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Hydrological Forecast of Soil Moisture

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Abstract : *A procedure for soil moisture forecast has been described. The implications of boundary conditions during storm up to ponding time and beyond ponding time and during inter storm period have been explained. The numerical methods which have been tested for their efficiency have been high lighted.*

1.0 Introduction

A hydrological forecast of soil moisture is the prior estimate of the future state of soil water in the zone of aeration. Similar to forecast of other hydrologic elements, the hydrological forecast of soil moisture will have the following main characteristics.

1.1 The forecast variable

This would include soil moisture status in the root zone depth, time of occurrence of permanent wilting condition, period for which the root zone depth will be in saturated state, time at which root zone will be at field capacity, depth fraction of root zone depth that would be at less than field capacity.

1.2 The lead time or forecast period

The specified time in advance, for which the calculation of definite elements of the soil water regime in the root zone depth is done, is known as the lead time.

A short term hydrological forecast is defined as forecast of future value of an element of the regime for a period ending up to two days from the issue of the forecast, Medium-term or extended hydrological forecast is the forecast of the future value of an element of the regime for a period ending between two and ten days from the issue of forecast. Long term hydrological forecast is the forecast of the future value of an element of the regime for a period extending beyond ten days from the issue of the forecast. Thus a lead time may be two days, 5 to 10 days or more than 10 days in case of soil moisture forecast.

1.3 Hydrologic Model

Techniques for forecasting range from the use of simple empirical formulae to the use of complex mathematical models.

For analytical studies on soil moisture regime critical review and accurate assessment of the different controlling factors is necessary. The controlling factors of soil moisture may be classified under two main groups viz. climatic factors and soil factors.

Climatic factors include precipitation data containing rainfall intensity, storm duration, interstorm period, temperature of soil surface, relative humidity, radiation, evaporation and evapotranspiration.

The soil factors include soil matric potential and water content relationship, hydraulic conductivity and water content relationship of the soil, saturated hydraulic conductivity, effective medium porosity. Besides these factors, the information about depth to water table is also required.

The vertical movement of soil moisture in the liquid phase between the surface and the water table can be subdivided into the following three categories according to predominant forces involved.

(i) Infiltration and exfiltration

Alternate wetting and drying of soil surface during consecutive storm and interactive periods will case a penetration of the medium by an unsteady wave like diffusion of liquid soil moisture into the soil during wet surface (storm) periods under the complementary effects of capillarity and gravity and out of the soil during dry surface (interstorm) periods when capillarity opposes gravity. With increasing depth of penetration, diffusion reduces the soil moisture gradients and thus reduces the effect of capilarity until moisture movement becomes dominated by gravity. The depth at which surface induced capillary forces becomes negligible determines the penetration depth of the surface process and is used to define the thickness of the zone of the soil moisture. The presence of transpiring vegetation adds another mechanism for moisture extraction distributed over a depth which is related to root structure.

(ii) Percolation

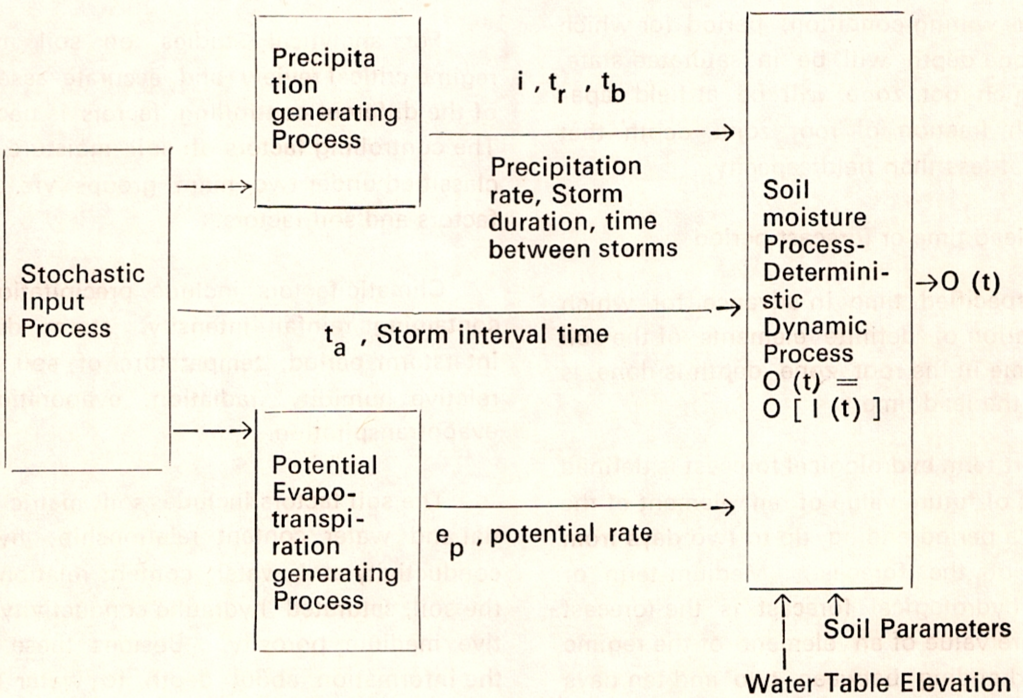
Liquid soil moisture moves out of the bottom of the zone of soil moisture and percolates downward under the domination of gravity forces until it encounters the increasing soil moisture gradients lying above the water table. At some depth upward capillary forces will be prominent defining the bottom of this intermediate zone.

(iii) Capillary rise

Between the water table and the intermediate zone there is a capillary fringe in which gravity and capillarity again jointly govern the liquid soil moisture movement.

1.4 Method of computation

The following general procedure has been suggested by Eagleson (1978) for soil moisture forecast.



The presentation of the forecast. O (t) may be in the form of a single value or in the form of probability distribution and its dissemination may be achieved by regular news bulletin.

2.0 The Mathematical Model

The continuous variation of soil moisture with time and depth in homogeneous bare soil can be known by solving the Richard's equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta) \frac{\partial \theta}{\partial t} \right] - \frac{\partial k(\theta)}{\partial z} \quad (1)$$

satisfying the initial condition. i.e. soil moisture variation with depth at a given time and the boundary conditions those prevail at the soil surface and at the lower boundary of the zone of aeration. In above equation θ is the effective volumetric moisture content, which is equal to the volume of active soil moisture divided by the total volume, t is the time, $K(\theta)$ is the effective hydraulic conductivity. $D(\theta)$ is the diffusivity defined as $D(\theta) = K(\theta) \frac{\partial \psi(\theta)}{\partial \theta}$ where $\psi(\theta)$ is the soil matrix potential z represents the vertical space co-ordinate and z is positive downward.

For vegetated surface the internal extraction of soil moisture by the plant roots needs to be incorporated. The local extraction rate will be a function of the plant species through root structure, and effective leaf area. It will be also a function of the climate through the potential rate of evaporation and will be sensitive to the soil moisture content. The root extraction is considered by including an appropriate sink term in Richard's equation and the final equation is expressed as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta) \frac{\partial \theta}{\partial z} \right] - \frac{\partial k(\theta)}{\partial z} - g_r(z.\theta) \quad \ldots (2)$$

Solution to the equation is governed by initial and boundary conditions. Since no exact analytical solution has yet been found, therefore, a numerical technique has to be adopted for solving the equation.

The soil water diffusivity term $D(\theta)$ is infinite for $\theta = \theta_s$ and $\theta = \theta_r$, in which θ_s = saturated moisture content, θ_r = residual moisture confent. It is therefore preferable to solve Richards equation in terms of soil water pressure h instead of volumetric wator content θ. Richards equation in terms of soil water pressure is given by

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h) \frac{\partial h}{\partial z} \right] - \frac{\partial K(h)}{\partial z} \quad \ldots (3)$$

in which C (h) is the specific water capacity defined as

$$C(h) = d\theta/dh \quad (4)$$

The value of C(h) is zero for $\theta = \theta_s$ and $\theta = \theta_r$.

3.0 Model Parameters

For a proper description of the unsaturated flow, a correct description of the two hydraulic functions. $K(\theta)$ and $\psi(\theta)$, is important. The hydraulic conductivity, $K(\theta)$, decreases strongly as the moisture content θ decreases from saturation. The experimental procedure for measuring $K(\theta)$ at different moisture contents is rather difficult and not very reliable. Alternatively procedures have been suggested to derive the $K(\theta)$ function from more easily measurable characterizing properties of the soil. In many studies, the hydraulic conductivity of the unsaturated soil is defined as product of a non-linear function of the effective saturation and hydraulic conductivity at saturation. The relation is given by $K(\theta) = K_{sat} S_e^n$ in which S_e = effective saturation equal to $(\theta - \theta_r) / (\theta_s - \theta_r)$, K_{sat} = hydraulic conductivity at saturation The value of n is found to be 3.5 for course textured soils. n will vary with soil type. In literature established empirical correlation between θ and soil characteristic is available.

The relationship between the suction head $\psi(\theta)$ and moisture content θ, which is usually termed as the water retention curve or the soil moisture characteristics, is basically determined by the textural and the structural composition of the soil. Also the organic matter content may have an influence on the relationship. A characteristic feature of the water retention curve is that ψ decreases fairly rapidly with moisture content. Hysteresis effects may appear, and, instead of being a single valued relationship, the ψ-θ relation consists of a family of curves. The actual curve will have to be determined from the history of wetting and drying.

Based on experimental findings the following relationship for diffusivity have been suggested by Miller and Bresler (1977).

$$D(S_e) = [\alpha \, m^2) \exp (\beta \, S_e) \quad (5)$$

In which α and β appear to be universal constants both dimensionless with suggested values : $\alpha = 0.001$ and $\beta = 8$. The third parameter m is a unique constant for each soil. Its value can be estimated from observations of the visual wetting front by infiltration in an air dry soil.

$$m = x_f / \sqrt{t} \quad (6)$$

where x_f is the distance of the wetting front at the time t.

4.0 Numerical Schemes

Haverkamo et al. (1977) have compared six different schemes in terms of execution time and accuracy. According to Haverkamp et. al all the following schemes yield good agreement with water content profiles measured experimentally at various times.

Scheme 1 : Explicit scheme (Eq. 3)

$$h_i^{j+1} = h_1^i + \frac{\Delta t}{C_i^j \, \Delta z}\left[K^j_{i+1/2}\left(\frac{h^j_{i+1} - h^j_i}{\Delta z} - 1\right) - K^j_{i-1/2}\left(\frac{h^j_i - h^j_{i-1}}{\Delta z} - 1\right)\right] \quad (7)$$

Scheme 2 : Implicit scheme with explicit linearization (Eq. 3)

$$C_i^j \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{\Delta z}\left[k_i^j{}_{+1/2}\left(\frac{h_{i+1}{}^{j+1} - h_i^{j+1}}{\Delta z} - 1\right) - K_i^j{}_{-1/2}\left(\frac{h_i^{j+1} - h^{j+1}{}_{i-1}}{\Delta z} - 1\right)\right] \quad (8)$$

Scheme 3 : Implicit scheme with implicit linearization (Eq. 3) (prediction-correction)

Prediction (estimation of C^j_i and K^j_i)

$$\frac{2C^j_i}{K^j_i} \; \frac{h_i^{j+1/2} - h^j_i}{\Delta t} = \Delta_z^2 h_i^{j+1/2} + \frac{1}{K^j_i} \Delta_z K^j_i \left(\Delta_z h^j_i - 1\right) \quad (9)$$

Correction (estimation of h^j_i)

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}} \; \frac{h_i^{j+1} - h^j_i}{\Delta t} = \frac{1}{2} \Delta_z^2 \left(h_i^{j+1} + h_i^j\right) + \frac{1}{K_i^{j+1/2}} \Delta_z K_i^{j+1/2} \left(\Delta_z h_i^{j+1/2} - 1\right) \quad (10)$$

where,

$$\Delta_z^2 h^j_i = \frac{h^j_{i+1} - 2h^j_{i-1}}{(\Delta z)^2}$$

$$\Delta_z h^j_i = \frac{h^j_{i+1} - h^j_{i-1}}{2\Delta z} \quad \text{and}$$

$$\Delta_z K^j_i = \frac{K^j_{i+1} - K^j_{i-1}}{2\Delta z}$$

By applying the boundary condition i.e. h(o,t) during storm and interstorm periods, the soil moisture distribution with depth can be ascertained at desired time.

5.0 Boundary Conditions

Appropriate boundary conditions are to be applied. i) during storm till ponding time, ii) during storm after ponding and iii) during inter storm period. For variable rainfall pattern the expression for ponding time Morel-Sevtoux, (1982) is

$$t_p = t_{j-1} + \frac{1}{R(J)}\left[\frac{(\theta - \theta_i) \, H_f}{R^*(J) - 1}\right.$$

$$- \sum_{\gamma=1}^{J-1} R(\gamma)\ (t_\gamma - t_{\gamma-1}) \quad (11)$$

where R(J) is the rainfall at J^{th} time $R^*(J) = R(J)/K$. K is the hydraulic conductivity at natural saturation, and HT = average capillary drive which can be determined by method suggested by Bouwer and it is given by

$$H_f = \int_0^{h_{ci}} k_{rw}(\theta)\, dh_c \quad (12)$$

where $k_{rw}(\theta) = K(\theta)/K$. h_c = capillary pressure head. h_{ci} = capillary pressure head corresponding to the initial soil moisture θ_i, prevailing at the onset of storm.

The finite difference equations (7), (8), (9) and (10) at the node next to the upper boundary node take the following forms respectively to satisfy the Neumann type boundary condition prevails till ponding time :

$$h_2^{j+1} = h_2^j + \frac{\Delta t}{J_{c2}\,\Delta z}\left[K^j_{2+1/2}\left(\frac{3h^j_2 - h^j}{\Delta z} - 1\right) + R(j)\right] \quad (13)$$

$$C_2^1\ \frac{h_2^{j+1} - h^j_2}{\Delta t} = \frac{1}{\Delta z}\left[K^j_{2+1/2}\left(\frac{h_3^{j+1} - h_2^{j+1}}{\Delta z} - 1\right) + R(j+1)\right] \quad (14)$$

Prediction (estimation of C_2^j and K_2^j)

$$\frac{2C_2^j}{K_2^j}\ \frac{h_2^{j+1/2} - h^j_2}{\Delta t} = \frac{1}{(\Delta z)^2}\left[(h_3^{j+1/2} - h_2^{j+1/2}) + R_{(j+1)}\frac{\Delta z}{J_{k_{2-1/2}}}\right] + \frac{1}{2\Delta z}\left[(h_3^j - h_2^j) - R(j)\frac{\Delta z}{K^j_{2-1/2}}\right] \quad (15)$$

Correction (estimation of h_2^j)

$$\frac{C_2^{j+1/2}}{K_2^{j+1/2}}\ \frac{h_2^{j+1} - h^j_2}{\Delta t} = \frac{1}{2(\Delta z)^2}\left[R_{(j+1)}\frac{\Delta z}{K^{j+1/2}_{2-1/2}} - h_2^{j+1} + h_3^{j+1} + R(j)\frac{\Delta z}{K^j_{2-1/2}} - h_2^j + h_3^j\right]$$
$$+ \frac{1}{K_2^{j+1/2}}\ \Delta_z K_2^{j+1/2}\left[\frac{3h^{j+1/2} - 2h^{j+1/2}}{2\Delta z} - \frac{R_{(j+1)}}{2K^{j+1/2}_{2-1/2}} - 1\right] \quad (16)$$

After ponding the upper boundary condition is satisfied by assigning h_1^j equal to the depth of ponded water on the ground surface.

During inter storm period the upper boundary condition is given by the relation (Hillel. 1977).

$$h_i^j = \frac{R\,T(j)}{Mg}\ \log_e f(j) \quad (17)$$

where R is the universal gas constant (ergs $mole^{-1}$. degree K^{-1}. T is the temperature in Kelvin, g is the acceleration due to gravity (cm per sec). M is the molecular weight of water (gm per mole) and f is the relative humidity of air.

6.0 Conclusion

Forecasting soil moisture would decide irrigation application, enable prediction of the annual evaporation loss from shallow water table, and recharge to ground water storage due to rainfall. Using the numerical scheme presented here, the soil moisture forecast can be made by making use of the forecast input parameter.

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