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LONG TERM PREDICTION OF GROUNDWATER REGIME IN AN INTERNALLY DRAINING BASIN

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PREFACE

The internally draining basins are usually found in the part of arid zones where natural streams and artificial surface drains are absent. If recharge component is more than the natural discharge it may lead to water logging. In the arid areas water logging which is usually accompanied by salinization of the soil is harmful to the land resources. The hydrologic equilibrium does not exist in these basins and there is wide scope to study the long term effect of such a hydrologic equilibrium does not exist in these basins and there is wide scope to study the long term effect of such a hydrologic non-equilibrium ground water regime on soil salinization. It is necessary to predict the position of water table at any time in future keeping in mind the present condition or the projected conditions for the future. The water table rise may be checked by taking appropriate planning and methodologies.

The present study is envisaged to review the methods to evaluate the efficiency of the each of the actions such as canal and channel lining, improved water management at farm level and afforestation in controlling water table rise in an internally draining basin. The different methods of solving groundwater problem are also discussed in detail.

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ABSTRACT

An internally draining basin is one which is entirely without well defined natural streams or artificial surface drain. An example is the internal basins occupying the western Haryana and north eastern Rajasthan with a constrained outlet to the western part of the Ghaggar Basin in the vicinity of Sirsa. Introduction of surface water irrigation in an internally draining basin changes the groundwater balance of the area which may lead to water logging and soil salinization. Any action which reduces deep percolation slows down the rate of water table rise. These actions are canal and channel lining, improved water application systems, improved water management at farm level and afforestation. In the present study different methods to evaluate the efficiency of each of the actions in controlling water table rise in an internally draining basin have been reviewed. Different methods of solving groundwater problem are also discussed in detail.

1.0 INTRODUCTION

The groundwater in a basin does not remain at rest but remains in a state of continuous movement. Its volume increases by the downward percolation of rain and surface water causing the water table to rise. At the same time, its volume decreases by evapotranspiration, by discharge to springs, and by outflow into streams and other natural drainage channels, causing the water table to fall. When considered over a long period, the average recharge equals the average discharge and a state of hydrological equilibrium exists.

An internally draining basin is one which is entirely without well defined natural streams or artificial surface drain. Hence the outflow from the internally draining basin would occur only due to evapotranspiration (natural) or abstraction (manmade). Other discharges in this type of basin will virtually be near zero. Naturally, the hydrologic equilibrium does not exist in these type of basins and it is essential to study the long term effect of such a hydrologic nonequilibrium on groundwater regime.

The internally draining basins are usually a part of arid zones of the world. In such basins, if recharge is scanty and evapotranspiration rate is high, then the hydrologic equilibrium may exist but if the recharge component is more than the natural discharge, it may lead to waterlogging in arid areas usually accompanied by salinization of the soil which can render once fertile land into waste land, to the detriment of local farmers and even of national economies. It is very essential to check the rise of water table. One must be able to predict the position of water table at any time in future keeping in mind the present conditions or the projected conditions for future. Accordingly, appropriate methodologies can be planned and adopted to check the water table rise.

1.1 Examples of Internal Draining Basin

There are several examples of internal draining basins throughout the world. Some of them are selected here for representation.

1.1.1 Biskra alluvial valley, ALGERIA

This valley is situated near the edges of Sahara desert in Algeria. The climate is semiarid with an annual rainfall of 250-400 mm. Most of the rain occurs during autumn and winter. The principal aquifer is an unconfined alluvial deposit. The boundaries of the aquifer are well defined. The depression shown in Figure 1 is an internally draining basin. The surface area of this aquifer is 5 km^2 and its thickness ranges from 30 to 40 m. Recharge occurs through direct infiltration of flood waters.

1.1.2 Plain of Ghazvin, IRAN

The plain of Ghazvin is situated about 120 km. west of Tehran, between the Elburz Mountains in the north and the central mountain ranges in the south. It forms the westernmost extension of the great Iranian plateau. The area is approximately rectangular, extending over a maximum of 100 km. from East to West and 70 km. from South to North; it covers about 5000 sq.km. The plain varies in elevation from 1,150 m. to 1,500m. above sea level while the mountain ranges reach elevations of 2,900 m. in the north-east and 2,600 m. in the south. The total areal extent of the basins draining into the area is about 13,000 sq.km. The two major streams, the Khar Rud and Abhar Rud, flow into a salt marsh on the eastern boundary of the area. There is only a small outflow from this swamp, drained by the Rud-E-Shur in an easterly direction (Figure 2).

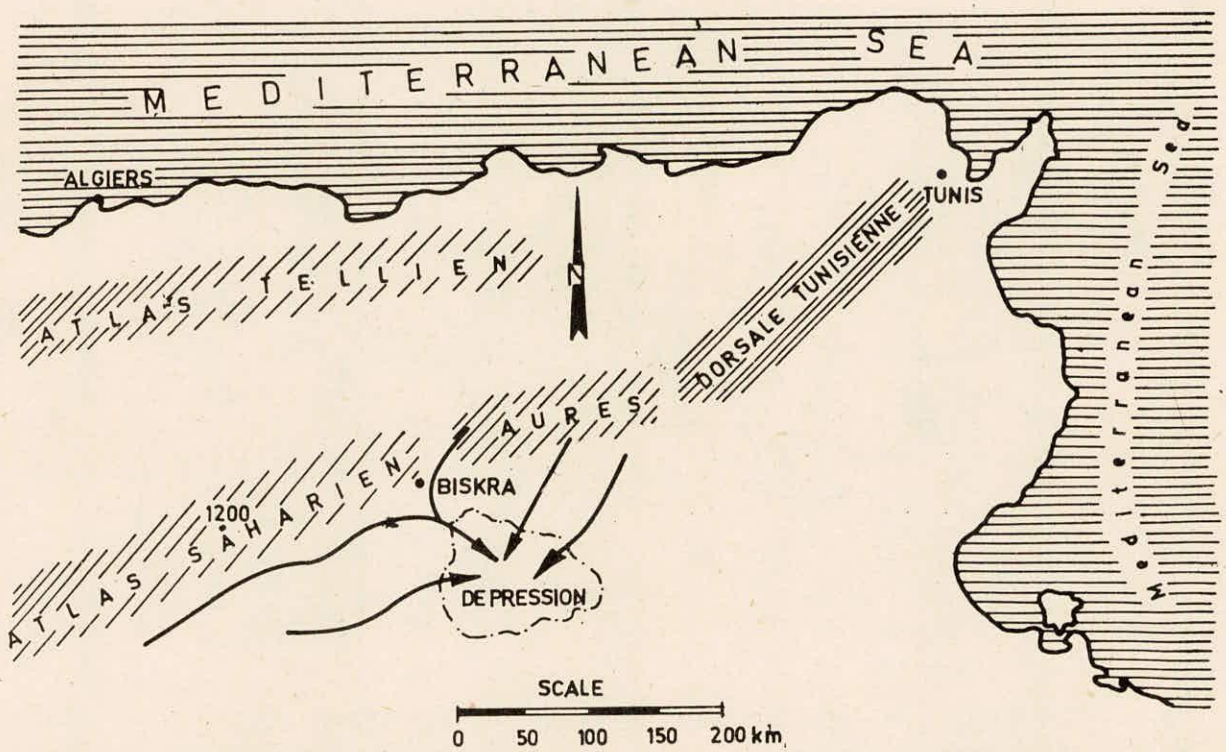

FIG. 1 — BISKRA ALLUVIAL VALLEY, ALGERIA

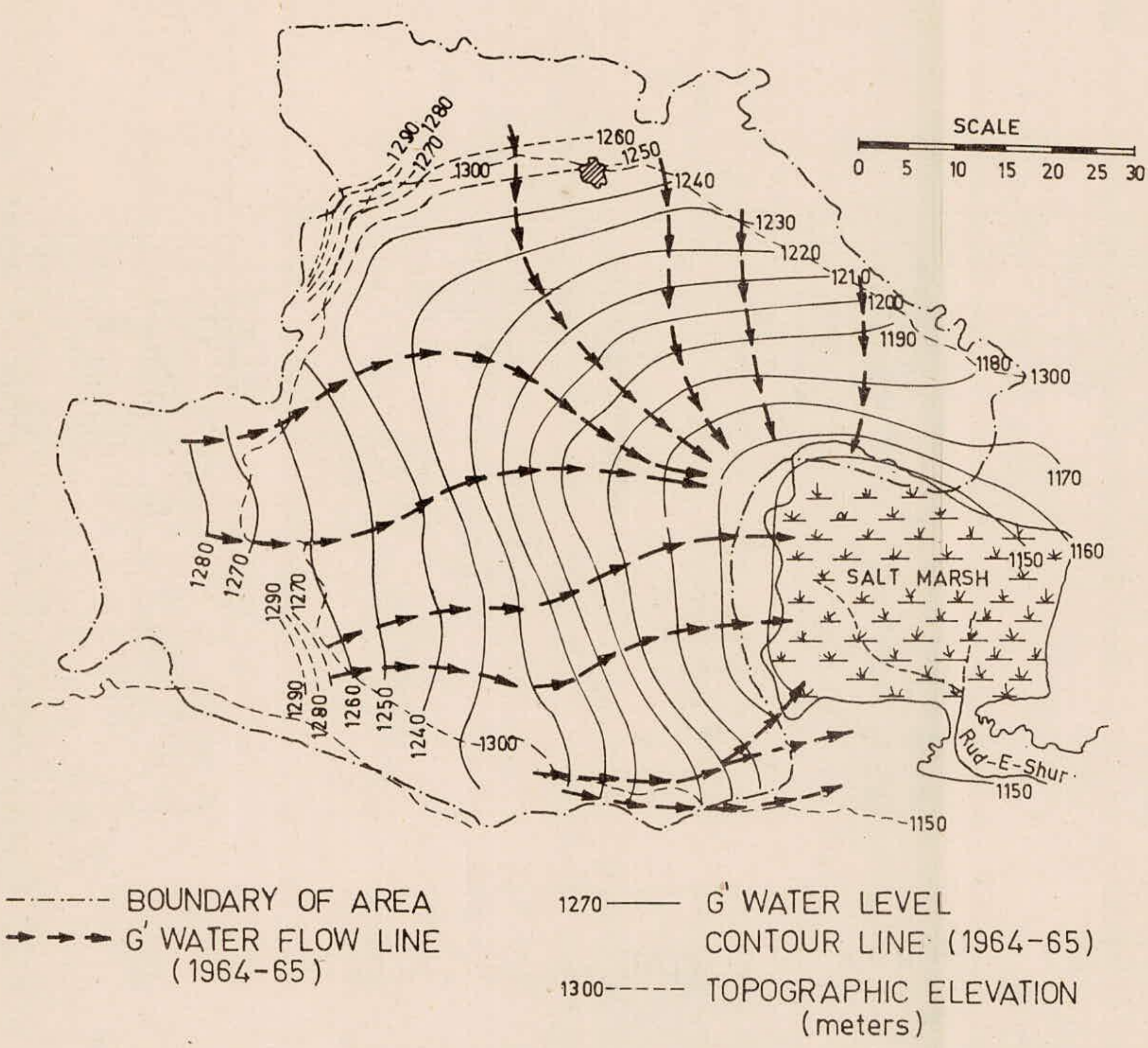

FIG. 2 — PLAIN OF GHAZVIN, IRAN

1.1.3 Lathi basin, western Rajasthan, INDIA

The Lathi basin forms an exclusive productive aquifer in Jaisalmer district, western Rajasthan. The areal extent of the Lathi formation is about 7500 sq.km. between latitudes N $26^{o}15'$ and N $27^{o}45'$: and longitude E $70^{o}45'$ and E $72^{o}30'$.

As in all deserts, the area is characterised by low and sporadic rainfall and heavy evaporation losses. Mean annual rainfall is 190 mm.

The drainage system is poor, disorganised and mainly of interior type. There are no rivers worth the name, nor are there any perennial streams in the area, the streams are ephemeral with the result that there is a lack of effective discharge when there is heavy precipitation. Sukri Nadi is the most prominent ephemeral stream of the region, which originates in the gently rising hills west of Sankara and follows a gently undulating north-westerly course until it disappears in the sand dunes north west of Chandan. Runoff is virtually non existent in the area except in the ridge areas underlain by hard sandstones.

1.1.4 Part of Ghaggar basin, HARYANA-RAJASTHAN

Another example of an internally draining basin is a basin occupying the western Haryana and north eastern Rajasthan with a constrained outlet to the western part of the Ghaggar basin in the vicinity of Sirsa (Figure 3).

1.2 Problems in an Internally Draining Basin

An internally draining basin can have several types of problems. Since the drainage system of such basins happens to be very poor, the inflow

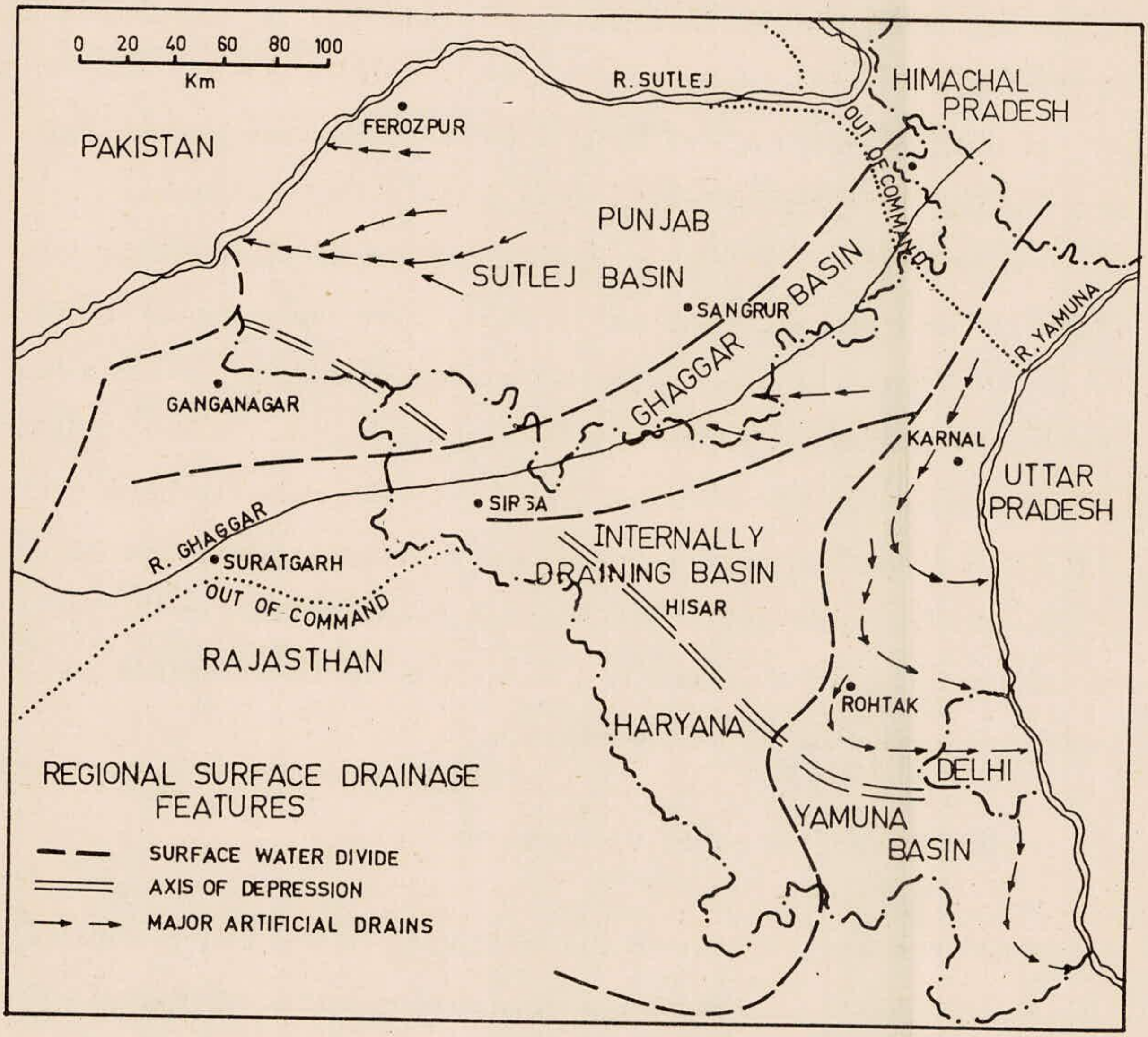

FIG. 3. INTERNALLY DRAINING BASIN-HARYANA

due to rainfall recharge gets accumulated and gives rise to water table which in turn creates waterlogging and salinization problems. If the draft from an internal draining basin is more than the annual recharge then it can result over exploitable groundwater.

1.2.1 Waterlogging

The water logging problem not only causes agricultural set back and decline in vegetative growth, but also would lead to destruction of human inhabitation and vegitation. The soils if continuously washed, also loose nutrients and become ineffective for agriculture except for meagre production of paddy and sugarcane in patches.

As a result of seepage from unlined canals of the irrigation systems, many areas in an internally draining basin become water logged either permanently or seasonally. If seepage from these surface bodies or other parts of the basin are allowed to continue, a serious waterlogging problem may occur in a later stage.

1.2.2 Soil Salinity:

Saline soils occur commonly in arid zones. The drier the climate, greater the intensity of the soil salinity. Maximum soil salinity is found in soils of deserts such as the Chile, the Sahara and Western China.

Salt accumulation takes place in soils by various geographical and geochemical processes. Few of such processes, known as cycles of salt accumulation, are described below.

Continental cyle is the process of movement, redistribution and accumulation of carbonic, sulphuric and chloric salts in inland regions that have no natural drainage. If the salt accumulation has taken place as a process of weathering and soil formation from igneous rocks, it is known as primary salt accumulation cycle. If the salt accumulation has been caused by the process of redistribution of salts formerly accumulated in the masses of sedimentary salt - bearing rocks, it is known as secondary salt accmulation cycle.

Marine cycle is the process of accumulation of marine salts on the coastal plains of dry low lands and along the shores of shallow bays.

Delta cycle is characterized by a complex combination of movement processes and accumulation of salts carried either from the continents by the rivers or from the sea.

Artesian cycle is the process of salt accumulation by evaporation of deep underground waters wedged upto the surface through tectonic fractures and destroyed structures.

Anthropogenic cycle is a result of formation of salt from the errors of the economic activity of man or from the ignorance of the laws of salt accumulation.

Salt accumulation processes are due to definite types of relief and certain geomorphological and hydrogeological conditions. Salt accumulation in different cycles is linked geomorphologically to low lands or component parts of low lands,- flood plains, deltas, troughs, low river terraces, lakes or coastal terraces. From the point of hydrogeology, the processes are related to regions with high water table (i.e. within the limit of the capillary rise of solutions). Hydrologically, salt accumulation occurs particularly in regions where runoff is low or virtually absent and where the groundwater storage is not regulated by runoff, but by evaporation and transpiration.

Natural evaporation is usually the only cause of salt accumulation and formation of gypsum and calcareous crusts. Capillary rise of groundwater table causes the salinisation of the upper soil horizons and causes decay of the plant growth. Higher the degree of mineralisation, greater the depth to which groundwaters can salanise the soil and thereby destroy the crop growth. Phisiological effects on crops due to salt concentration in soils play an important role in controlling plant growth. As such all the above mentioned points are to be kept in view while planning measures to control soil salinity.

As is seen from the above, accumulation of salts causes the soil salinity. In order to check the soil salinity, one should ensure better quality water supply to the agricultural fields and also take steps so as to prevent waterlogging conditions. Experience in the last few decades has shown that waterlogging and salinisation of the soils are the chief causes of failure of an irrigation project. Soluble salts can be removed by the process of leaching which needs of unsoluble salts cause more or less permanent loss of culturable lands as salts of such nature can not be removed by easy viable economic processes.

1.2.3 Over exploitation of groundwater

This phenomenon is just reverse of waterlogging problem and is quite common in internally draining basins. In areas, where pumping of groundwater exceeds the recharge, groundwater levels gradually decline. This creates acute shortage of drinking water and thus cause a threat to human life.

1.2.4 Deterioration in water quality

The water logged area in an internal draining basin also pollutes the ground water. Leaching of salts in water logged zones creates deterioration in water quality. When water table is within 2m. from surface, groundwater rises due to capillary action and gets evaporated which creates soil salinization.

Since there is no drainage pattern in such a basin, several pockets of water are found, where sodium concentration increases and causes futher deterioration the water quality.

1.3 Control Measures

To check the water logging problems in an internally draining basin, proper development of groundwater is needed. Groundwater is the major source of water in such a basin. Wise use of groundwater can solve most of the problems in a internally draining basin. Improvement in irrigation practices and a well planned redistribution of water can be a feasible solution.

To control the water logging problem in an internally draining basin, one has to take appropriate measures so that deep percolation does not take place. These measures involve canal lining, proper irrigation and afforestation.

1.3.1 Improvements to the irrigation systems and water use

Deep percolation losses to groundwater by seepage from canal and channel systems can be reduced to minimum through proper lining of channels. It should be noted that any savings on the conveyance and distribution system are transferred to the fields where they are again subject to deep percolation losses. Thus net savings of deep percolation due to canal and channel lining are not large and may be only locally important.

Sprinkler or drip irrigation can be operated with high field efficiencies, but these systems are not applicable to some of the field crops and would require high investments. Moreover, they would require more regular supplies of water from canals (continuous in the case of drip) or the creation of on-farm storage reservoirs. Improved delivery of surface water in terms of timeliness and quantity leads to more efficient use of water as this would reduce the tendency of farmers to over-irrigate in an attempt to compensate for the relative unreliability of supplies. However, any water application improvemnts must take account of the leaching requirement and the practicalities of operating large scale irrigation systems to serve a very large no. of farms, each with several crops requiring irrigation.

1.3.2 Use of brackish and saline groundwater for irrigation

It is often suggested that the solution to the drainage problems lies in encouraging farmers to use brackish or saline water for irrigation. Farmers are actually using groundwater of these quality categories, either directly or blended with surface water. The associated sodium absorption ratio of the water, the crops grown and the soil characteristics are critical in deciding the quality of the water which can be safely used. While it is accepted that the farmers should be encouraged to pump as much poor quality groundwater as possible, the question remains as to whether they can be expected to pump sufficient poor quality water to balance the drainage requirement. Given the facts that water tables rise throughout almost the entire poor quality groundwater zone and that zones are become water logged in an internally draining basin, the answer to the question appears to be that farmers may contribute to water level control by pumping out poor quality groundwater, but it cannot be expected that they will provide the full drainage requirement.

1.3.3 Afforestation

There is no doubt that trees can be used to control water levels and have been used to drain water logged areas. A number of tree species tolerate fairly saline groundwater at shallow depth, particularly if the profile is leached from time to time, as would happen with the monsoon or could be provided by irrigation. Most of the trees which are salt tolerant, and of commercial value, provide wood for fuel.

A closed canopy of trees could remove more than 1,000 mm. of water from the soil profile in a year and upto 600 mm. over a dry season. Thus a well distributed but relatively thin cover of tree could have a major effect on the groundwater balance, given that the drainage surplus is in the range of 30 to 120 mm./yr. in most of the affected area. The concept has such merit that it justifies serious investigation. It appears that trees should, wherever possible, be established before a critical drainage situation develops, as establishement of trees on water logged lands, though possible, presents a more difficult problem.

2.0 PREDICTION OF WATER REGIME

In order to plan a strategy for wise use of groundwater in an internally draining basin, one has to predict the future behaviour of groundwater regime. If one is able to model the physical state of subsurface conditions, then the proper planning for future use of groundwater can be made and the effect of such planning can be seen before hand.

Groundwater modelling is a tool that can help analyse many groundwater problems. Models are useful for reconnaissance studies preceding field investigations, for interpretive studies following the field program, and for predictive studies to estimate future field behaviour. In addition to these applications, models are useful for studying various types of flow behaviour by examining hypothetical aquifer problems. Before attempting such studies, however, one must be familiar with groundwater modelling limitations.

2.1 Modelling Approaches

Prickett defined a groundwater model as any system that can duplicate the response of a groundwater reservoir. The operation of the model and the manipulation of the results is termed as simulation. The popularity of the groundwater models has increased as a result of two factors. First, the increased availability of relatively cheap powerful digital computers, and secondly the progressive increase in the demand for more efficient aquifer management.

Although models are extremely numerous and quite diverse in features, it is possible to group them into certain categories according to their objectives or function as follows:

a) Prediction models : These models generally simulate groundwater flow in an aquifer. They require information on aquifer characteristics, boundary conditions and pumping rates, while they yield data regarding the direction and rate of groundwater flow, changes in water level, surface-groundwater interaction and the effects of abstraction. This is the most common type of model.

b) Resource management models : These can be used in tandem with prediction models; they include optimization as well as simulation techniques. This type of model is designed to indicate the best course of action to achieve a particular objective, such as minimizing costs or ensuring the maximum rate of supply.

c) Identification models : These models determine input parameters for both of the above types. Any model is only as good as the data upon which it is based. Thus, identification models are used to determine the hydrogeological input parameters for other models from observations of field data. For example given the rate of abstraction from a well and drawdown data for several nearby observation holes, it is a relatively simple matter to change the hydraulic characteristics of the aquifer in the model until it responds in a similar fashion to the prototype. These values can then be used in a prediction model, which would simulate the effect of pumping the well in a manner or at a rate for which no field data exist.

d) Data manipulation models: These handle the data collection networks,process the field data, identify critical data,determine the inputs to other models and store all relevant data.

The procedure for developing a deterministic,mathematical model of physical system can be generalized as shown in Fig.4. The first step is to understand the physical behaviour of the system. Cause-effect relationships are determined and a conceptual model of how the system operates is formulated. For groundwater flow, these relationships are generally

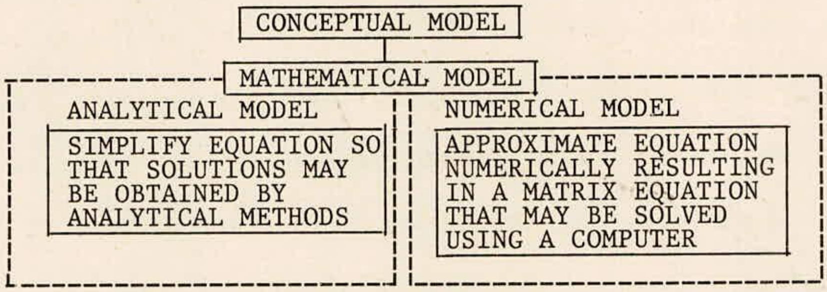

Fig.4 Logic diagram for developing a mathematical model

known, and are expressed using concepts such as hydraulic gradient to indicate flow direction. The next step is to translate the physics into mathematical terms that is, to appropriate simplifying assumptions and to develop the governing equations. This constitutes the mathematical model. The mathematical model for groundwater flow consists of a partical differential equation together with appropriate boundary conditions that express conservation of mass and that describe continous variable (for example hydraulic head) over the region of interest. In addition, it entails various phenomenological 'laws' describing the rate process active in the aquifer. An example is Darcy's law for fluid flow through porous media; this is generally used to express conservation of momentum. Finally, various assumptions may be invoked such as those of one or two-dimensional flow and artesian or water table conditions.

Once the mathematical model is formulated the next step is to obtain a solution using one of two general approaches. The groundwater flow equation can be simplified further, for example, assuming radial flow and infinite aquifer extent, to form a subset of the general equation that is amenable to analytical solution. The equations and solutions of this subset are referred to as analytical models. The familiar Theis type curve represents the solution of one such analytical model.

Alternatively, for problems where the simplified analytical models no longer describe the physics of the situation, the partial differential equation can be approximated numerically, for example, with finite difference method or with the finite element method. In so doing, one replaces continuous variables with discrete variables that are defined at grid blocks (or nodes). Thus, the continuous differential equation, defining hydraulic head everywhere in the aquifer, is replaced by a finite number of algebraic equations that defines the hydraulic head at specific points. This system of algebraic equations is generally solved using matrix techniques. This approach constitutes a numerical model, and generally, a computer program is written to solve the equations on a digital computer.

2.2 Data Requirements for Prediction Models

Any groundwater flow model is only as good as the data upon which it is based. In general, more the data are available initially, the better will be the completed model. This is true regardless of what type of model is constructed. Most groundwater flow models require data relating to the following if they are to effectively simulate the aquifer properties:

a) the extent of the aquifer and the location and nature of aquifer boundaries,

b) the flow of water into and out of the aquifer,

c) the rest water levels in the aquifer,

d) variations in the thickness and depth of the aquifer and any confining strata,

e) the spatial variation of the coefficients of transmissivity and storage,

f) data recorded during the pump testing of wells such as the discharge of the well and the drawdown recorded at various points in the aquifer,

g) water level fluctuations in the aquifer over a number of years,

h) the rate of infiltration to the aquifer in the recharge area during the same period,

i) the pumping schedules over the same period,

j) river base flows,

k) spring locations and spring flows, and

l) general background information regarding the hydrogeology of the region, such as areas of interconnection between surface and groundwater, interflow between aquifers, artificial recharge and so on.

It is extremely unlikely that all these data would be available at the beginning of a model study. Indeed, a model is often commissioned with the objective of supplying such information without having to incur the expense of a field investigation. Models which are to be used for predictive purposes can be based upon quite scanty data initially and updated as more field results become available.

During the construction of a model two distinct stages of development are often recognised: calibration and verification. The calibration stage requires information relating to items (a) and (b) above and usually involves, for a given rate of groundwater flow, adjusting the transmissivity of the model aquifer until the elevation of the water table or piezometric surface in the model is analogous to that in the prototype. As a first approximation, transmissivity values may be estimated from the spacing of the groundwater contours. Wide contour spacing is generally indicative of high transmissivity.

The verification stage of development uses information relating to items (e) and (f) and involves adjusting the model until it reproduces satisfactorily the recharge and discharge mechanisms of the prototype. In particular, the model must be able to duplicate adequately two sets of prototype data. First, the specific capacity data which is the ratio of yield to drawdown at the pumped well, and secondly the drawdown recorded in the observation holes for a given discharge from the nearby observation well. When trying to reproduce in the model the drawdown that occurs as a result of pumpage, inspection of well discharge equation gives several useful relationships that are of assistance.

2.3 Types of Groundwater Models

Because the number of groundwater models available today is large, when beginning a study the first question that may come to mind is, "which one should I use?" Actually, the first question one asks should be, "Do I need a numerical model study for this problem?" The answers to both of these questions can be determined by first considering the following: a) what are the study objectives? b) how much is known about the aquifer system; that is, what data are available? and c) does the study includes plans to obtain additional data?

Groundwater modelling begins with a conceptual understanding of the physical problem. The next step in modelling is translating the physical system into mathematical terms. In general, the final results are similar to groundwater flow equation. These equations, are often simplified, using site specific assumptions to form a variety of equation subsets. An understanding of these equations and their associated boundary and initial conditions is necessary before a modelling problem can be formulated.

2.3.1 Mathematical models

The derivation of equations used in groundwater applications are based on the conservation principles dealing with mass, momentum, and energy. These principles require that the net quantity (mass, momentum or energy) leaving or entering a specified volume of aquifer during a given time interval be equal to the change in the amount of that quantity stored in the volume. The derivation of the particular conservation equation involves representing the balance in terms of mathematical expressions. Once the balance equation is developed in mathematical terms, it is necessary to specify additional relationships among variables so that the equations can be solved. These include thermodynamic (e.g., the effect of fluid pressure on density) and constitutive (e.g., the effect of fluid pressure on porosity) relationships. The result of the derivation is usually a set of partial differential equations in three dimensional cartesian coordinate system.

An equation describing unsteady groundwater flow in three dimensions can be written as

$$\bar{\nabla} \cdot \bar{\bar{k}} \cdot \bar{\nabla} h + R = S_s \frac{\partial h}{\partial t} \qquad (2.1)$$

in which h is the hydraulic head (l); R is a general source or sink term (1/t), that is, volume of water injected per unit volume per unit time; S_s is the specific storage (1/l); and $\bar{\nabla}$ is a differential operator (1/l). This equation is known as a diffusion type equation and is derived by combining the mass conservation (water balance) and momentum conservation (Darcy's law) equations. In a more explicit form, equation (2.1) can be written as,

$$\frac{\partial}{\partial x}(k_{xx} \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(k_{yy} \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(k_{zz} \frac{\partial h}{\partial z}) + R = S_s \frac{\partial h}{\partial t} \qquad (2.2)$$

In developing this equation it was assumed that the principal components of the hydraulic conductivity tensor are colinear with the Cartesian coordinate system (that is, the directions of the anisotropy line up with coordinate system). To get an intuitive feel for what eqn.(2.2) expresses, consider a small control volume of the aquifer. The first three terms in eqn. (2.2) represent the difference in the rate of water flowing into and out of the volume. R represents the rate of water gained or lost from some source or sink within or along the boundary of the volume. The right hand side represents the change in the amount of water stored in the control volume expressed as a rate.

2.3.1.1 Boundary and initial conditions

In order to obtain unique solution of a partial differential equation corresponding to a given physical process, additional information about the physical state of the process is required. This information is described by boundary and initial conditions. For steady state problems only boundary conditions are required, whereas for unsteady state problems both boundary and initial conditions are required. Mathematically, the boundary conditions include the geometry of the boundary and the values of the dependent variable or its derivative normal to the boundary.

In physical terms, for ground-water applications the boundary conditions are generally of three types: (1) specified value; (2) specified flux; or (3) value-dependent flux, where the value is head, concentration or temperature, depending on the equation. These are shown in Table 1.

The initial conditions are simply the values of the dependent

Table 1. Ground-Water Boundary Conditions

Type	Description
Specified value	Values of head, concentration or temperature are specified along the boundary. (In mathematical terms, this is known as the Dirichlet condition)
Specified flux	Flow rate of water, concentration or temperature is specified along the boundary and equated to the normal derivative. For example, the volumetric flow rate per unit area for water in an isotropic media is given by $$q_n = - K \frac{\partial h}{\partial n},$$ where the subscript n refers to the direction normal (perpendicular) to the boundary. A no-flow (impermeable boundary is a special case of this type in which $q_n = 0$. (When the derivative is specified on the boundary, it is called a Neumann condition.)
Value-Dependent flux	The flow rate is related to both the normal derivative and the value. For example, the volumetric flow rate per unit area of water is related to the normal derivative of head and head itself by $$-K \frac{\partial h}{\partial n} = q_n(h_b),$$ where q_n is some function that describes the boundary flow rate given the head at the boundary (h_b).

variable specified everywhere inside the boundary. For example, in a confined aquifer for which the equations are linear, there is no need to impose the natural flow system since the computed drawdown can be superimposed on the natural flow system. In this case, the initial condition is drawdown (the dependent variable) equal to zero everywhere.

Just as the physical groundwater system is idealized as continuous in deriving the differential equations, it is also expedient to idealize the conditions on the boundaries of the system in order that they too can be given mathematical expression. The boundary conditions of groundwater systems in nature are of several kinds, perhaps the most common being those describing the conditions at a well. Since the porous media stops at the well face, the aquifer not only has a boundary around its perimeter, but the outline of each well is also considered a boundary to the aquifer. The boundary conditions at wells are treated as constant or variable specified flux, or constant head, depending on which best describes the actual physical conditions. If the well is discharging or recharging at a given concentration or temperature, then these may also be specified, if the transport equations are being considered.

Impermeable or nearly impermeable boundaries are formed by underlying or overlying beds of rock, by contiguous rock masses along a fault(or along the wall of a buried rock valley), or by dikes or similar structures. Permeable boundaries are formed by the bottom of rivers, canals, lakes, and other bodies of surface water. These permeable boundaries may be treated as surfaces of equal head (specified), if the body of surface water is large in volume, so that its level is uniform and independent of changes in ground-water flow. The uniform head on a boundary of this type may, however, change with time due to seasonal variation in the surface-water level. Other bodies of surface water, such as streams, may form boundaries with nonuniform distributions of head which may be either constant or variable with time. A small stream, for example, might be affected by a nearby withdrawal of ground water if that withdrawal occurred at a rate of the same order of magnitude as the flow in the stream. Then the boundary condition would not be independent of the ground-water flow; that is, it would be a head-dependent flux.

2.3.2 Numerical Models

Numerical models provide the most general tool for the quantitative analysis of ground water applications. They are not subject to many of the restrictive assumptions required for familiar analytical solutions (e.g. Theis' solution for radial flow to a pumping well in an infinite, confined aquifer). In spite of the flexibility of numerical models, their mathematical basis is actually less sophisticated than that of the analytical methods. Unfortunately, to the would be model user, numerical model seems complex. This perception results from two primary causes; the first is that the number of alternative methods appears to be very large. Actually, the number of basic alternative methods is few; only the number of minor variations is large. Each of these variations contributes to the second cause, unfamiliar terminology, by introducing new names and jargon.

To develop a numerical model of a physical system (in our case, an aquifer), it is first necessary to understand how that system behaves. This understanding takes the form of laws and concepts (e.g., Darcy's law and the concept of storage). These concepts and laws are then translated into mathematical expressions, usually partial differential equations, with boundary and initial conditions.

Numerical methods provide a means for solving these equations in their most general form.

Numerical solution normally involves approximating continuous (defined at every point) partial differential equations with a set of discrete equations in time and space. Thus, the region and time period of interest are divided in some fashion, resulting in an equation or set of equations for each subregion and time step. These discrete equations are combined to form a system of algebraic equations that must be solved for each time step. Finite-difference and finite-element methods are the major numerical techniques used in ground-water applications. The important components and steps of model development for the two alternative methods are shown in Figure 5.

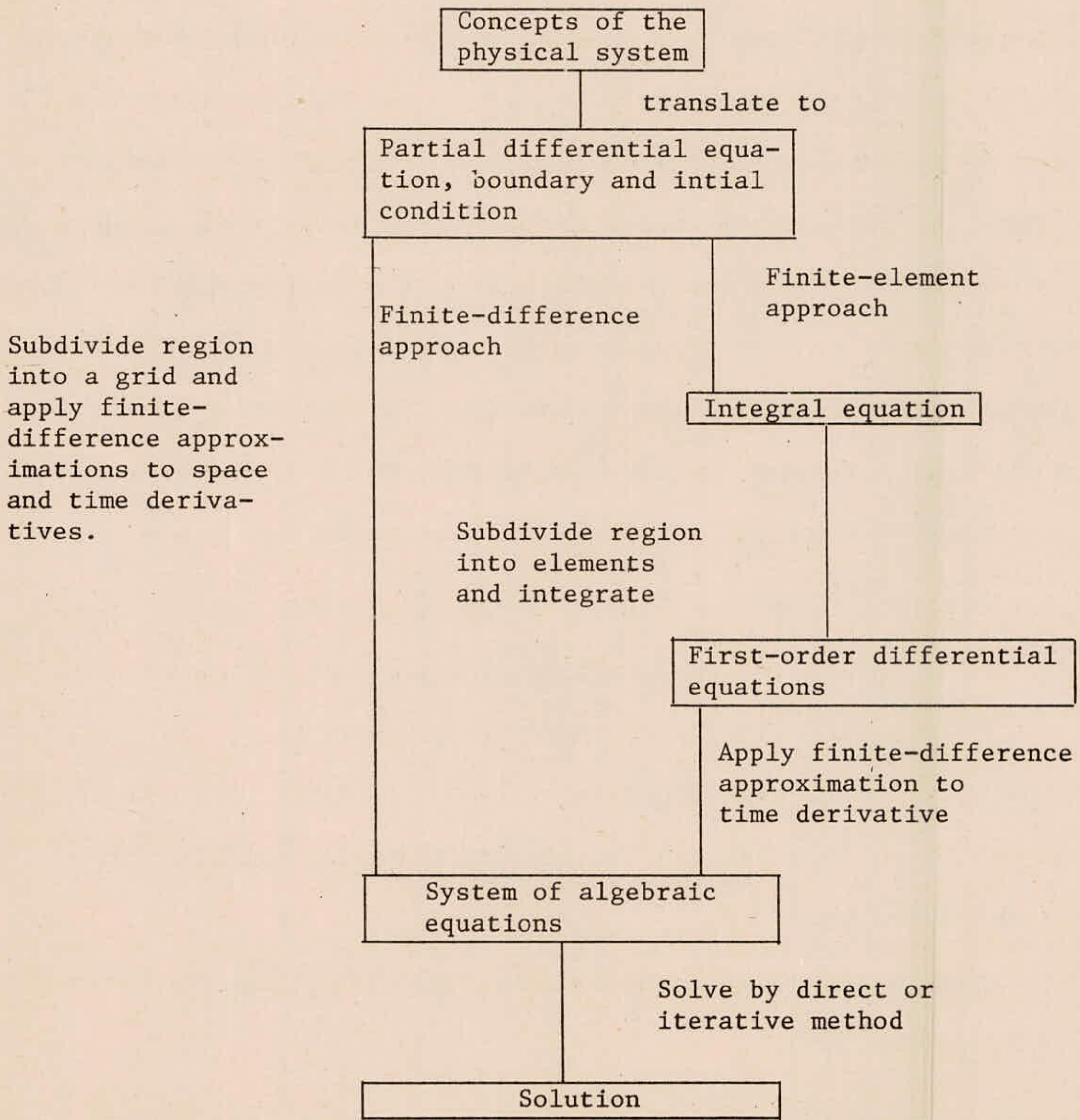

Fig.5 Generalized model development by FD and FE methods

2.3.2.1 Finite-difference methods (FDM)

One numerical approach that has been applied successfully to the ground-water flow equation involves finite difference approximations. When using FDM to solve a partial differential equation, a grid is first established throughout the region of interest. For two-dimensional, areal problems, we overlay a grid system on a map view of the aqufer. There are two common types of grids: mesh-centered and block-centered. Associated with the grids are node points that represent the position at which the solution of the unknown values (head, for example) is obtained. In the mesh-centered grid the nodes are located on the intersection of grid lines, whereas in the block-centered grid the nodes are centered between grid lines. The choice of the type of grid to use depends largely on the boundary conditions. The mesh-centered grid is convenient for problems where values of head are specified on the boundary, whereas the block-centered grid has an advantage in problems where the flux is specified across the boundary. From a practical point of view, the differences in the two types of grids are minor.

The final result is an algebraic equation for each node in the grid system. For a rectangular grid the form of a typical equation is

$$B_{i,j}h^{n}_{i-1,j} + D_{i,j}h^{n}_{i,j-1} + E_{i,j}h^{n}_{i,j} + F_{i,j}h^{n}_{i,j+1} + H_{i,j}h^{n}_{i+1,j} \approx Q^{n-1}_{i,j} \qquad (2.3)$$

The notation in equation (2.3) refers to the nodal locations, where h is the head at the designated node; the explicit definitions of the coefficients $B_{i,j}, D_{i,j}, E_{i,j}, F_{i,j}$, and $H_{i,j}$ are not given here, but can be found, for example, in Freeze and Cherry (1979). The main reason for presenting equation (2.3) is to demonstrate the form of the algebraic equation. This equation is for an arbitrary node (i,j) and as may be seen, it has contributions from four adjacent nodes. These are evaluated at the new time level (n) and are related to some known quantity, $Q_{i,j}^{n-1}$, which is computed from information at the old time level (n-1).

Writing an equation similar to (2.3) for each node results in N equations with N unknown head values to be determined, where N is the total number of nodes. This may be formulated in matrix form and solved using matrix methods.

2.3.2.2 Finite-element methods (FEM)

There are two fundamental problems in calculus: (1) examining the area under a curve, i.e., integration; and (2) examining the tangent of a curve at a point, i.e. differentiation. Both of these concepts were fairly well understood by the seventeenth century. For example, Archimedes demonstrated an understanding of integration by deriving his approximation for π. However, it was not until 1667 that Isaac Barrow, the teacher of Newton, discovered that integration and differentiation are essentially inverse to one another, which is the fundamental theorem of calculus.

Whereas FDM approximates differential equations by a differential approach, FEM approximates differential equations by an integral approach. Based on the fundamental theorem, one would expect the two methods to be related and to converge to the same solution, but perhaps from different directions.

The FEM actually refers to the numerial method whereby a region is divided into subregions called elements, whose shapes are determined by a set of points called nodes. Note that flexibility of elements enables consideration of regions with complex geometry; for example, a water-table aquifer with a meandering river can be outlined with elements fairly accurately.

For transient problems, the time domain may also be approximately using finite elements. In general, however, most studies use finite-difference approximation for the time derivatives.

The first step in applying the FEM is to derive an integral representation of the partial differential equation. This may be accomplished by several methods; two of the more popular ones include: (1) the method of weighted residuals and (2) the variational method. In the method of weighted residuals (Finlayson, 1972), one works directly with the differential equation and boundary conditions, whereas in the variational method (Zienkiewicz, 1971), one uses a functional (a function of a function) related to the differential equation and boundary conditions. The mathematics of both of these approaches is fairly straight forward, but not intuitive.

The next step is to approximate the dependent variables (head, concentration or temperature) in terms of interpolation functions. The interpolation functions are called basis functions, and are chosen to satisfy certain mathematical requirements for ease of computation. Although any system of independent functions can be chosen as the basis function, piecewise-continuous polynomial sets are often preferred because they are both easily integrated and differentiated. Since the element is usually small, the interpolation function can be sufficiently approximated by a low-order polynomial, for example, linear, quadratic, or cubic. As an example, consider a linear basis function for a triangular element. This basis function describes a plane surface including the values of the dependent variable (head) at the node points in the element.

Once the basis functions are specified and the grid designed, the integral relationship must be expressed for each element as a function of the coordinates of all node points of the element. Next the values of the integrals are calculated for each element. The values for all elements are combined, including boundary conditions, to yield a system of first-order linear differential equations in time. As previously mentioned, this is approximated using finite-difference techniques to produce a set of algebraic equations. As with finite-difference equations, matrix methods are required for solution.

2.3.2.3 Matrix solution techniques

As we have seen, each numerical approximation leads to an algebraic equation for each node point. These are combined to form a matrix equation, that is, a set of N equations with N unknown, where N is the number of nodes. The general form of these equations, written in matrix form is

$$\bar{\bar{A}}\bar{h} = \bar{d}, \qquad (2.4)$$

where $\bar{\bar{A}}$ is a matrix containing coefficients related to grid spacing and aquifer properties, such as transmissivity; $\bar{h}$ is a vector containing the dependent variables to be determined, for example, head values at each node; and $\bar{d}$ is a vector containing all known information, for example, specified pumpage and boundary condition information.

In general, a matrix equation may be solved numerically by one of two basic ways: (1) direct and (2) iterative. Some solutions may involve a combination of the two. In direct methods a sequence of operations is performed only once, providing a solution that is exact, except for machine round-off error. Iterative methods attempt solution by a process of successive approximation. They involve making an initial guess at the matrix solution, then improving this guess by some iterative process until an error criterion is attained. Therefore, in these techniques, one must be concerned with convergence, and the rate of convergence.

Although solving the matrix equation is a mathematical problem, the hydrologist must be aware of some of its important aspects, since generally the matrix solution is the most expensive part of the computer costs. In very general terms, iterative techniques are more efficient than direct solution techniques for matrix equations that contain more than 1,000 unknowns. It should also be pointed out that for some problems, the matrix $\overline{\overline{A}}$ does not have to be regenerated each time step. For a direct method, this means that $\overline{\overline{A}}$ is decomposed only once, and a subsequent time step requires only back substitution. Since back substitution is much less expensive than decomposition (elimination), this improves the efficiency of direct methods considerably.

(a) Direct Methods

Direct methods can be further subdivided into: (1) solution by determinants, (2) solution by successive elimination of the unknowns, and (3) solution by matrix inversion. According to Narasimhan and Witherspoon (1977), perhaps the most widely used direct approach for transient problems is that of successive elimination and back substitution, which includes the Gaussian elimination method (Scarborough, 1966) and the Cholesky decomposition method (Weaver, 1967).

Direct methods have two main disadvantages. The first problem deals with storage requirements and computation time for large problems. The matrix in equation (2.4) is sparse (contains many zero values) and in order to minimize computational effort, several techniques have been proposed. Various schemes of numbering the nodes have been studied; an efficient one for finite-difference nodes is alternating direction (D4) ordering (Price and Coats, 1974). Other methods have been attempted with the finite-element method. However, for both finite-difference and finite-element methods storage requirements may still prove to be unavoidably large for three-dimensional problems.

The second problem with direct methods deals with round-off errors. Because many arithmetic operations are performed, round-off errors can accumulate for certain types of matrices.

(b) Iterative Methods

Iterative schemes avoid the need for storing large matrices, which make them attractive for solving problems with many unknowns. Numerous schemes have been developed; a few of the more commonly used ones include successive over-relaxation methods (Varga, 1962), alternating direction implicit procedure (Douglas and Rachford, 1956), iterative alternating direction implicit procedure (Wachpress and Habetler, 1960), and the strongly implicit procedure.

Since iterative methods start with an initial estimate for the solution, the efficiency of the method is dependent on this initial guess. This makes the iterative approach less desirable for solving steady-state problems (Narasimhan and Witherspoon, 1977). To speed up the iterative process, relaxation and acceleration factors are used. Unfortunately, the definition of best values for these factors is often problem dependent. In addition, iterative approaches require that an error tolerance be specified to stop the iterative process. This, too, may be problem dependent.

According to Narasimhan and Witherspoon (1977), perhaps the greatest limitation of the iterative schemes is the requirement that the matrix be well conditioned. An ill-conditioned matrix can drastically affect the rate of convergence or even prevent convergence. An example of an ill-conditioned matrix is one in which the main diagonal terms are much smaller than other terms in the matrix. Such matrices can result from finite-element applications.

3.0 REMARKS

Probably the most frequent application of ground water models is that of history matching and prediction of site-specific aquifer behaviour. Of the various types of models discussed, the numerical model offers the most general tool for simulating aquifer behaviour. The effect of each kind of activity in an internally draining basin,e.g., revised pumping pattern, method of irrigation, afforestation,etc. can be studied through numerical models and behaviour of the aquifer in presence of these activities can be predicted. Accordingly, an effective plan can be made to make best possible use of ground water resources of an internal draining basin. Deterministic mathematical models (both analytical and numerical) retain a good measure of physical insight while permitting a large class of problems to be considered with the same model. Analytical methods, such as type curve analysis are easy to use. Numerical models, although more difficult to apply are not limited by many of the simplifying assumptions necessary for the analytical methods.

Each type of model has both advantages and disadvantages. Consequently, no single approach should be considered superior to others for all applications. The selection of a particular approach should be based on the specific aquifer problem addressed. Whichever approach is taken, the final step in modelling a ground water flow system is to translate the (mathematical) results back to their physical meanings. In addition, these results must be interpreted in terms of both their agreement with reality and their effectiveness in answering the hydrologic questions that motivated the model study.

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