Assessment of Artificial Ground-Water Recharge through a Vertical Shaft under Un-Steady-State-Flow Condition

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Statement of the Problem

A sedimentary groundwater basin consists of a confined aquifer overlain by an aquiclude and underlain by an impervious stratum. The aquifer is homogeneous, isotropic, and of infinite areal extent. The water level in the surface water body is at a height h_1 above the bottom impervious base chosen as the datum. The thickness of the upper clay layer beneath the surface water body is L. Prior to onset of recharge, the piezometric surface in the aquifer stands at a height h_2 above the datum. The height h_2 is lower than h_1.

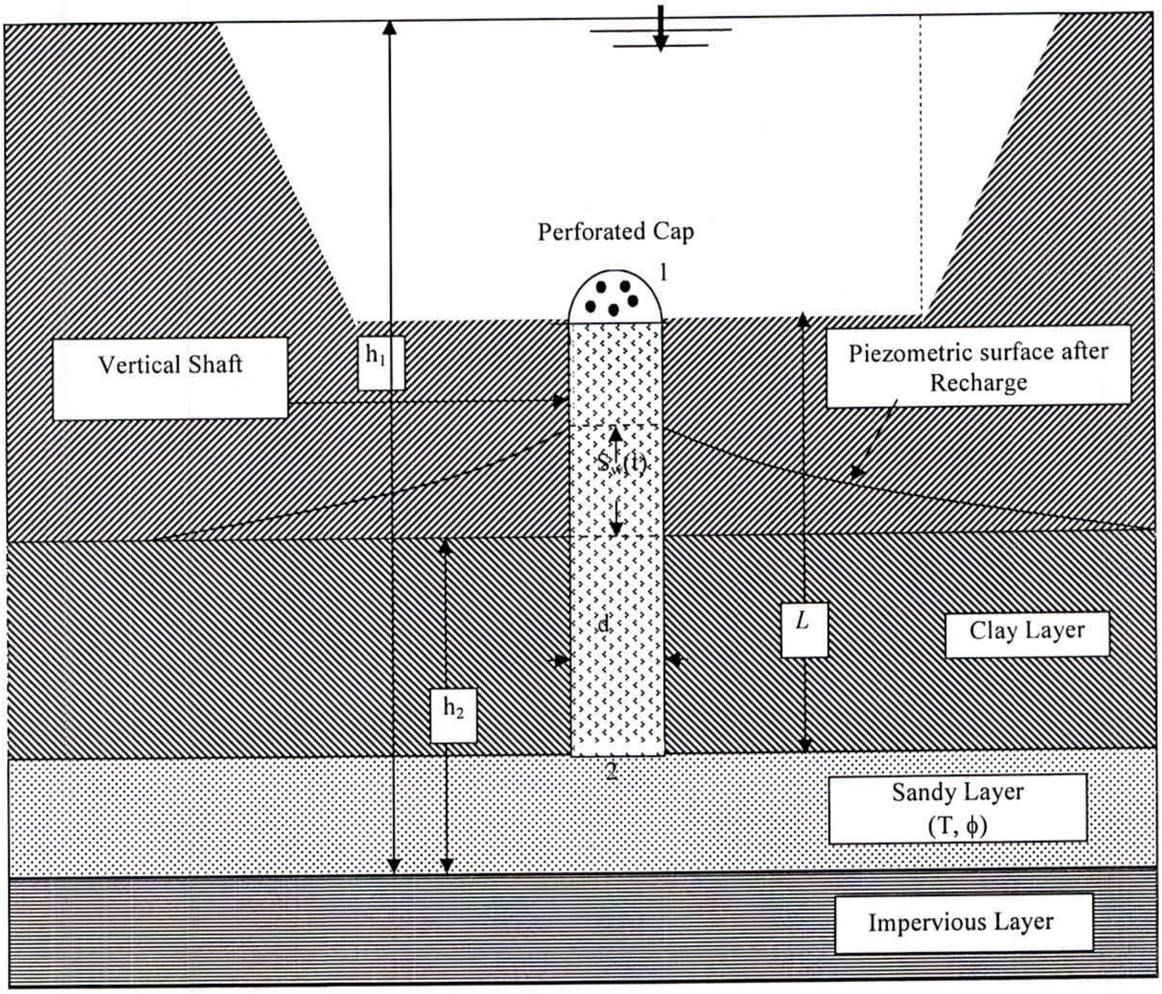

Fig. 1 A vertical shaft penetrating marginally into the aquifer and filled with a coarse material

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Case 1: A vertical shaft penetrating marginally into the aquifer and no filling coarse material

The aquifer can be recharged by constructing a vertical shaft in the bed of the surface water body through the intervening clay layer. The shaft may be filled with a filter material such as coarse sand to restrict contamination of groundwater. The aquifer can be recharged through a fully or partially penetrating recharge well. Quantification of recharge rate through a vertical shaft penetrating marginally into the aquifer, with no filling material, is sought.

An analytical expression for recharge is derived applying Bernoulli's equation.

Bernoulli's equation

The Bernoulli's equation applicable for steady-state-flow-condition in a non-viscous incompressible fluid is:

$$\frac{p}{\gamma_w} + z + \frac{\bar{v}^2}{2g} = \text{constant} = \text{h}$$

where, p=fluid pressure; γ_w =unit weight of water; $\bar{v}$ in case of ground water is seepage velocity which is the true velocity of fluid through pores; g=acceleration due to gravity, h= total head=energy per unit weight of fluid. To take into account loss of energy due to viscous resistance within pores, Bernoulli's equation is expressed as:

$$\frac{p_1}{\gamma_w} + z_1 + \frac{\bar{v}_1^2}{2g} = \frac{p_2}{\gamma_w} + z_2 + \frac{\bar{v}_2^2}{2g} + \Delta h$$

points 1 and 2 are two points on a stream tube at a distance Δs apart; Δh is the head loss between the two points 1 and 2. The above equation implies that the flow is in a steady state condition. Discretising the time-domain by uniform time steps, assuming that an unsteady state is succession of steady states, and boundary perturbations are constant within a time step that may vary from step to step, Bernoulli's equation is applied to find solution to unsteady well hydraulics problem. Computation of recharge capacity through a vertical shaft is described in the following example.

Let the time domain be discretised by uniform time step. Let within each time step, the flow through the vertical shaft gets stabilized. Such condition would prevail for shallow aquifer under lying clay layer. Accounting for the entry, exit, and friction losses and applying Bernoulli's equation between points 1 and 2 for mth time step:

$$h_1 = \frac{c_e v^2}{2g} + \frac{f\, L v^2}{4 g r_w} + \frac{v^2}{2g} + h_2 + s(r_w, m\Delta t) \tag{1}$$

$$\text{v} = \frac{R(m)}{(86400 \Delta t \pi\; r_w^{\,2})} \tag{2}$$

in which, c_e = coefficient of entry loss, f = friction loss factor, L = length of the shaft, γ_w = unit weight of water, g = acceleration due to gravity (m/sec^2), v = velocity of water in the shaft (m/sec) during mth time step, R(m) recharge volume (m^3) during mth time step, Δt= time step size (day). The first term in the right hand side of equation (1) accounts for entry loss, the second term accounts for friction loss in the shaft and the third term is the expansion loss at the exit of the shaft. s(r_w,mΔt) is the rise in piezometric surface at the recharge well consequent to the recharge taken place. The rise in piezometric surface, s(r_w,mΔt), at the recharge well face at time mΔt due to variable recharge, R(γ), γ= 1,2,...,m, is given by(Morel-Seytoux,1972):

$$s(r_w, m\Delta t) = \sum_{\gamma=1}^{m} R(\gamma)\delta_p(m-\gamma+1,\Delta t) \tag{3}$$

The kernel coefficient, δ_p (m,Δt), is:

$$\delta_p(m,\Delta t) = \frac{1}{\Delta t}\left[U(r_w, m\Delta t) - U(r_w,(m-1)\,\Delta t)\right] \tag{4a}$$

The step response function $U(r_w,t)$ of a partially penetrating well and a confined aquifer system is given by (Hantush, 1967):

$$U(r_w,t) = \frac{1}{4\pi T}\ \{W(u) + 2\sum_{n=1}^{\infty} W_n(u,\frac{n\pi r_w}{b})\} \tag{4b}$$

where

$$u = \frac{r_w^2\phi}{4Tt}, \qquad W(u) = \int_u^{\infty} \frac{e^{-y}}{y}dy, \quad \text{and}$$

$$W_n(u,\frac{n\pi r_w}{b}) = \int_u^{\infty} \frac{dy}{y} exp\left\{-y - \frac{1}{4y}\left(\frac{n\pi r_w}{b}\right)^2\right\}$$

T = transmissivity (m^2/day), ϕ=storativity, and b= thickness of the aquifer(m); r_w = radius of the well or shaft(m). Incorporating (2) and (3) in (1) the following quadratic equation in R(m) is obtained:

$$\frac{1}{2g}\left(\frac{1}{86400\pi r_w^2\Delta t}\right)^2\left(1+c_e+\frac{fL}{2r_w}\right)R^2(m) + \delta_p(1,\Delta t)R(m) + \sum_{r=1}^{m-1} R(\gamma)\delta_p(m-\gamma+1,\Delta t) + h_2 - h_1 = 0 \tag{5}$$

Considering the positive root of the equation

$$R(m) = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \qquad (6)$$

in which,

$$a = \frac{1}{2g}\left(\frac{1}{86400\pi r_w^2 \Delta t}\right)^2 \left(1 + c_e + \frac{fL}{2r_w}\right) \; ; \; b = \delta_p(1, \Delta t) \; ;$$

$$c = \sum_{r=1}^{m-1} R(\gamma)\delta_p(m - \gamma + 1, \Delta t) + h_2 - h_1$$

For m=1, $c = h_2 - h_1$. R(m), m =1,2, ...n, can be found in succession starting from m=1. The recharge rate during mth time step is equal to R(m)/Δt (m^3/day).

Case 2: A vertical shaft penetrating marginally into the aquifer and filled with coarse material

For safeguarding against pollution, the vertical shaft should be filled with a filter pack. The recharge capacity in that case can be computed applying simple Darcy's law as described here. The recharge during mth time step is given by:

$$R(m) = \pi\, r_w^2\, k_f \frac{\left[h_1 - h_2 - \sum_{r=1}^{m} R(\gamma)\delta_p(m - \gamma + 1, \Delta t)\right]}{L} \Delta t \qquad (7)$$

in which, k_f = hydraulic conductivity of the coarse material the shaft is filled with. The term within the bracket is the hydraulic head difference dissipated in length *L* of the shaft. Solving for the recharge during the mth time step from (2)

$$R(m) = \frac{\pi\, r_w^2 k_f \Delta t}{L} \frac{\left[h_1 - h_2 - \sum_{r=1}^{m-1} R(\gamma)\delta_p(m - \gamma + 1, \Delta t)\right]}{1 + \frac{\pi r_w^2 k_f}{L} \Delta t\ \delta_p(1, \Delta t)} \qquad (8)$$

R(m), m =1,2, ...n, can be found in succession starting from m=1 to n. The recharge rate during mth time step is equal to R(m)/Δt (m^3/day).

Case 3: A recharge well fully penetrating into the upper aquifer

The procedure for finding recharge through a fully penetrating well is same as that for the partially penetrating well described above except that the rise in piezometric surface at the well face is to be computed using discrete kernel coefficients pertaining to the fully penetrating well. The unit pulse response function coefficients $\delta_1(m,\Delta t)$ are obtained from the unit step response function derived by Hantush (   ) for a fully penetrating well of finite radius (Appendix II).

Results

The kernel coefficients are generated assigning values to the aquifer parameters. The thickness of the intervening clay layer is taken as 10m; the hydraulic conductivity of the packed porous medium is 10 times the hydraulic conductivity of the aquifer medium and is assumed to be 380m/day; friction factor f =0.02; entry loss coefficient c_e =0.05. Variations of non-dimensional recharge rate, $\frac{R(m)}{\Delta t\, T\,(h_1 - h_2)}$, with dimensionless time factor, $\frac{4Tm\Delta t}{\phi\, r_w^2}$, are presented in Fig.2. Since, the recharge rate is governed by the difference in the hydraulic heads at the entry and exit points of the shaft, and the head difference decreases with time, the recharge, therefore, decreases with time. When the shaft is filled with the coarse material, the rate of recharge at large time is half of the recharge rate that would occur without filling material in the shaft.

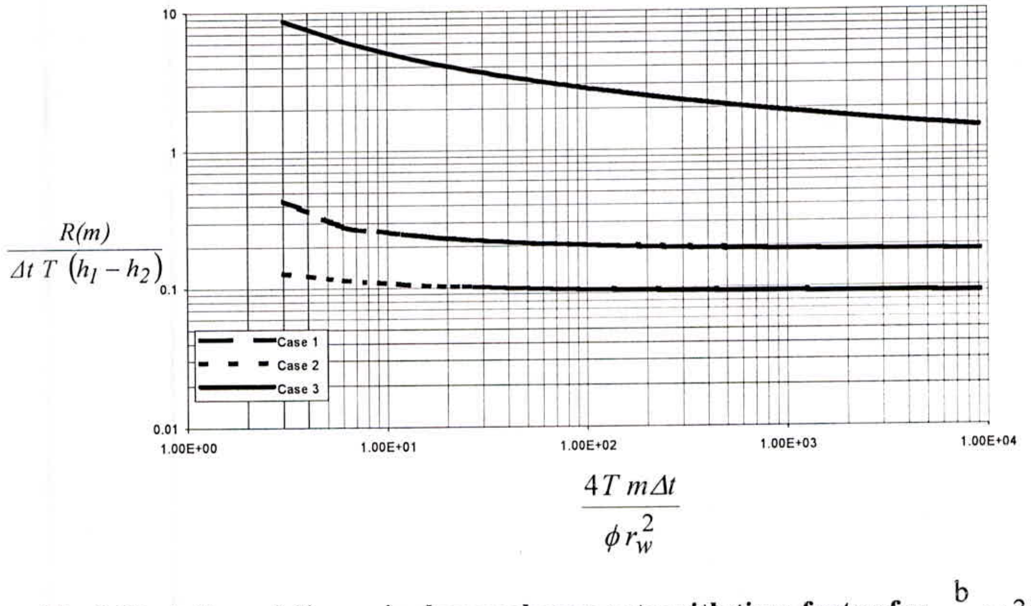

Fig.2 Variation of dimensionless recharge rate with time factor for $\frac{b}{r_w} = 34.5$

Numerical results are presented for the following aquifer parameters for various radii of the recharging structure. Transmissivity of the aquifer T=655.5 m^2/day ; storativity of the aquifer ϕ=0.01.Thickness of clay layer, L=10m, Initial hydraulic head difference $h_1 - h_2$=5m. The average recharge rates during 120 days for different well radii are presented in Table1. A vertical shaft with 2m radius, 10m length filled with a filter material having hydraulic conductivity of 380m/day, can recharge at an average rate of 700m^3/day under an initial hydraulic head difference of 5m.

Table 1. Average recharge rate for various radii

Radius of the Shaft (m)	Rate of Recharge (Case 2) (m^3/day)	Rate of Recharge (Case 1) (m^3/day)	Rate of Recharge (Case 3) (m^3/day
0.1	5.42	59.08	1979.7
0.2	19.82	116.85	2078.6
0.4	67.43	229.43	
0.5	97.84	284.15	
1	283.99	541.73	
2	700.75	991.76	

Appendix I

Discrete kernel, δ_p (m,Δt)

Let the unit step response function for piezometric rise at the well face of a marginally penetrating recharge well and confined aquifer system be designated as $U(r_w,t)$. According to Hantush(1962)

$$U(r_w,t)=\frac{1}{4\pi T_1}\{W(u)+2\sum_{n=1}^{\infty}W_n(u,\frac{n\pi r_w}{b})\} \tag{1}$$

in which, T_1 = transmissivity (m^2/day), ϕ_1=storativity, and b_1= thickness of the upper aquifer(m); r_w = radius of the well or shaft(m), $u=\frac{r_w^2\phi_1}{4T_1 t}$, $W(u)=\int_u^{\infty}\frac{e^{-y}}{y}dy$, and

$$W_n(u,\frac{n\pi r_w}{b})=\int_u^{\infty}\frac{dy}{y}\exp\left\{-y-\frac{\left(\frac{n\pi r_w}{b}\right)^2}{4y}\right\}.$$

Let the time domain be discretised by time steps of uniform size Δt. The unit pulse response function of the system, $\delta_p(m,\Delta t)$, is given by:

$$\delta_p(m,\Delta t)=\frac{1}{\Delta t}\left[U(r_w,m\Delta t)-U(r_w,\overline{m-1}\,\Delta t)\right]=$$

$$\frac{1}{4\pi T_1\Delta t}\left\{W(\frac{r_w^2\phi_1}{4T_1 m\Delta t})+2\sum_{n=1}^{\infty}W_n(\frac{r_w^2\phi_1}{4T_1 m\Delta t},\frac{n\pi r_w}{b})-W(\frac{r_w^2\phi_1}{4T_1(m-1))\Delta t})-2\sum_{n=1}^{\infty}W_n(\frac{r_w^2\phi_1}{4T_1(m-1)\Delta t},\frac{n\pi r_w}{b})\right\} \tag{2}$$

W(u) and W_n (u,nπr_w/b) are improper integrals as the upper limit of integration is infinite. They are evaluated using Gaussian quadrature after converting the improper integrals into proper integrals and changing the limit. The procedure is as follows.

$$W(u)=\int_u^{\infty}\frac{e^{-y}}{y}dy=\int_u^{1}\frac{e^{-y}}{y}dy+\int_1^{\infty}\frac{e^{-y}}{y}dy=\int_u^{1}\frac{e^{-y}}{y}dy+\int_0^{1}\frac{e^{-1/\zeta}}{\xi}d\xi$$

$$=\int_{-1}^{1}0.5(1-u)\frac{e^{-(1+u)/2-(1-u)x/2}}{(1+u)/2+(1-u)x/2}dx+\int_{-1}^{1}\frac{e^{-2/(1+x)}}{1+x}dx$$

$$W_n\left(u, \frac{n\pi r_w}{b}\right) = \int_u^{\infty} \frac{dy}{y} \exp\left\{ -y - \frac{\left(\frac{n\pi r_w}{b}\right)^2}{4y} \right\}$$

$$= \int_u^{1} \frac{dy}{y} \exp\left\{ -y - \frac{\left(\frac{n\pi r_w}{b}\right)^2}{4y} \right\} + \int_1^{\infty} \frac{dy}{y} \exp\left\{ -y - \frac{\left(\frac{n\pi r_w}{b}\right)^2}{4y} \right\}$$

$$= \int_u^{1} \frac{dy}{y} \exp\left\{ -y - \frac{\left(\frac{n\pi r_w}{b}\right)^2}{4y} \right\} + \int_0^{1} \frac{d\zeta}{\zeta} \exp\left\{ -\frac{1}{\zeta} - \frac{\zeta\left(\frac{n\pi r_w}{b}\right)^2}{4} \right\}$$

Range of Validity of Darcy's Law

Several investigators have found that flow in soils changes from laminar to turbulent for $1 \le R_e < 12$ (vide Harr,1962) where the Reynolds' number in case of flow through soils is given by $R_e = \frac{vd\rho}{\mu}$, v=discharge velocity, d=average of diameters of soil particles, $\rho =$ density of fluid, and μ =coefficient of viscosity. Darcy's law is accepted to be valid when Reynolds number R is equal to or less than 1.

For a fully turbulent condition, the relation between hydraulic gradient and velocity of flow is represented as:

$$-i = av + bv^2 \tag{1}$$

where 'a' , and 'b' are positive constants and $i = \frac{dh}{ds}$. Equation (1) is valid for which $\frac{dh}{ds}$ is negative and v is positive. Solving 'v' from above,

$$v = \frac{-a}{2b} + \frac{\sqrt{a^2 - 4bi}}{2b} = \frac{-a}{2b} + \frac{a}{2b}\left[1 - \frac{4bi}{a^2}\right]^{1/2} \cong -\frac{i}{a} - \frac{bi^2}{a^3} \tag{2}$$

Under turbulent flow condition the velocity for a given gradient is less than the velocity that would result under laminar flow condition. Under turbulent flow condition the specific capacity of a well is less than that under laminar flow condition. The following example explains the effect of turbulence on specific capacity.

Assessment of recharge through a well

A fully penetrating well with radius r_w in a confined aquifer at the center of a circular groundwater basin having a constant head boundary condition at the outer periphery is recharged maintaining a constant head at the well face. Find the recharge rate per unit water level rise at the well face?

Under steady state flow condition, at any radial distance 'r' from the well, the radial flow is given by:

$$Q_r = 2\pi\, rDv_r = Q_R \tag{3}$$

where D=thickness of aquifer, v_r = radial Darcy's velocity. From (1),

$$v_r = \frac{-a}{2b} + \frac{\sqrt{a^2 - 4b\frac{dh}{dr}}}{2b} \tag{4}$$

Incorporating v_r in (3) after simplification

$$\left(\frac{bQ}{\pi D}\right)^2 \frac{1}{r^2} - \left(\frac{2abQ}{\pi D}\right)\frac{1}{r} = -4b\frac{dh}{dr}$$

Integrating and applying the boundary conditions $h(r_w) = h_w$, and $h(R) = h_R$

$$h_w - h_R = b\left(\frac{Q}{2\pi D}\right)^2\left[\frac{1}{r_w} - \frac{1}{R}\right] + \frac{aQ}{2\pi D}\log_e\left(\frac{R}{r_w}\right)$$

The first part of head loss is due to turbulence and the second part is due to viscous resistance.

The specific recharge rate is thus decreased due to turbulence.

In case of a pumping well, for turbulent flow condition, the relation between hydraulic gradient and velocity is

$$i = -av + bv^2 \tag{5}$$

$$-Q_P = 2\pi r D v_r \tag{6}$$

$$v_r = \frac{a}{2b} - \frac{a}{2b}\left(1 + \frac{4bi}{a^2}\right)^{1/2} \approx \frac{-i}{a} + \frac{i^2 b}{a^3} \tag{7}$$

Incorporating (7) in (6)

$$-Q_P = 2\pi r D\left(\frac{a}{2b} - \frac{\sqrt{a^2 + 4b\frac{dh}{dr}}}{2b}\right) \tag{8}$$

or

$$\left(a + \frac{bQ_P}{\pi r D}\right)^2 = a^2 + 4b\frac{dh}{dr} \tag{9}$$

or

$$\left(\frac{Q}{2\pi D}\right)^2 b\frac{dr}{r^2} + \frac{aQ}{2\pi D}\frac{dr}{r} = dh \tag{10}$$

Integrating

$$h = -\left(\frac{Q}{2\pi D}\right)^2 \frac{b}{r} + \frac{aQ}{2\pi D}\ln r + A \tag{11}$$

Applying the boundary condition

$$h_R - h_w = b\left(\frac{Q}{2\pi D}\right)^2 \left(\frac{1}{r_w} - \frac{1}{R}\right) + \frac{aQ}{2\pi D}\ln\frac{R}{r_w} \tag{12}$$

The first part of the head loss is due to turbulence. The specific capacity of the pumping well is thus reduced due to turbulence.