forms the objective function for the LP formulation with constraints derived from the property of joint probabilities.

Stochastic Linear programming models can be used in place of SDP models. SLP models are used to solve multireservoir problems (Houck and Cohon, 1978). However SLP formulations are not popular because of enormous computational effort, time and storage, required for solving simple reservoir operating problems.

CHANCE CONSTRAINED DYNAMIC PROGRAMMING MODELS

Chance constrained models (Askew, 1974a) are formulated on the similar lines of stochastic dynamic programming models, but modified to incorporate a risk constraint. When SDP is formulated with an objective of maximizing net benefits (for a reservoir), the optimum operating policy derived may not ensure that the system will perform without any failure (demand is not met). Hence in order to have a control over the probability of failure of the system, a constraint on the probability of failure is included. In cases where the constraint can not be included directly, it is applied indirectly by using a hypothetical penalty whose value is 'W', The penalty is 'W' when the constraint is violated and zero otherwise. The stochastic dynamic programming model recursive relation can be modified to take the penalty.

The basic recursive relationship in SDP (Askew, 1974 a) is

$$F_i\,(S_i) = \text{MAX}\,[\sum_{i=1}^{n} P(Q_i) * [B\,(r) - C(x) + (1/1+d) * F_{i-1}\,(S_{i-1})\,]\,]$$

i : year number

d : annual discount rate

C (x) : cost incurred for annual release of 'x'

B (r) : benefit acquired from actual release 'r'

P (Q_i) : probability of obtaining annual inflow of Q_i.

The SDP relationship can be amended to take care of constraint as follows

Whenever the system fails (i.e. r is less than x) then a penalty of value 'W' is applied to the recursive relation

$$B\,(r) - C\,(x) - W + (1/1+d) * F_{i-1}\,(S_{i-1})$$

when $r \geqslant x$, the value of 'W' becomes zero.

Whenever a shortfall occurs in release, the SDP tends to choose a more conservative release as an optimum release, with the expected net benefit reduced. The probability of failure and the response of the system can be obtained by various runs of the model using different values of 'W'.

Chance constraints related to average number of failures of system can be included directly into these type of formulations (Askew, 1974b). The benefit and the average number of failures are represented as functions of storage. An iterative search is made to identify the value of 'W' that yields a maximum value of net benefit while the average number of failures is limited to a predetermined value.

CCDP models use iterative search methods to arrive at optimum operating policies at the same time restricting the probability of failure to an acceptable level. A modified CCDP model which penalizes the discount rate was proposed by Askew (1975). Many researchers believe that the chance constraints used in this type of models are not explicit to optimization. CCDP models are found to have limited application in reservoir management problems, since the iterative search procedures involved require enormous computation time and storage.

Implicit Stochastic Optimization models

Implicit stochastic models (Young, 1967) form a different class of optimization models where in the stochastic nature of hydrologic variables is included in the model implicitly. The stochastic nature of hydrologic variables is estimated from the past historical data to generate the time series realizations of system inputs The optimum decision sequence is then obtained for each input realization by application of any suitable optimization technique. The optimal decision sequences obtained from the model are analyzed and the relationship with system variables is established through multivariate analysis (usually regression analysis). The relationships are useful in defining the decision function which in turn are used to obtain the optimum decision for the entire system.

Usually a nonstationary Markov model (e.g. . e. ing & Jackson, 1971) is used for synthetic streamflow generation. A reservoir operating policy model is presented (Murali & P.P. Mujumdar, 1991) to help understand ISO technique. A linear optimization model is used for obtaining optimal decision sequences.

Inputs to the Model

The inputs to the model (i.e. streamflow values) are generated by synthetic streamflow generation model, considering past historical data of 24 years (of monthly inflows) to the reservoir. Twenty sequences of monthly inflows for 24 years are generated. Then these sequences are used as inputs to the optimization (here LP model) to obtain different decision sequences.

FORMULATION OF MODEL

The linear programming model can be formulated as follows :

$$\text{MIN} \left[\sum_t (D_t - R_t) - b^*S_t + L^*O_t\right]$$

S.T. :

$$R_t \leqslant D_t;$$

$$S_{t+1} = S_t + I_t - R_t - D_t - E_t$$

$$0 \leqslant S_{t+1} \leqslant S_{max};$$

$$R_t \geqslant 0;\ O_t \geqslant 0;\ S_t \geqslant 0$$

R_t : release made during time 't'

D_t : the demand (inclusive of both irrigation and power)

I_t : the generated

E_t : evaporation

O_t : overflow

S_{max} : maximum reservoir capacity

Sufficiently large value for L and low value for b is chosen to achieve the required objective. The large value of L penalizes the objective function thus making the LP model to achieve lower value of overflow. Similar reasoning will help understand that lower values of 'b' will result in high storage levels (as the objective function is minimized). The objective function is aimed at achieving two things :

(i) Optimal release rule to meet the demand.

(ii) High storages.

The LP model (Murali & Mujumdar, 1991) is solved for 20 sequences of 24 years each. The optimum solution for each sequence provides value of S_t (storage) and R_t (release) made during each period 't'. For each month under consideration the optimization model gives 20*24 sets of releases and storages.

In order to obtain the optimal release policy for each month, a multivariate analysis (Least square regression) is used to obtain a relationship between R_t, S_t and I_t. The relationship of the form $R_t = a_o + a_1 {}^*S_t + a_2 {}^*I_t$ can be achieved. Finding out the values of regression coefficients for each month, one can obtain the release (optimum) which has to be made in the month.

Table 2 gives the values of regression coefficients obtained by Multivariate analysis.

Table 2

month	a_0	a_1	a_2
1	215,00	0.000	0.000
2	464.23	-0.005	0.000
3	605.07	-0.003	0.005
4	639.78	-0.057	0.024
5	518.76	-0.026	0.130
6	691.48	-0.173	0.799
7	1413.44	-0.246	0.534
8	1697.26	-0.305	-0.249
9	1220.35	-0.221	-0.257
10	893.73	-0.115	-0.091
11	372.78	-0.002	-0.001
12	118.00	0.000	0.000

Reservoir capacity (live) : 3307 M. cum
Mean annual inflow : 1021 M. cum

Using the coeffieients, the reservoir operation can be achieved in real time with a suitable forecast of current period's inflow I_t.

ISO can be applied to wide variety of problems related to reservoir operation. The simulation procedure (synthetic streamflow generation) used facilitates the designer or decision maker to keep away from cumbersome analytical techniques required for formulation of problems (in slochastic environment). Modifications are possible to basic ISO model to bring out better results (one of the version of ISO is MCDP-Monte carlo dynamic programming; Young, 1967). One of the best known modifications of ISO methodology is ASO (Alternate Stochastic optimization) provided by Croley (1974 a).

These type of models require large amount of computational time or storage in the generation of synthetic streamflow sequences, multivariate analysis and optimization procedures. Sometimes the multivariate analysis used, may not depict the exact relationship between different parameters.

REFERENCES

1. Askew, A.J., 1974 a, "Optimum Reservoir Operating Policies and the Imposition of Reliability Constraint", Water Resources Research, Vol. 10, No. 1, pp. 51-56.

2, Askew, A.J., 1974b, "Chance Constrained Dynamic Programming and the Optimization of Water Resources Systems", Water Resources Research, Vol. 10, No. 6, pp. 1099-1106.

3. Askew, A.J., 1975, "Use of Risk Premiums in Chance Constrained Dynamic Programming", Water Resources Research, Vol. 11, No. 6, pp. 862-866.

4. Butcher, W.S., 1971, "Stochastic Dynamic Programming For Optimum Reservoir Operation' Water Resources Bulletin, Vol. 7, No. 1, pp. 115-123.

5. Croley, T.E., II, 1974 a, "Sequential Stochastic Optimisation for Reservoir System", Journal of the Hydraulics Division, Vol. 100, No. Hyl, pp. 201-219.

6. Eastman, J. and Revelle, C.S., 1973, "Linear Decision Rule in Reservoir Management and Design 3, Direct Capacity Determination and Intraseasonal Constraints", Water resources Research, Vol. 9, No. 1, pp. 29-42.

7. Fiering, M.B. and Jackson, B. B. 1971, "Synthetic Streamflows" Water Resources Monograph 1, American Geophysical Union, Washington D.C.

8. Gablinger, M. and Loucks, D.P.. 1970, "Markov Models for Flow Regulation", Journal of Hydraulics Division, Vol. 96, No. HY1, pp. 165-181.

9. Houck, M.H. and Cohon, J.L., 1978, "Sequential Explicitly Stochastic Linear Programming Models : A Proposal Method for Design and Management of Multipurpose Reservoir Systems", Water Resources Research, Vol. 14, No. 2, pp. 161-169.

10. Joeres, E.F., Liebman, J. C. and Revelle, C. S., 1971, "Operating Rules for Joint Operation of Raw Water Sources", Water Resources Research, Vol. 7, No. 2, pp. 225-235.

11. Karmouz, M. and Houck, M. H., 1987, "Comparison of Stochastic and Deterministic Dynamic Programming for Reservoir Operating Rule Generation", Water Resources Research, Vol. 23, No. 1, pp. 1-10.

12. Loucks, D.P. and Dorfman, P.J., 1975, "An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation", Water Resources Research, Vol. 11, No. 6, pp. 777-782.

13. Loucks, D.P. and Falkson, L.M., 1970, "A Comparison of Some Dynamic, Linear and Policy Iteration Methods for Reservoir Operation", Water Resources Bulletin, Vol. 6, No. 3, pp. 384-400.

14. Loucks, D. P., Stedinger, J.R. and Haith, D.A., 1981, Water Resources Systems Planning and Analysis. Prentice-Hall, Englewood Cliffs, N.J.

15. Manne, A.S., 1960, "Linear Programming and Sequential decisions", Management Science, Vol. 6, No. 3.

16, Murali, H. and Mujumdar, P. P., 1991 "Implicit and Explicit Stochastic Optimization for Reservoir Operation : A Case Study in India" Proc of International Conference On Computer Applications in Water Resources, China.

17. Nayak, S.C. and Arora, S.R., 1971, "Optimal Capacities for a Multireservoir System Using Linear decision Rule", Water Resources Research, Vol. 7, No. 3, pp. 485-498.

18. Nayak, S.C. and Arora, S.R., 1974, "Linear Decision Rule : A Note on Control Volume Being Constant", Water Resources Research, Vol. 10, No. 4, pp. 637-642.

19. Revelle, C., Joeres, E. and Kirby, W., 1969, "The Linear Decision Rule in Reservoir Management and Design 1, Development of the Stochastic Model", Water Resources Research, Vol. 5, No. 4, pp. 767-777.

20. Revelle, C. and Kirby, W., 1970, "Linear Decision Rule in Reservoir Management and Design 2, Performance Optimization", Water Resources Research, Vol. 6, No. 4, pp. 1033-1044.

21. Schwieg, Z. and Cole, J.A., 1968, "Optimal Control of Linked Reservoir", Water Resources Research, Vol. 4, No. 3, pp. 479-497.

22. Sniedovich, M., 1980, "Analysis of a Chance Constrained Reservoir Control Model, Water Resources Research, Vol. 16, No. 6.

23. Young, G.K., Jr., 1967, "Finding Reservoir Operating Rules", Journal of Hydraulics Division, Vol. 93, No. HY6, pp. 297-321.