TRAINING COURSE

ON

SOFTWARE FOR GROUNDWATER DATA MANAGEMENT

UNDER

WORLD BANK FUNDED HYDROLOGY PROJECT

LECTURE NOTES
ON

GROUNDWATER BALANCE
(UNIT-3)

BY

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STREAM AQUIFER INTERACTION

1.0 INTRODUCTION

Stream aquifer interaction has been studied in greater details in recent years. There are two aspects of the process: i) the exchange of flow between the stream and the aquifer during the passage of a flood wave; and ii) the effluent discharge during lean flow period.

The groundwater flow during the passage of a flood remains in an unsteady state where as the flow during the lean flow can be regarded as steady. Solution of Laplace equation, which satisfies the boundary conditions prevailing at the flow domain boundaries, enables quantification of steady state seepage from a stream. For unsteady state condition Boussinesq's equation is solved satisfying the initial and the boundary condition. The exchange of flow between a stream and an aquifer can be evaluated using analytical approach only for idealised stream aquifer system. Rigorous groundwater modelling can predict the exchange of flow in complex non-homogeneous anisotropic stream aquifer system for time varying boundary conditions.

The recharge from a stream can also be predicted from the observation of piezometric level in the vicinity of the stream. In the present lecture the parameter that is required to find the flow from a stream to an aquifer has been discussed. Using this parameter, the stream stage and piezometric level at a piezometer in the vicinity of the stream, the recharge from a stream can be computed for steady state groundwater flow condition.

The insitu hydraulic conductivity and storativity of an aquifer can be determined in various ways. The one which is mostly used is aquifer test. However aquifer test is expensive as well as time consuming. The hydraulic diffusivity is also determined using unsteady stream stages and corresponding water level fluctuations in an observation well. The transmissivity and storativity can not be identified separately from the study of unsteady flow in a stream aquifer system. Experimental method such as the dilution technique can be used to determine the insitu hydraulic conductivity. The dilution test is convenient and less time consuming.

1.1 Estimation of Insitu Hydraulic Conductivity:

It is possible to determine hydraulic conductivity in a piezometer or single well by the introduction of a tracer into the well bore. The tracer concentration decreases with time under the influence of the natural hydraulic gradient that exists in the vicinity of the well. This approach is known as the bore hole dilution. Bore hole dilution tests can be performed in relatively short periods of time in a single well or piezometer. The test provides an estimate of the horizontal average linear velocity of the groundwater in the formation near the well screen. The test is performed in a segment of a well screen that is isolated by packers from overlying and underlying portions of an observational well. Into this isolated well segment a tracer is quickly introduced and is then subjected to continual mixing as lateral groundwater flow gradually removes the tracer from the well bore. The combined effect of

(a)

(b)

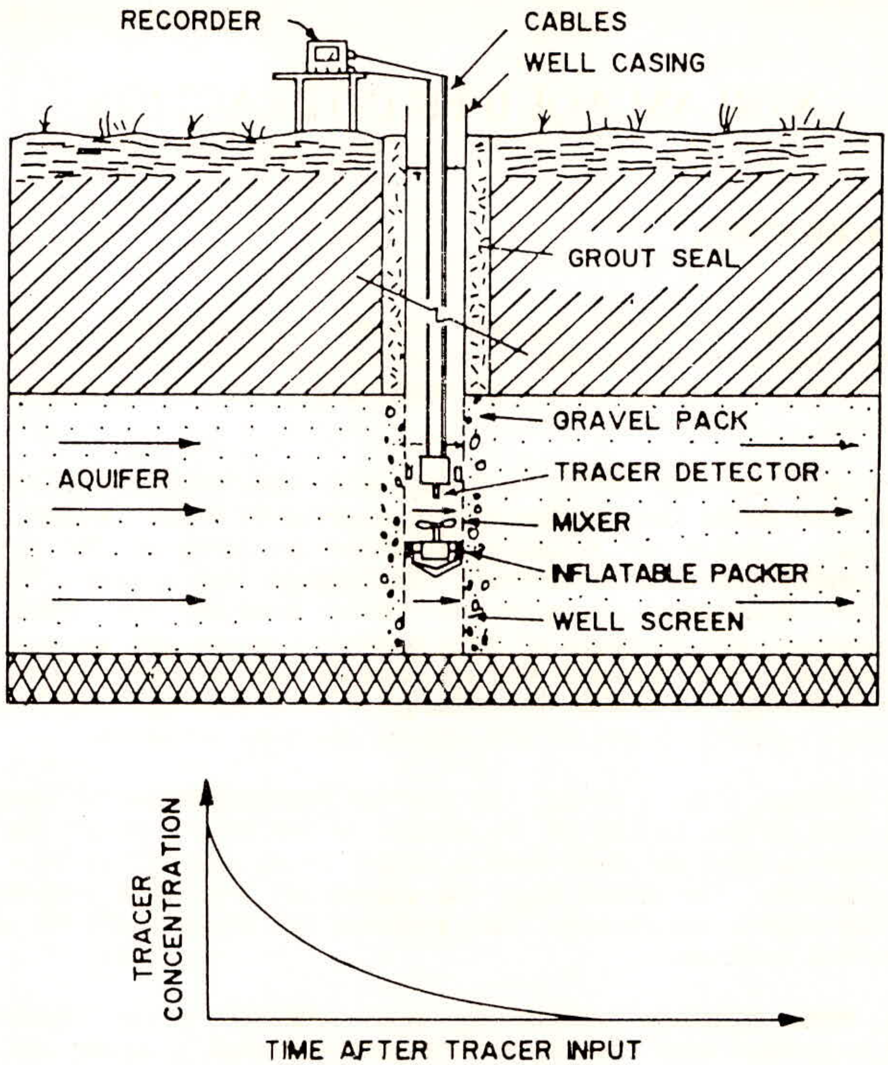

FIGURE I. BOREHOLE DILUTION TEST:
(a) SCHEMATIC DIAGRAM OF APPARATUS,
(b) DILUTION OF TRACER WITH TIME

groundwater through-flow and mixing within the isolated well segment produces a dilution versus time relation. From this relation the average horizontal velocity of groundwater in the formation beyond the sand or gravel pack but close to the well screen is computed. The theory on which the computational methods are based is described in text book (Freeze and Cherry, 1979) and it is described below.

The effect of the well bore and sand pack in a lateral flow regime is shown in Fig.1. The average linear velocity of the groundwater in the formation beyond the zone of disturbance is $\bar{v}$. The average bulk velocity across the center of the well bore is denoted by $\bar{v}^*$. It is assumed that the tracer is nonreactive and that it is introduced instantaneously at concentration C_0 into the isolated segment of the well screen. The vertical cross sectional area through the center of the isolated segment is denoted as A which is equal to the product of diameter of the bore hole and spacing of the packers. The volume of this well segment is W which is equal to cross sectional area of the bore hole multiplied by the distance between the two packers. At time $t>0$, the concentration C in the well decreases at a rate,

$$\frac{dc}{dt} = -\frac{A \,.\, \bar{v}^* \,.\, C}{W}$$

which, upon rearrangement, yields

$$\frac{dc}{C} = -\frac{A \,.\, \bar{v}^* \,.\, dt}{W}$$

Integration and use of the initial condition, $C= C_0$ at=0, leads to

$$\bar{v}^* = -\frac{W}{A \,.\, t} \ln (C/C_0)$$

From the concentration versus time data obtained during bore hole dilution tests values of $\bar{v}^*$ is computed.

The objective of the test, however, is to obtain estimates of $\bar{v}$. This is accomplished using the relation $\bar{v} = \bar{v}^* / \bar{\alpha}$, where $\bar{\alpha}$ is an adjustment factor that depends on the geometry of the well screen, and on the radius and hydraulic conductivity of the sand or gravel pack around the screen. The usual range of $\bar{\alpha}$ for tests in sand or gravel aquifer is from 0.5 to 4 (Drost et al. ,1968).

1.2 Reach Transmissivity Constant:

Using a simple potential theory Morel-Seytoux et al. (1979) have derived the following expression for influent seepage from a partially penetrating stream.

$$Q = L_r \, k \, [(0.5 \, W_p + e)/ (5 \, W_p + 0.5 \, e)] \, \Delta h$$

in which Q is exchange of flow between the stream and the aquifer per unit length of stream, k = hydraulic conductivity, W_p is the wetted perimeter of the stream, e is the saturated thickness of the aquifer below the stream bed, Δh is the difference in water level in the stream and in an observation well which is located at a distance of 5 W_p from the centre of the stream. If the stream stage is higher than the water level in the observation well the flow will take place from the stream to the aquifer, otherwise the aquifer contributes to the flow in the stream. This expression has been used in many stream aquifer interaction studies. The term $L_r k[(0.5W_p+e)/(5W_p+0.5e)]$ is known as reach the transmissivity which is the constant of proportionality between the exchange of flow and the potential difference between the stream and the aquifer in the vicinity of the stream. L_r is the length of the reach.

The reach transmissivity term can be derived rigorously using conformal mapping. In the present lecture the reach transmissivity has been derived for a partially penetrating stream of large width for a confined flow problem.

2.0 STEADY FLOW BETWEEN A STREAM AND AN AQUIFER

A partially penetrating stream of large width which forms the boundary of a semi infinite isotropic confined aquifer is shown in Fig.2(a) in z-plane ($z=x+iy$). The two dimensional flow is in a steady state condition. An observation well is located at a distance L from the stream bank in which the depth to water level is measured from a high datum. The level of water in the stream is also measured from the same high datum. It is required to estimate the seepage making use of the potential difference between the stream and the aquifer at the piezometer.

The complex potential plane w ($w=\phi+i\psi$) for the flow domain is shown in Fig.2(b). ψ is the stream function and velocity potential function, ϕ, is defined as $\phi = -k(p/\gamma_w+y) + c_1$ in which k=hydraulic conductivity, p=pressure, γ_w=unit weight of water, c_1 is an arbitrary constant (Harr, 1962). The complex potential plane has been drawn assuming the constant $c_1=0$. According to Schwarz-Christoffel transformation, the conformal mapping of the flow domain onto the lower half of the ξ plane is given by (vide, Harr, 1962),

$$dz = M \, d\xi \; / \; [(\xi-c)^{1/2}(1-\xi)\xi^{1-3/2} \,] \qquad \ldots(1)$$

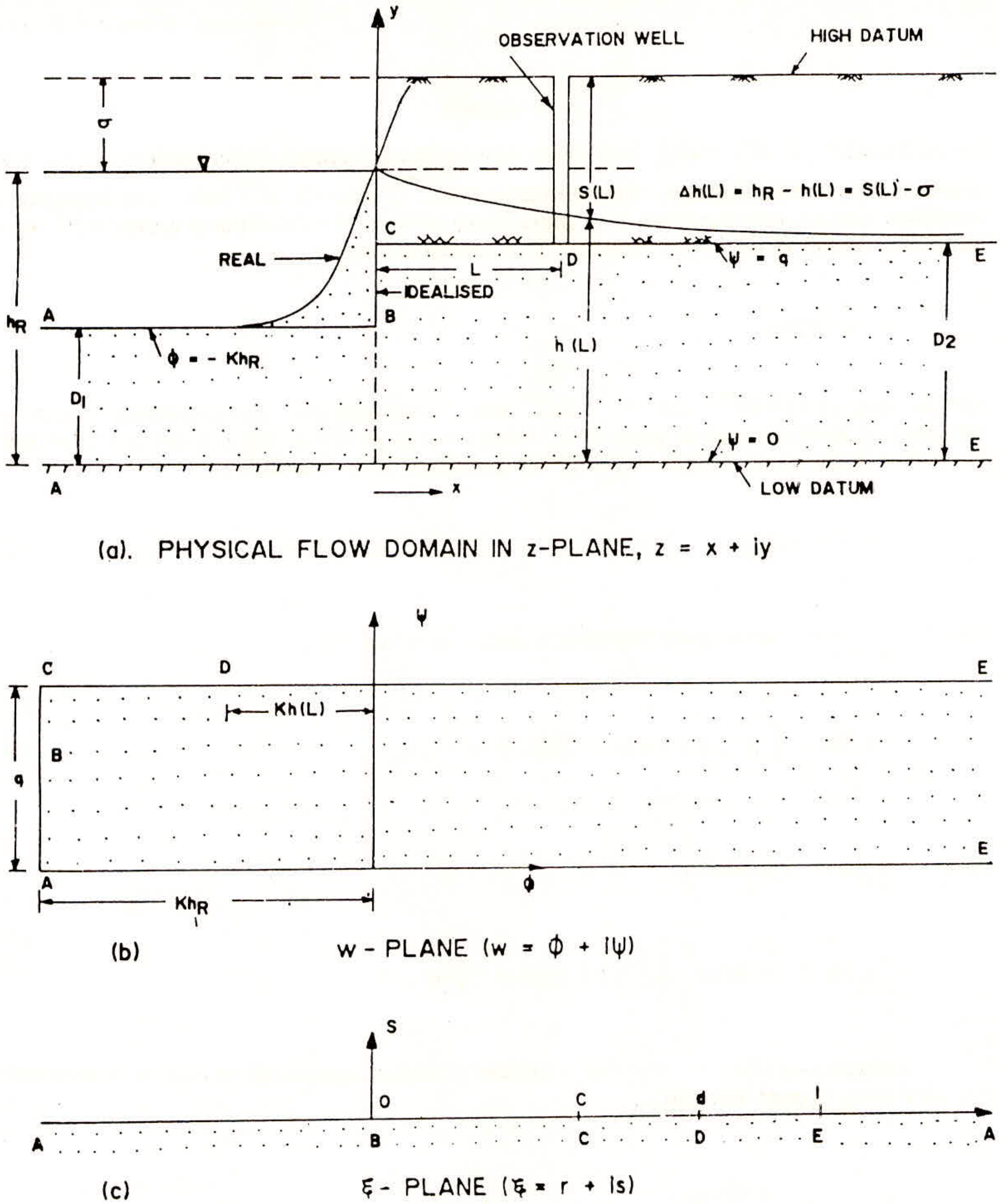

FIG.2 - PHYSICAL FLOW DOMAIN
COMPLEX POENTIAL (w) AND AUXILIARY PLANE (ξ)

the vertices A, B, C, and E in z-plane having been mapped onto points $-\infty$, 0, c and 1 respectively of the ξ-plane. Substituting $\xi = Re^{i\theta}$, $d\xi = R\, e^{i\theta}\, i d\theta$ and applying the condition that as one traverses in ξ-plane from $\theta = \pi$ to $\theta = 2\pi$ along a radius R, $R \to \infty$, the jump in z-plane is $-iD_1$, the constant M is found to be,

$$M = D_1/\pi \qquad \ldots(2)$$

Substituting $(1-\xi) = Re^{i\theta}$, $d\xi = -Re^{i\theta}\, i d\theta$, and applying the condition that as one traverses in ξ-plane around point $\xi = 1$ from $\theta = \pi$ to $\theta = 2\pi$ along a small circle of radius R, $R \to 0$, the jump in z-plane is $-iD_2$, the parameter c is found to be,

$$c = 1-(D_1/D_2)^2 \qquad \ldots(3)$$

For $c < \xi < 1$, the relation between z and ξ is given by:

$$z = M \int_c^{\xi} \left[\xi^{1/2}/\{(1-\xi)(\xi-c)^{1/2}\right] d\xi + z_c \qquad \ldots(4)$$

At $\xi = d$, $z = z_d$; hence,

$$z_d - z_c = L = M \int_c^{d} \left[\xi^{1/2}/\{(1-\xi)(\xi-c)^{1/2}\right] d\xi \qquad \ldots(5)$$

Substituting $\xi - c = v^2$: the improper integral appearing in (5) is converted to the following proper integral.

$$\frac{L}{D_1} = \frac{1}{\pi} \int_0^{\sqrt{(d-c)}} \left[2\sqrt{(c+v^2)}/(1-c-v^2)\right] dv \qquad \ldots(6)$$

The above integral can be evaluated using Gauss-Quadrature formula. For a given value of L, the parameter d can be obtained by an iterative procedure. The conformal mapping of the w-plane onto the lower half of the ξ-plane is given by the Schwarz-Chri stoffel transformation, as given below.

$$dw = M_1 d\xi/[(\xi-c)^{1/2}(1-\xi)] \qquad \ldots(7)$$

Substituting, $1-\xi=Re^{i\theta}$, and $d\xi=-Re^{i\theta}\, id\theta$ and applying the condition that as one traverses in ξ-plane from $\theta=\pi$ to $\theta=2\pi$ around point $\xi=1$ along a small circle of radius R, $R\to 0$, the jump in w-plane is -iq, the constant M_1 is found to be

$$M_1=(q/\pi)(1-c)^{1/2} \qquad ..(8)$$

For $c \le \xi \le d$

$$w = M_1 \int_c^{\xi} d\xi/[(1-\xi)(\xi-c)^{1/2}] -kh_R+iq \qquad ...(9)$$

Substituting $\xi-c=v^2$ in (9) and integrating,

$$w = \{M_1 /(1.-c)^{1/2}\}\log_e [\{(1-c)^{1/2}+\xi \}/ (1-c)^{1/2}-\xi \}]\Big|_o^{(\xi-c)^{1/2}}$$

$$-kh_R+iq \qquad ...(10)$$

Applying the condition that at $\xi=d$, $w=-kh(L)+iq$, we get,

$$k [h_R-h(L)]= q /\pi \log [\{(1-c)^{1/2}+d \}/ (1-c)^{1/2}-d \}] \qquad ...(11)$$

Solving for q,

$$q = \pi k [h_R-h(L)]/\ln\left[(\sqrt{1-c}+\sqrt{d-c})/(\sqrt{1-c}-\sqrt{d-c})\right] \qquad ...(12)$$

where, h_R and h(L) are the hydraulic heads in the stream and at the observation well respectively. The flow being steady, q at any section is constant. The flow can be known for a known value of h(L), which has to be recorded at an observation well. Equation (11) shows that the variation of head in the aquifer with distance from the stream bank is nonlinear. Equation(12) can be written as,

$$q = \Gamma_r \Delta h \qquad ...(13)$$

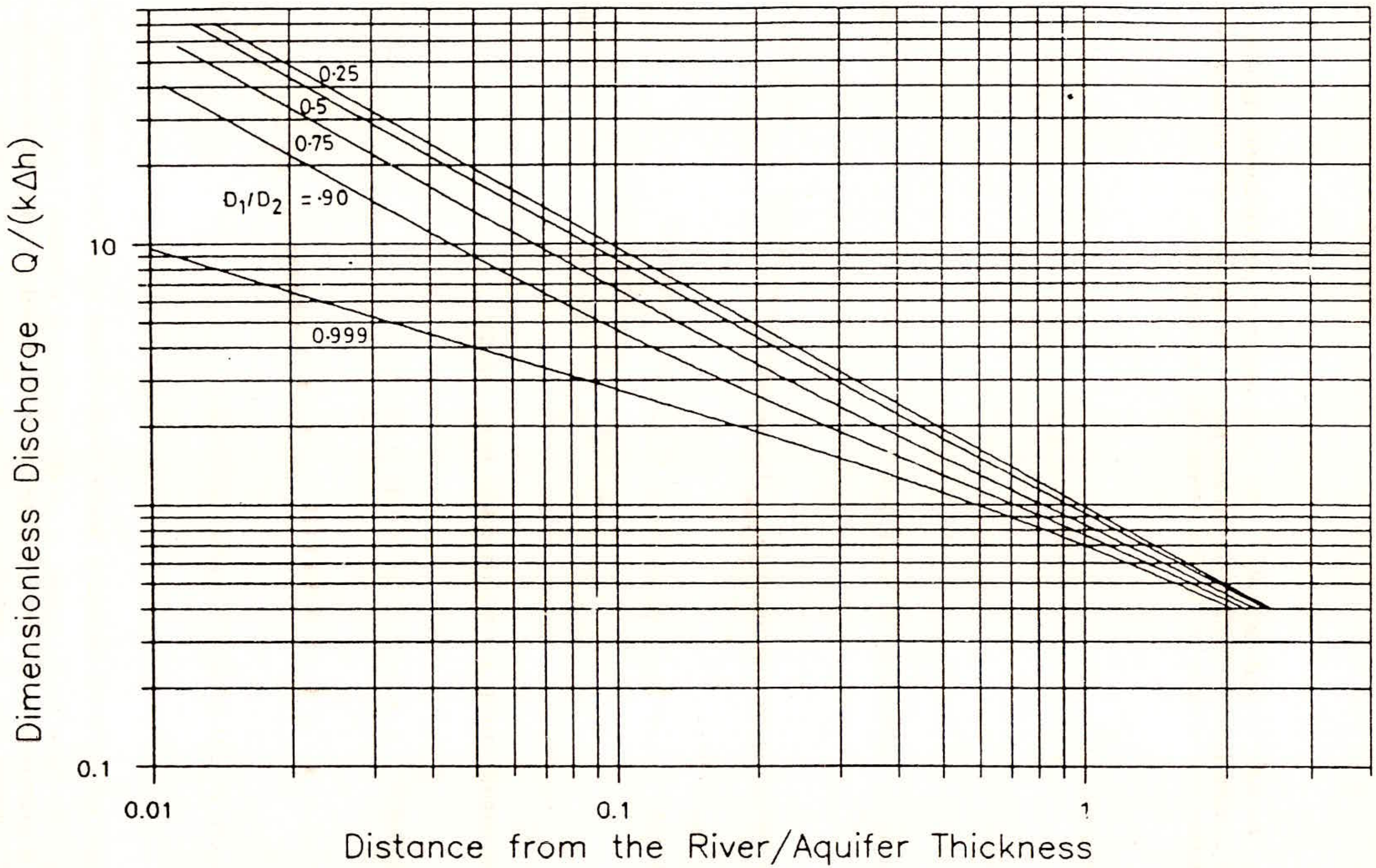

FIG. 3. VARIATION OF Q/(k h) WITH DISTANCE FROM THE RIVER FOR DIFFERENT PENETRATION OF THE RIVER

in which the potential difference $\Delta h = h_R - h(L)$ and the reach transmissivity per unit length of stream from equation(12) is given by,

$$\Gamma_r = \pi k / \ln\left[(\sqrt{1-c} + \sqrt{d-c})/(\sqrt{1-c} - \sqrt{d-c})\right] \quad \ldots(14)$$

The reach transmissivity is dependent on the location of the observation well besides the depth of penetration of the stream. The variation of $q/(k\Delta h)$ with distance of the observation well from the stream bank is shown in Fig.3 for different penetration depth of the stream. The reach transmissivity is found to vary with distance from the stream for any depth of penetration. The variation is linear for a fully penetrating stream only, otherwise the relationship is nonlinear. From the graph it can be seen that beyond 1.5 times the aquifer depth, the partial penetration has no influence on the reach transmissivity.

Using this graph the seepage from a stream can be ascertained provided the hydraulic conductivity, the stream stage and the piezometric level in the observation well are available. The program for computation of $q/(k\Delta h)$ is listed in Appendix I.

3.0 UNSTEADY FLOW BETWEEN A STREAM AND AN AQUIFER

The rise in piezometric surface in an initially steady state semi-infinite homogeneous and isotropic confined aquifer bounded by a fully penetrating straight stream due to a unit step rise in stream stage is given by (Carslaw and Jaeger, 1959),

$$K(x,t) = 1 - \text{erf}\left\{ \frac{x}{\sqrt{(4\beta t)}} \right\} \quad (15)$$

in which $K(x,t)$ = rise in piezometric surface due to a step rise in stream stage, x= distance from the bank of the stream, t = time measured since the onset of change in stream stage, $\beta = T/\Phi$, the hydraulic diffusivity of the aquifer, T = transmissivity, Φ = storage coefficient, and erf{.} = error function. Equation(1) is a good approximation for an unconfined aquifer if the changes in water level are small in comparison to the saturated thickness of the aquifer (Cooper and Rorabaugh, 1963).

The influent seepage through unit length from both sides of the stream bank due to a unit step rise in stream stage is given by,

$$K_q(t) = 2\sqrt{\frac{\Phi\, T}{\pi\, t}} \quad (16)$$

The cumulative flow in response to unit step rise is given by

$$K_V(t) = 4\sqrt{\frac{\Phi\ T\ t}{\pi}} \tag{17}$$

For varying stream stage, $\sigma(t)$, the influent seepage rate at time $t = m\Delta t$ according to the Duhamel's theorem is given by (Morel-Seytoux, 1975):

$$Q(m\Delta t) = \int_0^{m\Delta t} \frac{d\sigma}{d\tau}\, d\tau\ K_q(t-\tau)$$

$$= \sum_{\gamma=1}^{m} \left[[\ \sigma\{\gamma\ \Delta t\} - \sigma\{(\gamma-1)\ \Delta t\}]/\Delta t \quad \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} K_q(t-\tau)d\tau \right] \tag{18}$$

Let, $\delta_q(n,\Delta t) = \frac{1}{\Delta t} \int_0^{\Delta t} K_q(n\Delta t-\tau)d\tau$,

Substituting the expression of $K_q(n\Delta t-\tau)$ from eq. (16) and integrating,

$$\delta_q(n,\Delta t) = 4\sqrt{\frac{\Phi\ T}{\pi\Delta t}} \left\{ \sqrt{n} - \sqrt{n-1} \right\} \tag{19}$$

$$Q(m\Delta t) = \sum_{\gamma=1}^{m} \left\{\sigma(\gamma) - \sigma(\gamma-1)\right\}\delta_q(m-\gamma+1,\Delta t) \tag{20}$$

Similarly for varying stream stage, $\sigma(t)$, the cumulative influent seepage volume at time $t = m\Delta t$ according to the Duhamel's theorem (vide Thomson, 1950, p 37) is given by:

$$V(m\Delta t) = \int_0^{m\Delta t} \frac{d\sigma}{d\tau}\, d\tau\ K_v(t-\tau)$$

$$= \sum_{\gamma=1}^{m} [\ \sigma\{\gamma\ \Delta t\} - \sigma\{(\gamma-1)\ \Delta t\}]/\Delta t \quad \int_{\gamma}^{\gamma\Delta t} K_v(t-\tau)d\tau$$

Let, $\delta_v(n,\Delta t) = \frac{1}{\Delta t} \int_0^{\Delta t} K_v(n\Delta t-\tau)d\tau$, Substituting the expression of $K_V(n\Delta t-\tau)$ from eq. (17) and integrating,

$$\delta_V(n,\Delta t) = \frac{8}{3}\sqrt{\frac{\Phi\ T\ \Delta t}{\pi}}\left\{ n^{3/2} - (n-1)^{3/2} \right\} \tag{21}$$

$$V(m\Delta t) = \sum_{\gamma=1}^{m} \left\{\sigma(\gamma) - \sigma(\gamma-1)\right\}\delta_V(m-\gamma+1,\Delta t) \tag{22}$$

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Morel-Seytoux,H.J. 1975. A combined model of water table and river stage evolution. Water Resources Research. 11(6), 968-972.

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COMPUTER PROGRAM: ESTIMATION OF REACH TRANSMISSIVITY

```
      DIMENSION W(100),GX(100),CQBKHI(500)
      OPEN(1,STATUS='OLD',FILE='GANESH.DAT')
      OPEN(2,STATUS='NEW',FILE='GANESH.OUT')
      NX=96
      READ(1,*) D1,D2 C
      PART=THICKNESS OF AQUIFER BELOW STREAM/THICKNESS OF AQUIFER
C     D2=THICKNESS OF AQUIFER
C     D1=THICKNESS OF AQUIFER BELOW STREAM BED
C     GW(I) AND GX(I) ARE GAUSSIAN WEIGHT AND ABSCISSAS (REF: ABRAMOWITZ AND
C.    STEGUN, 1970)
      READ(1,*) (GW(I),I=1,NX) READ(1,*) (GX(I),I=1,NX)
      PAI = 3.14159265
200   CONTINUE
      D1BYD2 = D1/D2
      WRITE(2,3)
3     FORMAT(5X,'D1',8X,'D2',9X,'D1BYD2')
      WRITE(2,4)D1,D2,D1BYD2
4     FORMAT(3E16.7)
      WRITE(2,33)
33    FORMAT(3X,'AL',9X,'QBYKH')
      C=1. - D1BYD2**2
      DELC = (1.0-C)*0.001
      D=C+DELC
300   CONTINUE
      SUM1=0.
      SUM5=0.
      DO 100 I=1,NX
      X = GX(I)
      V = 0.5*SQRT(C) + 0.5*SQRT(C)*X
      F5 = SQRT(C)*SQRT(C-V*V)/(1.-C + V*V)
      F1=SQRT(C*(D - C)+((1. + X)*(D-C))**2/4.0)/
     1(1. - C - (1. + X)**2*(D - C)/4.)
      SUM1 = SUM1 + F1*GW(I)
      SUM5 = SUM5 + F5*GW(I)
100   CONTINUE
      AL = D1/PAI*SUM1
      D2MD1 = D1/PAI*SUM5
      TERM1 = SQRT(1. - C)+SQRT(D - C)
      TERM2 = SQRT(1. - C)-SQRT(D - C)
      TERM3 = ALOG(TERM1/TERM2)
      QBYKH = PAI/TERM3
      WRITE(2,2) AL,QBYKH
2     FORMAT(2E12.4)
      D = D + DELC
      IF(D.LT.1.0)
      GO TO 300
      STOP
      END
```