TRAINING COURSE

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LECTURE NOTES
ON

PUMP-TEST DATA AND DETERMINATION OF AQUIFER PARAMETERS

BY

S K SINGH

ORGANISED BY

NATIONAL INSTITUTE OF HYDROLOGY
ROORKEE - 247 667
INDIA

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1.0 INTRODUCTION

The better management of available groundwater resources and the creation of additional potential are the main issues which have drawn the attention of groundwater hydrologists. In the present era of high speed computers, groundwater modelling on a regional scale, has been proved to be a valuable tool for assessing ground-water potential and for providing the management alternatives. The esti-mation of aquifer parameters are pre-requisite for solving any type of groundwater problems including ground-water modelling. Therefore, the accuracy of the solutions of groundwater flow problem is very much dependent on the accuracy in the determination of aquifer parameters. Pump test occupies a prominent position among the various method available for determination of aquifer parameters. In the present lecture, the collection of pump test data and determination of aquifer parameters have been discussed.

2.0 AQUIFER PARAMETERS

The two most important aquifer parameters are transmissivity (T) and storage coefficient (S). The transmissivity or transmissibility is the product of the average hydraulic conductivity (or permeability) and the thickness of the aquifer. It may be defined as the rate of flow through a cross-section of unit width over the whole thickness of the aquifer under unit hydraulic gradient. It has the dimensions of L^2T^{-1} and generally expressed in the units m^2/day or m^2/hr.

The storage coefficient and the specific yield are both defined as the volume of water released or stored per unit surface area of the aquifer per unit change in the component of head normal to that surface. The storage coefficient refers only to the confined parts of an aquifer and depends on the elasticity of the aquifer material and the fluid. It has an order of magnitude of 10^{-4} to 10^{-6}. The specific yield (S_y) refers to the unconfined parts of an aquifer. In practice, it may be considered to equal the effective porosity or drainable pore space because in unconfined aquifers the effects of the elasticity of aquifer material and fluid are generally negligible. Both storage coefficient and specific yield are dimensionless. Small pores do not contribute to the effective pore space because in small pores the retention forces are greater than the weight of the water. For sands the specific yield may be of the order of 0.1 to 0.2.

3.0 PUMP-TESTS

3.1 Objective:

Pumping tests may be conducted to determine,

i) to determine the hydraulic characteristics of aquifers or water- bearing layers. Such a test is often called an 'aquifer test' because it is the aquifer, rather than the pump or well, which is tested. Carefully conducted aquifer tests may provide reliable drawdown data. It is with this objective that we are mainly concerned in this lecture.

ii) to determine the performance characteristics of a well and pump. For a well-performance test, yield and drawdown are recorded. These data can be used to determining the specific capacity or the discharge drawdown ratio of the well, for selecting the type of pump, and for estimating the cost of pumping. The specific capacity gives a measure of the effectiveness or productive capacity of the well. Such a pumping test is sometimes called a 'well test' because it is the well, rather than the aquifer, which is tested. An accurate test of a well before the pump is purchased pays for itself by assuring selection of a pump that will minimize power and maintenance costs.

3.2 Selection of Test Site:

The analysis of data collected during a pump-test will not produce accurate aquifer parameters unless the tests are carried out at proper sites which fulfill the conditions assumed in the development of the theoretical equations that are used in one or other form for determination of the aquifer parameters.

In certain cases the site of the test is pre-determined and there is no possibility of moving to another (more suitable) site. This applies, for example, when existing wells have to be used or when formation factors at a specific location are required. In the case of regional groundwater studies one may be more or less free to select a suitable site. For example groundwater modelling requires the aquifer parameters at every grid locations.

In selecting the site of an aquifer test, the following points should be kept in mind.

i) the hydrogeological conditions of the site should not change over short distances and should be representative of the area or a large part of the area under consideration;

ii) the site should preferably not be selected near rail-roads or highways where passing trains and heavy traffic may produce measurable fluctuations in the piezometric surface of confined aquifers;

iii) the pumped water must be discharged in such a way that it does not return to the aquifer;

iv) the gradient of the water table or piezometric surface should be low;

v) manpower and equipment must be able to reach the site easily.

Obviously, a careful selection of the test site will prevent many difficulties during the evaluation of aquifer parameters from pump-test data.

3.3 Design and Construction:

The major points are discussed below :

i) It is desirable that the well be completed to the bottom of the aquifer because then more of the aquifer thickness can be utilized as the intake portion of the well, resulting in a higher yield.

ii) The general rule is to screen 70 to 80 percent of the thickness of the aquifer because this makes it possible to obtain 90 percent or more of the maximum yield that could be obtained if the entire aquifer were screened. Another great advantage of this screen length is that the groundwater flow towards the discharging well may be assumed to be horizontal, an assumption which underlies nearly all pumping test formulae. Vertical flow components in the vicinity of the well, causing an extra drawdown can thus be avoided and no correction need be made for partial penetration.

iii) There are of course some exceptions to this rule. In water table aquifers it may be sufficient to screen the lower half or lower third of the aquifer because if appreciable drawdown occurs the upper part of the well screen may fall dry. Therefore measured drawdowns have to be corrected before they can be used to calculate the aquifer characteristics.

iv) With non-homogeneous aquifers, having continuous intercalated clay beds, it seems useful to make separate tests in the different aquifer parts. This may give a double check on the question whether the clay beds are impervious or leaky. The screen should have a sufficient total area so that the entrance velocity is low, say less than about 3 cm per sec. At this velocity the friction losses in the screen openings will be negligible.

3.4 Discharge of Pumped Water:

The water delivered by the well should be prevented from re-entering the tested aquifer. This can be done by conveying the water through a large-diameter pipe over a convenient distance, say 100 or 200 m, and then discharging it into a canal or natural channel which is not in hydraulic connection with the tested aquifer. The pumped water may also be conveyed through a shallow ditch but precautionary measures should be taken to seal the bottom of such a ditch with clay, or plastic sheets, in order to prevent seepage.

3.5 Piezometers:

The principle of an aquifer test is pump well at a certain rate and to record the effect of this pumping on the water table in the pumping well and in nearby observation wells at specific times. For this purpose a number of piezometers should be available near the discharging well. Therefore, after the discharging well is completed one has to decide on the number and depth of these piezometers and how far they should be located from the discharging well. Single piezometer often permit calculation of the average values of transmissivity and storage coefficient.

The advantage of two or more piezometers placed at different distances from the discharging well is that the drawdowns measured in these piezometers can be analyzed in two ways, by studying both the time-drawdown and the distance-drawdown relationships. Obviously, the results of calculations thus obtained are more accurate and are representative of a large area. It is always best to have as many piezometers as conditions permit, while on the other hand it is recommended that at least three be employed.

The piezometers should be placed neither too far nor too near to the pumped well. In placing the piezometers the following points should be considered.

i) In confined aquifers, the loss of hydraulic head caused by pumping propagates fast because the release of water from storage is entirely due to the compressibility of the aquifer material and that of water. Therefore, the loss of head is measurable at great distances, for instance a few hundred meters from the pumped well.

ii) In unconfined or water table aquifers the propagation of hydraulic head losses is rather slow because the release of water from storage is mostly due to dewatering of the zone through which the water is moving, and only partly due to the compressibility of water and aquifer material in the saturated zone. Unless the period of pumping is extended for several days, the loss of hydraulic head caused by pumping is only measurable within rather short distances of the pumped well, for instance not much farther away than about 100 m.

iii) Semi-confined aquifers have an intermediate position. The propagation of loss of hydraulic head depends on the hydraulic resistance of the semi-pervious layer. It may resemble either a confined aquifer or a unconfined aquifer more closely.

iv) When the hydraulic conductivity of the aquifer material is high, the cone of depression induced by pumping will be wide and flat. When the hydraulic conductivity is low the cone of depression will be steep and narrow. Therefore, in the first case piezometers can be placed further away from the pumped well than they can in the second.

v) If the discharge rate of the pumped well is high the cone of depression induced by pumping will be larger than that with a low rate. Therefore, in the first case greater distance between the piezometers are allowed than in the second.

vi) If the discharging well is a fully penetrating one, i.e. a well whose screen penetrates the entire thickness of the aquifer, or at least 80 percent of it, the flow of water to the pumped well will be horizontal. Therefore, drawdowns, measured in piezometers placed even at short distances from the pumped well can be used for the analysis. If the length of the well screen is considerably less than the saturated thickness of the aquifer, a distorted drawdown pattern is induced near the well, due to vertical flow components. These difficulties can be avoided if the piezometers are placed further away from the pumped well, where these abnormal effects do not appear. As a general rule it may be recommended that the nearest piezometers be placed at a distance which is at least equal to the thickness of the aquifer. At such a distance it may be assumed that the flow is horizontal.

In a homogeneous and isotropic aquifer, the piezometers should be installed at about the same depth as the middle of the well screen in the pumped well. Generally the length of the screen of the piezometers are 0.5-1m. However, longer screens are necessary in stratified aquifers. In aquifers with intercalated clay beds, screens should be installed above and below these clay beds to see if there is any hydraulic inter-connection between the sand layers. The recommended position of the screens are a few meters away from the upper and lower boundaries of the clay beds.

4.0 PERFORMANCE OF A PUMPING TEST

Several days before the test is to be conducted, the test well should be pumped for several hours to determine i) the maximum anticipated drawdown, ii) the best method to measure the yield, iii) whether the discharge from the pump is piped far enough away to avoid recharge, iv) whether the observation wells are so located as to record sufficient drawdown to produce usable data. Never begin the actual pumping test until the water level in the aquifer has returned to the normal static level.

The performance of a pumping test depends upon the care full recording of time, discharge, and piezometric levels. Range of time intervals between water level measurements in the pumped well and in the piezometers, are equally important. The pumping rate should be measured at least once every hour, however, for varying discharge rate frequent observations are necessary. The piezometric heads in pumping well as well as in the piezometers should be recorded preferably by automatic water level recorders. Water level measurements in the pumped well should be recorded at times suggested in table-1. The time intervals for recording piezometric surface in the piezometers are given in table-2. The format to record the pumping test data is given in Annexure A-1. The formats to record Well and Pump data; and for Well Maintenance Record is given at Appendices A-2 and A-3 respectively.

Table-1: Time Interval for Measurements in Pumped Well

Time Since Pumping Started(min.)	Time Intervals (min.)
0-10	0.5-1
10-15	1
15-60	5
60-300	30
300-1440	60
1440-termination of pumping	480

Table-2: Time Interval for Measurements in Piezometers

Time Since Pumping Started(min.)	Time Intervals (min.)
0-60	2
60-120	5
120-240	10
240-360	30
360-1440	60
1440-termination of pumping	480

4.1 Duration of the Pumping Test:

Better and more reliable results are obtained if pumping continues till the depression cone has reached a stabilized position and does not seem to expand further as pumping continues. At the beginning of the test the cone develops fast as the pumped water is initially derived from the aquifer storage immediately surrounding the well. But as pumping continues the cone expands and deepens at a decreasing rate with time because with each additional meter of horizontal expansion a larger volume of stored water becomes available.

Under average conditions a steady-state flow situation in semi-confined aquifer is generally reached after 15 to 20 hours of pumping. If a pumping test is performed in a confined aquifer the cone of depression expands slowly, a longer period of pumping is required and it is common practice to pump the well for three days.

5.0 METHODS OF ANALYZING PUMPING TEST DATA

The assumptions underlying all methods discussed in this lecture are given below:

i) the aquifer has a seemingly infinite areal extent,

ii) the aquifer is homogeneous, isotropic and of uniform thickness over the area influenced by the pumping test,

iii) prior to pumping, the piezometric surface and/or phreatic surface are (nearly) horizontal over the area influenced by the pumping test,

iv) the aquifer is pumped at a constant discharge rate

v) the pumped well penetrates the entire aquifer and thus receives water from the entire thickness of the aquifer by horizontal flow.

5.1 Steady State Flow in Confined Aquifers:

Thiem(1906) was first to utilize two or more piezometers to determine the transmissivity of an aquifer. He showed that for an aquifer satisfying the above conditions, the well discharge can be expressed as:

$$Q = \frac{2\pi T(h_1 - h_2)}{\ln(r_2/r_1)} \qquad (1)$$

Where, Q is the pumped discharge, r_1 and r_2 are the respective distances of the piezometers from the pumped well in meters, h_1 and h_2 are the respective steady state elevations of the water levels in the piezometers. Eq. (1) is commonly written as:

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln(r_2/r_1)} \qquad (2)$$

where, s_1 and s_2 are the respective steady-state drawdowns in the piezometers.

For the cases where only the piezometer at a distance r from the pumped well is available, the following equation can be written.

$$Q = \frac{2\pi T(s_w - s_1)}{\ln(r_1/R)} \qquad (3)$$

where s_w is the steady-state drawdown in the pumped well, and r_w is the radius of the pumped well.

Eq. (3) has limited use because local hydraulic conditions in and near the well strongly influence the value of s_w , e.g., well losses caused by the flow through the well screen and flow inside the well to the pump intake. Therefore, eq. (3) should be used with great caution and only when other methods can not be applied. In order to avoid errors, preferably two or more piezometers should be used.

Procedure I:

Knowing the values of the steady-state drawdowns s_1 and s_2 observed respectively at the two piezometers, the distances of which from the pumped well, i.e., r_1 and r_2 are known; T can be calculated using eq. (2).

Procedure II:

Plot on semi-logarithmic paper the observed steady-state drawdown s of each piezometer against r. s should be referred on simple axis (y-axis) and r should be referred on log-axis (x-axis).

i) Draw the best fit straight line through the plotted points: this is the so-called distance-drawdown graph.

ii) Determine the slope of this line Δs_m , i.e., the difference of maximum drawdown per log cycle of r, giving $r_2/r_1 = 10$. In doing so eq.(2) reduces to,

$$Q = \frac{2\pi T}{2.303} \Delta s_m \tag{4}$$

iii) Substitute the known values of Q and Δs_m into eq.(4) and solve for T.

5.2 Unsteady State Flow in Confined Aquifer:

The unsteady state or Theis equation, which was derived from the analogy between the flow of groundwater and the conduction of heat, may be written as:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{\exp(-x)}{x} dx = \frac{Q}{4\pi T} W(u) \tag{5}$$

where,

$$u = \frac{r^2 S}{4Tt} \tag{6}$$

$$W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \ldots \tag{7}$$

in which,

s = drawdown in m measured in a piezometer at a distance r in from the pumped well;

t = time since pumping started.

5.2(a) Theis Method:

The procedure is outlined below.

i) Prepare a type curve of the Theis Well-function on double logarithmic paper by plotting values of W(u) against the arguments u. This is the normal type curve. However, it is frequently more convenient to use the reversed type curve which is obtained by plotting values of W(u) against 1/u.

ii) Plot the values of s against t/r^2 on another sheet of double logarithmic paper of the same scale as that used for the type curve. For this the data from all available piezometers are used. If the normal type curve is used, s should be plotted against r^2/t. Notice that if the well discharge Q is constant, then the drawdown s is related to r^2/t in a manner which is similar to the relation of W(u) to u, and the curve of the observed data will be similar to the type curve.

iii) Place the observed data plot over the type curve and keeping the coordinate axes of both, the data plot and the type curve, parallel. Locate the position of best match between the data plot and the type curve.

iv) Select an arbitrary match point A on the overlapping portion of the two sheets of graph paper and determine the values of W(u), 1/u, s and t/r^2 for this match point. Notice that it is not necessary for the match point to be located along the type curve. In fact, the calculations are simplified if the point is selected where the coordinates of the type curve are W(u) = 1, and 1/u = 10.

v) Substitute the values of W(u), s and Q into eq. (5) and solve for T.

vi) Calculate S by substituting the values of T, t/r^2 and u into eq.(6).

It is obvious that the values of T and S can be calculated in the same way by using a plot of s versus r^2/t and a type curve between W(u) versus (u). When the hydraulic properties have to be calculated separately for each piezometer a plot of s versus t or s versus 1/t of each piezometer is used in conjunction with a type curve W(u) versus 1/u or W(u) versus u, respectively.

An experiment will show that plotting of 1/t or t versus s is of little importance since the paper with data merely needs to be turned over so that the data curve aligns properly with the type curve. The match point can be marked with a pin or read through the paper. The double logarithmic paper must of course, be on the same scale for both the curves.

It should be remembered that in applying the Theis-curve matching method, less weight in general should be given to the earliest part of the data, since these data may not be closely represented by the theoretical drawdown equation on which the type curve is based.

5.2(b) Cooper and Jacob Method:

The Cooper and Jacob method is based on the Theis equation, however, the conditions for its application are somewhat more restricted than for the Theis method. They proposed the approximation of the Well-function as $W(u)=-0.5772-\ln(u)$ if $u<0.01$. Substituting this approximate value of the Well-function in eq.(5) and rewriting after changing into decimal logarithms, we get,

$$s = \frac{2.303Q}{4\pi T} \log\left\{ \frac{2.25Tt}{r^2 S} \right\} \tag{8}$$

Therefore, when s versus t is plotted on a semi-log graph with s on simple axis (y-axis) and t on logarithmic axis (x-axis), data will fall on a straight line. This line is extended till it intercepts the time-axis where $s=0$, so the interception point has the coordinates $s = 0$ and $t = t_0$. Substitution of these values into eq.(8) gives:

$$S = \frac{2.25Tt_0}{r^2} \tag{9}$$

If two points on the straight line is selected such that $t_2/t_1=10$ and hence $\log(t_2/t_1)=1$, s in eq.(8) can be replaced by the following equation.

$$T = \frac{2.303Q}{4\pi \, \Delta s} \tag{10}$$

Where, Δs is the drawdown difference per log cycle of time. It should be noticed that Δs is the expression for the slope of the straight line. The condition that u is small will be satisfied in confined aquifers for moderate distances from the pumped well and for observations for moderately large time (greater than 20 min. to 1 hr.). But for unconfined conditions it may take 12 hours or more of pumping.

Procedure 1:

i) Plot for one of the piezometers (r = constant) the values of s versus the corresponding time t on a semi-logarithmic paper, and draw a straight line through the plotted points.

ii) Extend the straight line till it intercepts the time-axis where $s = 0$, and read the value of t_0.

iii) Determine the slope of the straight line, i.e. the drawdown difference Δs per log cycle of time.

iv) Substitute the values of Q and Δs into eq.(10) and solve for T. With the known values of T and t_0 , calculate S from eq.(9).

v) When the values of T and S are determined they are introduced into eq.(6) to check if $u < 0.01$ (which is condition for the applicability of the Jacob method).

Procedure 2:

A more or less identical procedure can be followed by plotting on semi-logarithmic paper s versus r (r on log scale, i.e. x-axis) for t = constant. Again a straight line is fitted through the plotted points and extended till it intercepts the r-axis where s = 0. The interception point has the coordinates s = 0 and $r = r_0$ (=radius of influence at the chosen moment). Following the same line of reasoning as outlined in procedure 1 above, the following equation is derived.

$$S = \frac{2.25Tt}{r_0^2} \tag{11}$$

As in Procedure 1 the values of r_0 and Δs are read from the graph (here, Δs is the difference in drawdown per log-cycle in r). From eqs.(10) and (11), the values of T and S can be calculated for t=constant. This procedure should be repeated for several values of t. The values of T calculated for different values of t should agree closely and the same holds true for the value of S.

Procedure 3:

All the data of all piezometers can be used in one graph if on a semi-logarithmic paper s is plotted versus t/r^2 (t/r^2 on the log-axis, i.e. x-axis). A straight line is drawn through the plotted points and intercept with the log-axis is determined. The coordinates of this interception point are s = 0 and $(t/r^2)_0$. Following the same line of reasoning as in Procedure 1, the following equation is used:

$$S = 2.25\ T \left\{ \frac{t}{r^2} \right\}_0 \tag{12}$$

The values of $(t/r^2)_0$ and Δs are determined from the graph (Δs is the difference n drawdown per log-cycle of t/r^2). T and S are calculated using eqs.(10) and (12).

5.3 Unsteady State Flow in Unconfined Aquifer:

Same methods may be used as used for confined aquifers. The drawdown should be small in relation to the saturated thickness of the aquifer, D; otherwise the assumption that the thickness of the aquifer is constant is no longer satisfied. The symbol S means here specific yield. If s is large, in the equations for confined aquifer, the drawdown s should be replaced by $s' = s - s^2/2D$, and the aquifer parameters can be determined.

6. AQUIFER PARAMETERS USING MARQUARDT ALGORITHM

The reliability of ground water models (predictive model/ management model) is enhanced by better aquifer parameter identification. Pump-test occupies a prominent position among the various methods available for identification of aquifer parameters. Graphical methods of identifying the aquifer parameters have been proposed by many investigators(Theis, 1935; Cooper and Jacob, 1946; and Chow, 1952). These methods involve considerable subjectivity on the part of analyst as personal errors are involved. It becomes practically difficult to determine the parameters by curve matching when observed data points show a flat curvature. In Jacob method, the observed drawdown data do not follow straight line if argument of the well function is greater than 0.01 and also if the data contain observational errors. In order to overcome these problems associated with traditional curve matching/graphical method, numerical methods that use the objective criteria for the matching of theoretical and observed drawdowns are preferred (Chander et al., 1981). These methods generally require a computer. In This section Marquardt Algorithm has been described for the determination of T and S by analyzing the observed drawdown in a confined aquifer due to pumping a tube-well at a variable rate (Often the pumped discharge vary with time due to variation in suction head of pump-motor, and in power supply to the pump-motor).

6.1 Marquardt Algorithm:

Let the general problem is to determine the parameters of a nonlinear equation $\bar{y} = f(x_1, x_2, \ldots, x_K; a_1, a_2, \ldots, a_M)$ utilizing N data points for y_i and $x_{k,i}$, $i=1,2, \ldots, N$; $k=1,2, \ldots, K$. The parameters to be determined are $a_1, a_2, \ldots, a_M$. In this method a least square objective function of the following form is utilized.

$$\text{Objective function} = S = \sum_{i=1}^{N} (y_i - \bar{y}_i)^2 \tag{13}$$

The nonlinear equation is linearized by expanding $\bar{y}_i$ in the form of Taylor series about current trial values of the parameters and neglecting the terms having power ≥ 2. Thus, only the linear term is retained as given in the following expression.

$$\bar{y}_i = \bar{y}_i^* + \left[\frac{\partial \bar{y}_i}{\partial a_1}\right]^* \Delta a_1 + \left[\frac{\partial \bar{y}_i}{\partial a_2}\right]^* \Delta a_2 + \ldots + \left[\frac{\partial \bar{y}_i}{\partial a_M}\right]^* \Delta a_M \tag{14}$$

Where, $\Delta a_j = \left[a_j - a_j^*\right]$, $j=1,2, \ldots M$, and asterisk designates quantities evaluated at the initial trial values. Minimization of the objective function requires that the partial derivatives of S w.r.t. each parameter is equal to zero, therefore,

$$\left[\partial S/\partial a_j\right] = 0, \quad j=1,2, \ldots, M \tag{15}$$

Substituting eq. (14) into eq (13) and applying the condition contained in eq.15, the following normal equation is obtained.

$$\left[\mathbf{A}^T\mathbf{A}\right] \Delta\mathbf{A} = \mathbf{A}^T \left[y - \bar{y}^*\right] \tag{16}$$

where, $\mathbf{A}^T$ is transpose of matrix $\mathbf{A}$ and,

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \bar{y}_1}{\partial a_1} & \frac{\partial \bar{y}_1}{\partial a_2} & \cdots & \frac{\partial \bar{y}_1}{\partial a_M} \\ \frac{\partial \bar{y}_2}{\partial a_1} & \frac{\partial \bar{y}_2}{\partial a_2} & \cdots & \frac{\partial \bar{y}_2}{\partial a_M} \\ \vdots & \vdots & & \vdots \\ \frac{\partial \bar{y}_N}{\partial a_1} & \frac{\partial \bar{y}_N}{\partial a_2} & \cdots & \frac{\partial \bar{y}_N}{\partial a_M} \end{bmatrix}^*$$

$$\Delta \mathbf{A} = \begin{bmatrix} (a_1 - a_1^*) \\ (a_2 - a_2^*) \\ \cdot \\ \cdot \\ \cdot \\ (a_M - a_M^*) \end{bmatrix} \qquad [y - y^*] = \begin{bmatrix} (y_1 - \bar{y}_1^*) \\ (y_2 - \bar{y}_2^*) \\ \cdot \\ \cdot \\ \cdot \\ (y_N - \bar{y}_N^*) \end{bmatrix}$$

The eq. (16) is the normal equation for the Gauss-Newton method. In Marquardt algorithm this normal equation is modified by adding a factor λ to allow for convergence with restively poor initial guesses for the unknown parameters, thus,

$$[\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I}]\ \Delta\mathbf{A} = \mathbf{A}^T\ [y-\bar{y}^*] \tag{17}$$

Where, **I** is the identity matrix. Initial values of λ are large and decreases towards zero as the optimum is approached. $\mathbf{A}^T$ is the transpose of matrix **A**. Normal equations are solved for $\Delta\mathbf{A}$. If the convergence is achieved, the final parameters are calculated using the following equation.

$$\mathbf{A}_j = \mathbf{A}_j^* + \Delta\mathbf{A}_j, \qquad j=1,2, \quad \ldots, M \tag{18}$$

If convergence is not achieved, $\mathbf{A}^*$ is updated by replacing the old values by the new values and the process is repeated. When the convergence is achieved, $\Delta\mathbf{A}$ and S will approach zero. Further details of the algorithm is given in the paper by Marquardt(1963).

6.2 Determination of Transmissivity and Storativity:

Determination of aquifer-parameters using drawdown data obtained during a pump-test, is known as inverse problem in general term of ground water modelling. For solving an inverse problem, the associated direct problem need to be necessarily solved. The 'direct problem' is the determination of the response when the parameters are known.

6.2(a) Direct Problem:

The differential equation governing the axis symmetric unsteady flow of water to a well in a homogeneous, isotropic and confined aquifer of infinite areal extent is (Bear, 1972),

$$\frac{\partial^2 s(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial s(r,t)}{\partial r} = \frac{S}{T}\frac{\partial s(r,t)}{\partial t} \tag{19}$$

Where, s(r,t) is the drawdown at a radial distance r from the well at the end of time t. The solution of the above differential equation when unit quantity of water is withdrawn instantaneously from the aquifer at r=0, with boundary condition as, $s(\infty,t)=0$ and initial condition as, $s(r,0)=0$, is given by (Bear, 1972),

$$s(r,t) = \frac{1}{4\pi Tt}\exp\left\{\frac{r^2S}{4Tt}\right\} \tag{20}$$

Since, s(r,t) in the above equation is the response of aquifer (drawdown) due to unit impulse pumping, a unit impulse response function for pumping, i.e., u(r,t) may be expressed as:

$$u(r,t) = \frac{1}{4\pi Tt}\exp\left\{\frac{r^2S}{4Tt}\right\} \tag{21}$$

The rate of pumping may vary with time due to various reasons such as increasing discharge head, fluctuation in power supply to the pump-motor, etc. In order to take into account the temporal variation in the rate of pumping, the time span is discretized into a number of uniform time steps of size Δt. The rate of pumping is assumed to be constant during a time step. The drawdown at the end of n^{th} time step using Duhamel's principle is given by,

$$s(r,n\Delta t) = \sum_{\gamma=1}^{n} Q(\gamma)\delta(r,n-\gamma+1) \tag{22}$$

Where, $Q(\gamma)$ is the rate of pumping during γ^{th} time step and $\delta(r,m)$ is the discrete pulse kernel for the drawdown at a radial distance r from the well at the end of m^{th} time step and is expressed as,

$$\delta(r,m) = \frac{1}{4\pi T}\left[W\left\{\frac{r^2S}{4Tm\Delta t}\right\} - W\left\{\frac{r^2S}{4T(m-1)\Delta t}\right\}\right] \tag{23}$$

where,

$$W(u) = \int_u^\infty \frac{\exp(-u)}{u} \tag{24}$$

Eq. (22) along with eq. (23) is dimensionally homogeneous and does not require careful attention in conversion of transmissivity in respect of time step size. With the above formulation, (eq.(22) & eq.(23)), the dimensional inconsistency is automatically avoided.

6.2(b) Inverse Problem:

Having obtained the solution of direct problem, the inverse problem can be attempted for its solution making use of an optimization technique. Since, the well function or the discrete pulse kernels are nonlinear functions of T and S , only a non linear optimization technique can be used for identification of parameters. Such an algorithm developed by Marquardt(1963) has been used for the present analysis. A brief description of the algorithm for a general problem has already been discussed at section 6.1.

The present inverse problem is to identify the aquifer parameters T and S utilizing the pump-test data observed at an observation well at a distance r from the pumped well. For the present problem $a_1 = T$; $a_2 = S$; $y = s_w$ = drawdown observation at the well; N= number of drawdown observations; M= number of parameters = 2.

The derivatives of calculated drawdown appearing in eq.(16) w.r.t. the parameters can either be calculated numerically or analytically. $\bar{y}$ can be calculated using eq. (22) & (23). Expressions for analytical derivatives have been given below.

6.2(c) Expressions for Analytical Derivatives:

Substituting eq. (23) in eq. (22) and differentiating w.r.t. T and S respectively, the following expressions are obtained.

$$\frac{\partial s(r,n\Delta t)}{\partial T} = \sum_{\gamma=1}^{n} Q(\gamma)\delta_1(r,n-\gamma+1) \tag{24}$$

and,

$$\frac{\partial s(r,n\Delta t)}{\partial S} = \sum_{\gamma=1}^{n} Q(\gamma)\delta_2(r,n-\gamma+1) \tag{25}$$

Where, $\delta_1(m)$ is the discrete pulse kernel for derivative of drawdown w.r.t. T at the end of m^{th} time step and is given by,

$$\delta_1(m) = \frac{1}{4\pi T^2}\left[\exp\left[-r^2\phi/(4Tm\Delta t)\right]-\exp\left[-r^2\phi/\left\{4T(m-1)\Delta t\right\}\right]\right]-\delta(m)/T \tag{26}$$

and, $\delta_2(m)$ is the discrete pulse kernel for derivative of drawdown w.r.t. ϕ at the end of m^{th} time step and is given by,

$$\delta_2(m) = \frac{1}{4\pi T\phi}\left[\exp\left[-r^2\phi/\left\{4T(m-1)\Delta t\right\}\right] - \exp\left[-r^2\phi/(4Tm\Delta t)\right]\right] \tag{27}$$

6.3 A New Graphical Method : Vide Annexure A-4.

6.4 Determination of Aquifer Diffusivity:

Aquifer diffusivity for a confined aquifer is defined as the ratio of aquifer transmissivity to aquifer storativity. The aquifer diffusivity can be determined using the records of groundwater observation during a flood wave in the stream. This is an inverse problem. The associated direct problem is the stream aquifer interaction. Stream-aquifer interaction for a fully penetrating stream in a confined aquifer consequent to a step rise in the stream-stage, can be expressed

$$h(x,t) = H\ \mathrm{erfc}\left\{\frac{x}{\sqrt{4\beta t}}\right\} \tag{28}$$

Where,

$h(x,t)$ = piezometric head in the aquifer,

x = distance from the stream,

β = aquifer diffusivity,

H = step rise in the stream stage,

erfc(.)= 1 - erf(.) = complementary error function.

The error function is defined as,

$$\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-\alpha^2) d\alpha \tag{29}$$

From eq. (28), the unit step response function may be expressed as,

$$U(x,t) = \text{erfc}\left\{ \frac{x}{\sqrt{4\beta t}} \right\} \tag{30}$$

If the stream stage varies with time, the piezometric head in the aquifer is given by the following equation.

$$h(x,n\Delta t) = \sum_{\gamma=1}^{n} H(\gamma)\delta_3(x,n-\gamma+1) \tag{31}$$

Where, $H(\gamma)$ is the stream stage during γ^{th} time step. The discrete kernel coefficient, $\delta_3(x,m)$ is given by,

$$\delta_3(x,m) = U(x,m\delta t) - U(x,m\delta t) \tag{32}$$

Eqs. (30), (31) and (32) solves the 'direct problem' (to determine the piezometric head in the aquifer when the parameter β is known.

The present inverse problem is to identify the aquifer diffusivity, β, utilizing the piezometric head data in an observation well at a distance x from the stream, consequent to the variation in the stream-stage. The parameter β can be estimated using Marquardt Algorithm discussed in section 6.1. For the present problem $a_1 = \beta$; $y=h(x,n\Delta t)$; N = number of drawdown observations; M= number of parameters= 1.

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AQUIFER TEST DATA

Owner ______________________ Address ______________________ County ______________ State __________

Date ______________ Company performing test ______________________ Measured by ______________

Well No ______________ Distance from pumping well __________ Type of test ______________________ Test No __________

Measuring equipment ____

Time Data	Water Level Data	Discharge Data	
Pump on Date ______ Time ______ (t) Pump off Date ______ Time ______ (t') Duration of aquifer test Pumping ________ Recovery ______	Static water level ______________ Measuring point ______________ Elevation of measuring point ________	How Q measured ______________ Depth of pump/air line ______________ Previous pumping? Yes ____ No ____ Duration __________ End ________	Comments on factors affecting test data

Date	Clock time	Time since pump started t	Time since pump stopped t'	t/t'		Water level measurement	Correction or Conversion	Water level	Water level change s or s'		Discharge measurement	Rate		

WELL AND PUMP DATA

Location of Well

County	Township Number	Range Number	Section No.	Fraction	
Township	N or S	E or W			¼ ¼ ¼

Street Address and City or Distance and Direction from Road Intersections

Show exact location of well in section grid with an 'x' — N — S — 1 mile

Sketch map of well location

Addition Name

Block Number

Lot Number

Remarks, Elevation, Source of Data, etc.

Borehole data

Formation Log	Color	Hardness	From	To

Contractor Name and Address

Name of Driller ____________

State License Number ____________

Property owner's name and address

Well depth | **Datum point from which all measurements are taken**

Method of Drilling
☐ Cable tool ☐ Hollow rod ☐ Driven ☐ Dug
☐ Direct rotary ☐ Air rotary ☐ Bucket auger ☐ ____
☐ Reverse rotary ☐ Jetted ☐ Flight auger

Use
☐ Domestic ☐ Public supply ☐ Industrial ☐ ____
☐ Irrigation ☐ Municipal ☐ Commercial
☐ Test Well ☐ Heating or cooling ☐ Monitoring

Casing Type
☐ Steel ☐ Threaded Height above/below
☐ Galv ☐ Welded surface ____
☐ PVC ☐ Solvent welded Drive shoe? Yes ___ No ___
☐ SS

____ in to ____ ft Wgt ____ lb/ft Sch No ____ | Hole diameter ____ in to ____ ft
____ in to ____ ft Wgt ____ lb/ft Sch No ____ | ____ in to ____ ft
____ in to ____ ft Wgt ____ lb/ft Sch No ____ | ____ in to ____ ft

Intake Portion of Well
Screen type ____ or open hole from ____ ft to ____ ft
Manufacturer ____
Material ____ Dia ____
Fittings ____ Length ____
Set between ____ ft and ____ ft Slot ____
____ ft and ____ ft Slot ____
____ ft and ____ ft Slot ____
Method of installation ____

Filter Pack
Source ____ Gradation ____
Method of installation ____ Composition ____
Volume used ____ Depth to top of f.p. ____

Grout
Used? ☐ Yes ☐ No Volume used ____
☐ Neat Cement ☐ Bentonite ____
Method of installation ____
Depth from ____ ft to ____ ft
from ____ ft to ____ ft

Development
Method ____ Duration ____
Dates ____ Sand content after ___ hrs ____
Chemicals used ____

Static Water Level
____ ft ☐ below ☐ above grade
Date measured ____

Pumping Water Level
____ ft ☐ below ☐ above grade Date ____
After ____ hrs pumping at ____ gpm

Specific Capacity
____ gpm/ft of drawdown at ____ hours
Date ____

Pump
Date installed ____ Type ____
Manufacturer ____ Model No ____
H.P. ____ volts ____ Capacity ____
Depth of pump/intake setting ____ No. of stages ____
☐ Oil ☐ Water lubrication Power source ____
Material of drop pipe ____ bowls ____
shafting ____ impellers ____ Bowl dia ____
Column pipe dia ____ Length ____ Modifications ____

Well Head Completion
☐ Pitless adaptor ☐ Basement offset Distance above grade ____

Nearest Sources of Possible Contamination
____ ft Direction ____ Type ____
Well disinfected upon completion? ☐ Yes ☐ No

Geophysical Logs Run

Water Quality
Sample taken? Yes No
Where analyzed ____

Date well completed ____

A NEW GRAPHICAL METHOD FOR IDENTIFICATION OF AQUIFER PARAMETERS

S.K. Singh[1] Shobha Ram[2]

ABSTRACT

The Jacob straight line method and the Theis curve- matching method are most widely usea methods for the identification of aquifer parameters from pump test data. Jacob method is not applicable when the argument of the well function is greater than 0.01. In Theis method, considerable subjectivity is involved in curve-matching. Both Jacob method and Theis method give less weight to the early drawdown data. In this paper, a straight line method applicable for full range of well function argument, has been evolved. The method enables the identification of aquifer parameters from short duration pump test data.

INTRODUCTION

Pump-test occupies a prominent position among the various tools (tracer test, bail test and slug test) available for identification of aquifer parameters. The important aquifer parameters are transmissivity, T and storage coefficient, S. Graphical methods for the identification of the aquifer parameters give more insight to the observed drawdowns and observational errors. Several graphical methods (Theis 1935; Jacob, 1946; etc.) are available for identification of aquifer parameters from pump test data on tube-wells in confined aquifers having infinite areal extent. A detailed description of these methods has been given by Kruseman DeRidder (1970).

The drawdown in an infinite confined aquifer due to pumping a fully penetrating tube well at constant rate, is given by (Bear, 1972);

$$s = \frac{Q}{4\pi T} W(u) \tag{1}$$

$$W(u) = \int_u^\infty (e^{-x}/x)\, dx \tag{2}$$

and

$$u = \frac{r^2 S}{4 T t} \tag{3}$$

where,

W(u)= well function, (dimensionless);

u = argument of well function, (dimensionless);

s = drawdown in the aquifer at a distance r from the centre of the well, (L);

t = time since the commencement of pumping, (T), and;

Q = rate of pumping, ($L^3 T^{-1}$).

Jacob (1950), using approximation to well function, has given the following expressions for T and S.

$$T = \frac{Q}{4\pi m} \tag{4}$$

1 Scientist 'C' at National Institute of Hydrology, Roorkee.

2 Senior Research Assistant, National Institute of Hydrology, Roorkee

$$S = \frac{2.25\, T\, t_o}{r^2} \quad (5)$$

Where, m is the slope of the straight line drawn through the drawdown data plotted on a semi-log graph with t on logarithmic scale and t_o is the time at which the extended straight line meets the time axis. He assumed the following approximation for the well function, for u≤0.01.

$$W(u) = -0.5772 - \ln(u) \quad (6)$$

Aquifer parameters T and S can be determined from eqn. (4) and (5). This is popularly known as Jacob straight line method. When u ≥ 0.01, the approximate expression for well function given by eq. (6) is not valid. Therefore, for large values of r and small values of t, the observed drawdown do not follow a straight line on a semi-log graph. Thus, in Jacob method early drawdown data should not be considered. The departure from straight line may also be due to observational errors. Departure from straight line due to above factors, introduces errors in the estimated aquifer parameters, particularly when observation well is at large distance from the pumping well and the pump-test is conducted for short duration. Though the Theis method of curve-matching is applicable for the full range of well function argument, more subjectivity is involved in the procedure of curve-matching than that in fitting a straight line.

This paper deals with the development of a new graphical (straight line) method for the identification of aquifer parameters. The method is applicable for the full range of well function argument.

METHODOLOGY

Differentiating eq. (1) w.r.t. t, we get,

$$t\frac{\partial s}{\partial t} = \frac{Q}{4\pi T}\exp(-r^2 S/(4Tt)) \quad (7)$$

$$\text{or} \quad \log\left(t\frac{\partial s}{\partial t}\right) = \log\left(\frac{Q}{4\pi T}\right) - \frac{r^2 S}{9.212\, T t} \quad (8)$$

The above equation represents a straight line when plotted on a semi-log graph with $t\frac{\partial s}{\partial t}$ on log- axis and 1/t on natural axis. The numerical derivative of drawdown at t_i is obtained by i fitting a parabola through the three consecutive points. i.e., s_{i-1}, s_i, and s_{i+1}. The equation for such a parabola can be written as,

$$s(t) = s_i + a_i(t - t_i) + a_2(t - t_i)^2 \quad (9)$$

The drawdowns, respectively at time t_{i-1} and t_{i+1} will satisfy the eq. (9), hence, a_1 and a_2 can be determined and, therefore, the following expression for numerical derivative of drawdown at t_i is obtained by differentiating eq. (9).

$$\left.\frac{\partial s}{\partial t}\right|_{t_i} = k_i s_{i-1} + k_2 s_i + k_3 s_{i+1} \quad (10)$$

where,

$$k_1 = \frac{t_i - t_{i+1}}{(t_{i-1} - t_i)(t_{i-1} - t_{i+1})}$$

$$k_2 = \frac{2t_i - t_{i-1} - t_{i+1}}{(t_i - t_{i-1})(t_i - t_{i+1})}$$

$$k_3 = \frac{t_i - t_{i-1}}{(t_{i+1} - t_{i-1})(t_{i+1} - t_i)}$$

The drawdown at each time should satisfy eq. (8). Using eq. (10), $t\frac{\partial s}{\partial t}$ at each time can be calculated. When drawdown data are plotted on a semi-log paper with $t\frac{\partial s}{\partial t}$ on log-axis and a straight line is fitted with C as the intercept on log-axis and m' as the slope of the straight line, then, the aquifer parameters can be obtained using following equations.

$$T = \frac{Q}{4\pi C} \quad (11)$$

$$\text{and} \quad S = 9.212\frac{Tm'}{r^2} \quad (12)$$

APPLICATION

The application of present methodology to the drawdown data of pump-test conducted by C.G.W.B., Govt. of India in a confined aquifer at 'Mathana' site located in upper Yamuna river basin, Haryana, India (C.G.W.B., 1982) has been illustrated below. The pump test was conducted at a constant rate of 1.8924 m^3/min. for 7000 min. on a fully penetrating tube well in alluvial aquifer under lain and overlain by confining clays 24m thick and 50m thick, respectively. The drawdown data were observed at two observation wells located at a distance of 99.90m and 199.80m respectively.

The drawdown data observed at both the observation wells were analyzed separately to get the aquifer parameters. Sample calculations for r=99.9m are given in Table 1. In plotting the graph, those data have not been considered for which $t\frac{\partial s}{\partial t}$ shows a decreasing or fluctuating trend. This decreasing or fluctuating trend is possibly due to the fact that for large t, slope of s vs. t graph becomes small and the numerically calculated values of $\frac{\partial s}{\partial t}$ may contain errors. Fig.1 shows the straight line fitted to the observed data on a semi-log graph.

Table 1. Calculations for r = 99.8 m

S.N.	t (min)	s (m)	k_1	k_2	k_3	$t\frac{\partial s}{\partial t}$	$\frac{1000}{t}$
1.	2	.024	-	-	-	-	-
2.	4	.084	-.2500	.0000	.2500	.1120	250.0
3.	6	.136	-.3333	.2500	.0833	.1455	166.7
4.	10	.219	-.1389	.0500	.0889	.1864	100.0
5.	15	.299	-.1000	.0000	.1000	.2070	66.7
6.	20	.357	-.1333	.1000	.0333	.2147	50.0
7.	30	.447	-.0500	.0000	.0500	.2310	33.3
8.	40	.511	-.0500	.0000	.0500	.2340	25.0
9.	50	.564	-.0667	.0500	.0167	.2425	20.0
10.	70	.643	-.0300	.0167	.0133	.2462	14.3
11.	100	.729	-.0133	-.0167	.0300	.2677	10.0
12.	120	.780	-.0300	.0167	.0133	.2764	8.3
13.	150	.838	-	-	-	-	-

The scatter of data (fig 1.) show that using present method, the aquifer parameters can be determined accurately. It is worth mentioning here that in Jacob method the early drawdown data are neglected and also in Theis method, generally more weigtage is given to later part of the drawdown data. Therefore, for short duration pump-tests, neither Jacob method nor Theis method is expected to give realistic values of the aquifer parameters. The present method is more reliable for the analysis of short duration pump-test data.

The values of aquifer parameters obtained utilizing the drawdown data at both the observation wells are given in Table 2. The values reported for Theis method and that for Jacob method are taken from CGWB (1982) and are based on the drawdown data up to 7000 min. In the present analysis, drwdown data only up to 150 min. have been used.

Table 2. Values of T and f Obtained Using Various Methods

Method of Analysis	*Observation well no.*	T (m^2/min.)	S
Theis Type-Curve	1	0.5764	7.40×10^{-4}
	2	0.5764	5.80×10^{-4}
Jacob Method	1	0.5694	8.30×10^{-4}
	2	0.5694	6.20×10^{-4}
Present Method	1	0.573	8.11×10^{-4}
	2	0.563	6.10×10^{-4}

CONCLUSION

A straight line method valid for full range of well function argument has been evolved for idetification of aquifer parameters. Application of the method to a field example has been illustrated. The method is reliably accurate for aquifer parameter identification from short duration pump-test data.

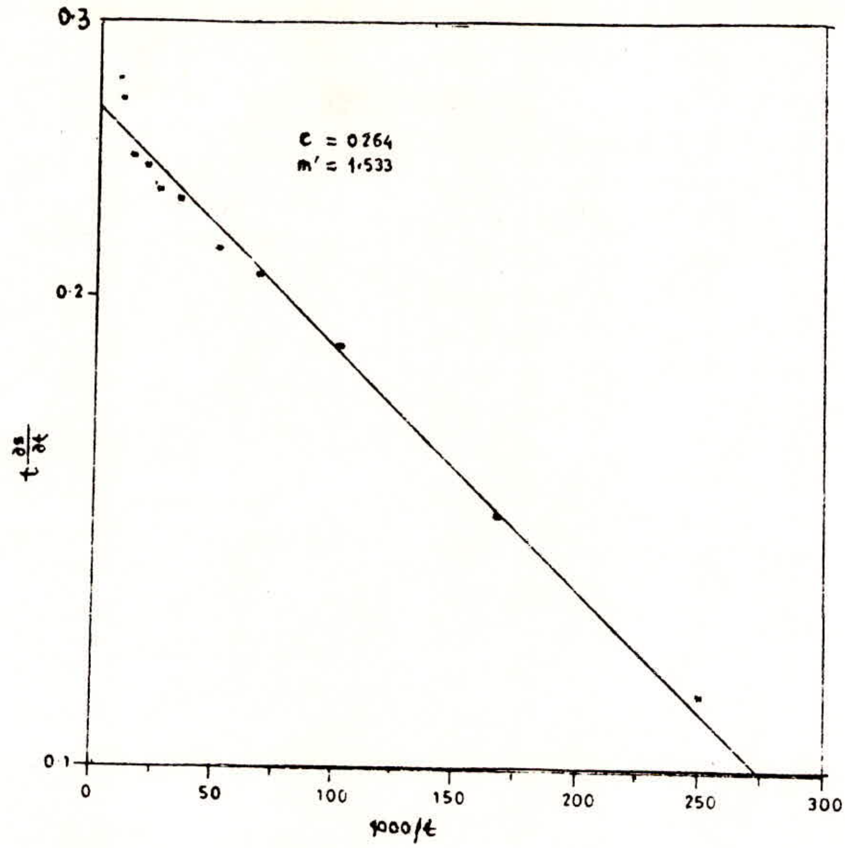

Fig. 1 Semi-log graph for r=99.9 m

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