# ROLE OF NUMERICAL METHODS IN ANALYSIS OF PUMPING TEST DATA FROM UNCONFINED AQUIFERS

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ABSTRACT

Many of the assumptions of Boulton's theory/type curves may be severely violated in case of long duration pumping tests on unconfined aquifers. Prominent among them may be the assumptions of fully penetrating wells, high enough $S_y/S$ ratio and absence of any other pumping in the vicinity of the test. Application of numerical methods coupled with Boulton's and Neuman's theories is suggested for a more rational analysis of data of such pumping tests.

## INTRODUCTION

Conventionally, the pumping test data from unconfined aquifers are analyzed by Boulton's type curves. However, these curves are based upon many assumptions which may not always hold. The assumption of fully penetrating wells is hard to satisfy especially in case of thick aquifers. Theoretically it is possible to eliminate effects of the partial penetration by placing the observation wells sufficiently distant from the pumped well. However, the drawdown at such a distance may be too small to be measured accurately. It is possible to generate type curves for partially penetrating pumped/observation wells by Neuman's theory

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(de Marsily, 1986), provided the well assemblies and the initial saturated thickness are known. However, many a time the initial saturated thickness may not be known uniquely. The assumption of exclusiveness of the test pumping (i.e., absence of any other pumping in vicinity of the test) may be difficult to satisfy in case of the long duration tests. Stallman's procedure (Kruseman and De Ridder (1983) can be employed to generate specific type curves accounting for such uncontrolled withdrawals, provided all the withdrawals commence and end at the same times as the main pumping. However, in practice the uncontrolled pumpings are intermittent in nature. The assumption of specific yield ($S_y$)/Storage coefficient (S) ratio tending to infinity holds nearly provided $S_y$ is at least 30S. This assumption is mostly satisfied for good unconfined aquifers which are characterized by high enough specific yields. However, these days it is becoming increasingly necessary to develop even not so good aquifers.

In the present paper, it has been demonstrated that computer assisted numerical analyses (which could be viewed as 'numerical matching') of the data can lead to more rational estimates of the parameters by eliminating the assumptions cited above.

NUMERICAL MATCHING

The numerical alternative to the type curve analysis essentially involves arriving at such estimates of the parameters which minimize the 'mismatch' between the observed drawdowns and the corresponding drawdowns computed numerically by a chosen theory (Kashyap et. al., 1988). The mismatch is generally quantified by sum of squares of differences (SSD) between the observed and the computed drawdowns. The minimization of the mismatch can be accomplished by computing successively the time drawdown curves and the associated SSD for various sets of the parameter values. The change of parameter values from the preceding to the following set can be made either heuristically or by a systematic mathematical search. The two procedures are known as Simulation and Optimization respectively. In both the procedures, the intention is to arrive at such a change in the parameter values which will correct the mismatch to the maximum possible extent. For example: if the computed drawdowns in all the three segments are lower than the observed drawdowns, a decrease in the transmissibility may improve the match and the vice versa; if the computed drawdowns in the second segment are lower but match fairly well in the first segment or if the span of the second segment of the computed dradowns is larger, an increase in B (drainage factor) or $K_z/K_r$ (vertical anisotropy) may help; if the computed drawdowns in the first two segments match well but are lower in the third segment a decrease in the specific yield is called for. Such changes are to be implemented by visual inspection of the computed and observed time-drawdown curves in case of simulation. In optimization procedures these are automatically implemented through a gradient procedure (i.e., if SSD increases by increasing a particular parameter (+ve gradient) the parameter should be decreased and vice versa). The magnitude of change in case of simulation procedures is to be decided subjectively while in optimization procedures, this is decided by the relative magnitudes of the gradients. It may be inferred that the optimization methods have the advantage of being more objective. However, the computer time requirement for applying such methods to unconfined aquifers with uncontrolled pumpings may become prohibitively large.

SOFTWARE REQUIREMENT

The simulation procedure requires a computer code (simulator) capable of simulating drawdown, at a given space-time point, in accordance with the chosen theory. The other input data are the discharge, the parameters and other information in accordance with the theory (e.g., initial saturated thickness, depth and length of screens of the pumped and observation wells in case of the Neuman's theory). In case of the optimization procedures, the software requirement is a simulator and an optimization code. A number of optimization codes are available (e.g., Kuester and Mize, 1973; Rao 1978). However any such code will require a subroutine capable of computing the objective function (SSD in the present case) for any values of the parameters. Thus, an appropriate simulator is to be incorporated in a subroutine to compute drawdowns at all the space-time point where the observed drawdowns are available and hence to compute the sum of squares of the differences.

NUMERICAL MATCHING BY NEUMAN'S THEORY

THICK AQUIFERS

Conventionally, the pumping test analyses for unconfined aquifers are aimed at estimating T, $S_y$, S and a parameter accounting for delayed yield. In Boulton's theory delayed yield is described in terms of an empirical parameter - delay index. Neuman (1972) attributes the delayed yield to sluggish vertical flow caused by the relatively lower vertical hydraulic conductivity. Thus, the delayed yield is implicitly accounted for by vertical anisotropy $K_z/K_r$. The analysis of pumping test data by Neuman's theory (Neuman, 1974, 1975) can yield estimates of T, $S_y$, S and $K_z/K_r$ provided the drawdown s(r,t) can be computed in accordance with theory for any trial values of the parameeters. Such a computation requires numerical values of $(b-Z_1)$, $(b-Z_2)$, L, d and b (Fig.1). Of these $(b-Z_1)$, $(b-Z_2)$, and d are fixed during the stage of selecting (or drilling) pumped and observation wells. However, as has been discussed, b may not be known uniquely. This problem can be overcome by replacing the transmissibility by $K_r$ and b and treating them as two independent aquifer properties to be estimated along with other properties i.e., S, $S_y$ and $K_z/K_r$. Such an approach is all the more logical since the horizontal lateral flow towards a partially penetrating well is governed by $K_r$ and not by T. In the final solution T appears merely as a product of $K_r$ and b, the latter defined as the depth below the initial position of watertable at which the vertical flow is reduced to zero. Thus, in the absence of definite data on b, the parameters to be estimated are S, $S_y$, $K_z/K_r$, $K_r$ and b. Such an estimation of aquifer thickness can be viewed as a 'hydraulic estimation'. A standard type curve procedure is ruled out due to large number of parameters. The parameters can be estimated by computer assisted minimization of an objective function comprising of sum of squares of differences between the observed drawdowns and the corresponding drawdowns computed by the Neuman's theory.

Field Example

A pumping test was conducted on an alluvial unconfined aquifer of large thickness for a period of 840 minutes during which 103 time drawdown measurements were made in an observation well located at a distance of 31 m from the pumped well. The discharge was 2658 $m^3$/day. The vertical dimensions of the pumped and the observation wells (Fig.1) are g = 6 m, $(b-Z_1)$ = 42 m $(b-Z_2)$ = 25.1 m, d = 24.5 m, $L_s$ = 21.3 m and L = 45.8 m. An analysis of the time-drawdown data by Boulton's type curves yields $S = 0.001$, $S_y = 0.18$, $T = 880\ m^2$/day and $\alpha = 0.2\ day^{-1}$ (Hamoudi, 1985).

Recognizing that these estimates may not be reliable on account of the partial penetration of the pumped and the observation wells, the same data were reanalyzed by the Neuman's theory employing the optimization technique. The sum of squares of the differences between the observed and the computed drawdowns was minimized with respect to S, $S_y$, $L_s/b$, $K_r$ and $K_z/K_r$ where $L_s$ is length of the screen. The thickness b was replaced by the parameter $L_s/b$ to make computed drawdowns and hence the sum of the squares adequately sensitive to even a small change in the parameter value. In the absence of such a sensitivity, the numerical differentiation carried out during optimization may not be accurate enough to lead to correct estimates of the parameters. A simulator developed by Khaled (1984) was used for the analysis. The minimization was carried out by employing a non-linear optimization code based upon SUMT (sequential unconstrained minimization technique). This technique was invented by Flacco and McCormick (1968) and has been very extensively programmed since then. From the available information that b is at least 55.8 m, a permissible range of variation (0 to 0.382) was defined for the variable - $L_s/b$. The lower limit corresponds to a possibly very large b while the upper limit is derived from the stipulated minimum possible value of b. The permissible range was incorporated in the optimization process in the form of two constraints i.e. $L_s/b > 0$ and $L_s/b < 0.382$. The estimated (optimized and implied) parameters are given in Table 1 (S.N.2). The associated computed time-drawdown curve is shown in Figure 2. The estimates lead to an inference that the aquifer extends up to a depth of 133.4 m below ground and that there is considerable vertical anisotropy.

The estimated depth (133.4 m) can be compared with inferences of two independent studies (borehole logging and resistivity sounding) carried out in the vicinity of the test pumping site. The litholog of a 152 m deep borehole shows medium to fine sand upto a depth of 136 m and silty clay below that. The soil classification reported in the litholog has been based upon proper sieve analyses of the collected materials (Singh, Raj Pal, Executive Engineer, U.P. State Ground Water Investigation Organisation, personal communication). Thus, the aquifer thickness can be considered as 136 m with a fair degree of confidence. This corroborates the thickness estimated by the suggested analysis. The resistivity sounding has indicated the presence of a clay layer in the depth range of 188.5 to 241 m (Mathew, 1983; Singhal, D.C., Reader, Dept. of Hydrology, University of Roorkee, personal communication). However, this estimate is subject to variations on account of the coefficient of anisotropy. Therefore, the contradiction is not too pronounced. $K_z$ has been estimated as ($0.151\ K_r$) or about one seventh of $K_r$. Such an extent of vertical anisotropy is quite common in stratified alluvial aquifers due to the presence of thin clay layers.

OTHER STUDIES

A pumping test of 5760 minutes was conducted on an unconfined alluvial aquifer. The discharge from the pumped well was 4620 $m^3$/day. However, three privately owned wells are known to have ben operated during the test at discharge rates of 571,857 and 1000 $m^3$/day. The wells are located at distances of 230 m, 255 m and 210 m respectively from the observation well. They were operated concurrently thrice from -455 minutes (i.e., 455 minutes before the beginning of the pumping test) to 355 minutes; 2455 minutes to 3235 minutes and finally from 4675 minutes to 5335 minutes. These uncontrolled withdrawals distorted the time-drawdown curve leading to an inclined second segment and short-lived declining trends (i.e., drawdowns decreasing with time) from 500 minutes to 900 minutes and at around 4000 minutes. Further, there was a considerable scatter towards end of the pumping test. It can be easily concluded that inclination of the second segment of observed time-drawdown curve was due to superposition of the first segments of the uncontrolled withdrawals over the second segment of the test pumping. The other possible reason i.e., low $S_y/S$ was ruled out since $S_y$ is known to be high enough. The short lived declining trends and the scatter towards the end were due to recovery of watertable after the three spells of the uncontrolled withdrawals. The curve did not match well with the Boulton's type curves. The parameters were estimated by the first author and one of his students (Kashyap and Sarkar, 1989) by the simulation technique employing the Neuman's theory. Uncontrolled withdrawals were conceptually replaced by eighteen pumping and recharge series (6 for each private well), starting from various times, but all continuing till the end of the test. The series were so stipulated that their superposition leads to the known intermittent pumping pattern of each of the private wells. The drawdown at any discrete time was computed by algebraically adding the drawdown due to the test pumping and the dradowns/buildups due to all such pumping/recharge series which commence after that time. The principle of superposition, though strictly not applicable to unconfined aquifers, does nearly hold in this analysis since the drawdowns were quite small in comparison to the initial saturated thickness. Each individual drawdown and buildup was computed by the Neuman's theory. Thus, computation of a single composite dradown required upto eighteen drawdown and buildup computation. This determined the choice of a simulation procedure in preference to the optimization. By successive trials it was possible to reproduce closely the entire time-drawdown curve including the scatter and the declining trends (Kashyap and Sarkar, 1989).

NUMERICAL MATCHING BY BOULTON'S THEORY

LOW $S_y/S$ RATIO

Boulton's theory (Boulton, 1954, 1963) yields drawdowns in response to a point abstraction from an unconfined aquifer for any $S_y/S$. However, the type curves based upon the Boultons theory have been generated making an additional assumption of $S_y/S$ tending to infinity. Such an assumption results in horizontal second segment. In practice $S_y/S$ has to be finite. Nevertheless, the error on this count may be negligible provided $S_y/S$ is atleast 30. For lower $S_y/S$, the second segment of the observed time drawdown curve shows an upwards inclination. It is hard to match such a curve with the standard type curves. Such data can be analyzed by the optimization technique described earlier.

Field Example

A pumping test of 72 hours was conducted on an alluvial unconfined aquifer of Wadi Tuban area of PDR Yemen (Saadan, 1987). During the test a discharge of 2315 $m^3$/day was maintained. The drawdowns were monitored in one observation well at a distance of 93.5 m from the pumped well. The observed time-drawdown curve does not display horizontal second segment (Fig.3). The data were analyzed by the Boulton's theory employing the optimization technique. A simulator developed by Hamoudi (1985) was used in the analysis. The minimization was carried out by SUMT (Fiacco and McCormick, 1968). The optimized parameters are S=0.003, $S_y$=0.004, T=922 $m^2$/day and $\alpha$=15.462 $day^{-1}$. These estimates lead to an inference that $S_y/S$ for the aquifer is 13.33. This explains the inclination of the second segment of the observed time-drawdown curve. The reproduction of the observed drawdowns by the optimized parameters is shown in the Fig.3.

CONCLUSION

The pumping test data from unconfined aquifers may not always be amenable to a standard type-curve analysis. Generation of specific type curves may be possible only under certain conditions. Numerical methods of Simulation or Optimization, coupled with an appropriate chosen theory, can be alternative mode of analysis. Such analyses of the data from the aquifers of large and poorly known thicknesses, can yield an estimate of the aquifer thickness and vertical hydraulic conductivity, apart from the conventional parameters. Uncontrolled intermittent withdrawals which are quite likely to occur during a long duration test can also be accounted for. The data from the aquifers having low specific yields may not display a horizontal segment due to low $S_y/S$ ratio. Such data can be numerically analyzed by the Boulton's theory which unlike the type curves is not based upon the assumption of large $S_y/S$.

Table 1 - Parameter Estimation

| S.N. | Theory | Mode of Analysis | Parameters | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Estimated | | | | | | | Implied | | |
| | | | S | Sy | T ($m^2$/day) | α ($day^{-1}$) | Kz/Kr | Kr (m/day) | ls/b | b (m) | T ($m^2$/day) | Kz (m/day) |
| 1. | Boulton's | Graphical | 0.00112 | 0.1849 | 880 | 0.198 | - | - | - | - | - | - |
| 2. | Neuman's | Optimization | 0.0033 | 0.1481 | - | - | 0.151 | 10.13 | 0.1672 | 127.4 | 1291 | 1 53 |

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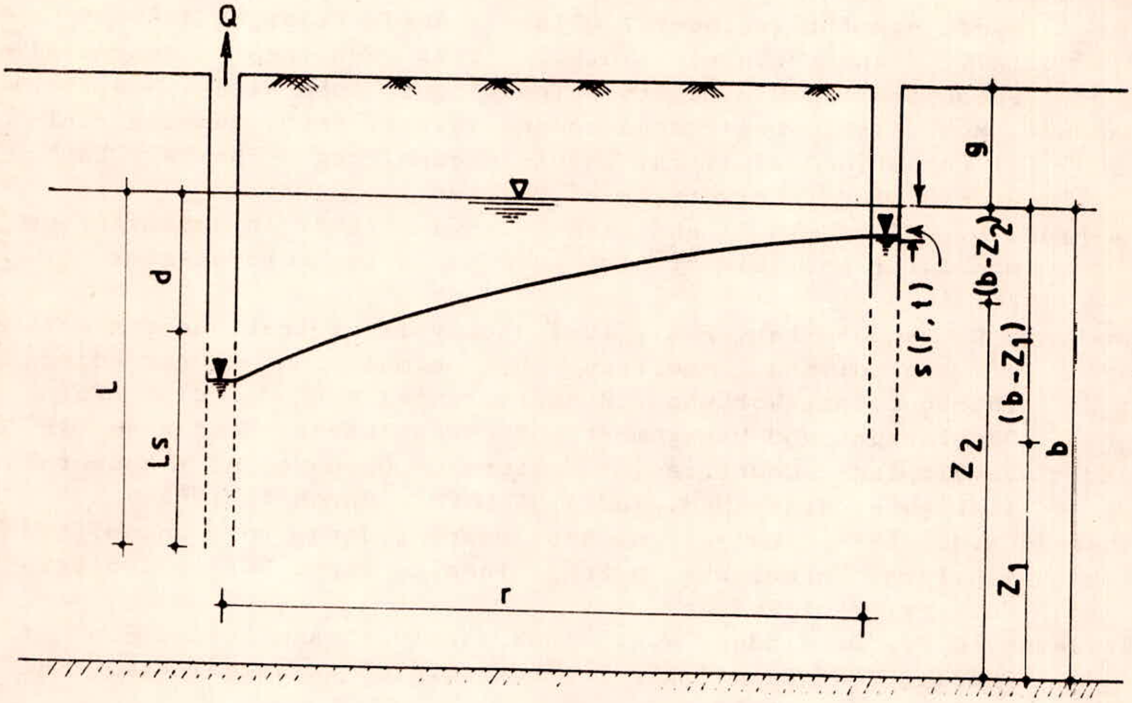


Fig.1 Definition Sketch

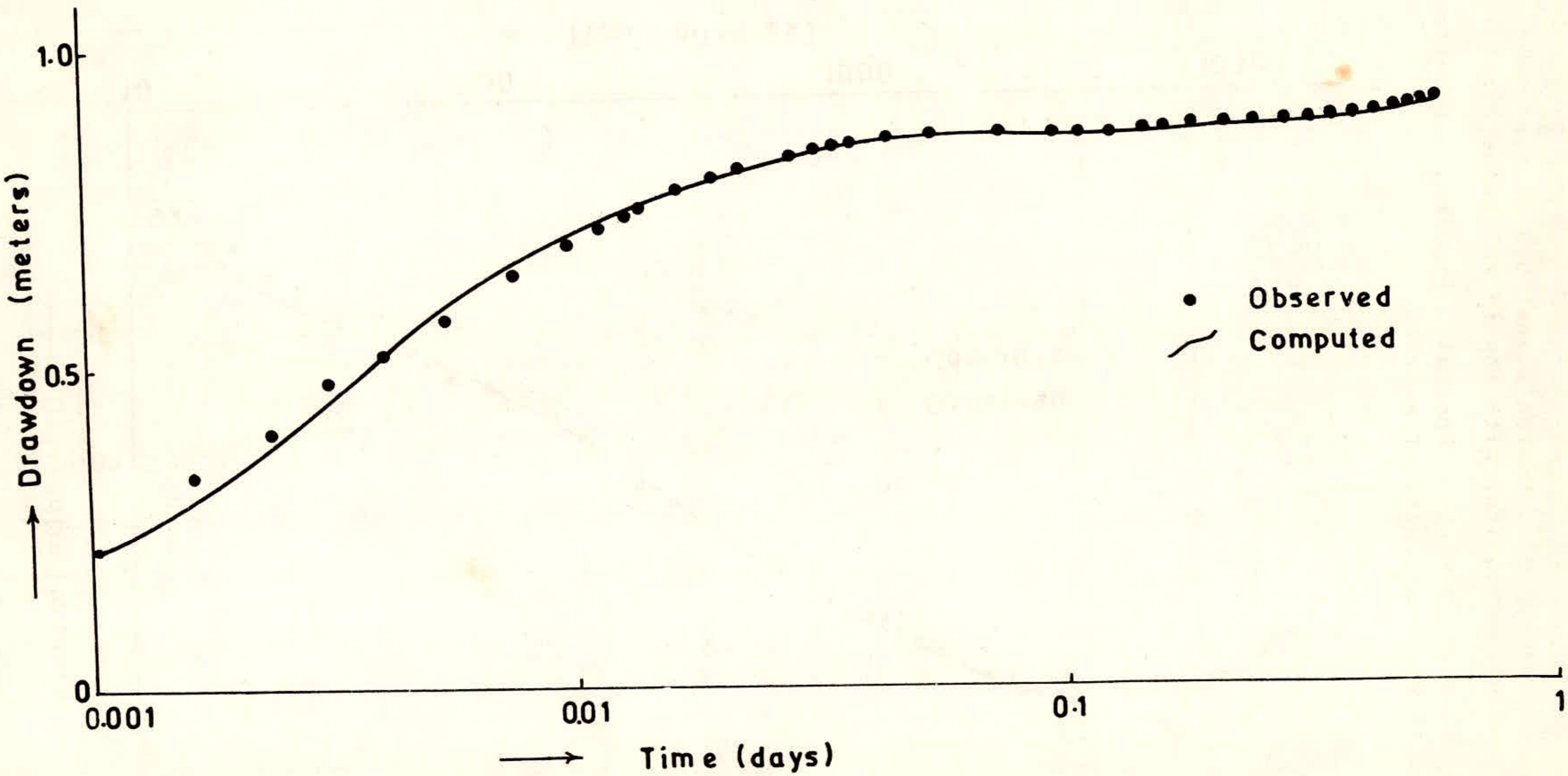


Fig.2 Optimal Reproduction of Time Drawdown Data (Partially Penetrating Wells)

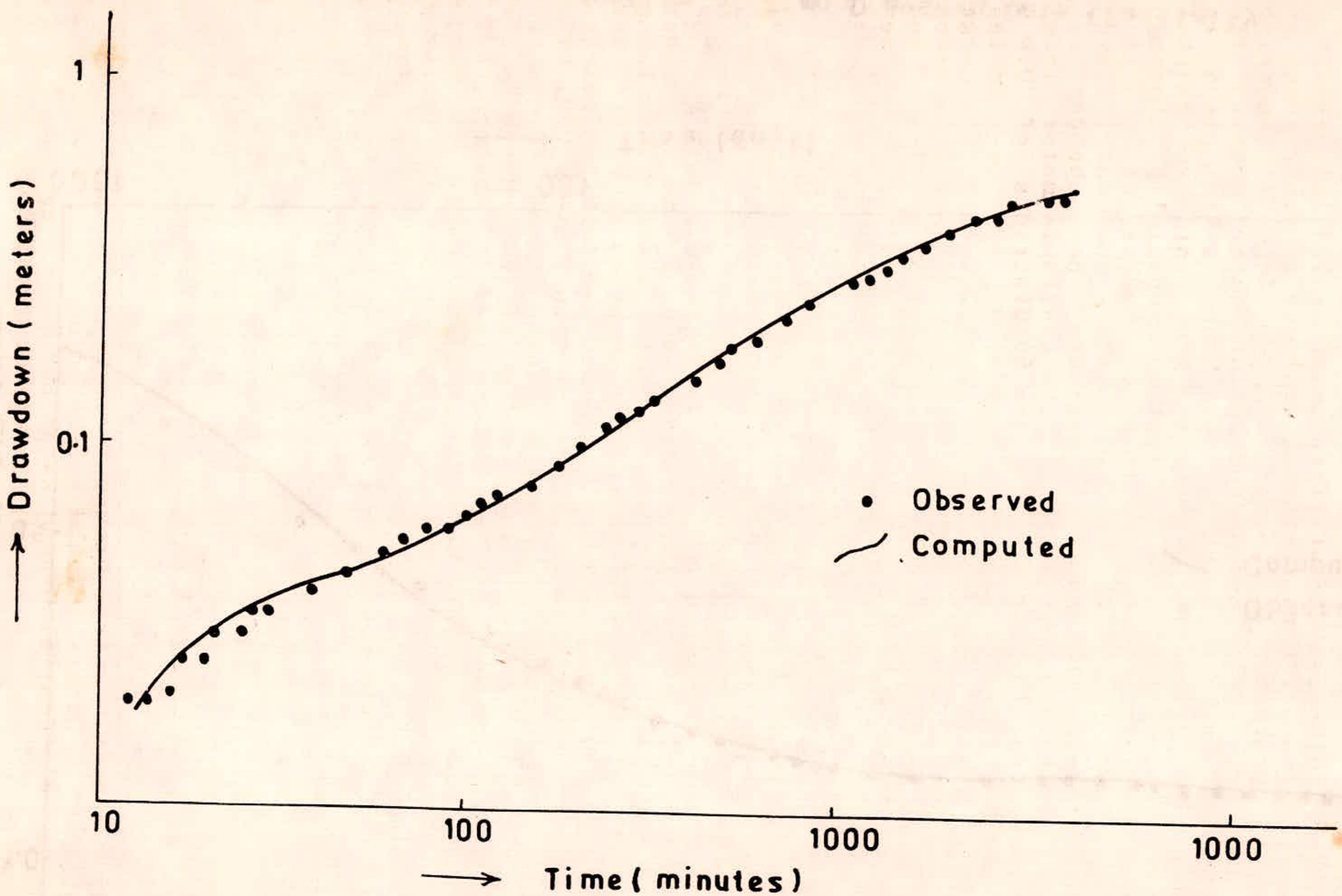


Fig. 3 Optimal Reproduction of Time Drawdown Data (low specification yield Storage Coefficient)