Jalvigyan Sameeksha—a publication of
Indian National Committee on Hydrology
Vol. V, No. 1 & 2, June 1991

An Event Based Overlapped Flow Model Considering Rainfall Infiltration

M.S. Ahluwalia Subhash Chander P.N. Kapoor
Department of Civil Engineering, Indian Institute of Technology, Delhi, India

S.R. Singh
Water Technology Centre for Eastern Regions, Bhubaneswar, Orissa, India
Department of Soil and Water Engineering Punjab Agricultural University
Ludhiana-141 004, INDIA

Abstract : *A distributed mathematical model to simulate the overland flow from fallow upland phase of the small watershed subjected to rainfall conditions has been presented. Overland flow has been simulated using St.–Venant equations with kinematic wave approximation. Mein and larson approach based on original Green and Ampt method has been used to estimate the rainfall excess rate. All the parameters of the model have physical significance and can be easily measured in the field or the laboratory by conducting experiments. The solutions to the governing partial differential equations of flow have been obtained using finite element technique and the model validated using reported runoff data.*

Introduction

Many hydrologic simulation models utilize hydrologic technique to route the flow (e.g., Stanford Watershed Model-IV and USDA HL-73). This technique employs the equation of continuity (an ordinary differential equation) with either an analytic or an assumed relationship between storage and discharge within the system. Therefore, it cannot best represent the dynamics of flow (Viessman et al.. 1972). Also, these models do not predict the peak values to the desirable satisfaction (Subramanya, 1985) which is of tremendous importance to those interested in sediment yield estimation. Moreover, these models are lumped parameter models (Woolhiser, 1973 and Heatwale et. al. 1982) in which non-uniform parameters such as rainfall, soil, land use and topographic features of the watershed are weighted to obtain some representative values of the entire watershed. These type of models can only be used when necessary data requirements are made which require continuous gauging of the watersheds. These type of models beside being less accurate requires a lot of effort to collect such an enormous data and their applicability may not be feasible in developing countries due to the lack of the relevant data.

A mathematical model that employ hydraulic routing to the flow, describes the dynamics of the flow more accurately to the one utilizing the hydrologic flood routing (Viessman, et. al. 1972). Also, a mathematical model that integrates the spatial variability of the controlling parameters on an ungauged watershed can provide more reliable prediction of spatial and temporal distributiod of watershed response. A distributed parameter model possesing the above characteristics is described in the following paragraphs.

Development of the Model

Water Routing

Overland flow can be simulated using hydrodynamic equations of continuity and momentum.

Continuity equation :

The continuity equation can be expressed as :

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (1)$$

where

Q=discharge per unit width of the overland flow plane, m^2/sec.

A=Cross-sectional area of the flow per unit width of the overland flow plane, m.

x=distance in the direction of flow,

t=time, sec., and

q=lateral inflow rate per unit length per unit length per unit width of the flow plane (rainfall rate-infiltration rate) m/sec.

The lateral inflow rate can be computed by estimating the infiltration rate of the soil.

Estimation of infiltration

Most of the infiltration equations (Kostiakov, 1932; Horton, 1940; Holtan, 1961; Huggins and Manke, 1966; Smith, 1972; and Smith, 1976); are strictly empirical and the parameters must be calibrated from the field data. The equation developed by Philip (1957 a) consist of parameters that have physical significance but its accuracy is questionable for long durations as it does not have any parameter that accounts for the equilibrium infiltration rate. Mein and Larson (1973) considered infiltration process to be a two-stage event and applied the original Green and Ampt model (1911) for rainfall conditions for determining the cumulative infiltration at the time of the surface ponding. The Green and Ampt model was modified to make it applicable to post-surface ponding conditions while accounting for the volume of water infiltrated before the ponding began. The equation, hereafter for brevity termed as Green-Ampt-Mein-Larson (GAML), consist of the parameters that have physical significance and can be easily measured in the laboratory or the field. The parameters, saturated hydraulic conductivity and the suction at the wetting front, are easily measurable in the laboratory from the soil samples brought from the field or directly in the field itself. The other input of the GAML model, saturation moisture deficit can either be computed by measuring the initial moisture content of the soil or estimated.

The GAML model considers infiltration process to belong to either pre-ponding stage or post-ponding stage. For a steady rain, infiltration may first start with an unponded surface and afterwords it may change to a stage with surface ponding and will continue to be in this stage till the cessation of the rainfall event.

Pre-ponding stage : The equation employed in the GAML to predict the infiltration amount prior to ponding is given by :

$$F_p = \frac{SW.\ M}{(1/Ks) - 1} : \ I \geqslant Ks \qquad (2)$$

and the time to surface ponding is given by

$$t_p = F_p/I \qquad (3)$$

where

F_p = Cumulative infiltration at the time of ponding, m,

SW = Average suction at the wetting front. m,

M = Initial soil moisture deficit, m/m,

Ks = saturated Hydraulic conductivity, m/sec.,

I = rainfall intensity, m/sec.

t_p = time to surface ponding, sec.

Equations (2) and (3) are used only when rainfall intensity exceeds the saturated hydraulic conductivity. When the rainfall intensity is less than Ks, all of the rain infiltrates. Mathematically, it is expressed as :

$$F = I.\, t \qquad : \; I \leqslant Ks \tag{4}$$

where

F = cumulative infiltration at time t, m and

t = time, sec.

Post-ponding stage : The equation used for estimating infiltration at any time durieg post-ponding stage is obtained using the following equation :

$$Ks\,(t - t_p + t/_p) = F - M\,SW\,\ln\left(1 + \frac{F}{M\,SW}\right) \tag{5}$$

where

t_p = equivalent time to infiltrate volume F_p under ponded surface condition, sec.

This equation can be solved directly for t and by iteration for the cumulative infiltration amount F. The pseudotime t_p can be calculated from

$$Ks\, t_p = F_p - SW\,M\,\ln\left(1 + \frac{F_p}{SW\,M}\right) \tag{6}$$

Surface ponding indicators : Chu's (1978) surface ponding indicators for identifying pre-ponding, post-ponding conditions that may arise during unsteady rainfall infiltration has been used in this study. For both no ponding and ponding conditions at the beginning of a uniform rainfall intensity interval, two possibilities arise, that is, after the interval the surface conditions may correspond to ponding or no-ponding situations.

(1) No surface ponding at the beginning of a period :

The surface indicator has been expressed by :

$$CU = CR\,(tn) - CRO\,(t_{n-1}) - \frac{Ks.\,SW.M}{(I - Ks)} \qquad I \geqslant Ks \tag{7}$$

where

CU = surface ponding indicator, when there is no surface ponding at the beginning of a period.

CR = cumulative precipitation amount at time, t_n, mm,

CRO = cumulative rainfall excess amount at time, t_n, mm

t_n = present time period, and

t_{n-1} = previous time period.

For the case of no surface ponding at the beginning of a period, a value of CU less than zero shows that there will be no ponding at the end of the period.

(2) Surface ponding at the beginning of a period : The surface indicator has been expressed by :

$$CP = CR\,(t_a) - F(t_n) - CRO\,(t_{n-1}) \tag{8}$$

For the case of surface ponding at the beginning of a uniform intensity interval, a value of CP greater than zero shows that there will be ponding at the end too but a value CP less than zero indicates that there will be no ponding at the end of the period.

The complete working of the GAML, model to estimate the infiltratian and the rainfall excess under unsteady rainfall conditions is illustrated in the flow diagram in Fig. (1).

Momentum equation :

The momentum equation can be expressed as :

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial h}{\partial x} = gA\,(S - S_f) \tag{9}$$

where,

S = bed slope of flow plane, m/m,

S_f = friction slope, m/m

h = depth of flow, m, and

g = acceleration due to gravity, m/sec².

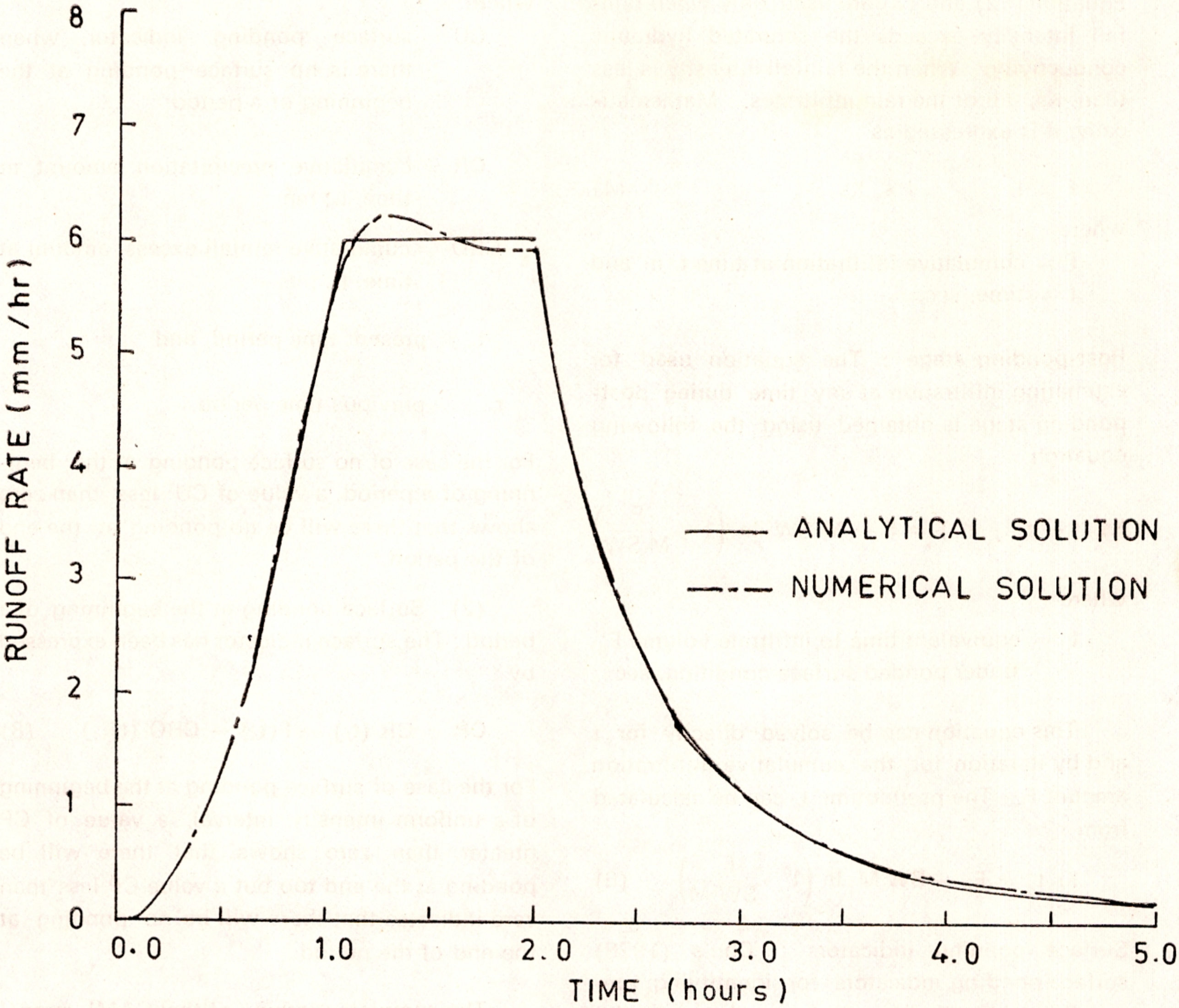

Fig. 1. Comprasion of the Hydrograph of Numerical Solution (Using Linear Interpolation Function) with that of the Analytical Solution)

Kinematic wave approximation :

The assumption of the kinematic wave approximation (Lighthill and Whitham, 1955) is that the friction slope is equal to the bed slope. In other words, the gradients due to local and convective accelerations are assumed to be negligible and the water surface slope is assumed to be equal to the bed slope implying uniform flow for which Manning's equation can be used.

$$Q = V\ A = R^{2/3}\ S^{1/2}\ A/n \qquad (10)$$

where,

V = velocity of flow, m/sec

R = hydraulic radius, m, and

n = Manning's roughness coefficient.

Initial and Boundary Conditions

To solve the equations of flow on a sloping plane subject to rainfall and infiltration the following initial and boundary conditions can be assumed :

$$A(x,0)=0\ ;\ Q(x,0)=0\quad 0 \leqslant x \leqslant L \tag{11}$$

$$A(0,t)=0\ ;\ Q(0,t)=0\quad t \geqslant 0 \tag{12}$$

where,

L = Length of the flow plane, m.

Finite Element Formulation

Overland flow :

Using Galerkin residual method to develop the algebraic equations from the partial differential equation of flow, following finite element equation is obtained for a linear element.

$$1\,[KM]\,\{A\} + (LM)\,\{Q\} - 1\,q\,\{m\} = 0$$

where

$$[KM]=\begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix};\ [LM]=\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\{M\}=\begin{Bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{Bmatrix};\ \{A\}\ \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix};\ \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

and **1** = length of the element, m

If the time differential of the area is represented by a simple explicit time integration procedure.

$$A(t) = \frac{A(t+dt)-A(t)}{dt} \tag{14}$$

where dt = time increment, the equation for one linear element becomes :

$$\frac{1}{dt}[KM]\{A\}_{t+dt} - \frac{1}{dt}[KM]\{A\}_t + [LM]\{Q\}_t - 1\,q\{M\} = 0 \tag{15}$$

For the element equation to be adopted to a finite element grid consisting of more than one element, it must be arranged to cover the total number of elements. The Direct Stiffness Method (Desai and Abel, 1972) has been used in this formulation to obtain the assembled matrices.

Similarly, one can derive finite element equation of continuity equation using cubic interpolation function as follows :

$$[KM]=\frac{1}{1680}\begin{bmatrix} 128 & 99 & -36 & 19 \\ 99 & 648 & -81 & -36 \\ -36 & -81 & 648 & 99 \\ 19 & -36 & 99 & 128 \end{bmatrix}$$

$$[LM]=\frac{1}{240}\begin{bmatrix} -120 & 171 & -72 & 21 \\ -171 & 0 & 243 & -72 \\ 72 & -243 & 0 & 171 \\ -21 & 72 & -171 & 120 \end{bmatrix}$$

$$\{M\}=\frac{1}{8}\begin{Bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{Bmatrix};\ \{\dot{A}\}=\begin{Bmatrix} \dot{A}_1 \\ \dot{A}_2 \\ \dot{A}_3 \\ \dot{A}_4 \end{Bmatrix}$$

$$\text{and}\quad \{Q\}=\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

Time increment for solution convergence :

Courent condition has been used as a criterion in selecting time increment, dt, which states

$$dt \leqslant 1/c \tag{16}$$

where,

c = kinematic wave speed, m/sec. which is equal to (5/3 v).

VERIFICATION OF FINITE ELEMENT METHOD WITH ANALYTICAL SOLUTION

Since the analytical solutions to the kinematic wave approximation are not available, the verification of the finite element solution (FES) has been made with analytical solution (Eagleson, 1970) based on certain assumptions. It has been assumed that rainfall excess is invariant in time and space and the catchment is a uniform plane whose surface roughness, slope and flow regime are invariant in time and space. The duration of rainfall excess has been considered as finite. The analytical solution (Eagleson, 1970) for three situations is presented as follows :

Case (1) $t \leqslant t_c \leqslant t_r$

$$h = i\,t \tag{28}$$

Case (2) $t_c \leqslant t \leqslant t_r$

$$h = \left(\frac{L\,i}{\alpha}\right)^{1/m} \tag{29}$$

for the computation of flow profile

$$h = (x\, i/\alpha)^{1/m},\ O \leqslant x \leqslant L \qquad (30)$$

Case (3) $t \geqslant tr \geqslant t_c$

h is given by the implicit relation

$$L = \alpha\, h^{m-1} (h\, i^{-1} + m\, (t - t_r))$$

where

x = distance along the plane, m,

h = depth of flow, m,

L = Length of the plane, m,

i = rainfall excess, m/sec

$\alpha = s^{1/2}/n$

m = 2 in flow equation. $Q = \alpha h^m$

t_r = duration of rainfall excess, sec.

t_c = time of concentration, sec, which is equal to $(L\, i^{1-m} \alpha)^{4/m}$

Comparison of FES (Using Linear Interpolation function) with analytical solution

Finite element solution was obtained using linear interpolation function for a hypotheticai case of flow over a flow plane of length 100 metres. The parameters assumed were :

(i) Slope = 0.01

(ii) Manning's n = 0.025

(iii) tjme step = 30 sec.

A rainfall excess rate of 6 mm/hr was assumed for a period of 2hr and the flow plane was divided into three elements of equal length. Solution was obtained using explicit time integration scheme. The comparison is shown in Fig. 2. Comparison of FES (Using Cubic Interpolation Function) with Analytical Solution

A smilar comparison was made using cubic interpolation function in the finite element solution. In this case the flow plane has been considered to be consist of one cubic element and rest of the conditions and values of parameters were assumed to be same as that in the solution obtained using linear interpolation function. The result is shown in Fig. 3. It illustrates that both the interpolation functions, give good agreement with the analytic solution. A comparison of flow profile using three cubic elements give good agreement between analytic and numerical solution, (Fig. 4).

VALIDATION OF THE MODEL

The mathematical model has been verified by a set of observed data reported by Akan and Ezen (1982). Eight different flow conditions examined together with the assumed saturated moisture deficit (SMD) and Mannings n values are summarized in Table 1. In the first four cases, the overland flow length is 23m with a bottom slope of 0.10. In the remaining cases the overland flow plane is 21.9m long and has a bottom slope of 0.17. Also shown in the table 1 are the observed and the computed results.

A perusal of the tabulated values shown that good agreement exist between the simulated values and the observed results for all the flow cases examined.

CONCLUSIONS

The event based overland flow model, utilizing distributed parameter approach, gives good agreement with the reported runoff data. The model requires only limited data input which can be collected easily. All the parameters of the model have physical significance and can be measured in the field or in the laboratory. The model needs to be further evaluated with the observed runoff hydrograph to test its validity in the temporal domain.

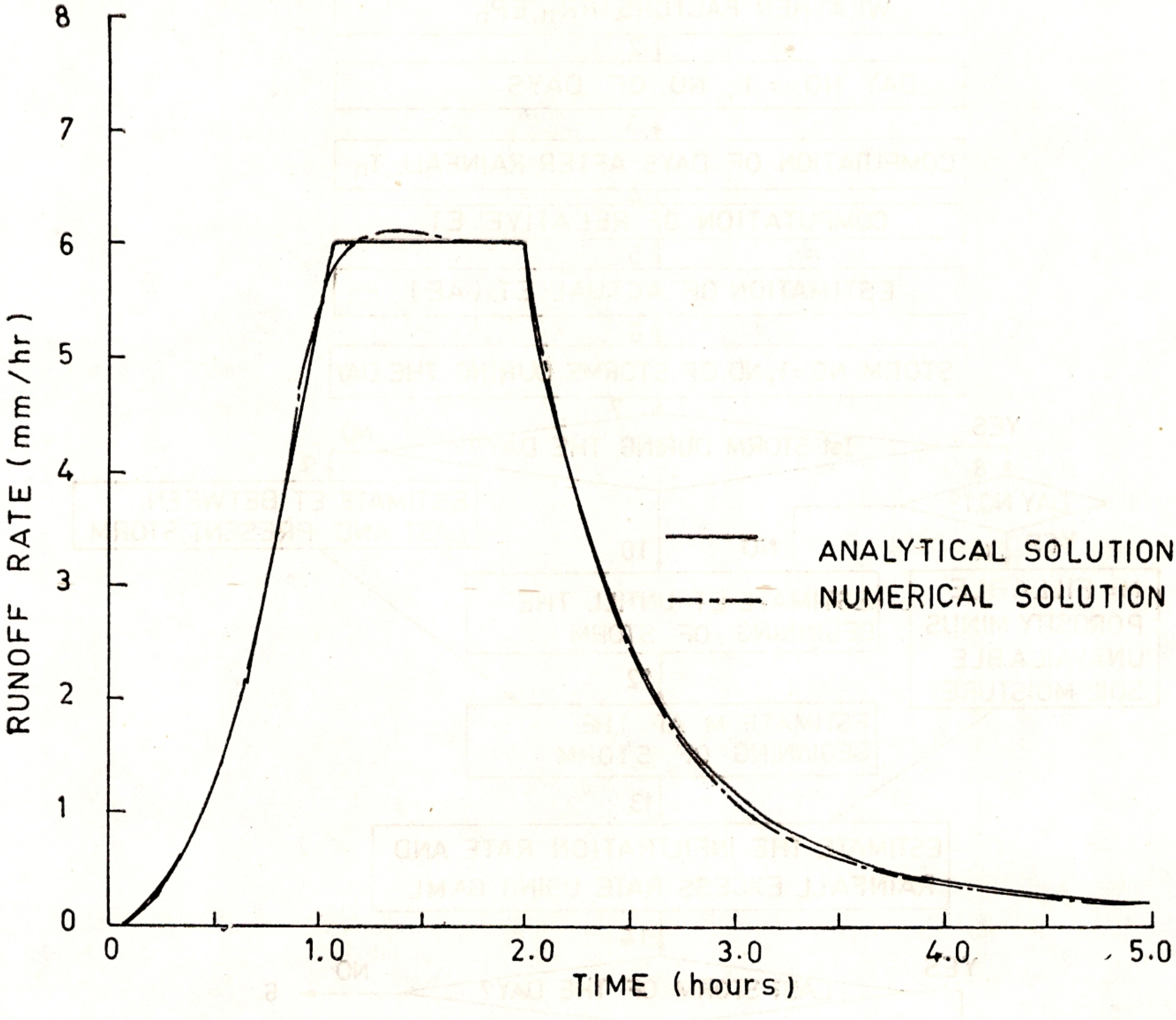

Fig. 2. Comparison of the Hydrograph of Numerical Solution (Using Cubic International Function) with that of the Analytical Solution.

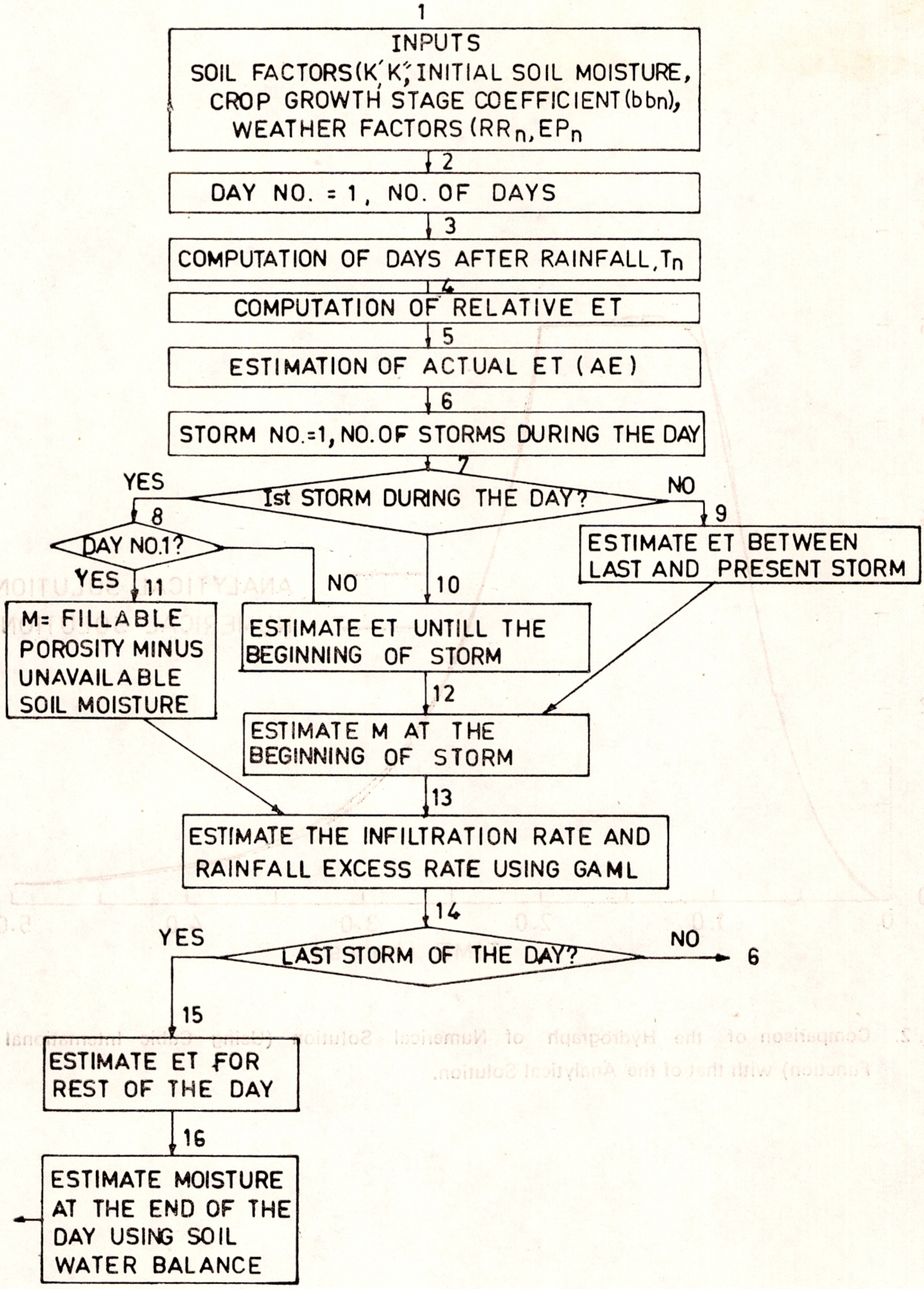

Fig. 3. A Flow Chart Showing Computation Steps to Calculate Soil Water Balance.

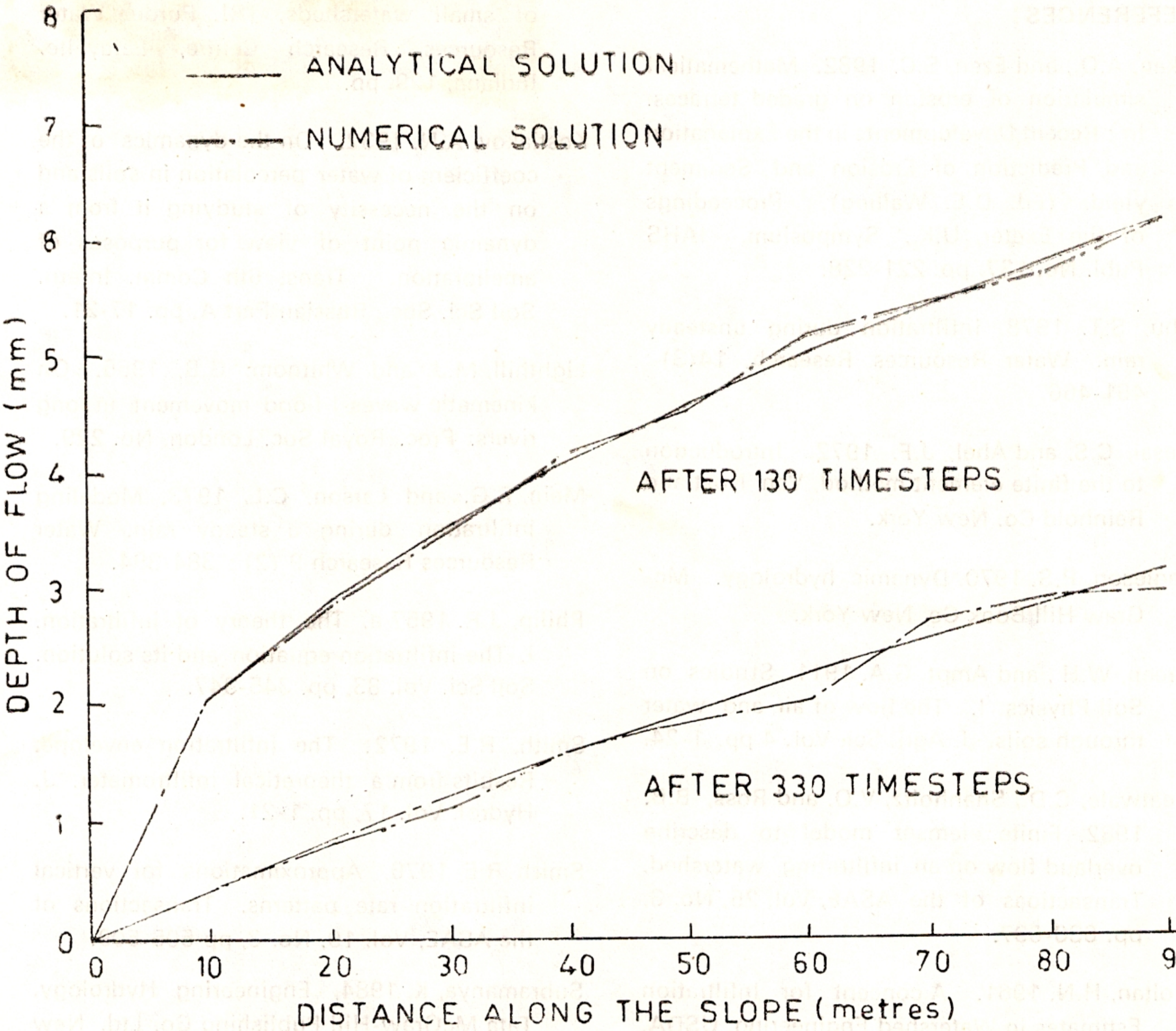

Fig. 4. Comparison of Flow Profile using Cubic Interpolation Function.

Table 1 Comparison of simulated and observed runoff volumes.

Case No.	M	n	Rainfall Intensity (mm/h)	Rainfall Duration (min)	Runoff volume (m^3) Observed	Runoff volume (m^3) Simulated
1	0.310	0.18	18.0	10	0.001	0.002
2	0.255	0.012	23.8	15	.040	0.051
3	0.155	0.006	37.8	12	.100	0.111
4	0.220	0.020	10.8	35	.026	0.026
5	0.310	0.180	18.0	10	.002	0.002
6	0.255	0.012	23.8	15	.048	0.051
7	0.155	0.006	37.8	12	.110	0.111
8	0.220	0.02	10.8	35	.027	0.026

REFERENCES

Akan, A.O., and Ezen, S.C. 1982. Mathematical simulation of erosion on graded terraces. In : Recent Developments in the Explanation and Prediction of Erosion and Sediment yield. (Ed. D.E. Walling). Proceedings of the Exeter, U.K., Symposium. IAHS Publ. No. 137. pp. 221-228.

Chu, S.T. 1978. Infiltration during unsteady rain. Water Resources Research, 14(3) : 461-466.

Desai, C.S. and Abel, J.F. 1972. Introduction to the finite element method, Van Nostrand Reinhold Co. New York.

Eagleson, P.S. 1970. Dynamic hydrology. Mc-Graw Hill Book Co. New York.

Green, W.H , and Ampt. G.A. 1911. Studies on Soil Physics. I. The flow of air and water through soils. J. Agri. Sci. Vol. 4 pp. 1-24.

Heatwale, C.D., Shanholtz, V.O. and Ross, B.B. 1982. Finite elemsnt model to describe overlaud flow on an infiltrating watershed. Transactioas of the ASAE, Vol. 25, No. 3, pp. 630-637.

Holtan, H.N. 1961. A concept for Infiltration Estimates in Watershed Engineering, USDA. ARS Bulletin 41-51, 25 pp.

Horton, R.E. 1940. An approach towards a physical interpretation of infiltration capacity. Soil Sci. Soc. Am. Proc., Vol. 5, pp. 399-417.

Huggins, L.F., and Monke, E.J. 1966. The mathematical simulation of the hydrology of small watersheds, TRI, Purdue Water Resources Research Centre, Eafayelte, Indiana, 129. pp.

Kostiakov, A.N. 1932. On the dynamics of the coefficient of water percolation in soils and on the necessity of studying it from a dynamic point of view for purposes of amelioration. Trans. 6th Comm. Intern. Soil Sci. Soc., Russian Part A, pp. 17-21.

Lighthill, M.J. and Whitnom, G.B, 1955. On kinematic waves-I flood movement in long rivers. Proc. Royal Soc. London. No. 229.

Mein, R.G. and Larson, C.L. 1973. Modeling infiltration during a steady rain. Water Resources Research 9 (2) : 384-394.

Philip, J.R. 1957 a. The theory of infiltration. I. The infiltration equation and its solution. Soil Sci. Vol. 83, pp. 345-347.

Smith, R.E. 1972. The infiltration envelope. Results from a theoretical infiltrometer. J. Hydrol. Vol. 17, pp. 1-21.

Smith, R E 1976. Approximations for vertical infiltration rate patterns. Transactions of the ASAE, Vol. 19, No. 3, pp 505-509.

Subramanya, K. 1984. Engineering Hydrology. Tata McGraw-Hill Publishing Co. Ltd. New Delhi.

Viesaman, W., Harbaugh, I.E. and Krupp, J.W. 1972. Introduction to hydrology. Intext Educational Publishers, New York.

Woolhiser, D.A., 1973. Hydrologic and Watershed modeling states of art. Transactions of the ASAE 16(3) : 553-559.