FLOWING WELLS

by

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Abstract

Using discrete kernel approach an analytical solution has been obtained to determine temporal variation of discharge of a flowing well in a confined aquifer of finite areal extent. The quantities of water that remain in the aquifer storage at any time after the onset of flow, which will be subsequently drained by the flowing well, have been quantified. Type curves have been prepared to enable determination of aquifer parameters, such as: storage coefficient, transmissivity ,distance of the no flow boundary from the flowing well, and the initial hydraulic head.

Introduction

When a permeable bed sandwiched in between impermeable strata is warped into synclinal fold with the permeable bed exposed at the surface along an out crop, condition favourable for flowing well may develop. Recharge due to precipitation to the aquifer may take place along the out crop and the permeable layer may contain water under artesian condition. In such a case when a well is sunk to the permeable bed a flowing well can be obtained. Flowing wells are uncommon occurrence resulting from erratic geological process. Discharge characteristics of a flowing well and certain spring are comparable. An artesian aquifer is drained by a flowing well. The discharge of a flowing well depends on the difference between the elevation of the piezometric surface in the vicinity of the flowing well and the elevation of the flowing well's threshold. A flowing well's discharge is derived from the water stored in the aquifer. Hence, piezometric head in the aquifer gradually declines and the flowing well's discharge slowly reduces to zero.

In the present paper an analysis of discharge of a flowing well in an aquifer of finite areal extent has been made. Using the solution it is possible to predict the temporal dynamic storage and the time at which the discharge from a flowing well becomes insignificant.

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Review

Many investigators have analysed the unsteady flow associated with a flowing well. Notable among them are Nicholson, Smith, Goldstein, Carslaw and Jaeger (vide Glover,1974). Assuming that up to a finite distance b from the centre of the flowing well the excitation has not propagated, the following solution to the differential equation

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad \ldots(1)$$

for the initial condition $s(r,o)=o$, $r>r_w$, and for the boundary condition $s=s_o$ at $r=r_w$ for $t>o$, has been given (vide Glover,1974):

$$s = s_o\,[1-\sum_{n=1}^{\infty} A_n\, U_o(\beta_n r)\, \exp\{-k^2(\beta_n b)^2\, Tt/(\phi\, r_w^2)\}]$$

where

$A_n=\{2k/(\beta_n b)\, U_o'\,(\beta_n r_w)\,\}/[\{U_o(\beta_n b)\}^2 - k^2\,\{U_o'(\beta_n r_w)\}^2]$,

$U_o(\beta_n r) = J_o(\beta_n r_w)Y_o(\beta_n r)-Y_o(\beta_n r_w)J_o(\beta_n r)$,

$k = r_w/b$,

$U'(\beta_n r\,) = dU_o(\beta_n r)/\,d(\beta_n r)$,

$(\beta_n b)$ are the roots of the equation $U_o'\,(\beta_n b)=\emptyset$,and J_o and Y_o are Bessel's function of the first and the second kind of zero order respectively.

The flow from the well is given by

$Q(t)= 2\pi T\, s_o\, G\,[\sqrt{(4Tt/\phi)}/r_w]$

where

$$G[\sqrt{(4Tt/\phi)}/r_w] =\sum_{n=1}^{\infty} A_n(\beta_n r_w)\, U_o'(\beta_n r_w)\, \exp\{-k^2(\beta_n b)^2\, Tt/(\phi\, r_w^2)\}]$$

When the excitation propagates to the farthest boundary, the above solution will be no longer valid for aquifer of finite areal extent.

Analysis of unsteady flow to a well in an aquifer of finite areal extent has been presented by Muskat(1937) and Kuiper(1972). The differential equation (1) has been solved for the following boundary and initial conditions:

$$\frac{\partial s}{\partial r}\Big|_{r=a}=0$$

$$\lim_{r\to 0} r\,\frac{\partial s}{\partial r} = -Q/(2\pi T) \quad \text{, and}$$

$$s(r,0)=0 \quad ,$$

in which 'a' is the radial distance to the impermeable boundary, Q is the constant pumping rate. The solution that has been given by Muskat is

$$s=-Q/(2\pi T)\ [3/4 + \ln\ (r/a)\ -1/2\ \{(r/a)^2 + 4Tt/(\phi\ a^2)\}$$

$$+2\sum_{m=1}^{\infty}\{\alpha_m J_o(\alpha_m a)\}^{-2} J_o(\alpha_m r)\exp\ \{-(\alpha_m^2)Tt/(\phi\ a^2)\}] \qquad \ldots(2)$$

$(\alpha_m a)$ values, m=1,2,3...., are zeros of J_1, the Bessel's function of the first kind and of first order. $(\alpha_m a)$ values have been tabulated for values of m up to 20 (Abramowitz and Stegun, 1970). $(\alpha_m a)$ values for higher values of m can be evaluated using the following formula of McMahon's expansions for large zeros:

$$(\alpha_m a) \simeq \beta - \frac{\mu-1}{8\beta} - \frac{4(\mu-1)\ (7\mu-31)}{3(8\beta)^3} - \frac{32(\mu-1)(83\mu^2 - 982\mu + 3779)}{15(8\beta)^5}$$

$$- \frac{64(\mu-1)(6949\mu^3 - 153855\mu^2 + 158743\mu - 6277237)}{105(8\beta)^7} \qquad \ldots(3)$$

in which,

$\beta = (m + \frac{1}{4})\pi$, and $\mu=4$.

In case of a flowing well the discharge rate, Q, varies with time and drawdown at the well point is constant and equal to the difference between the initial piezometric level and level of the flowing well's threshold. The solution given by Muskat can be used to solve the unsteady flow to a flowing well in an aquifer of finite areal extent.

Statement of the Problem

A schematic section of a flowing well in a confined aquifer of finite areal extent is shown in figure 1. The level of the flowing well's threshold is at a height H_2 above the datum. Prior to the sinking of the well, the piezometric level was at a height H_1. The time is reckoned from the instant the well is sunk and it starts flowing. It is required to determine (i) discharge of the flowing well at various time,(ii) temporal and spatial variation of drawdown in piezometric surface, and (iii) quantities of water that remain in the storage of the aquifer.

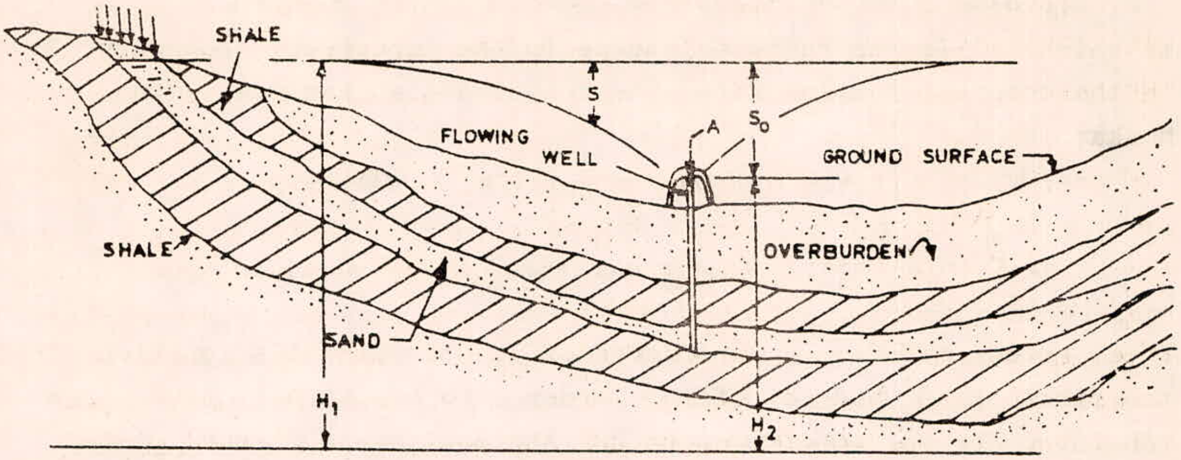

Fig.1-SCHEMATIC SECTION OF A FLOWING WELL

Analysis

The following assumptions have been made in the analysis:

i) The time parameter is discrete. Within each time step, the discharge of the flowing well is constant but it varies from step to step.

ii)Though the aquifer near the out crop is unconfined, the entire aquifer has been assumed to be confined and the position of the no flow boundary is assumed to be stationary.

The solution of differential equation (1) needs to satisfy the following initial and boundary conditions for the flowing well under consideration:

$$s(r,o)=o$$

$$s(r_w,t)=H_1-H_2$$

$$\left.\frac{\partial s}{\partial r}\right|_{r=a}=o$$

Let K(t) be the drawdown in piezometric surface of a confined aquifer of finite areal extent at a radial distance r from the well due to a unit step excitation. Expression for K(t) can be obtained from equation(2) substituting Q by 1. Let $\delta_r(N)$ be the response of the aquifer at the end of time step N due to a unit pulse excitation. $\delta_r(N)$ is recognised as discrete kernel (Morel Seytoux, 1975) and it has the following relation with unit step response:

$$\delta_r(N) = K(N) - K(N-1) \qquad \ldots(4)$$

Substituting for K(N) and K(N-1) in equation (4) and simplifying, the following expression for discrete kernel for drawdown for N >1 in a confined aquifer of finite areal extent is obtained:

$$\delta_r(N)=\frac{N}{\pi\phi a^2} - \frac{1}{\pi T}\sum_{m=1}^{\infty}\{(\alpha_m a)J_o(\alpha_m a)\}^{-2} J_o(\alpha_m r)\exp\{-\alpha_m^2 TN/\phi\}$$
$$-\frac{(N-1)}{\pi\phi a^2} + \frac{1}{\pi T}\{\sum_{m=1}^{\infty}\{(\alpha_m a)J_o(\alpha_m a)^{-2}\}J_o(\alpha_m r)\exp\{-\alpha_m^2 T(N-1)/\phi\} \qquad \ldots(5)$$

For N = 1, $\delta_r(1)$ is given by

$$\delta_r(1) = -\frac{1}{2\pi T}[\frac{3}{4} + \ln(r/a) - \frac{1}{2}\{(r/a)^2 + 4T/(\phi a^2\}$$
$$+ 2\sum_{m=1}^{\infty}\{(\alpha_m a)J_o(\alpha_m a)\}^{-2}\exp\{-(\alpha_m)^2 T/\phi\}] \qquad \ldots(6)$$

Let $Q(\gamma)$, γ = 1,2,3,.....I, be the discharge of the flowing well during time step γ. The drawdown in piezometric surface at the end of time step I at any point in the aquifer is governed by the discharges of the flowing well up to time step I. The relation between drawdown at flowing well and the well discharge is

$$s(r_w,I) = \sum_{\gamma=1}^{\infty} Q(\gamma)\delta_{rw}(I-\gamma+1) \qquad \ldots(7)$$

The discrete kernel coefficient $\delta_{rw}(.)$ can be obtained from equations (5) and (6) replacing r by r_w.

Since the drawdown at the flowing well at any time step is H_1-H_2, therefore,

$$H_1-H_2 = \sum_{\gamma=1}^{I} Q(\gamma)\delta_{rw}(I-\gamma+1) \qquad \ldots(8)$$

Equation (8) can be rewritten as

$$H_1-H_2 = \sum_{\gamma=1}^{I-1} Q(\gamma)\delta_{rw}(I-\gamma+1)+ Q(I)\delta(1) \qquad \ldots(9)$$

Thus, the discharge of the flowing well during time step I is given by

$$Q(I) = \frac{1}{\delta(1)} [H_1 - H_2 - \sum_{\gamma=1}^{I-1} Q(\gamma)\delta_{rw}(I-\gamma+1)] \quad \ldots(10)$$

Q(I) can be found in succession starting from time step 1.

In particular for time step 1, $Q(1) = [H_1-H_2]/\delta(1)$. Once Q(I) values are known the drawdown at any point can be evaluated using the relation

$$s(r,I) = \sum_{\gamma=1}^{I} Q(\gamma)\delta_r(I-\gamma+1) \quad \ldots(11)$$

Results and Discussion

The discrete kernel coefficients $\delta_r(I)$ are generated for r equal to r_w and for other radial distances at which drawdown calculations are sought for a known set of aquifer parameters T, ϕ and radial distance 'a'. The large zeros ($\alpha_m a$) of the Bessel's function $J_1(.)$ required for the evaluation of discrete kernels have been obtained making use of equation (3). The first one hundred zeros(m=100) have been considered for evaluation of discrete kernel coefficients. A plot of $\delta_r(I)$ versus I is presented in Fig.2 for different values of r. Since the discrete kernels are the response of the aquifer to a unit withdrawal during the first time period, and the aquifer has a finite radius equal to 'a', at large time step I, $\delta_r(I)$ tends to a limiting value equal to $1/(\pi a^2 \phi)$. After generating the discrete kernel coefficients, the discharge of the flowing well Q(I) have been solved in succession starting from time step 1. The variation of dimensionless discharge rate $Q(t)/\{T(H_1-H_2)\}$ with nondimensional time factor $\phi\, r_w^2/(4Tt)$ is shown in Fig.3 for values of a/r_w ranging from $0.5x10^4$ to $5x10^4$. Fig.3 shows that at large value of $\phi\, r_w^2/(4Tt)$ i.e in the beginning when the well starts flowing, the graphs for two different values of a/r_w merge with each other indicating that the presence of the no flow boundary has not affected the well discharge until the time of merger. The discharge of the flowing well decreases with time and reduces to a negligible quantity at large time.

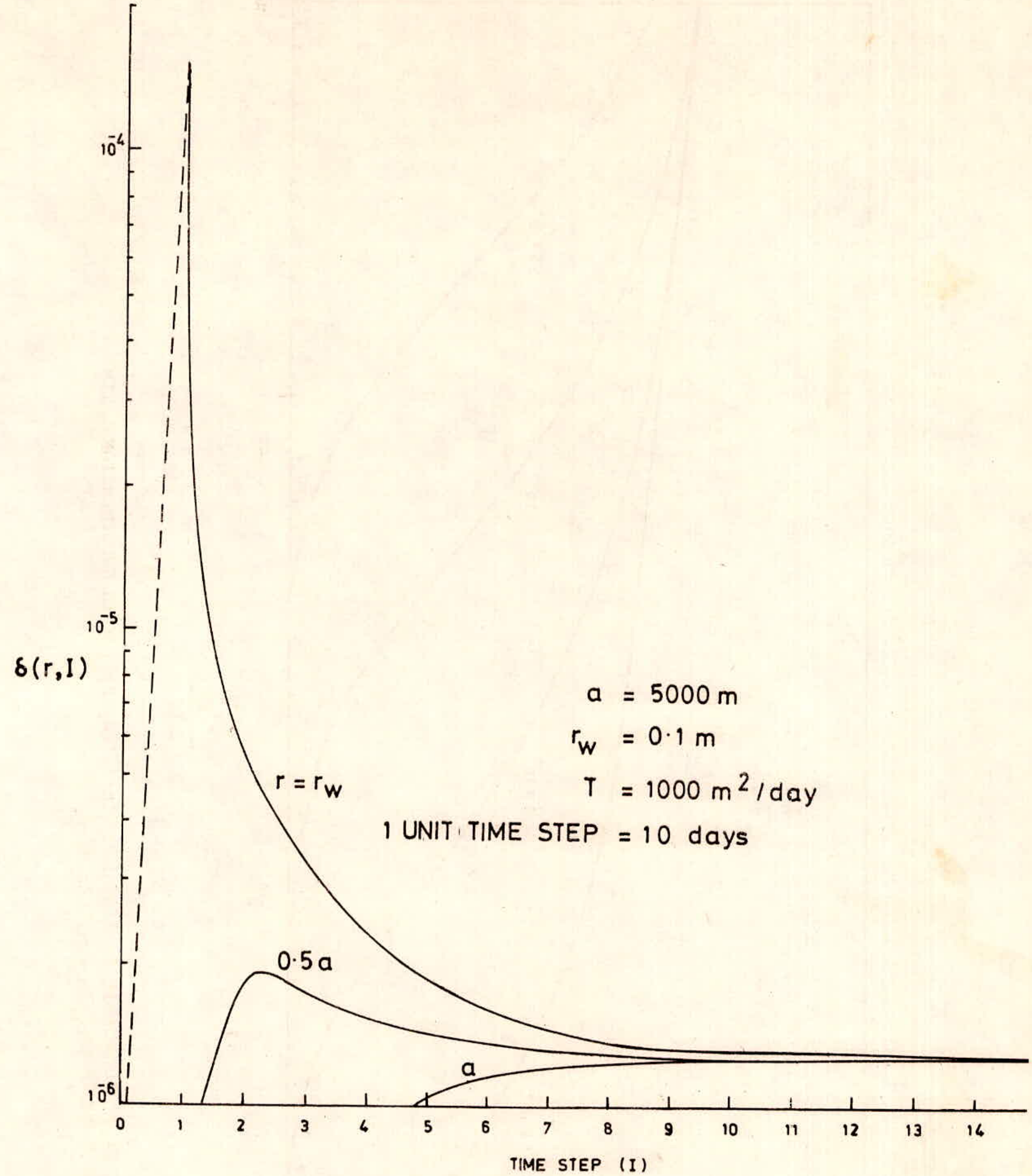

Fig.2-DISCRETE KERNEL COEFFICIENT $\delta_r(I)$ AT TIME STEP I

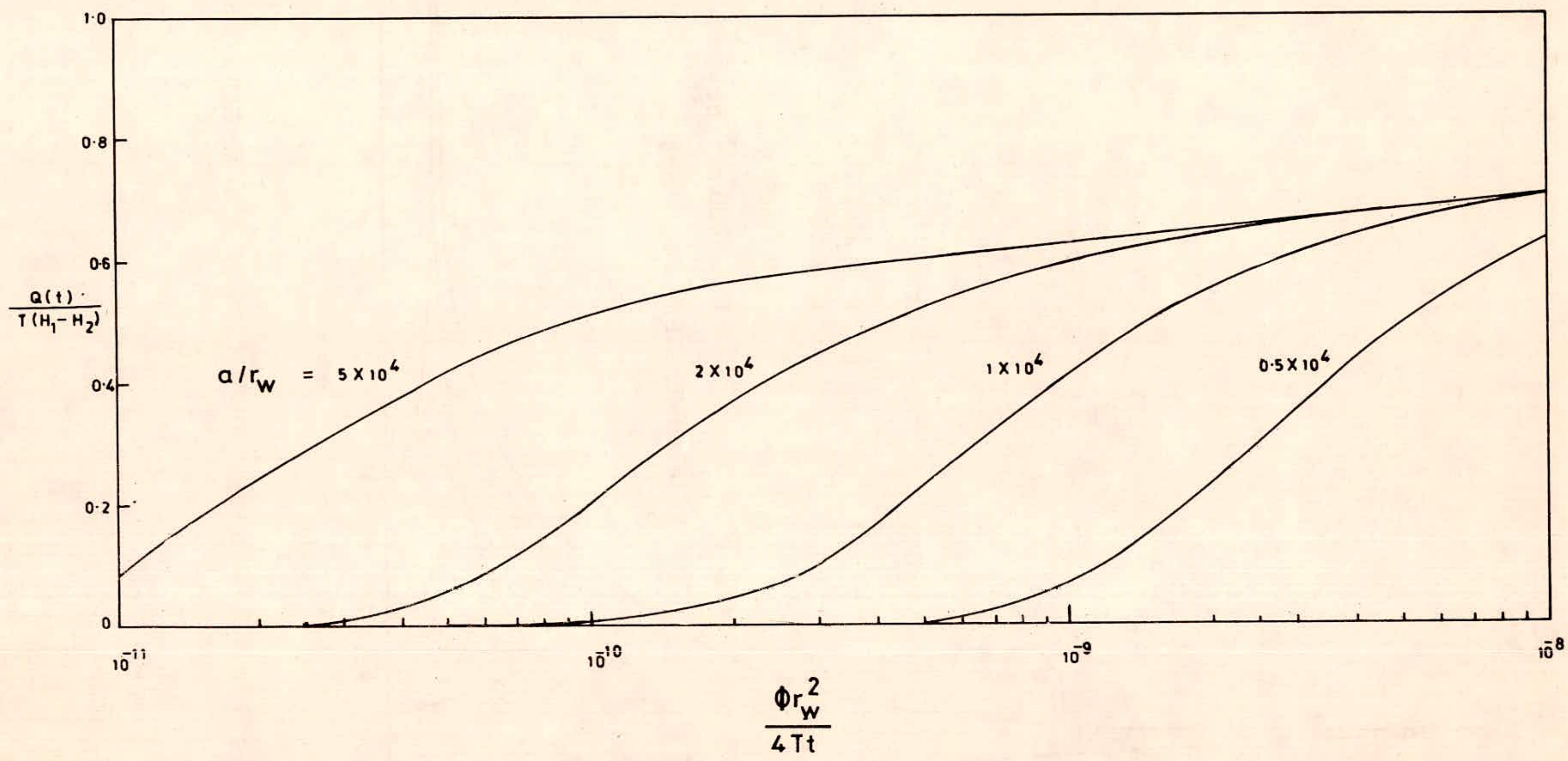

Fig.3- VARIATION OF DIMENIONLESS DISCHARGE $Q(t)/[T(H_1-H_2)]$ WITH NONDIMENSIONAL TIME PARAMETER $\phi\ r_w^2/(4Tt)$ IN SEMI LOG GRAPH

The variation of dimensionless discharge with non dimensional time, $4Tt/(\phi r_w^2)$, is shown Fig.4 in a log-log scale. Fig.4 shows that larger the value of 'a' longer is the life of the flowing well. For $a/r_w = 10^4$, at nondimensional time 3.4×10^9, the dimensionless discharge is 0.1. For $a/r_w = 2\times10^4$ and 5×10^4, the corresponding time is 15.8×10^9 and 95.2×10^9 respectively.

The variation of dimensionless drawdown, $s(r,t)/(H_1-H_2)$, with $\phi r^2/(4Tt)$ is shown in Figs 5(a) and 5(b) for $a/r_w=1.0\times10^4$ and 5.0×10^4 for a set of r/a. $s(r,t)/(H_1-H_2)$ can be regarded as the well function for a flowing well in an aquifer of finite areal extent. The type curve presented in Figs.5(a) and 5(b) can be used to find the parameters (H_1-H_2), T/ϕ, a, and r_w. If the variation of drawdown with r^2/t at a piezometer plotted in a log -log paper which has the same scale as that of the type curve presented in Figs.5(a) and 5(b) matches with any of the type curves, it is then possible to estimate the parameter (H_1-H_2), T/ϕ, a, r_w. If the discharge of the flowing well is measured and its variation with time is plotted in a double log paper T and ϕ can be estimated by matching this graph with the curve presented in Fig.4.

The total quantity of water that can be drained by the flowing well is equal to $\pi a^2\phi (H_1 - H_2)$. In Fig.6 the variation of the ratio of cumulative discharge to the total discharge of the flowing well with time has been presented. The graph also shows the the quantity of water that remains in the aquifer storage at any time to be drained by the well.

Conclusion

A solution of unsteady flow to a flowing well in a confined aquifer of finite areal extent has been obtained by discrete kernel approach using Muskat's basic solution of unsteady flow to a well in a confined aquifer of finite areal extent. The temporal variation of the flowing well's discharge has been predicted. Type curves for prediction of the parameters, and radius of the circular aquifer, and the head difference which causes the flow have been presented. The quantity of water that remains in the aquifer's storage at any time has been assessed.

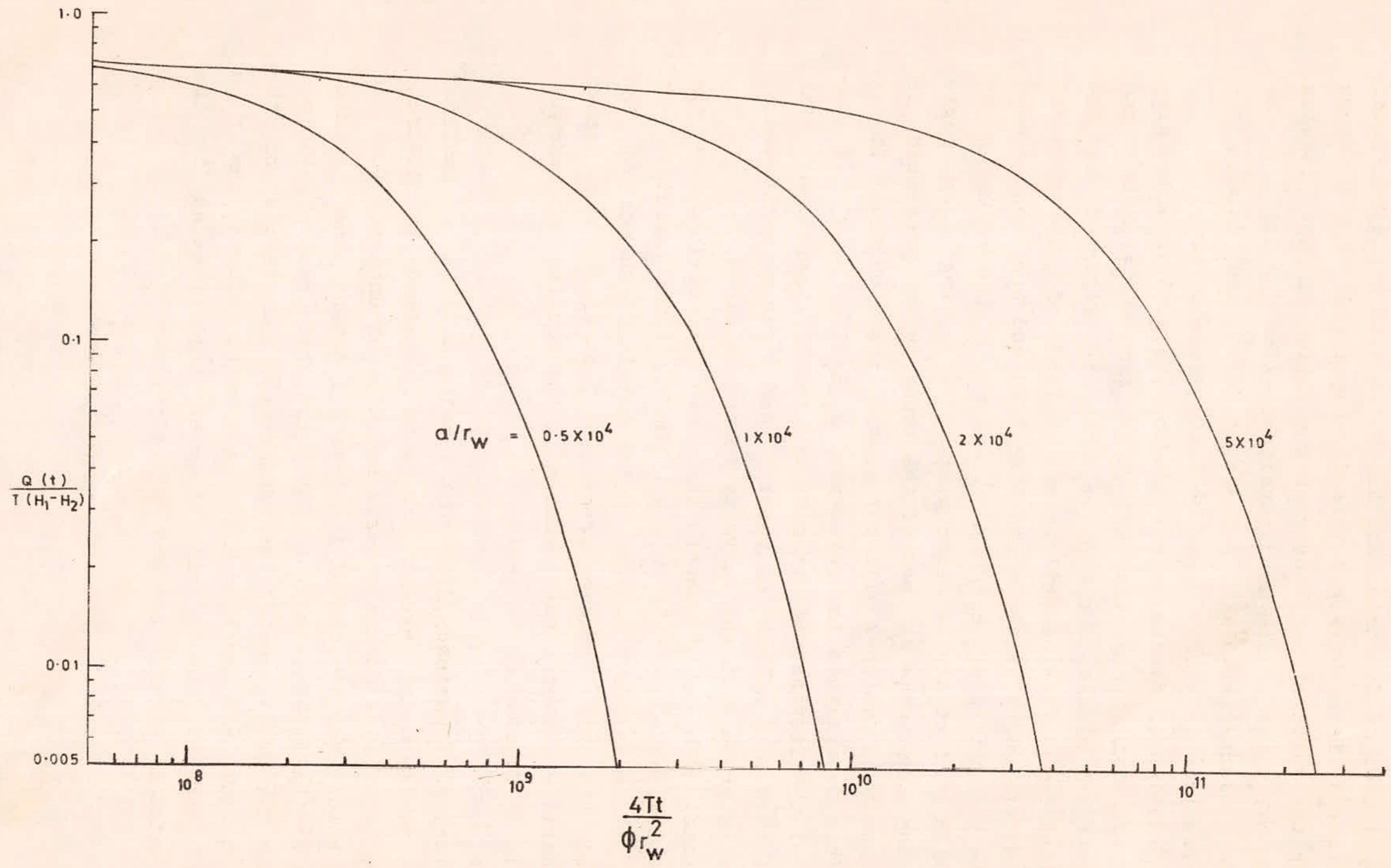

Fig.4-VARIATION OF DIMENSIONLESS DISCHARGE $Q(t)/\{T(H_1 - H_2)\}$ WITH NONDIMENSIONAL TIME PARAMETER $\phi\ r_w^2/(4Tt)$ IN DOUBLE LOG GRAPH

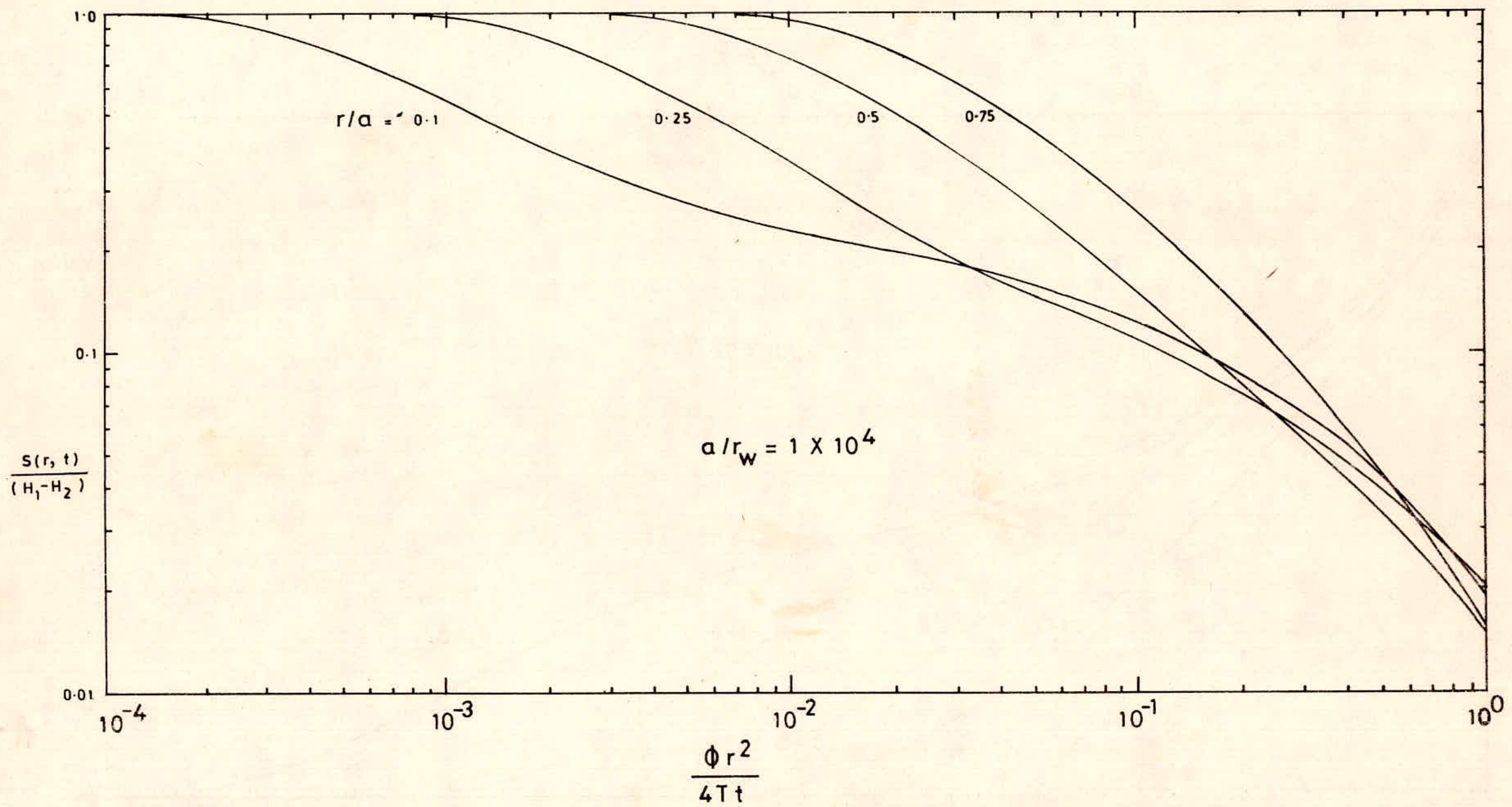

FIG.5(a) - VARIATION OF $s(r,t)/(H_1-H_2)$ WITH $\emptyset\ r^2/(4Tt)$ FOR a/r_w = 10000

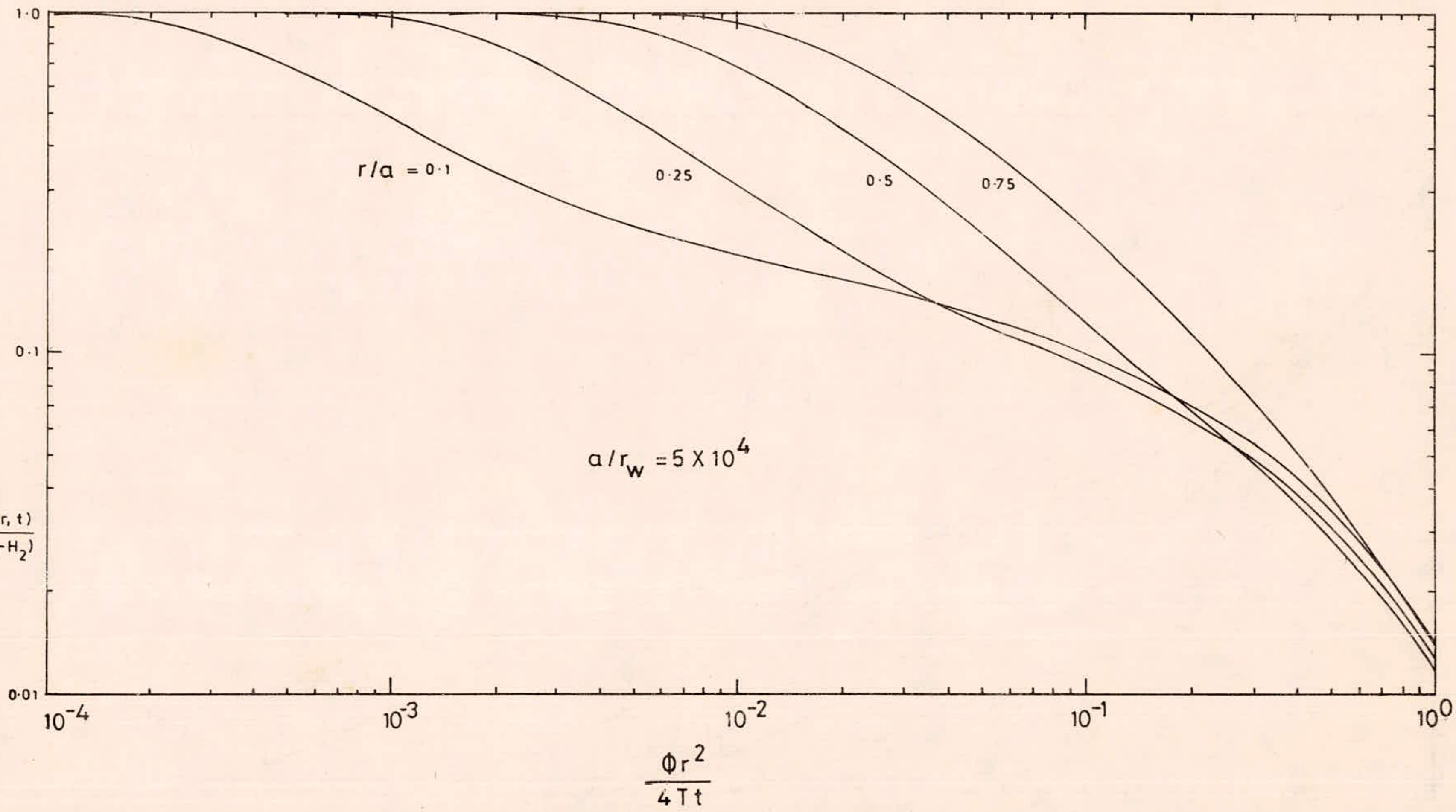

FIG.5(b) - VARIATION OF $s(r,t)/(H_1-H_2)$ WITH $\phi\ r^2/(4Tt)$ FOR a/r_w = 50000

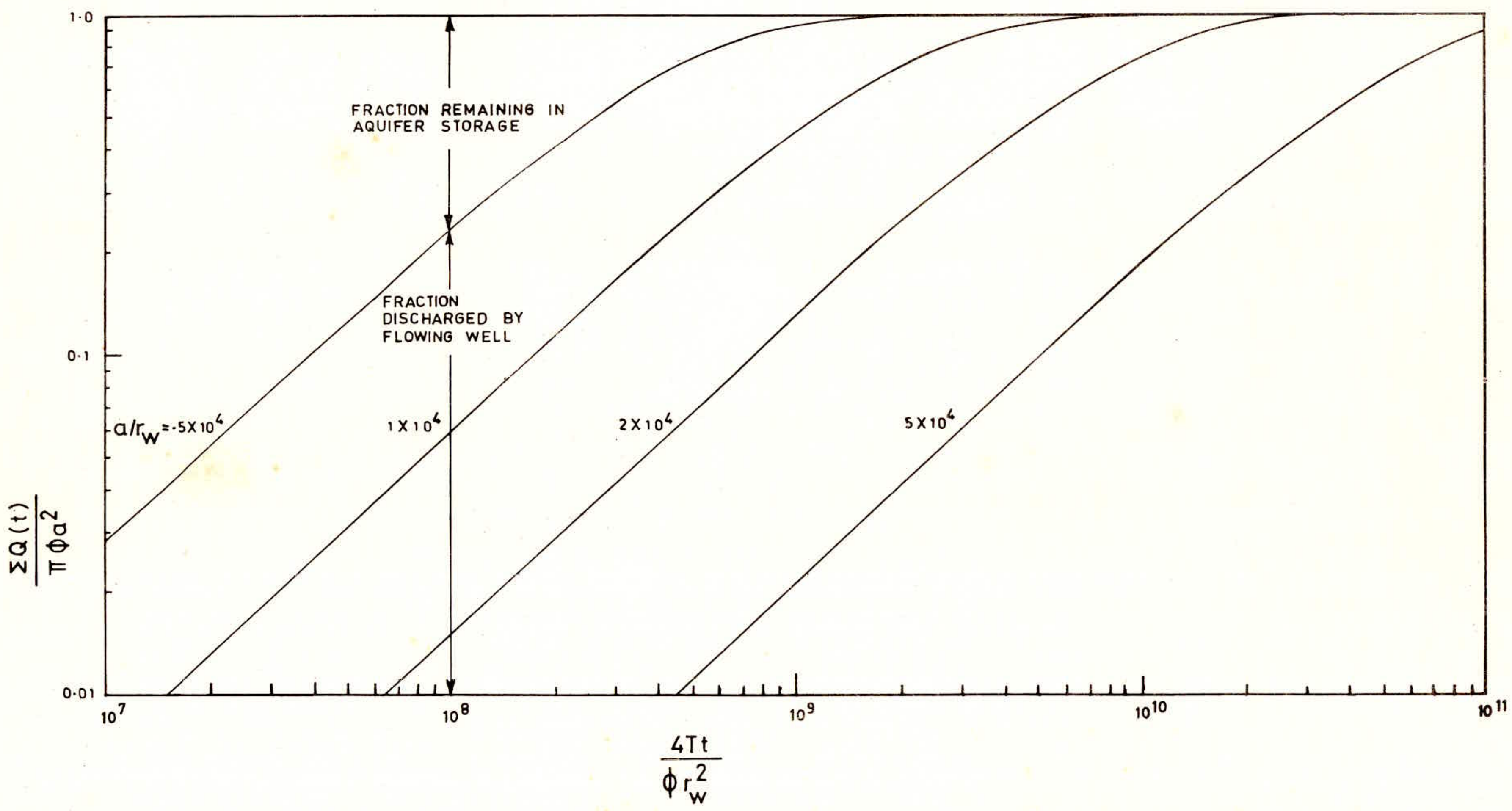

Fig.6-VARIATION OF $\Sigma Q(t)/(\phi \pi a^2)$ WITH $4Tt/(\phi r_w^2)$

References

1. Abramowitz, M., and I.A. Stegun, 1965. Handbook of Mathematical Function, Dover Publication, Newyork.

2. Glover, R.E., 1974. Transient Ground Water Hydraulics, Water Resources Publication.

3. Kuiper, L.K., 1972. Drawdown in a Finite Circular Aquifer with Constant Well Discharge, Water Resources Research, Vol.8, No.3, pp.734-736.

4. Morel-Seytoux, H.J., 1975. Optimal Legal Conjunctive Operation of Surface Water and Ground Water, Second World Congress, International Water Resources Association, New Delhi, Vol.IV, pp.119-129.

5. Muskat, M., 1937. The Flow of Homogeneous Fluid through Porous Media, p.657, McGraw Hill, New York.

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