# LAKE-AQUIFER-RIVER INTERACTION IN AN ANISOTROPIC AQUIFER

by

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Abstract

Lake-aquifer-river interaction has been studied using a three dimensional ground water flow model. The interaction has been studied for idealized boundary conditions. A family of non-dimensional time and lake recharge curves for different degree of anisotropy has been developed. Using these curves and the aquifer parameters, the transient recharge from the water body can be determined from known head changes in an observation well situated within the area of influence of the water body. Ideal location for such an observation well has also been suggested.

## Introduction

Lakes are natural reservoirs of water and are as important as man made surface water reservoirs. In spite of early recognition of their water management studies, very few attempts were made to understand the interaction of lake with aquifer. The assessment of recharge from lake has drawn the attention of many investigators during recent past because very often the recharge from a lake forms a significant component of ground water balance. Mayboon(1967) was among the early investigators who stressed upon the groundwater flow pattern around a lake. In most of the lake water balance the groundwater component has been calculated either as the residual of water balance equation or by making direct measurement of hydraulic gradients. Thus, a wide variation had been observed between the different estimates of seepage rates from the same lake indicating the need of a refined method for determining the groundwater flux from a lake.

Some insight into the groundwater regime of discharge estimates was provided by Winter(1976, 1978). He applied two and three dimensional steady state models to hypothetical groundwater lake systems. He showed that the movement of groundwater to and from a lake depends on the continuity of the flow domain of the lake with the intermediate and regional flow systems passing at

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depth beneath the lake. Based on simulation study, he suggested a field method and some new approaches for the study of the interaction of lakes and groundwater along with presentation of a critique of the commonly used approaches. He stressed that the studies, in which one or many wells are placed near a lake to determine the interaction of lakes and groundwater, must be scrutinized carefully, because placement and construction of wells are critical to a proper understanding of the interrelationship of a lake and groundwater.

Using a numerical model, McBride and Pfannkuch(1975) evaluated the vertical component of groundwater flow into a lake for a number of alternate boundary conditions. Munter and Anderson(1982) showed that two and three dimensional groundwater flow models provide flexible and effective means of calculating flow rates in well defined but complex natural flow systems around lakes. Bhar and Mishra(1988) developed a mathematical model based on discrete-kernel approach to study the interaction between depression storage and a shallow water table aquifer.

From the critical review of the past studies it is observed that in most of the water balance studies, the quantification of seepage from large water bodies have been done as a residual of water balance equation, thus leading to erroneous values because the uncertainties in the estimation of other components get incorporated into seepage component. In most of the model studies, the influences of the various parameters, specially the influences of anisotropy and the size of the lake, on the recharge rate from water bodies have not been clearly analysed and thus, no guidelines or methods have been evolved for the assessment of recharge from a lake. Recently Singh and Seethapathi (1988) have analysed the problem of lake aquifer river interaction for a hypothetical lake of square cross-section in a homogeneous and isotropic aquifer. They have presented a type curve which enables the assessment of transient recharge from a lake in homogeneous aquifer. The purpose of the present study is to quantify the recharge from a lake in a homogeneous and anisotropic aquifer.

## Statement of the Problem

A schematic plan and cross-sectional view of a lake and two fully penetrating rivers in a homogeneous and anisotropic aquifer are shown in fig.1 and 2 respectively. The lake is assumed to be at the centre of the aquifer in plan and the aquifer extends to a distance of 3 km from the centre of the water body in both positive and negative x and y directions. The boundaries of the aquifer parallel to one side of lake are assumed to be fully penetrating constant head boundaries(rivers) and boundaries parallel to the other orthogonal sides of the lake are taken as no flow boundaries. The depth of water in the lake is assumed to be uniform. The lake aquifer and the rivers were initially at rest condition. There is a step rise of $\Delta H$ in the lake water level.

The problem is to find the time variant recharge from the lake consequent to the step rise in lake water level. It is intended to develop a family of type-curves between non-dimensional drawdown and non-dimensional time due to seepage occurring from the lake for different values of $K_h/K_v$ under above mentioned setting of boundaries. Using these type curves and the water level fluctuations in an observation well situated within the influence area of the lake, it is intended to assess the recharge from the lake. It is also aimed to find suitable location for such an observation well.

## Mathematical Model

The three dimensional ground water flow model developed by Mcdonald and Harbaugh(1985) has been used for the present study.

The partial differential equation governing an unsteady and three dimensional incompressible groundwater flow through heterogeneous and anisotropic saturated porous medium is given by

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - w = S_s\frac{\partial h}{\partial t} \qquad \ldots(1)$$

where,

x,y,z are the cartesian coordinates aligned along the major axes of conductivity $K_{xx}$, $K_{yy}$, $K_{zz}$;

h is the piezometric head (L);

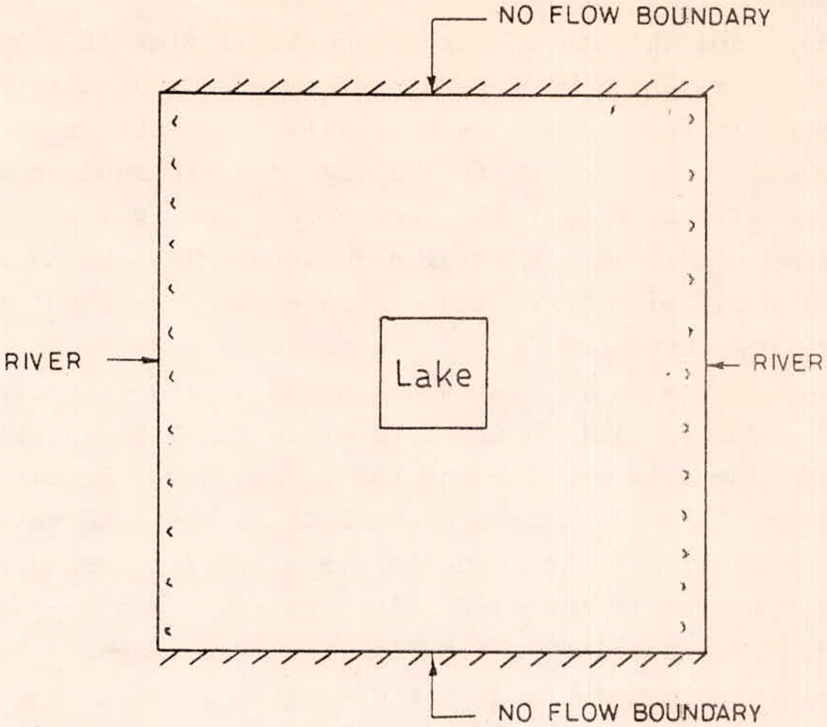


Fig.1-PLAN VIEW OF THE PROBLEM

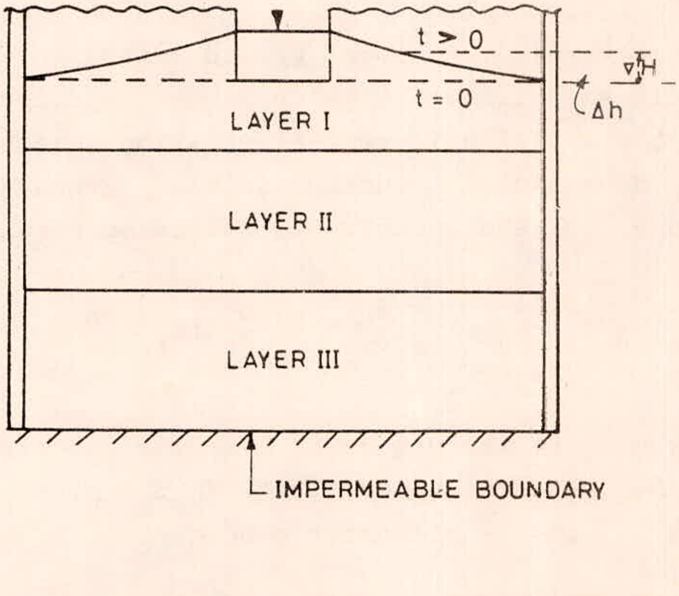


Fig.2-CROSS-SECTIONAL VIEW OF THE PROBLEM

w is the volumetric flux per unit volume and represents source and/or sinks ($t^{-1}$);

$S_s$ is the specific storage of the porous material of the aquifer ($L^{-1}$); and

t is time (t).

This equation when combined with boundary conditions (flow and/or head conditions at the boundaries of the aquifer system) and initial conditions ( in case of transient flow, specification of head conditions at t=Ø), constitutes the a mathematical model of transient groundwater flow.

The aquifer system has been discretized into a mesh of points termed nodes, forming 32 rows, 32 columns and 3 layers. Finer grids were taken close to the lake and coarser grids away from the lake. The spatial discretization for the quarter portion of the aquifer and the lake is shown in fig.3. The width of cells along rows is designated as $\Delta r_j$ for the $j^{th}$ column; the width of cells along columns are designated as $\Delta C_i$ for $i^{th}$ row; and the thickness of layers in vertical are designated as $\Delta V_k$ for the $k^{th}$ layer. Block-centered formulation was adopted with the nodes at the centre of the cells. The flow into the cell i,j,k in row direction from cell i,j-1,k is given by

$$q_{i,j-1/2,k} = CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) \quad \ldots(2)$$

where,

$CR_{i,j-1/2,k} = KR_{i,j-1/2,k} \Delta C_i \Delta V_k / \Delta r_{j-1/2}$

$q_{i,j-1/2,k}$ =the volumetric flow discharge through the face between the cells i,j,k and i,j-1,k ($L^3t^{-1}$);

$KR_{i,j-1/2,k}$ =the hydraulic conductivity along the row between nodes i,j,k and i,j-1,k;

$\Delta r_{j-1/2}$ =the distance between nodes i,j,k and i,j-1,k (L)

$CR_{i,j-1/2,k}$ =the conductance in $i^{th}$ row and $k^{th}$ layer between nodes i,j-1,k and i,j,k [$L^2t^{-1}$].

Here, C represents the conductance and R represents for row direction. Similar expressions can be written approximating the flows into or out of the cell i,j,k through the remaining five faces. Applying continuity equation of flow to the cell i,j,k the following equation is obtained:

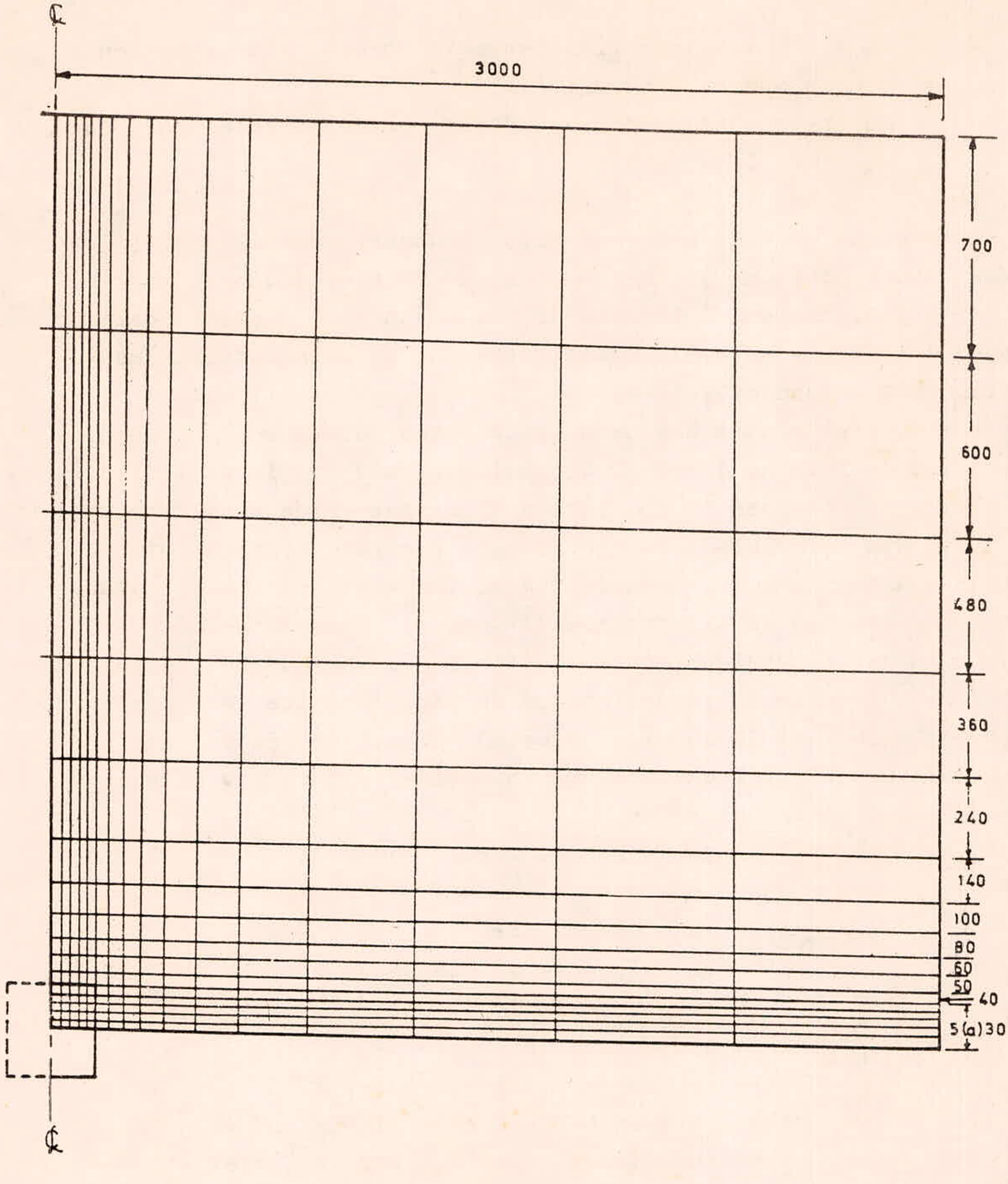


Fig.3-SPATIAL DISCRETIZATION FOR A QUARTER PORTION OF THE LAKE AND THE AQUIFER

$$CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) + CR_{i,j+1/2,k}(h_{i,j+1,k} - h_{i,j,k}) +$$

$$CC_{i-1/2,j,k}(h_{i-1,j,k} - h_{i,j,k}) + CC_{i+1/2,j,k}(h_{i+1,j,k} - h_{i,j,k}) +$$

$$CV_{i,j,k-1/2}(h_{i,j,k-1} - h_{i,j,k}) + CV_{i,j,k+1/2}(h_{i,j,k+1} - h_{i,j,k}) \pm$$

$$QS_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta C_i \Delta V_k).(\Delta h_{i,j,k}/\Delta t) \quad \ldots(3)$$

where, $\Delta h_{i,j,k}/\Delta t$ is a finite difference approximation for head change with respect to time $[LT^{-1}]$

$SS_{i,j,k}$ is the specific storage of cell i,j,k $[L^{-1}]$

$\Delta r_j \Delta C_i \Delta V_k$ is the volume of cell i,j,k $[L^3]$; and

$QS_{i,j,k}$ is the recharge from the $(i,j,k)^{th}$ rectangular cell of lake bottom (as an external source$[L^3T^{-1}]$

In order to simulate the interaction of lake and aquifer, the term representing the leakage ,i.e., $QS_{i,j,k}$ has been added to the equation 3. Due to the discretization, the bed of the lake consists of a number of rectangular cells. The recharge from one of such rectangular cell has been expressed as

$$QS_{i,j,k} = CL_{i,j,k}(H_{i,j,k} - h_{i,j,k}) \quad \ldots(4)$$

where,

K,L,and W are the hydraulic conductivity, length, and width of the bed material of lake contained in the cell i,j,k ;

D is the thickness of the bed material of the lake contained in the cell i,j,k ;

$H_{i,j,k}$ is the water level in the lake in the cell i,j,k.

$CL_{i,j,k} = \frac{K.L.W}{D}$ = conductance of the lake bed contained in the cell i,j,k

After substituting equation 4 in equation 3, equation 3 is written in backward difference form by specifying flow term at $t_m$, the end of the time interval, and approximating the time derivative of head over the interval $t_{m-1}$ to $t_m$ for each of the 'n' cells in the system. There is only one unknown head for each cell. Hence, there are 'n' equations and n unknowns. For the solution of these equations strongly implicit procedure has been adopted.

## Result and Discussion

The head distribution in the aquifer system at discrete nodes at each time step over a period of time was obtained. The simulation was carried out for different value of $\Delta H$ keeping the head in the rivers constant for every value of $K_h/K_v$. In each simulation the difference between the head at discrete points (nodes) and the river water level was computed for each time-step and was designated as $\Delta h$. $(\Delta h)_s$ is the value of $\Delta h$ when steady state has reached.

Variation of $((\Delta h)_s - (\Delta h))/\Delta H$ with X/L and t was observed for all values of t for each simulation ($\Delta H$ = 3.0m, 5.0m, 7.0m and 9.0m; $K_h/K_v$=1,10,100,250,500,750; Here, X is the perpendicular distance from the lake to the observation well and L is the perpendicular distance between centre of the water body to the constant head boundary). For same value of $K_h/K_v$, the variation of $((\Delta h)_s - (\Delta h))/\Delta H$ with X/L and t was found to be the same for different value of $\Delta H$ and for each lake size. Analysis of the above plots show that with increasing time, the maxima of the curve shift in a positive X-direction and shift of the maxima for all value of t is ranged between X/L = 0.15 to 0.25 (this range of X/L was found to be the same for different values of $\Delta H$ and $K_h/K_v$). Fig. 4 shows such plot for $K_h/K_v$ = 100 . Same range of X/L, i.e., 0.15 to 0.25 has also been reported by Singh and Seethapathi,1988. Therefore, if an observation well is located within the above range, it will observe a comparatively rapid change of head and thus it will be more sensitive and less liable to errors.

The further analysis of results showed that the parameters $X^2S/T.t$ and $T.\Delta h/Q_R$ are uniquely related for every value of $K_h/K_v$ and the relation was found to be the same for different values of $\Delta H$(similar observation were reported by the author,1988), but the relation is different for different values of $K_h/K_v$ (Here, T is transmissivity of the aquifer, $Q_R$ is the rate of seepage from the water body and t is the time since the start of simulation). The above relation was found to be the same for the two different sizes of the lake considered for. Fig. 5 shows

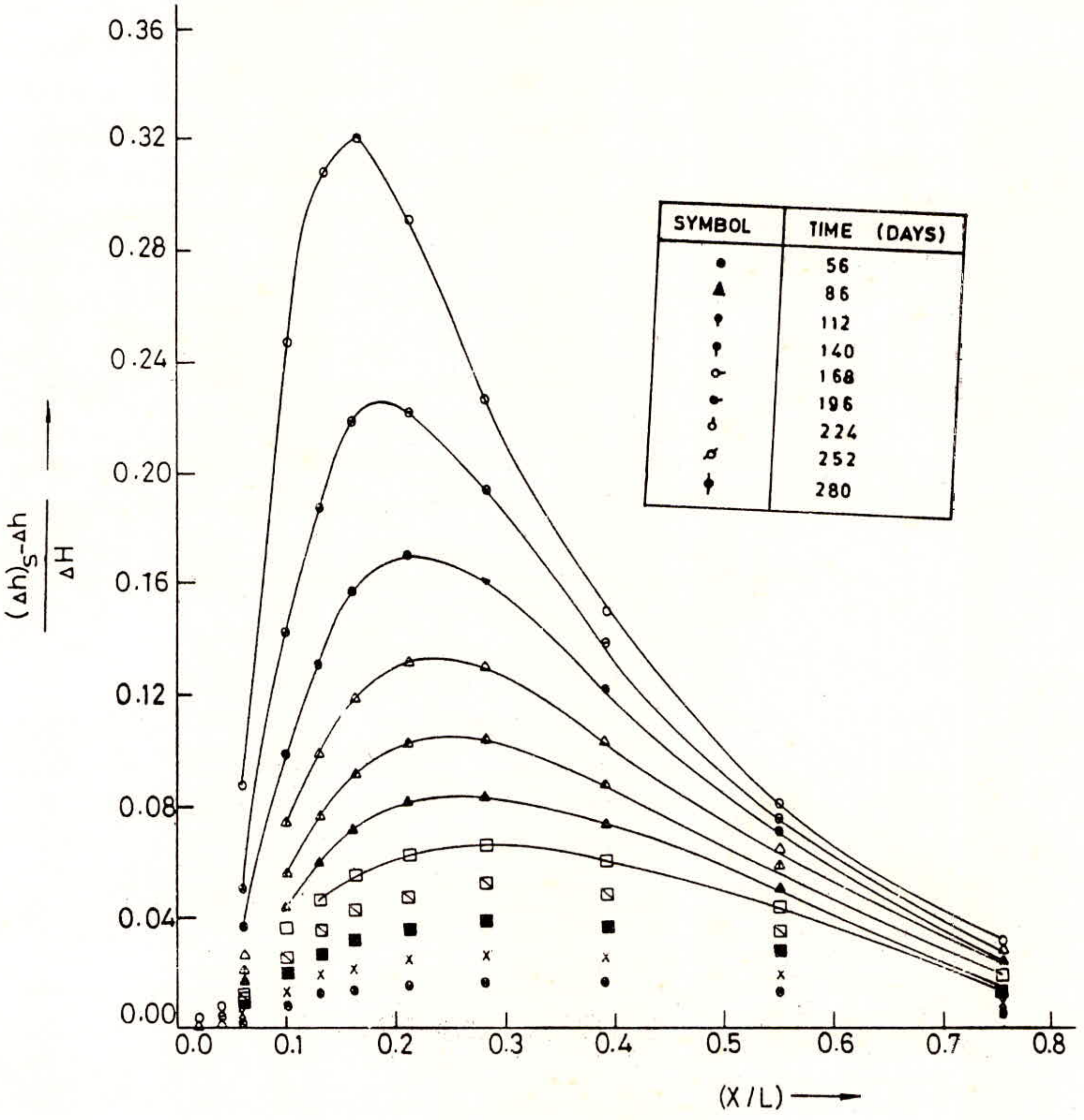


Fig.4-VARIATION OF $((\Delta h)_s - \Delta h)/\Delta H$ WITH X/L FOR $K_h/K_v$=100

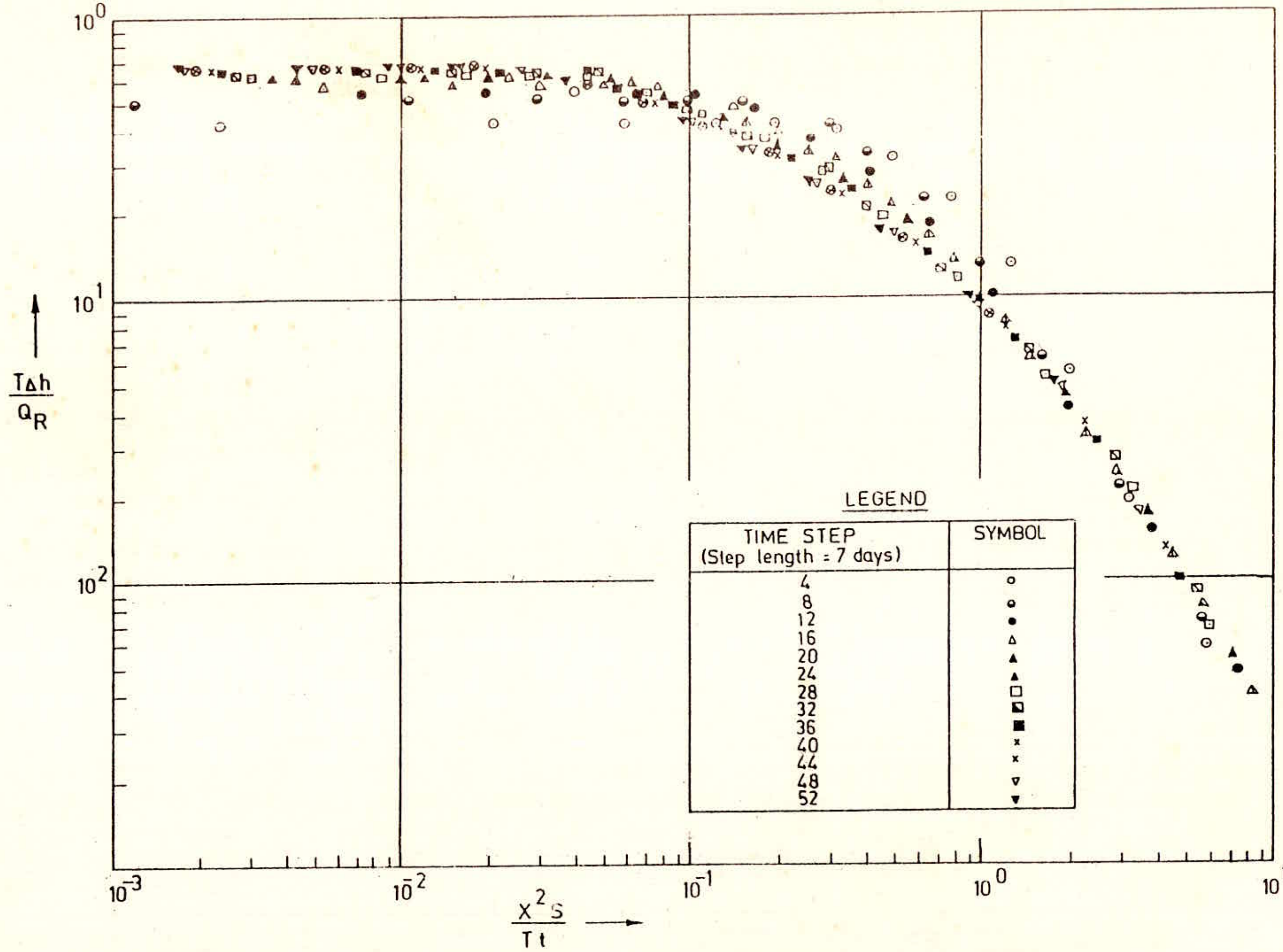


Fig.5-VARIATION OF $T.\Delta h/Q_R$ WITH $X^2S/T.t$ for $K_h/K_v=100$

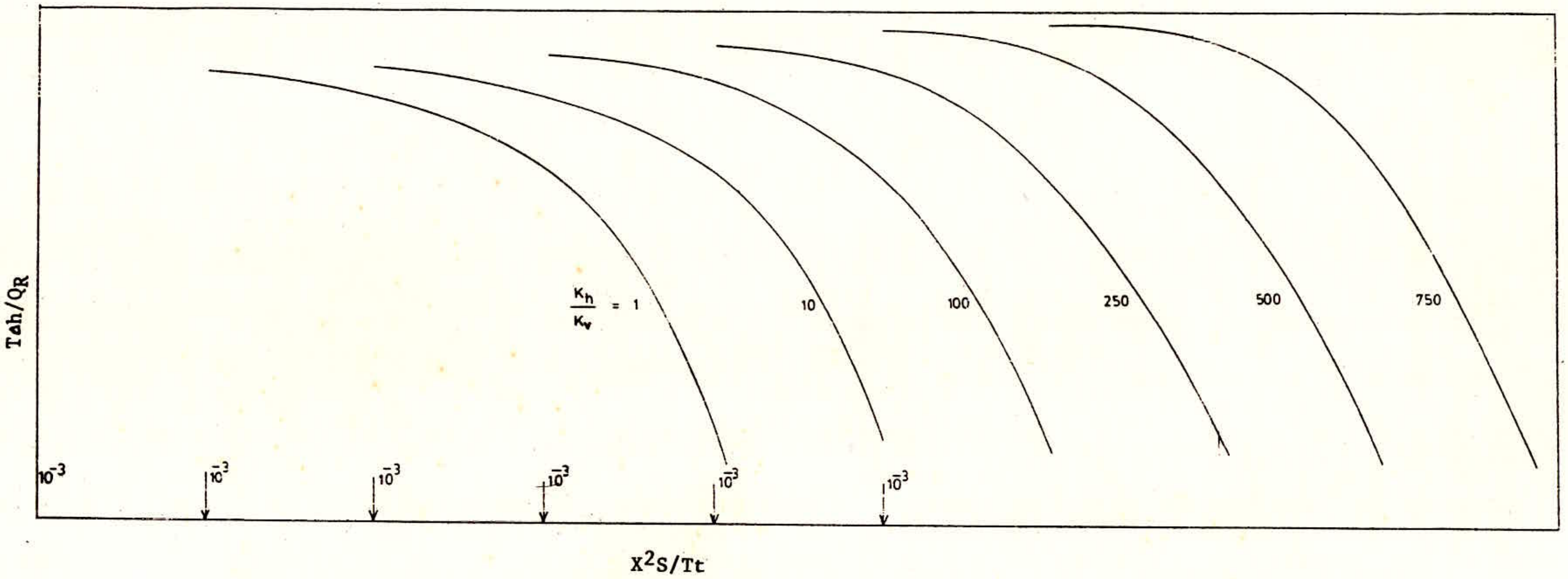


Fig.6-VARIATION OF $T.\Delta h/Q_R$ WITH $X^2S/T.t$ and $K_h/K_v$

such relationship for $K_h/K_v$ = 25Ø along with the data. The mean lines for different values of $K_h/K_v$ are shown in fig. 6. If the head changes in an observation well, $\Delta h$ and aquifer parameters are known, the recharge from the lake can be determined for known value of $K_h/K_v$ making use of fig 6.

Conclusions

A three dimensional model study of the lake-aquifer-river interaction have been done for lakes having square cross sections and uniform depths in a homogeneous and anisotropic aquifer with constant head boundaries on one sides and no flow boundaries on the other sides of the lake each at a distance of 3km. from the centre of the water body. The conclusions drawn from the study are as given below.

1. The parameters ($X^2S/T.t$) and $T.\Delta h/Q_R$ are found to be uniquely related for every value of $K_h/K_v$ irrespective of the value of $\Delta H$. This relation is found to be the same for the two different size of the water body. This relation has been expressed in the form of a family of type curves for different value of $K_h/K_v$ .

2. With the help of the type curves developed , the recharge from a lake, i.e., $Q_R$ can be determined provided the aquifer parameters and the water level fluctuations in the observation well situated within the influence area, are known.

3. The proper location of an observation well to record the rapid change of head has also been suggested, i.e., X/L=Ø.15 to Ø.25.

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