TRAINING COURSE

ON

SOFTWARE FOR GROUNDWATER DATA MANAGEMENT

UNDER

WORLD BANK FUNDED HYDROLOGY PROJECT

LECTURE NOTES
ON

GROUNDWATER MODELLING SOFTWARES
(UNIT-4)

BY

G C MISHRA
&
KAMAL

ORGANISED BY

NATIONAL INSTITUTE OF HYDROLOGY
ROORKEE - 247 667
INDIA

BLOCK LEVEL WATER BALANCE

1.0 INTRODUCTION

Some times it is required to carry out water balance of a block which may form part of a groundwater basin. Part of the block boundary may be comprised of (i) a river conforming to a constant head boundary, (ii) an impervious zone conforming to a no-flow boundary and (iii) a pervious zone conforming to a flow boundary. If the block happens to be at the central region of the groundwater basin it is possible to carry out water balance analytically considering the subsurface inflows and outflows.

2.0 METHODOLOGY

The aquifer is hydrologically decomposed into a number of circular zones as shown in Fig. 1. The centre of the each circle defines a local origin of a polar co-ordinate system. Let the radius of a circular zone be 'R'. Let 'i' be an abstraction point whose polar co-ordinates are (r_i,θ_i). The cumulative inflow to the aquifer from the adjacent area across the circle upto time 't' due to continuous withdrawal at unit rate at point 'i' is given by

$$K(r_i,\theta_i,R,t) = \int_0^{2\pi}\int_0^{t} \frac{R[R-r_i\mathrm{Cos}(\theta_i-\theta)]}{2\pi[R^2+r_i^2-2Rr_i\mathrm{Cos}(\theta_i-\theta)]}\exp\left\{-\frac{R^2+r_i^2-2Rr_i\mathrm{Cos}(\theta_i-\theta)}{4\beta\tau}\right\}d\tau d\theta \quad (1)$$

in which

β = T/Φ, the hydraulic diffusivity of the aquifer,
T = transmissivity of the aquifer, and
Φ = storage coefficient.

Integrating with respect to τ

$$K(r_i,\theta_i,R,t) = \int_0^{2\pi} \frac{R[R-r_i\mathrm{Cos}(\theta_i-\theta)]}{2\pi[R^2+r_i^2-2Rr_i\mathrm{Cos}(\theta_i-\theta)]}\left\{t\,\exp(-C_i/t) - C_iE_i(C_i/t)\right\}d\theta \quad (2)$$

in which,

$$C_i = \frac{R^2 + r_i^2 - 2Rr_i Cos(\theta_i - \theta)}{4\beta}$$

$$E_1(C_i/t) = \int_{C_i/t}^{\infty} \frac{e^{-u}}{u} du = \text{an exponential integral.}$$

A numerical integration has to be carried out to evaluate the integral appearing in eq.(2).

If unit withdrawal takes place at the first unit time at the i^{th} abstraction point, and no withdrawal after that, the cumulative flow at the end of the n^{th} time step across the circle is given by,

$$\delta_{Ii}(n) = K(r_i, \theta_i, R, n) - K(r_i, \theta_i, R, n-1)$$

and,

$$\delta_{Ii}(1) = K_i(r_i, \theta\ , R, 1)$$

The cumulative inflow at the end of n^{th} time-step across the circle, $CUMI_i(n)$, due to varying abstraction from the i^{th} well is given by

$$CUMI_i(n) = \sum_{\gamma=1}^{n} QP_i(\gamma)\delta_{Ii}(n-\gamma+1)$$

in which,

$QP_i(\gamma)$ = Quantity of water pumped during γ^{th} time-step at i^{th} well.

If there are 'M' abstraction points, then total inflow up to the end of n^{th} time-step, CUMI(n), due to all withdrawal is expressed by

$$CUMI(n) = \sum_{i=1}^{M} \sum_{\gamma=1}^{n} QP_i(\gamma)\delta_{Ii}(n-\gamma+1)$$

Similarly total outflow from the circular zone at the end of n^{th} time-step due to recharge $QR_j(\gamma)$ occurring during γ^{th} time-step at j^{th} recharge point, $j=1,2,..,N$,and $\gamma=1,2,..,n$, is given by

$$CUMO(n) = \sum_{i=1}^{N} \sum_{\gamma=1}^{n} QR_j(\gamma)\delta_{Oj}(n-\gamma+1)$$

$\delta_{Oj}(n)$ and $\delta_{Ii}(n)$ are identical.

If the number of abstraction points and recharge points inside the circular zone are M_C and N_C respectively, then the amount of draft and recharge at the end of n^{th} time-step will be,

$$CUMQP(n) = \sum_{i=1}^{M_C} \sum_{\gamma=1}^{n} QP_i(\gamma)$$

$$CUMQR(n) = \sum_{j=1}^{N_C} \sum_{\gamma=1}^{n} QR_j(\gamma)$$

The groundwater availability in that particular zone at the end of nth time-step can be expressed by,

$$GW(n) = CUMQR(n)-CUMQP(n)-CUMO(n)+CUMI(n)$$

or

$$GW(n) = \sum_{j=1}^{N_C} \sum_{\gamma=1}^{n} QR_j(\gamma) - \sum_{i=1}^{M_C} \sum_{\gamma=1}^{n} QP_i(\gamma) - \sum_{j=1}^{N} \sum_{\gamma=1}^{n} QR_j(\gamma)\delta_{Oj}(n-\gamma+1)$$

$$+ \sum_{i=1}^{M} \sum_{\gamma=1}^{n} QP_i(\gamma)\delta_{Ii}(n-\gamma+1) \qquad (3)$$

Similar equations for all the circular zones, by which the area is hydrologically decomposed, is obtained and algebraic addition of all these availability of groundwater will give the groundwater storage available for the district. A software for block level water balance is given in Appendix I.

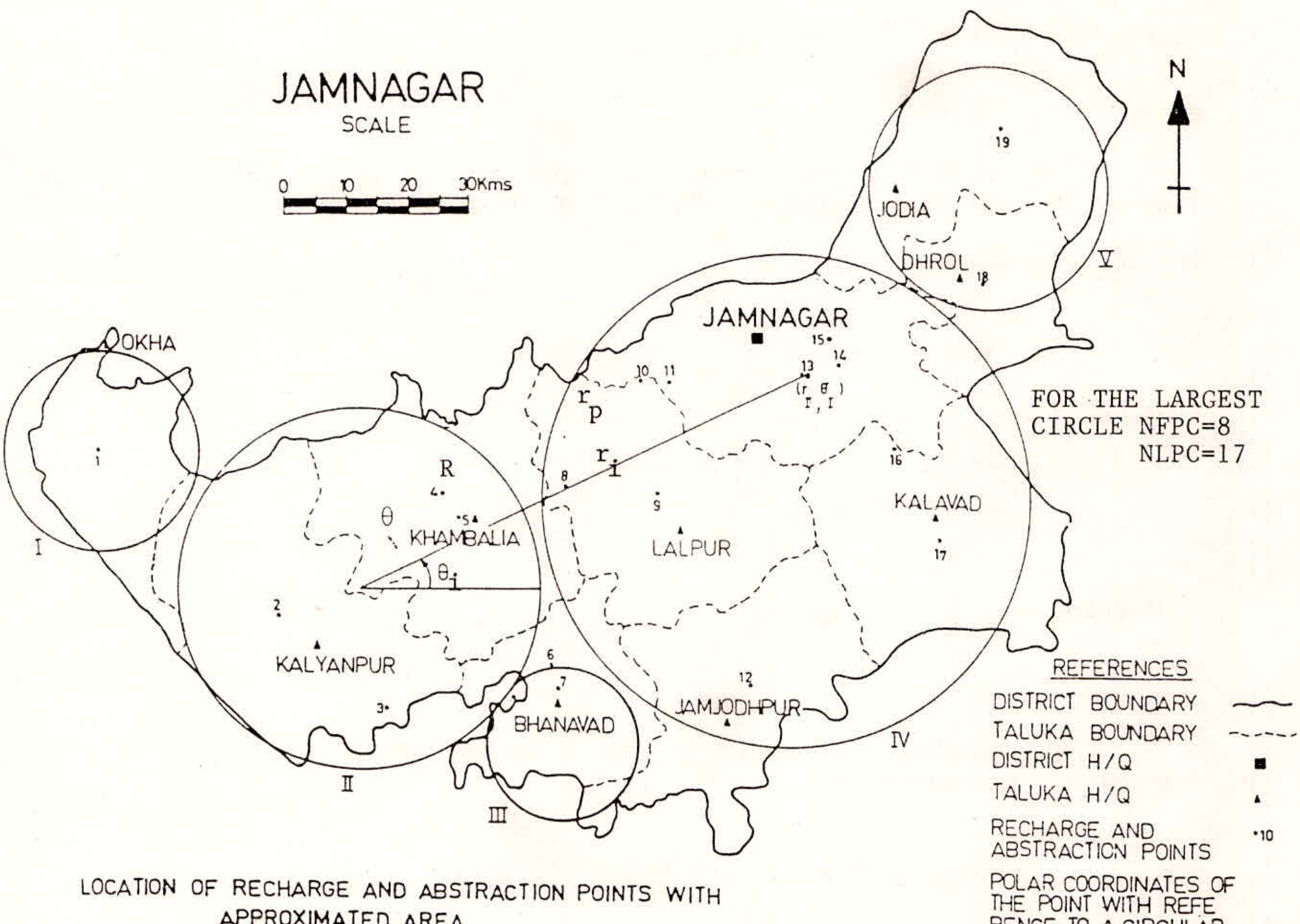

FIG.1: INFLOW TO AND OUTFLOW FROM A CIRCULAR ZONE OF AN INFINITE AQUIFER DUE TO VARIOUS RECHARGE AND ABSTRACTION

```
C       SOURCE PROGRAM: G11.FOR
C       GROUND WATER AVAILABILITY STUDY
C       ANNUAL GROUND WATER BALANCE
C       15 SPACE SHOULD BE ASSIGNED FOR EACH BLOCKNAME IN DATA
C       FILE
C       6 SPACE SHOULD BE ASSIGNED FOR EACHMONTH NAME IN DATA
C       FILE
C       10 SPACE FOR NAME OF INPUTFILE AND OUTPUT FILE
C
        CHARACTER*15 BLNAME
        CHARACTER*6 MONTH
C       CHARACTER*10 INPUT,OUTPUT
C
        DIMENSION RABST(50),THABST(50),RRECH(50),THRECH(50),
     1  QP(50,20),QR(50,20),SUMQP(20),SUMQR(20),SUMQPI(20)
     2  ,SUMQRO(20),GAVAIL(20),DELP(50,20),DELPF(50,20)
     3  ,DELR(50,20),DELRF(50,20),MONTH(20),BLNAME(20),BLAREA(20)
     4  ,BLWATER(20)
C
C       WRITE (5,30)
C30     FORMAT(4X,' INPUT FILE NAME:  '$)
C       READ(5,29)INPUT
C       WRITE(5,31)
C31     FORMAT(4X,'OUTPUT FILE NAME:  '$)
C       READ(5,29)OUTPUT
C29     FORMAT(A)
C
C       OPEN(UNIT=1,FILE=INPUT,STATUS='OLD')
C       OPEN(UNIT=2,FILE=OUTPUT,STATUS='NEW')
        OPEN(UNIT=1,FILE='G11.DAT')
        OPEN(UNIT=2,FILE='G11.OUT')
        READ(1,*)NABSTP,NRECHP,NTIME,NFPC,NLPC
        READ(1,*)R,T,PHI,NBL,AVFL
C
C       R=RADIUS OF THE CIRCULAR ZONE
C       T= AQUIFER TRANSMISSIVITY IN SQUARE METER PER MONTH
C       PHI=STORAGE COEFFICIENT
C       NBL=NUMBER OF BLOCKS INSIDE THE CIRCLE
C       AVFL=AVERAGE WATER LEVEL HEIGHT ABOVE THE DATUM IN
C       METER
C       AVFL REPRESENTS THE INITIAL STORAGE PRIOR TO ONSET OF
C       MONSOON
C
        PAI=4.*ATAN(1.)
        AREA=PAI*R*R/1000000.
C       AREA IS  IN SQ KM
        DYASTI=PAI*R*R*PHI
        READ(1,46)(MONTH(N),N=1,NTIME)
```

```
      DO K=1,NBL
      READ(1,42)BLNAME(K),BLAREA(K)
C     BLAREA SHOULD BE READ IN SQ KM
      ENDDO
      READ(1,*)(RABST(I),I=1,NABSTP)
      READ(1,*)(THABST(I),I=1,NABSTP)
C     RABST(I)= RADIUS OF THE ITH ABSTRACTION POINT
C     THABST(I)=THETA OF THE ITH ABSTRACTION POINT
      DO 1 I=NABSTP
      THABST(I)=THABST(I)*PAI/180.
1     CONTINUE
      READ(1,*)(RRECH(I),I=1,NRECHP)
      READ(1,*)(THRECH(I),I=1,NRECHP)
C     RABST(I)= RADIUS OF THE ITH RECHARGE POINT
C     THABST(I)=THETA OF THE ITH RECHARGE POINT
      DO 2 I=1,NRECHP
      THRECH(I)=THRECH(I)*PAI/180.
2     CONTINUE
      DO I=1,NABSTP
      READ(1,*)(QP(I,J),J=1,NTIME)
C
C     QP(I,J)= PUMPING RATE AT ITH WELL DURING JTH MONTH
C     IN CUBIC METER PER MONTH
C
      ENDDO
      SUM1=0.
      SUM2=0.
C     NFPC=FIRST(LOWEST) NUMBER ASSIGNED TO WELL (RECHARGE OR
C     DISCHARGE)
C     NLPC=LAST(HIGHEST) NUMBER ASSIGNED TO WELL (RECHARGE OR
C     DISCHARGE)
      DO 50 I=NFPC,NLPC
      DO 51 J=1,4
      SUM1=SUM1+QP(I,J)
51    CONTINUE
      DO 52 J=5,12
      SUM2=SUM2+QP(I,J)
52    CONTINUE
50    CONTINUE
      QMP=SUM1
      QNMP=SUM2
C
      DO I=1,NRECHP
      READ(1,*)(QR(I,J),J=1,NTIME)
C     QR(I,J)= RECHARGE RATE AT ITH WELL DURING JTH MONTH
C     IN CUBIC METER PER MONTH
      ENDDO
```

```
      SUM1=0.
      SUM2=0.
      DO 53 I=NFPC,NLPC
      DO 54 J=1,4
      SUM1=SUM1+QR(I,J)
54    CONTINUE
      DO 55 J=5,12
      SUM2=SUM2+QR(I,J)
55    CONTINUE
53    CONTINUE
      QMR=SUM1
      QNMR=SUM2
      WRITE(2,32)AVFL
      WRITE(2,49)DYASTI
      WRITE(2,33)AREA
      WRITE(2,34)PHI
      WRITE(2,35)T
      WRITE(2,36)QMR
      WRITE(2,37)QNMR
      WRITE(2,38)QMP
      WRITE(2,39)QNMP
      WRITE(2,40)
      WRITE(2,41)
      WRITE(2,28)
      WRITE(2,26)
      DO 6 I=1,NABSTP
      THETA=THABST(I)
      AAR=RABST(I)
      DO 7 J=1,NTIME
      AJ=J
      CALL DEL(THETA,AAR,AJ,R,T,PHI,RES)
      DELP(I,J)=RES
7     CONTINUE
6     CONTINUE
      DO 8 I=1,NABSTP
      DELPF(I,1)=DELP(I,1)
      DO 9 J=2,NTIME
      DELPF(I,J)=DELP(I,J)-DELP(I,J-1)
9     CONTINUE
8     CONTINUE
      DO 10 I=1,NRECHP
      THETA=THRECH(I)
      AAR=RRECH(I)
      DO 11 J=1,NTIME
      AJ=J
      CALL DEL(THETA,AAR,AJ,R,T,PHI,RES)
      DELR(I,J)=RES
11    CONTINUE
10    CONTINUE
```

```
      DO 12 I=1,NRECHP
      DELRF(I,1)=DELR(I,1)
      DO 13 J=2,NTIME
      DELRF(I,J)=DELR(I,J)-DELR(I,J-1)
13    CONTINUE
12    CONTINUE
      DO 14 N=1,NTIME
      SUM=0.
      DO 15 I=1,NABSTP
      DO 16 NGAMA=1,N
16    SUM=SUM+QP(I,NGAMA)*DELPF(I,N-NGAMA+1)
      IF(SUM.LT.0.0)SUM=0.0
15    CONTINUE
      SUMQPI(N)=SUM
14    CONTINUE
      DO 27 N=1,NTIME
      SUM=0.
      DO 17 I=1,NRECHP
      DO 18 NGAMA=1,N
18    SUM=SUM+QR(I,NGAMA)*DELRF(I,N-NGAMA+1)
      IF(SUM.LT.0.0)SUM=0.0
17    CONTINUE
      SUMQRO(N)=SUM
27    CONTINUE
      DO 19 N=1,NTIME
19    GAVAIL(N)=0.
      DO 20 N=1,NTIME
      SUMQP(N)=0.
      SUMQR(N)=0.
      DO 21 I=NFPC,NLPC
21    SUMQP(N)=SUMQP(N)+QP(I,N)
      DO 22 I=NFPC,NLPC
22    SUMQR(N)=SUMQR(N)+QR(I,N)
20    CONTINUE
      DO 23 N=1,NTIME
      SUM=0.
      DO 24 NGAMA=1,N
      SUM=SUM+SUMQPI(NGAMA)-SUMQRO(NGAMA)-
     1 SUMQP(NGAMA)+SUMQR(NGAMA)
24    CONTINUE
      GAVAIL(N)=SUM+DYASTI
      WRITE(2,25)N,MONTH(N),GAVAIL(N),SUMQR(N),SUMQP(N),SUMQRO(N),
     1 SUMQPI(N)
23    CONTINUE
      WRITE(2,47)
      DO 99 K=1,NBL
      WRITE(2,43)BLNAME(K)
      WRITE(2,48)
      WRITE(2,45)
```

```
C
      DO 98 N=1,NTIME
      BLWATER(N)=GAVAIL(N)
      BLWATER(N)=BLWATER(N)*BLAREA(K)/AREA
      WRITE(2,44)MONTH(N),BLWATER(N)
98    CONTINUE
      WRITE(2,47)
99    CONTINUE

C     DO I = 1, 6
C     WRITE(2,*) (DELPF(I,J),J=1,12)
C     ENDDO
C     DO I = 1,4
C     WRITE(2,*) (DELRF(I,J),J=1,12)
C     ENDDO

25    FORMAT(1X,I2,1X,A7,2X,E10.2,4(2X,E12.4))
26    FORMAT(17x,'(cum)',8x,'(cum)',9x,'(cum)',9x,'(cum)'
    1 ,9X,'(cum)'//)
28    FORMAT(1X,'N',2X,'MONTH',2X,'WATERBALANCE',3X,'RECHARGE',
    1 5X,'WITHDRAWAL',7X,'OUTFLOW',7X,'INFLOW')
32    FORMAT(10X,'Average water level height above datum =',1X,F5.2,
    1 1X,'m',/)
33    FORMAT(10X,'Area of the circular zone ='1X,F7.2,1X,'sq.km.',/)
34    FORMAT(10X,'Storage coefficient =',1X,F7.2,/)
35    FORMAT(10x,'Transmissivity =',1X,E7.2,1X,'sqm/month',/)
36    FORMAT(10X,'Monsoon recharge ='1X,E10.2,1X,'cum',/)
37    FORMAT(10X,'Nonmonsoon recharge ='1X,E10.2,1X,'cum',/)
38    FORMAT(10X,'Monsoon draft ='1X,E10.2,1X,'cum',/)
39    FORMAT(10X,'Nonmonsoon draft =',1X,E10.2,1X,'cum',/)
40    FORMAT(10X,'TEMPORAL GROUNDWATER AVAILABILITY:')
41    FORMAT(10X,34('*'),/)
42    FORMAT(A10,F8.2)
43    FORMAT(26X,'BLOCK:',1X,A10)
44    FORMAT(24X,A7,4X,E10.3)
45    FORMAT(24X,'Month',5X,'WATER BALANCE(in cum)',/)
46    FORMAT(12A6)
47    FORMAT(/)
48    FORMAT(25X,18('*'))
49    FORMAT(10X,'INITIAL DYNAMIC STORAGE =',2X,E12.4, 2X,'cum',/)
      STOP
      END
C
```

```
C       ****************************************************************
C                       Numerical Integration
C       ****************************************************************
C
        SUBROUTINE DEL(THETA,AAR,AJ,R,T,PHI,RES)
        PAI=4.*ATAN(1.)
        N=100
        A=0.
        B=2.*PAI
        H=B/FLOAT(N)
        CALL EVAL(A,THETA,AAR,AJ,R,T,PHI,FINAL)
        FA=FINAL
        CALL EVAL(B,THETA,AAR,AJ,R,T,PHI,FINAL)
        FB=FINAL
        SUM=FA+FB
        NN=N-1
        DO 10 K=1,NN
        X=A+FLOAT(K)*H
        CALL EVAL(X,THETA,AAR,AJ,R,T,PHI,FINAL)
        SUM=SUM+2.*FINAL
10      CONTINUE
        RES=H*SUM/2.
        RETURN
        END
C
C       ****************************************************************
C                   Computation of the Integrand
C       ****************************************************************
C
        SUBROUTINE EVAL(TH,THETA,AAR,AJ,R,T,PHI,FINAL)
        PAI=4.*ATAN(1.)
        F1=AAR*COS(THETA-TH)
        F2=R*R+AAR*AAR
        F3=R-F1
        F4=F2-2.*R*F1
        BETA=4.*T/PHI
        F5=F4/(BETA*AJ)
        CALL EXI(F5,EXFN)
        F6=AJ*EXP(-F5)-EXFN*F5*AJ
        F7=R*F3/(2.*PAI*F4)
        FINAL=F7*F6
        RETURN
        END
C
```

```
C       ************************************************************
C               Computation of Exponential Integral
C       ************************************************************
C
        SUBROUTINE EXI(X,EXFN)
        IF(X-1.0)1,1,22
1       EXFN=-ALOG(X)-0.57721566+0.99999193*X-0.24991055*X**2
     1  +0.05519968*X**3-0.00976004*X**4+0.00107857*X**5
        GO TO 3
22      CONTINUE
        IF(X-80.)5,4,4
5       CONTINUE
        EXFN=((X**4+8.5733287*X**3+18.059017*X**2+8.6347608*X
     1  +0.26777373)/(X**4+9.5733223*X**3+25.632956*X**2
     2  +21.099653*X+3.9584969))/(X*EXP(X))
        GO TO 3
4       EXFN=0.
3       CONTINUE
        RETURN
        END
```

INPUT FILE: G11.DAT

```
6 4 12 1 5
3000. 9000. 0.01 2 2.
JUL. AUG. SEP. OCT. NOV. DEC. JAN. FEB. MAR. APR. MAY JUN.
WASIS        7.0
SPRING       3.0
1000. 2000. 1500. 4000. 5000. 3500.
15.  30.  45.  90.  60.  65.
700.  2500.  5000.  6000.
5.   20.  35.  70.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  1000. 4500. 1000. 1000. 1000.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  2000. 2500. 1000. 1000. 1000.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  3000. 1500. 1000. 1000. 1000.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  4000. 2000. 1000. 1000. 1000.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  5000. 3500. 1000. 1000. 1000.
 0.0  0.0 0.0 0.0 1000. 2000.  750.  6000. 4000. 1000. 1000. 1000.

1000.   1500.   1400.  4300.  0.   0.   100.  200. 300. 300.
 100. 100.
2000.   2500.   2400.  3300.  0.   0.   100.  200. 300. 300.
 100. 120.
3000.   3500.   3400.  2300.  0.   0.   100.  200. 300. 300.
 100. 130.
4000.   4500.   4400.  1300.  0.   0.   100.  200. 300. 300.
 100. 140.
```

OUTPUT FILE: G11.OUT

Average water level height above datum = 2.00 m

INITIAL DYNAMIC STORAGE = .2827E+06 cum

Area of the circular zone = 28.27 sq.km.

Storage coefficient = .01

Transmissivity = .90E+04 sqm/month

Monsoon recharge = .45E+05 cum

Nonmonsoon recharge = .45E+04 cum

Monsoon draft = .00E+00 cum

Nonmonsoon draft = .63E+05 cum

TEMPORAL GROUNDWATER AVAILABILITY:
****

N	MONTH	WATER BALANCE (cum)	RECHARGE (cum)	WITHDRAWAL (cum)	OUTFLOW (cum)	INFLOW (cum)
1	JUL.	.29E+06	.1000E+05	.0000E+00	.6228E+03	.0000E+00
2	AUG.	.30E+06	.1200E+05	.0000E+00	.1707E+04	.0000E+00
3	SEP.	.31E+06	.1160E+05	.0000E+00	.3022E+04	.0000E+00
4	OCT.	.32E+06	.1120E+05	.0000E+00	.4915E+04	.0000E+00
5	NOV.	.31E+06	.0000E+00	.5000E+04	.6608E+04	.0000E+00
6	DEC.	.29E+06	.0000E+00	.1000E+05	.7885E+04	.3588E+03
7	JAN.	.28E+06	.4000E+03	.3750E+04	.8902E+04	.1569E+04
8	FEB.	.26E+06	.8000E+03	.1500E+05	.9786E+04	.1783E+04
9	MAR.	.23E+06	.1200E+04	.1400E+05	.1062E+05	.2763E+04
10	APR.	.22E+06	.1200E+04	.5000E+04	.1141E+05	.5321E+04
11	MAY	.22E+06	.4000E+03	.5000E+04	.1210E+05	.8051E+04
12	JUN.	.21E+06	.4900E+03	.5000E+04	.1272E+05	.1070E+05

BLOCK: WASIS
****

Month	WATER BALANCE(in cum)
JUL.	.723E+05
AUG.	.749E+05
SEP.	.770E+05
OCT.	.785E+05
NOV.	.757E+05
DEC.	.713E+05
JAN.	.687E+05
FEB.	.632E+05
MAR.	.581E+05
APR.	.556E+05
MAY	.535E+05
JUN.	.519E+05

BLOCK: SPRING
****

Month	WATER BALANCE(in cum)
JUL.	.310E+05
AUG.	.321E+05
SEP.	.330E+05
OCT.	.337E+05
NOV.	.324E+05
DEC.	.306E+05
JAN.	.294E+05
FEB.	.271E+05
MAR.	.249E+05
APR.	.238E+05
MAY	.229E+05
JUN.	.222E+05