RIVER AQUIFER INTERACTION- A PROBLEM WITH VARYING STAGE AND WATER SPREAD AREA

by

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Abstract

A mathematical groundwater flow model has been developed to predict the exchange of flow between a partially penetrating river and a homogeneous infinite aquifer. The model considers the changes in river stage and corresponding changes in river width while predicting the exchange of flow between the river and the aquifer. Knowing the aquifer parameters, the transmissivity and the storage coefficient, the saturated thickness below the river bed, the initial width of river at the water surface, depth of water in the river, and an equation describing river cross section, the model developed can predict the exchange flow rate between the aquifer and the river consequent to passage of a single or several successive floods. From the study it is found that in case of a partially penetrating river the exchange flow rates are reduced significantly in comparison to those of a fully penetrating river due to river resistance. In case of a partially penetrating river the peak recharge tends to occur simultaneously with the occurrence of peak stage. It is found that about 25% of the aquifer recharge comes back to river after the recession of a typical flood. A five times abrupt increase in river width during passage of a flood results in doubling the maximum recharge rate.

Introduction

The river aquifer interaction process has been examined in some detail in recent years. There are two main aspects of this process: i) the flow from the aquifer to support river flow, and ii) the flow from the river to the aquifer. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. A single aquifer river interaction problem has ben studied analytically by several investigators (Todd, 1955, Cooper and Rorabough, 1963 Morel-Seytoux and Daly, 1977,) for finite and infinite aquifer. The expressions for aquifer recharge in the time of varying river stages have been derived by these investigators.

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Cooper and Rorabaugh (1963) have studied flow into and out of an aquifer of finite length ,L, in response to changes in river stage of a fully penetrating river. They solved one dimensional Boussinesq's equation under the conditions.

$$H(x,0) = 0,\ 0 \le x \le L\ ;\ \frac{\partial H}{\partial x}(L,t)=0\ ,t \ge 0\ ;$$

and

$$H(0,t) = \begin{cases} NH_0\ (1-\cos\omega t)\ e^{-\delta t} & \text{for } 0 \le t \le \tau \\ 0 & \text{for } t > \tau \end{cases}$$

where H is the rise above initial water level in the river, τ is the duration of the flood wave ($\omega = 2\pi/\tau$). $\delta = \omega \cot(0.5\omega t_c)$ determines the asymmetry of the flood wave, t_c is the time of flood crest, and $N = \exp(\delta t_c) / (1-\cos\omega t_c)$ adjusts all curves of a given δ to peak at the same H_o. The solution has been carried out in two steps, one for $t \le \tau$ and another for $t \ge \tau$. Cooper and Rorabough have also solved the Boussinesq's equation for a semi-infinite aquifer, ($L = \infty$) excited by a symmetrical flood wave ($\delta = 0$). The analysis, made by Cooper and Rorabough, is for a fully penetrating river. Therefore the influence of the river width on river aquifer interaction cannot be ascertained from their analysis. Morel-Seytoux and Daly (1977) have analysed the river aquifer interaction problem for varying river stage in a partially penetrating river. Assuming that the exchange of flow between a river and an aquifer is linearly proportional to the difference in potentials at the periphery of the river and in the aquifer below the river bed and accordingly making use of the relation $Q_r(n) = \Gamma_r[\sigma_r(n)-S_r(n)]$, in which Γ_r is the reach transmissivity, $\sigma_r(n)$ is the river stage measured from a high datum during time n, $S_r(n)$ is the depth to piezometric surface below the river at the time n measured from the same high datum, the flow to a partially penetrating river from an aquifer has been derived by Morel-Seytoux and Daly. The drawdown in the piezometric surface is a function of the unknown return flow taking place at time t and also function of all the return flow taken place from all the reaches prior to t. The integral equation derived by Morel-Seytoux and Daly for solving the aquifer contribution is:

$Q_r(t)+\Gamma_r \int_0^t Q_r(\tau)k_{rr}(t-\tau)d\tau = \Gamma_r \sigma_r(t)$,where $k_{rr}(.)$ is the reach kernel (Morel-Seytoux,1975). This expression is valid for the case in which the interaction is taking place through a single reach. In case of several pervious reaches the generalized equation has been given as : $Q_r(t) + \Gamma_r \sum_{\rho=1}^{R} \int_0^t Q_\rho(\tau)k_{r\rho}(t-\tau)d\tau = \Gamma_r \sigma_r(t)$, where R is the number of reaches. The above relation is a system of R integral equations to be solved simultaneously. Discretising the time parameter and assuming the river flow to be uniform within a time step, the following solution to the above integral equation has been given by Morel-Seytoux and Daly ;

$$Q_r(n) + \Gamma_r \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} \delta_{r\rho}(n-\gamma+1)\, Q_\rho(\gamma) = \Gamma_r \sigma_r(n)$$

in which

$$\delta_{r\rho}(n)= 1/(4\pi T)[E_1\{\phi\, d_{r\rho}^2 /(4Tn)\}-E_1\{\phi\, d_{r\rho}^2 /(4T(n-1))\}] ,r=\rho$$

$$\delta_{rr}(n)= 1/(\phi ab) \int_0^1 \mathrm{erf}\left[\frac{a}{2} \left\{\frac{\phi}{4T(n-\tau)}\right\}^{1/2}\right] \mathrm{erf}\left[\frac{a}{2} \left\{\frac{\phi}{4T(n-\tau)}\right\}^{1/2}\right]d\tau$$

ϕ = storage coefficient , T = transmissivity, a = length of the river reach, b = width of the river reach, $d_{r\rho}$ =distance from the centre of the r^{th} reach to the ρ^{th} reach ,and $\sigma_r(n)$=river stage of the r^{th} reach during time period n measured from the high datum.

In the analysis presented by Morel-Seytoux and Daly the river width has been assumed to remain invariant during the variation of stage. During passage of a flood, the stage as well as the river width changes. It is therefore pertinent to analyze the river aquifer interaction during passage of a flood incorporating both the changes in river stage and width.

Statement of the Problem

A schematic section of a partially penetrating river in a homogeneous and isotropic aquifer of infinite areal extent is shown in Fig.1. The aquifer is initially at rest condition. During passage of a flood, the river stages change with time. The changes are identical over a long reach of the river. The width of the

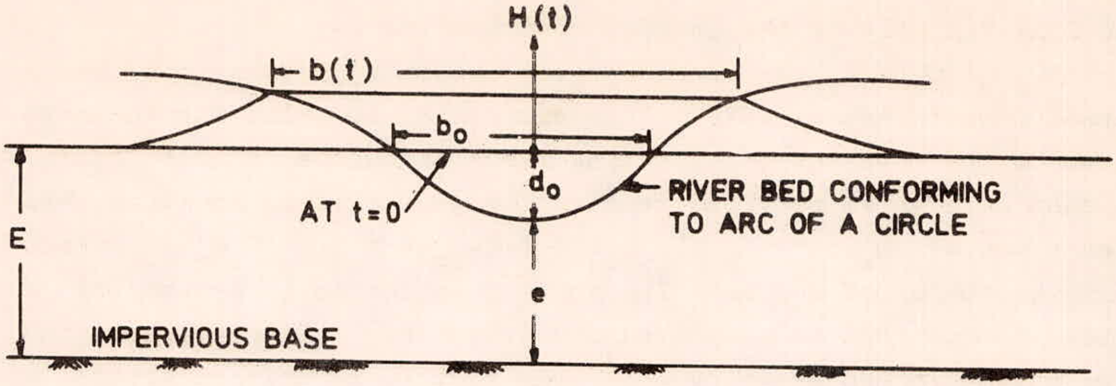

Fig.1(a) Schematic section of a river whose width changes gradually during the passage of a flood.

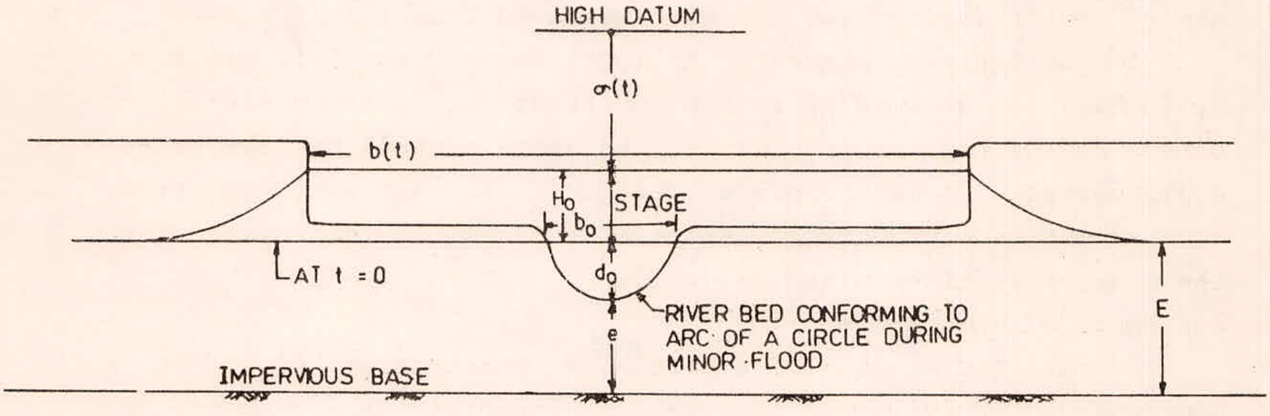

Fig.1(b) Schematic section of river whose width changes abruptly during the passage of a high flood.

river at the water surface changes with change in river stage. The change may be gradual or abrupt. It is required to find the recharge from the river to the aquifer and the flow from the aquifer to the river after the recession of the flood.

Anlysis

The following assumptions are made for the analysis:

i) The flow in the aquifer is in the horizontal direction and is governed by one dimensional Boussinesq's equation.

ii)The time parameter is discrete. Within each time step, the river stage, width at the water surface and the exchange flow rate between the river and the aquifer are separate constants but they vary from step to step.

iii)The exchange of flow between the river and the aquifer is linearly proportional to the difference in the potentials at the river boundary and in the aquifer below the river bed.

The differential equation which governs the flow in the aquifer is

$$T \frac{\partial^2 s}{\partial x^2} = \phi \frac{\partial s}{\partial t} \qquad \ldots(1)$$

in which

s = the water table rise in the aquifer, T = transmissivity and ϕ= storage coefficient of the aquifer.

The aquifer being initially at rest condition, the initial condition to be satisfied is : $s(x,0) = 0$.

The boundary conditions to be satisfied are: $s(\infty ,t) = 0$.

At the river and the aquifer interface recharge from the river to the aquifer takes place in a manner similar to that from an overlying bed source to an underlying aquifer through an intervening aquitard. The river resistance and the aquitard resistance are analogous. The recharge, which can be assumed to be linearly proportional to the potential difference between the river and the aquifer under the river bed, is to be incorporated at the river boundary.

Solution to the problem has been obtained using a discrete kernel approach. The basic solution given by Polubrinova-Kochina for rise in piezometric surface due to continuous recharge from a strip source has been used in the analysis. If recharge takes place at unit rate per unit length of the river and if the width of the river is W ,the rise in piezometric surface at a distance x from the centre of the river is:

$$s(x,t)=F(T,\ \phi,\ W,\ x,\ t)-(x^2+0.25W^2)/(2WT) \quad \text{for } x \le W/2;$$
$$=F(T,\ \phi,\ W,\ x,\ t)-\surd(x^2)/(2T) \quad \text{for } |x| \ge W/2 \quad \ldots(2)$$

in which $\alpha=T/\phi$ and

$$F(T,\phi,W,x,t)=t/(2\phi W)\ [\ \mathrm{erf}\{(x+0.5W)/\surd(4\alpha t)\}-\mathrm{erf}\{(x-0.5W)/\surd(4\alpha t)\}\ ]$$
$$+1/(4TW)\ [(x+0.5W)^2\mathrm{erf}\{(x+0.5W)/\surd(4\alpha t)\}-$$
$$(x-0.5W)^2\mathrm{erf}\{(x-0.5W)/\surd(4\alpha t)\}\ ]$$
$$+(\alpha t/\pi)^{1/2}/(2TW)\ [(x+0.5W)\ \exp\ \{-(x+0.5W)^2/(4\alpha t)\}-$$
$$(x-0.5W)\ \exp\ \{-(x-0.5W)^2/(4\alpha t)\}]$$

Let the rise in piezometric surface at x at the end of n^{th} unit time step due to recharge that occurred at unit rate from unit length of river during the first time step only be designated as $\delta\ [x,W(1),n\ ]$. W(1) is the width of the river at the water surface during the first time step. The discrete kernel coefficients are related to the unit step response function as given below:

$$\delta\ [x,W(1),n\ ]=F(T,\phi,W(1),x,n)-F(T,\phi,W(1),x,n-1) \quad ;n>2$$
$$\delta\ [x,W(1),1\ ]=F(T,\phi,W(1),x,1)-\surd\ (x^2)/(2T) \quad ;|x|>W(1)/2$$
$$\delta\ [x,W(1),1\ ]=F(T,\phi,W(1),x,1)-[.25W(1)^2+x^2]/[2TW(1)] \quad ;|x|<W(1)/2$$
$$\ldots(3)$$

Dividing the time span into discrete time steps, and assuming that, the recharge per unit length is constant within each time step but varies from step to step, the rise in piezometric surface below the centre of the river due to time variant recharge taking place through varying width can be written as

$$s(0,n)=\Sigma_{\gamma=1}^{n}\ q\ (\gamma)\delta\ (0,W(\gamma),n-\gamma+1\ ) \quad \ldots(4)$$

in which $q(\gamma)$ is the recharge rate per unit length per unit time which is taking place through a width of $W(\gamma)$ during time step γ.

The recharge from the r^{th} reach during n^{th} time step can be expressed as:

$$q(n) = \Gamma_r(n)\ [H(n)-s(0,n)] \qquad \ldots(5)$$

in which H(n) is the stage in the river measured from the initial water level position in the river.

Substituting s(0,n) in equation (5) and simplifying

$$q(n) = \Gamma_r(n)\ [\ H(n)-\Sigma_{\gamma=1}^{n}\ q\ (\gamma)\delta\ (0,W(\gamma),n-\gamma+1\ )] \qquad \ldots(6)$$

Splitting the temporal summation into two parts, one part containing the summation up to (n-1)th time step and the other part containing the term of n^{th} time step and rearranging

$$q(n)=[H(n)-\Sigma_{\gamma=1}^{n-1}\ \{q(\gamma)\ \delta(0,W(\gamma),n-\gamma+1)\}]/[1/\Gamma_r(n)+\delta(0,W(n),1)] \qquad \ldots(7)$$

In particular for the first time step

$$q(1)=H(1)/\ [1/\Gamma_r(1)+\delta(0,W(1),1)] \qquad (8)$$

Making use of equation (7), q(n) can be solved in succession starting from the first time step .

Results and Discussion

Exchange of flow between the river and the aquifer consequent to passage of a flood has been presented for the following cases :

Case 1:Change in river width is gradual with change in river stage;

Case 2:Change in river width is gradual during low flood and the width abruptly attains a high value during high flood.

The width of the river for Case 1 corresponding to any stage has been determined assuming that the cross section of the river conforms to part of a circle.

For numerical computation the following data are required :

i) Initial saturated thickness at large distance from the river

ii) Initial width of the river at the water surface and the initial depth of water in the river.

iii) Thickness of aquifer below the river bed section for each stage has to be assigned for numerical computation.

The reach transmissivity constant which changes with change in river width has been evaluated using the following relation given by Bouwer (1969)

$$\Gamma_r(n)=k\pi /[\log_e\{(e+d)/w.p\}+0.5\pi L/(e+d)]$$

in which e is the saturated thickness below the river bed and d is the depth of water in the river which changes during the passage of flood. The distance L specifies the zone of influence on each side of the river and it has been assumed to be b(n)/2 +200m. The wetted perimeter, w.p., and the characteristic length ,L, change with change in river stage.

The variations of $Q(t)/[0.5H_o(2\pi T\phi /\tau)^{1/2}]$ with $kt/(2 \phi E)$ are presented in Fig.2(a) for three floods of different durations. The dimensionless exchange flow rate term, $Q(t)/[0.5H_o(2\pi T\phi/\tau)^{1/2}]$ has been formed following Cooper and Rorabaugh. The time to peak $kt_c/(2 \phi E)$, for each flood has been assumed to be 0.015. It could be seen from Fig.2(a) that, corresponding to τ/t_c = 6,4,2, the dimensionless peak recharge rates are 0.62, 0.515 and 0.365 respectively. The dimensional peak recharge rates are in proportion to 0.253: 0.257 : 0.259 . Thus with decrease in the duration of flood, the peak recharge rate increases though the time to peak are same and maximum depth of water for all the three floods are same. This is because of the fact that the river stage at any time except at peak is higher for flood of longer duration. Higher stage in the beginning would lead more recharge in the beginning which in turn will dampen the recharge at a later time. It is seen that the time to peak discharge matches with the time to peak stage. The variation for τ/t_c = 2 corresponds to a symmetrical flood wave. Variations of $Q(t)/[0.5 H_o(2\pi T\phi/\tau)^{1/2}]$ with t/τ during passage of a symmetrical flood wave have been presented by Cooper and Rorabaugh (1963) for a fully penetrating river. It is found that there is distinct difference between the variations in exchange flow rates for a partially penetrating and a fully penetrating river. In case of a partially penetrating river the occurrence of peak recharge rate coincides with time of peak stage. In case of a fully penetrating river the time of peak flow rate from the river to the aquifer precedes the occurrence of maximum river stage. In case of a partially penetrating river, the magnitude of maximum recharge rate to the aquifer is greatly reduced. For example in case of a fully penetrating river, the dimensionless peak flow rate from one side of the river to the aquifer is about 1.35, whereas in case of a partially penetrating river the peak flow rate from both sides of the river to the aquifer is 0.365. The river resistance reduces the flow rate significantly. It is further seen that for fully penetrating river the peak out flow rate from the aquifer to the river during the recession of the flood is comparable to the maximum recharge rate. For a fully penetrating river the maximum out flow rate, $Q(t)/[0.5H_o(2\pi T\phi/\tau)^{1/2}]$ is about 1.15. The maximum outflow rate for a partially penetrating river obtained from the present analysis is 0.02. Thus the inflow and outflow are reduced significantly due to river resistance in case of a partially penetrating river.

The exchange of flow between a river having a large cross section with $b_o/E = 0.5$ and an aquifer has been presented in Fig.2(b) for $d_o/E = 0.002$. It could be seen that between two rivers which have same depth of water ,the one with larger width contributes more towards the aquifer recharge .The recharge is not proportionate to the river width. For the river with $b_o/E = 0.2$ and $d_o/E = 0.002$,the maximum recharge rate, $Q(t)_{max}/[0.5H_o(2\pi T\phi/\tau)^{1/2}]$ is 0.365. For the river with $b_o/E = 0.5$, the maximum recharge rate is 0.525.

The variation of exchange of flow with time is presented in Fig.2(c) for a flood wave whose peak occurs at $kt_c/(2\phi E) = 0.025$. Comparing the results presented in Figs.2(a) and 2(c), it is found that for symmetrical flood waves with same peak stage, if time of occurrence of the peak stage increases, the maximum inflow rate decreases marginally. For example for $kt_c/(2\phi E)= 0.015$, $H_o/E = 0.006$, $b_o/E=0.2$, $d_o/E=0.002$, $\tau/t_c=2$, the peak inflow rate is $11.24 m^3$/day corresponding to E=1000m, $T=1000 m^2$/day, $\phi=0.1$ and $t_c=3$ days. For $kt_c/(2\phi E)=0.025$, the corresponding peak inflow rate is $10.92\ m^3$/day.

The variation of cumulative flow from the river to the aquifer with time has been shown in Fig.3 for different values of τ/t_c. The cumulative flow reaches a maximum value rapidly and then decreases sluggishly. The commencement of decline in cumulative flow indicates the commencement of reverse flow from the aquifer to the river. The monotonic decreasing trend at large time indicates that if the aquifer is of infinite length, the volume of water which would flow from the river to the aquifer during the passage of a flood will not return back to the river after the recession of the flood. For $\tau/t_c = 2$, 26% of the total aquifer recharge returns to the river by dimensionless time $kt/(2\phi E)= 0.2$.

Using the present model, the exchange of flow that takes place between the aquifer and the river during several successive floods could be ascertained from continuous record of the river stages. The exchange of flow between a river and an aquifer has been determined for three successive floods occurring over a time span of 72 days and the results are shown in Fig.4. The river stages and the corresponding river widths are also shown in the figure. The river stage has been assumed to attain the same maximum height during each of the floods. During the first flood the peak recharge rate is $6.3\ m^3$/day. During the second and the third flood, the peak recharge rates are $6 m^3$/day and $5.8 m^3$/day respectively. The recharge that would take place corresponding to the river stages indicated in Fig.4, if the river width changes abruptly to attain a width of 1000m when its stage crosses a height of 5m, has been shown in Fig.5. Though the width changes by

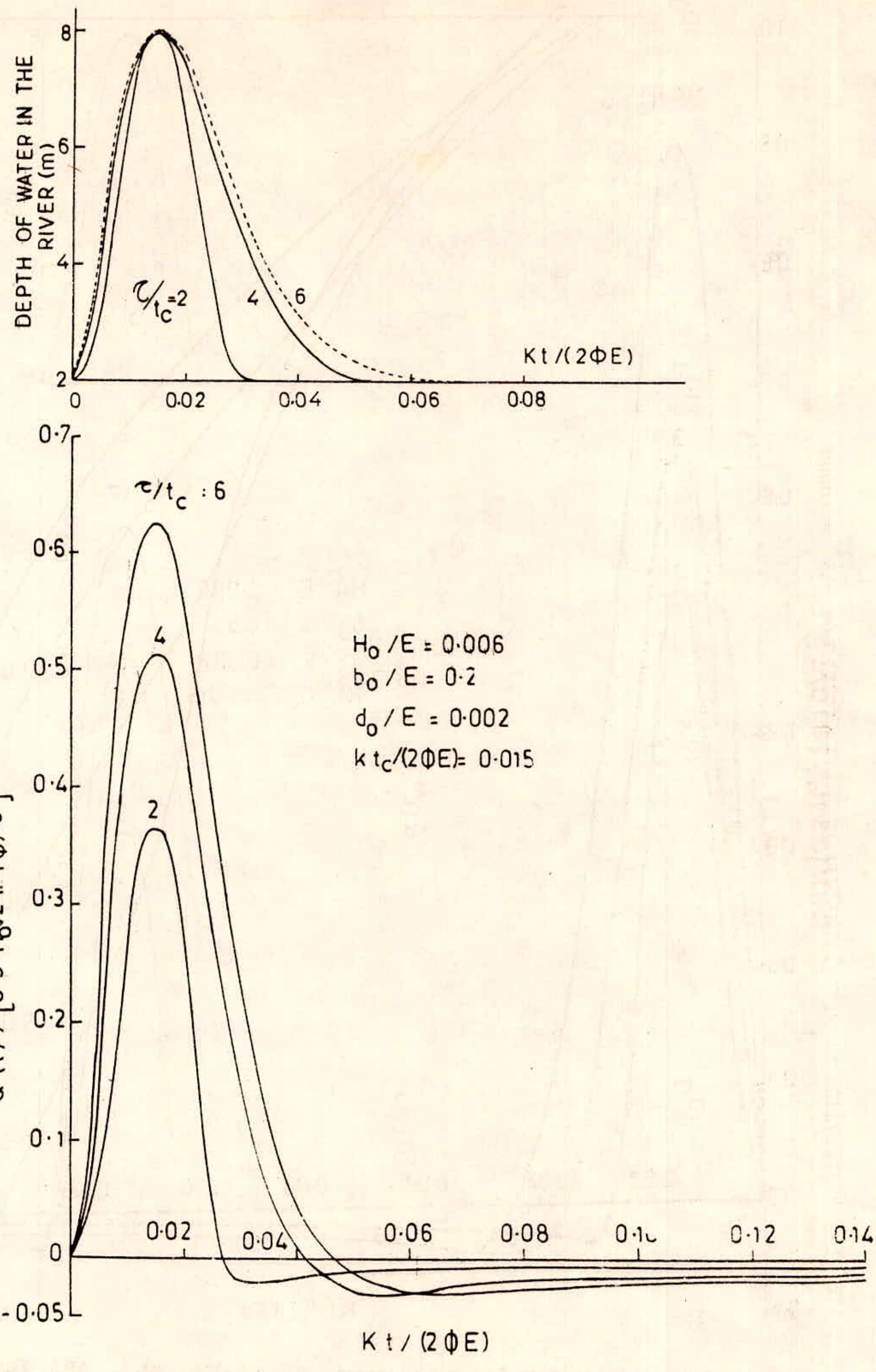

Fig.2(a) Exchange of flow between a river and an aquifer during the passage of flood of different durations; the flood peak occurs at $kt/(2\phi E)$ = 0.015; initial width of the river $b_o/E=0.2$.

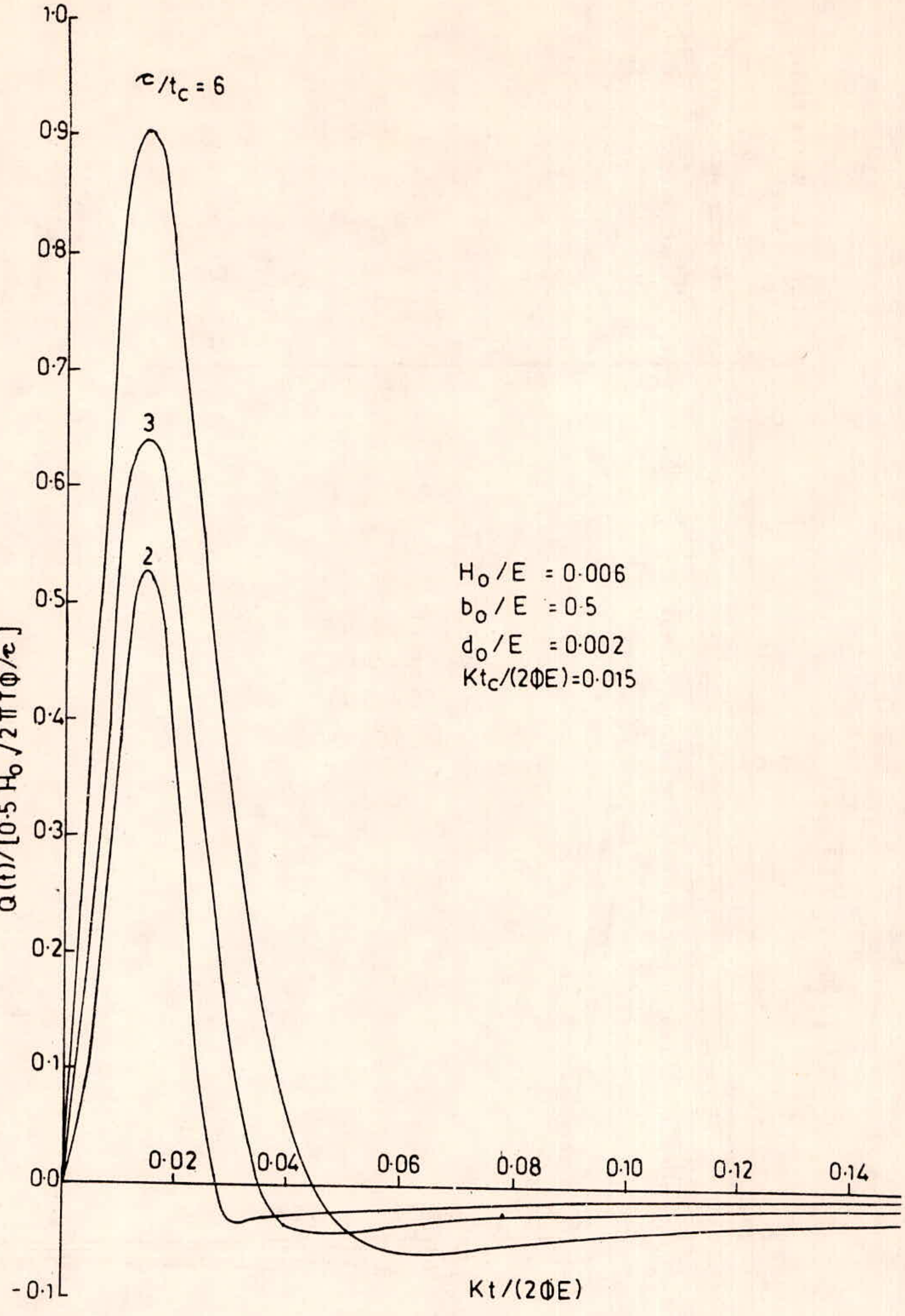

Fig.2(b) Exchange of flow between a river and an aquifer during passage of floods of different durations, the flood peak occurs at $kt/(2\phi E)=0.015$, initial width of the river, $b_o/E=0.5$

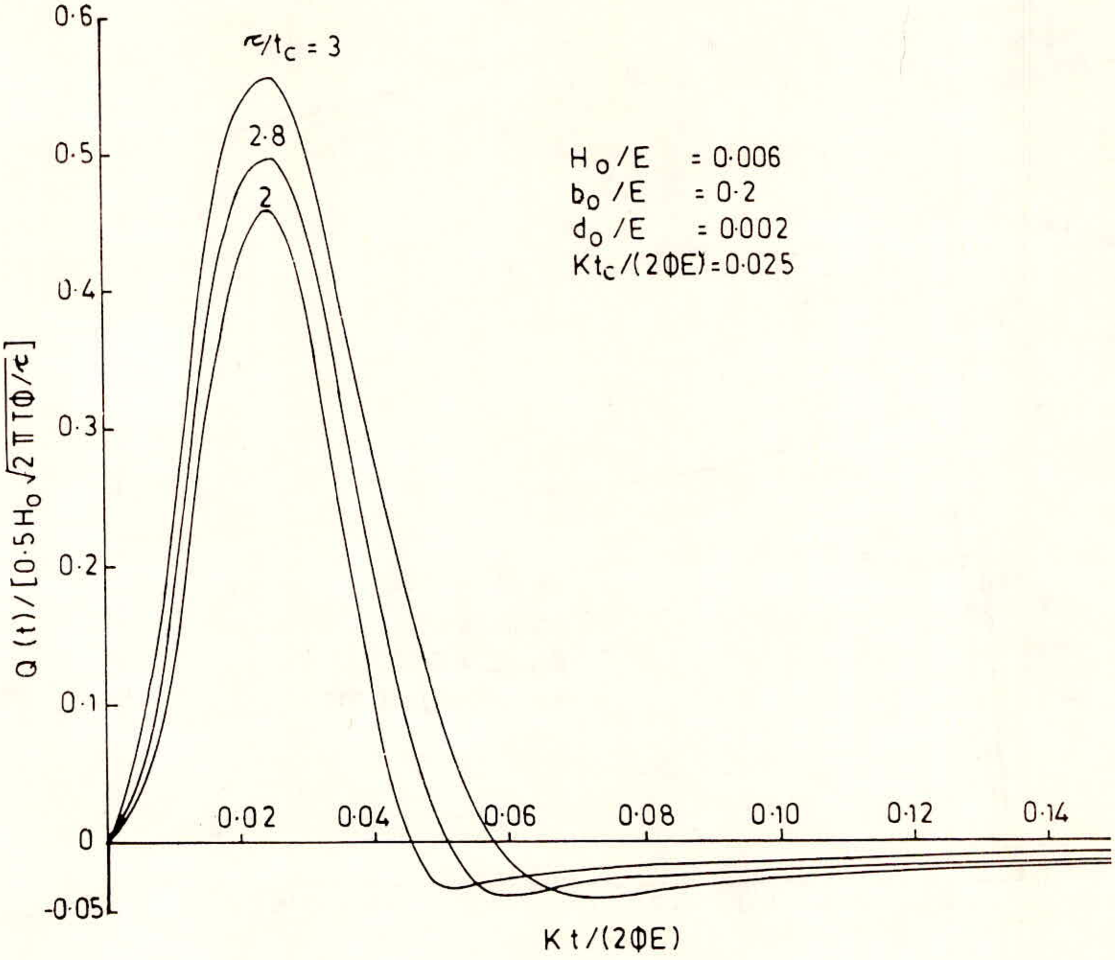

Fig.2(c)Exchange of flow between a river and an aquifer during the passage of flood for different durations, the flood peak occurs at $kt/(2\phi E)=0.025$, and initial width of the river, $b_o/E=0.2$.

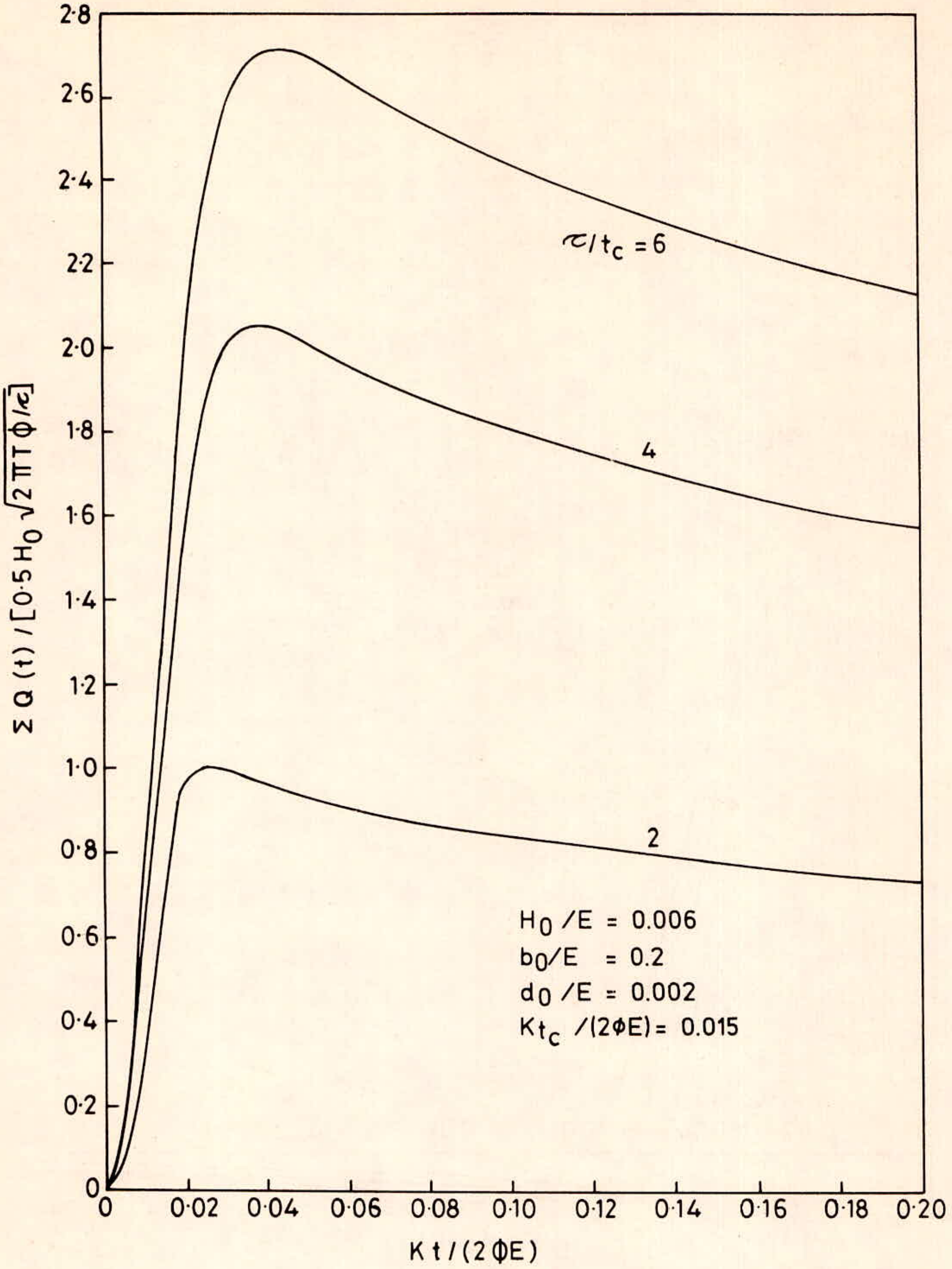

Fig.3 Variation of cumulative flow with time consequent to passage of floods of different duration.

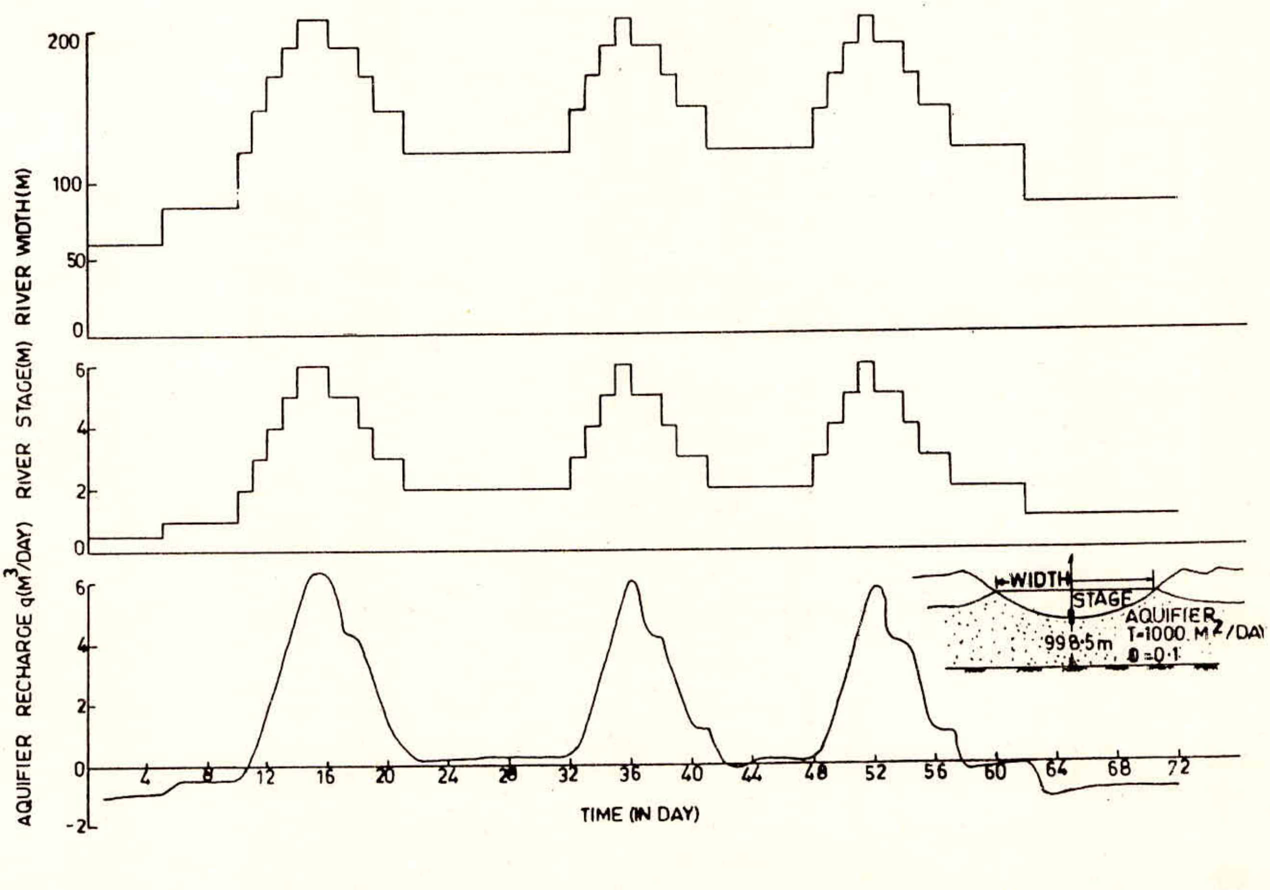

Fig.4 Variation of flow from a river whose stage and width vary with time.

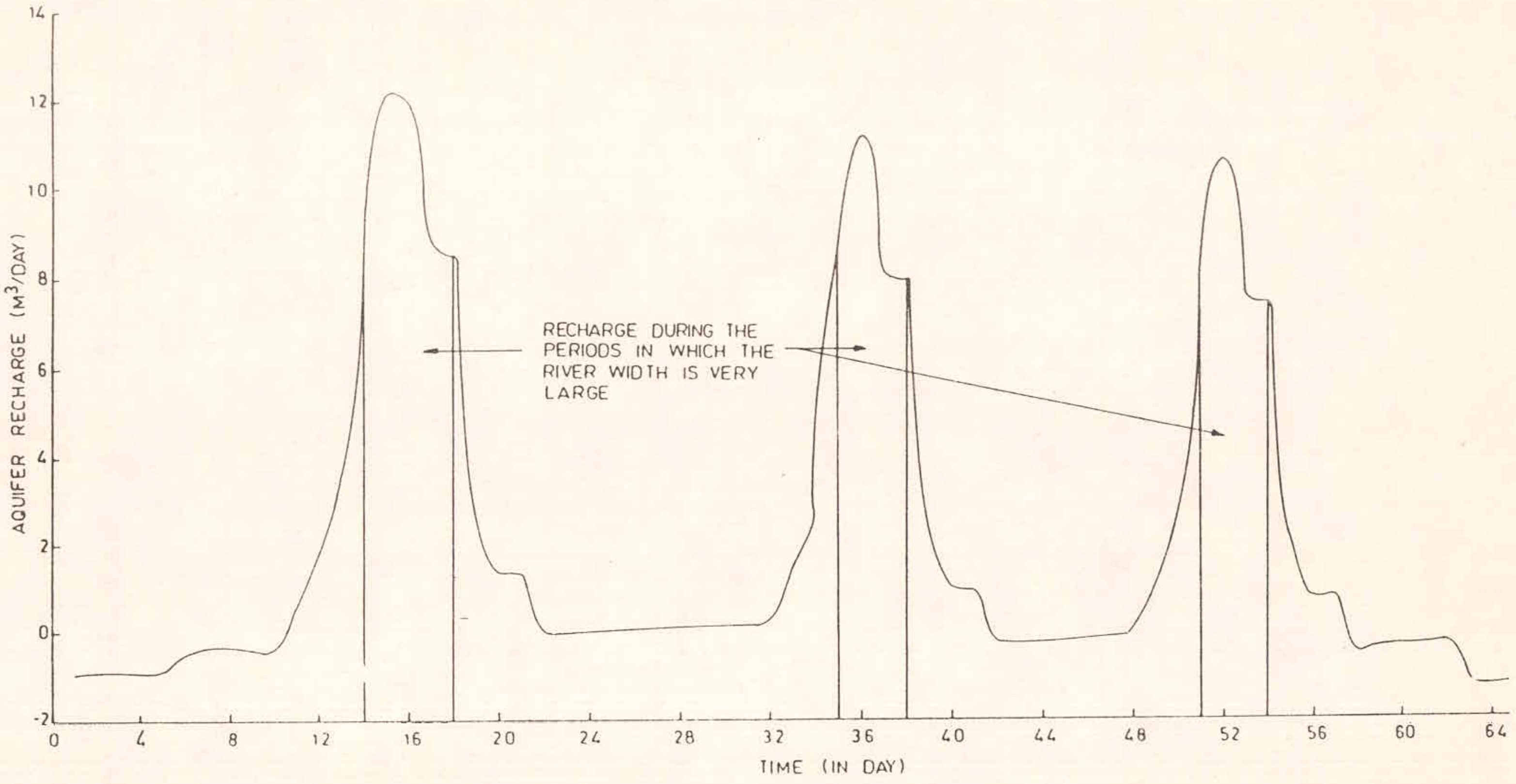

Fig.5 Variation of flow from a river whose width varies abruptly with stage

the peak recharge rate is 6.3 m^3/day. During the second and the third flood, the peak recharge rates are $6m^3$/day and $5.8m^3$/day respectively. The recharge that would take place corresponding to the river stages indicated in Fig.4, if the river width changes abruptly to attain a width of 1000m when its stage crosses a height of 5m, has been shown in Fig.5. Though the width changes by 5 times, the changes in peak discharges are about two time. The recharges rates, being governed primarily by the difference in potentials between the river and the aquifer, do not change in proportion to the change in river width.

Conclusion

A mathematical model has been developed to predict the exchange of flow between an aquifer and a partially penetrating river whose width at water surface changes with the change in its stage during the passage of a flood. Based on the study the following conclusions have been derived :

1. For partially penetrating river, the river resistance reduces the maximum inflow and outflow rates which are very much less than those of a fully penetrating river.
2. In case of a partially penetrating river the peak inflow tends to occur simultaneously with the occurrence of peak stage.
3. In case of aquifer of infinite length about 25% of the aquifer recharge comes back to river after the recession of a typical flood.
4. A five times increase in river width during the passage of a flood may cause the maximum recharge rate to increase by two times

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