SEEPAGE FROM TWO PARALLEL CANALS

by

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Abstract

Interference of seepage from two parallel canals, which are hydraulically connected with an aquifer, has been investigated using discrete kernel approach. Results of interference of two parallel canals have been presented when both the canals run continuously. From the study the following conclusions have been drawn:(i) the unsteady seepage losses from the canals and the reductions in seepage due to interference are linearly proportional to the initial potential difference that causes the flow; (ii) in case of two continuously running parallel canals, the reduction in seepage from one canal due to interference of the other is zero in the beginning of seepage, the interference increases with time, attains a maximum value and then decreases and the decrease is monotonic at large time; (iii)the maximum reduction in seepage due to interference decreases with increase in the spacing between the canals; (iv) the occurrence of maximum interference is delayed for canals with larger spacing;(iv) in case of two unequal parallel canals the interference of the larger canal on the smaller one is more than that of the smaller canal on the larger one.

Introduction

Canals continue to be the major conveyance system for delivering water for irrigation in most parts of the world. The main canals, in an irrigation project, are designed keeping in view the water availability and the irrigation water requirement in the canal command areas. It is observed that in some canal systems in Northern India, the capacities of the main canal are inadequate for conveying the required irrigation water for paddy and sugar cane crops during the months of May to October. Since, during this period, additional water is available in the rivers from which the canals take off, parallel canals have been constructed along the existing canals to augment supplies to the

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respective command areas. If the water table is present at a shallow depth, the seepage from a parallel canal system would remain in an unsteady state condition. The difference between seepage loss from a canal of the parallel canal system and the seepage loss from the same canal had the other canal not been present quantifies the interference of the latter on the former. In the present paper, the interference of seepage of two parallel canals, which are hydraulically connected with the aquifer, has been analysed using a discrete kernel approach.

Statement of the Problem

Two parallel canals have been constructed in a homogeneous, isotropic, porous medium of finite depth and infinite lateral extent. The widths of canals at water surface during an n^{th} time period are $B_1(n)$ and $B_2(n)$ and the depths of water in the canals are $H_1(n)$ and $H_2(n)$ as shown in Fig.(1). The depths $H_1(n)$ and $H_2(n)$ may vary with time n depending on the supply from the source. The canal section being trapezoidal, $B_1(n)$ and $B_2(n)$ vary with $H_1(n)$ and $H_2(n)$ respectively. Initially the water table is at equilibrium condition and it is at a shallow depth below the canal beds. The canals get hydraulically connected with the underlying aquifer soon after the water is conveyed in them. It is required to find the variation of seepage from the canals with time and the evolution of water table after the the onset of seepage

Anlysis

The following assumptions have been made in the analysis. (i)The time span is divided by uniform time-steps. (ii) Within each time-step the seepage rate from each canal is a constant but it varies from step to step. (iii) The seepage rate from a canal reach, which is hydraulically connected with the underlying aquifer, is linearly proportional to the difference in the potentials at the periphery of the canal reach and in the aquifer below the canal bed (Herbert,1970; Morel-Seytoux, 1975 ; Besbes et al.1978; Flug et al.1980).

Let $Q_1(n)$ and $Q_2(n)$ be the seepage rates per unit length of the first and the second canal and $\Gamma_1(n)$ and $\Gamma_2(n)$ be the reach transmissivity constants of the first and the second canal respectively. The reach transmissivity of a canal is the constant of proportionality between seepage per unit length of a canal and the potential difference. Let the origin be chosen at the centre of the first canal and the drawdown, s(x,n), be measured from a high datum. According to the linear relationship $Q_1(n)$ and $Q_2(n)$ are given by (Morel-Seytoux,1975):

$$Q_1(n) = -\Gamma_1(n)[\sigma_1(n)-s(0,n)] \qquad \ldots(1)$$

$$Q_2(n) = -\Gamma_2(n)[\sigma_2(n)-s(D,n)] \qquad \ldots(2)$$

in which, D = the distance between the canals,and $\sigma_I(n)$= the depth to water surface in the I^{th}canal from the high datum.

The reach transmissivity constants per unit length of canal reaches are given by (Herbert,1970):

$$\Gamma_I(n) = \pi k/\log_e\{(e_I+H_I(n))/(2\ r_{Ir})\}\ ;\quad I=1,2 \qquad \ldots(3)$$

in which, k is the hydraulic conductivity, e_1 and e_2 are the saturated thicknesses below the first and the second canal, and r_{1r} ,r_{2r} are the radii of the equivalent semi circular section of the first and the second canal respectively.

The drawdown s(0,n) and s(D,n) can be expressed in terms of recharge as:

$$s(0,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} Q_1(\gamma)\delta_1[0,B_1(\gamma),n-\gamma+1] - \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_2[-D,B_2(\gamma),n-\gamma+1] \qquad \ldots(4)$$

$$s(D,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} Q_1(\gamma)\delta_1[D,B_1(\gamma),n-\gamma+1] - \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_2[0,B_2(\gamma),n-\gamma+1] \qquad \ldots(5)$$

in which, D_i=depth to impervious stratum from the high datum, $\bar{H}$=the initial saturated thickness of the aquifer. The discrete kernel coefficients for rise in water table are given by:

$\delta_I[0,B_I(N),M\ ] = F\ [0,B_I(N),M\ ] - F\ [0,B_I(N),M-1];I=1,2;\ M \geq 2$

$$\delta_I[0,B_I(N),1] = F[0,B_I(N),1] - B_I(N)/(8T); I=1,2; \; T=k\bar{H}$$

$$\delta_2[-D,B_2(N),M] = F[-D,B_2(N),M] - F[-D,B_2(N),M-1]; \; M \geq 2$$

$$\delta_2[-D,B_2(N),1] = F[-D,B_2(N),1] - \frac{1}{2T}\gamma(-D)^2$$

$$\delta_1[D,B_1(N),M] = F[D,B_1(N),M] - F[D,B_1(N),M-1]; \; M \geq 2$$

$$\delta_1[D,B_1(N),1] = F[D,B_1(N),1] - \frac{1}{2T}\gamma(D)^2;$$

$$F(x,B,t) = \frac{\alpha t}{2BT}\left[\mathrm{Erf}\left\{\frac{x+0.5B}{\sqrt{4\alpha t}}\right\} - \mathrm{Erf}\left\{\frac{x-0.5B}{\sqrt{4\alpha t}}\right\}\right]$$

$$+ \frac{1}{4BT}\left[(x+\tfrac{B}{2})^2\mathrm{Erf}\left\{\frac{x+0.5B}{\sqrt{4\alpha t}}\right\} - (x-\tfrac{B}{2})^2\mathrm{Erf}\left\{\frac{x-0.5B}{\sqrt{4\alpha t}}\right\}\right]$$

$$+ \frac{\sqrt{\alpha t}}{2BT\sqrt{\pi}}\left[(x+\tfrac{B}{2})\exp\left\{-\frac{(x+.5B)^2}{4\alpha t}\right\} - (x-\tfrac{B}{2})\exp\left\{-\frac{(x-.5B)^2}{4\alpha t}\right\}\right]$$

$\alpha = T/\phi$; T=transmissivity, and ϕ =storativity of the aquifer. The discrete kernel coefficients are the response of a linear system to a unit pulse excitation given to the system during the first unit time-step. In the present case, the coefficients are the values of rise in water table height at a point due to the recharge that takes place at a unit rate per unit length of a canal during the first time step. The discrete kernel coefficients can be derived from the response of a linear system to a unit step excitation ,K(t), using the relation, $\delta(n)=K(n)-K(n-1)$. Polubarinova Kochina (1962) has derived an expression of the response function to a step excitation for a straight recharging strip which has been used for deriving the discrete kernel coefficients.

Incorporating Equations (4) and (5) in Equations (1) and (2) respectively, $Q_1(n)$and $Q_2(n)$ are solved which are given by:

$$\begin{bmatrix} Q_1(n) \\ Q_2(n) \end{bmatrix} = [A]^{-1}.\,[C] \qquad \ldots(6)$$

The elements of the matrices are:

$A(1,1) = -1/\Gamma_1(n)-\delta_1[0,B_1(n),1]$; $\quad A(1,2) = -\delta_2[-D,B_2(n),1]$;

$$A(2,1) = -\delta_1[D,B_1(n),1]; \qquad A(2,2) = -1/\Gamma_2(n)-\delta_2[0,B_2(n),1];$$

$$C(1,1) = \sigma_1(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\delta_1[0,B_1(\gamma),n-\gamma+1] + \sum_{\gamma=1}^{n-1} Q_2(\gamma)\delta_2[-D,B_2(\gamma),n-\gamma+1] ;$$

$$C(2,1) = \sigma_2(n) - D_i + H + \sum_{\gamma=1}^{n-1} Q_1(\gamma)\delta_1[D,B_1(\gamma),n-\gamma+1] + \sum_{\gamma=1}^{n-1} Q_2(\gamma)\delta_2[0,B_2(\gamma),n-\gamma+1].$$

Once the seepage losses at different times are obtained, the rise in water table can be computed by making use of the equation given below:

$$s(x,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} Q_1(\gamma)\delta_1[x,B_1(\gamma),n-\gamma+1] - \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_2[x-D,B_2(\gamma),n-\gamma+1] \qquad \ldots(7)$$

Results and Discussion

Numerical results have been presented for interference of two parallel canals which run continuously with constant depth of water in them. It has been assumed that their beds are at same level. Making use of the solution presented here, results could be obtained for canals in case their beds are at different levels and they run intermittently with varying depth of water. The results for seepage losses and water table rise have been obtained for assumed canal dimensions, aquifer parameters, k, $\bar{H}$, ϕ, and an initial potential difference between the canals and the aquifer. The discrete kernels for water table rise have been generated for assumed values of canal dimensions, spacing between the canals, and the aquifer parameters. Since both the canals run continuously with constant depth of water, and the widths of the canals at water surface do not change with time, the reach transmissivity constant for each canal also does not change with time. The seepage losses have been solved in succession, starting from time-step 1, using equation (6).

The unsteady seepage losses from one of the two identical parallel canals for $B_1/\bar{H}=B_2/\bar{H}=0.03$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $D/\bar{H}= 0.18$ and for initial water table positions, $H/\bar{H} =0.005$ 0.007 and 0.009, are presented in Fig.(2). The seepage loss from a canal at time t=0 is a finite quantity and it is given by the product of the corresponding reach transmissivity and the initial potential difference. The seepage loss from one canal that would occur if the other canal is at $D=\infty$, has also been shown in the figure. The difference between seepage loss from a canal of the parallel canal system and the seepage loss from the same canal, if the other canal does not exist, quantifies the interference of the latter on the former. It could be seen from the figure, that in the beginning, the reduction in seepage in each canal due to interference of the other is zero; the interference increases with time and attains a maximum value.

The variations in reduction of seepage with time due to interference for different values of initial potential difference, are also shown in Fig.(2). It could be seen from the figure that the interference between the parallel canals, at any time increases with increase in the initial potential difference. At nondimensional time factor, $kt/(2\phi \bar{H})=0.50$, for $(H-\sigma_1)/\bar{H}=0.004$, the reduction in seepage due to interference is 0.180. If the initial potential difference, $(H-\sigma_1)/\bar{H}=0.008$, the corresponding reduction due to interference is 0.360. It could be seen that the interference is linearly proportional to the initial potential difference. At non-dimensional time factor 0.5, the reductions in seepage losses due to interference are 0.360, 0.270, and 0.180 for $(H-\sigma_1)/\bar{H}=0.008$, 0.006,and 0.004 respectively. The ratios of the reduction in seepage due to interference and the corresponding initial potential difference are equal to 0.045. Thus the reduction in seepage from any canal due to interference of the other is linearly proportional to the initial potential difference.

The reduction in seepage loss due to interference at large times for various distances between two identical canals has been shown in Fig.3, in a semi log plot, for $B_1/\bar{H}$=0.06. It is seen from the figure that the reduction in seepage due to interference reaches a maximum value at very large time and then decreases. The reason for the decline is as follows:

The seepage from a canal decreases with the decrease of the potential difference between the canal and the aquifer. In a parallel canal system the potential difference under a canal decreases with time partly due to its own seepage and partly due to seepage from the other canal. Ultimately, the seepage loss from a canal at large time would tend to zero whether it runs alone or it runs along with the other canal. Since the seepage loss tends to zero in either case, the reduction in seepage loss due to interference will also tend to zero. Because the interference ultimately tends to zero, it would decline after reaching a maximum value.

It is seen that the occurrence of maximum interference is delayed for larger spacing between the canals. For $D/\bar{H}$=0.08, the maximum interference occurs at $kt/(2\phi\bar{H})$=0.80. For $D/\bar{H}$=0.18 the maximum interference occurs at $kt/(2\phi\bar{H})$=0.90. The maximum value of interference declines with increase in spacing between the canals. It could be seen from the figure that for $D/\bar{H}$=0.08, the maximum value of interference is 0.480, where as for $D/\bar{H}$=.48, the maximum value of interference is 0.430

The variation of reduction in seepage due to interference for two unequal parallel canals, which run continuously , is presented in a semi log plot in Fig.4 for different values of $D/\bar{H}$ for $B_1/\bar{H}$=0.06, $B_2/\bar{H}$=0.03, $H_1/\bar{H}=H_2/\bar{H}$=0.003, and $\bar{H}/H$=0.009. The figure shows that, at any time, the reduction in seepage from each canal due to interference is more for smaller spacing between the canals. The reductions in seepage due to interference attain different maximum values at different times for a given spacing between the canals. After reaching a maximum, the interference reduces with time. The occurrence of maximum reduction in seepage due to interference takes place at earlier time for the larger canal. For example, for $D/\bar{H}$=0.24, the maximum reduction in seepage for the first canal, which is larger, occurs at $kt/(2\phi \bar{H})= 1.1$, whereas, for the smaller canal the maximum interference occurs at a nondimensional time 1.3. The effect of the larger canal on the smaller canal is more than the effect of the smaller canal on the larger one. For example, the maximum reduction in seepage from the larger canal due to interference of the smaller canal is 0.403 for $D/\bar{H}=0.24$, whereas, the maximum reduction in seepage from the smaller canal, due to interference of the larger one, is 0.432.

The rise in water table due to seepage from two equal parallel canals have been evaluated at different locations across the canals for $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, initial potential difference, $(H-\sigma_1)/\bar{H} =0.008$, and $D/\bar{H}=0.18$. The results are shown in Fig.(5). It could be seen from the figure that in the beginning of seepage, at non-dimensional time 0.005, well defined water mounds are formed under the centre of each canal. As the time passes, the ridges get dissipated and the points of maximum rise move towards each other, indicating higher fraction of seepage flow from each canal going towards the outer sides of the canals. The points of maximum rise of water table, however, do not go beyond a distance of half the respective width of the canals at the water surface.

Conclusion

Based on the results presented in this chapter, the following conclusions are drawn :i) The unsteady seepage losses from the canals and the reduction in seepage due to interference are linearly proportional to the initial potential difference that initiates the flow. ii) In case of two continuously running parallel canals, the reduction in seepage from one canal, due to interference of the other is zero in the beginning of seepage. The interference increases as the time passes and attains a maximum value and then decreases. The decrease is monotonic at large time. iii) The maximum reductions in seepage due to interference decrease with increase in the spacing between the canals. Also, the occurrence of maximum interference is delayed for canals having larger spacing. iv) The interference of a bigger canal on smaller canal is more than that of the smaller canal on the bigger one. v) For the parallel canals of equal dimensions, distinct water mounds of equal height are formed under the canals. In the beginning of seepage, the ridges lie under the centre of the canals. With lapse of time, as seepage continues, the points of maximum water table height move towards each other; but they do not cross the width of the respective recharging strips. With passage of time, the zone in between the canals becomes a stagnant zone.

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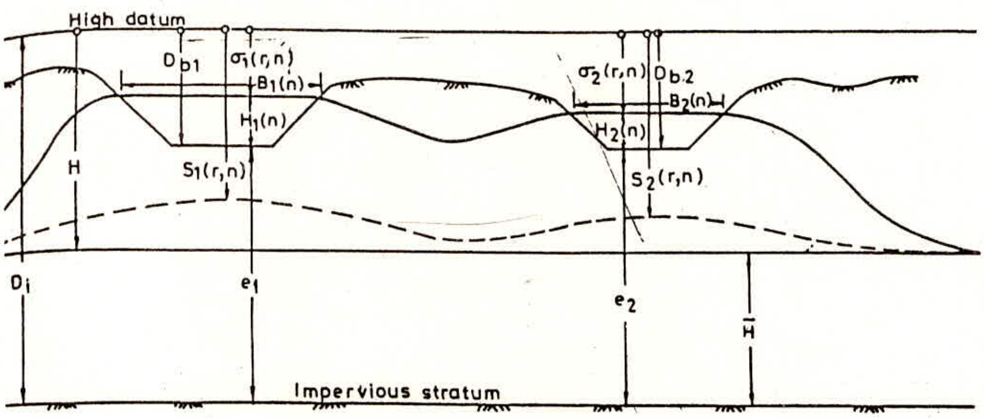

Fig.1 - Schematic section of a parallel canal system

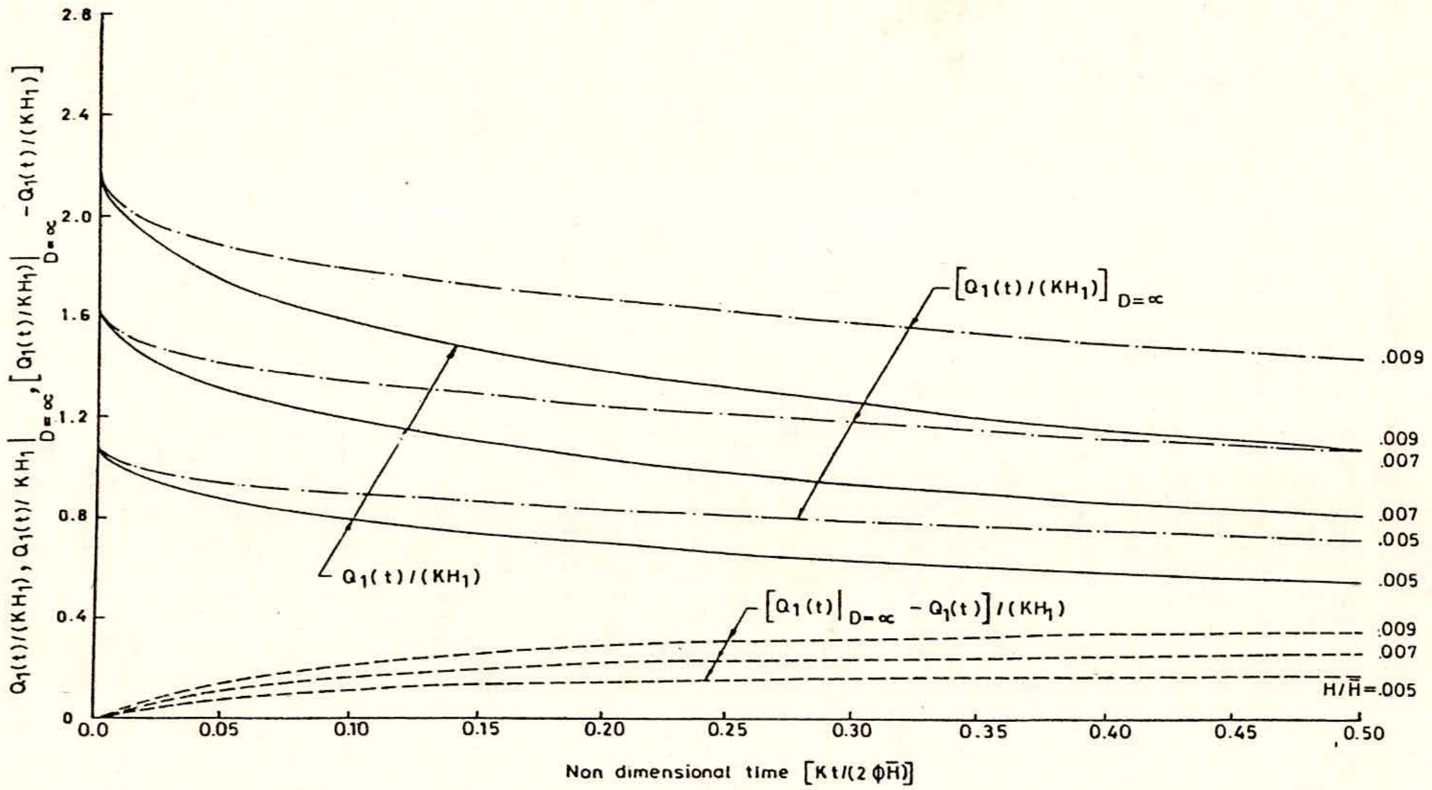

Fig. 2 –Interference of seepage losses from two continuously running identical parallel canals, evaluated for $D/\bar{H} = 0.18$, $B_1/\bar{H} = B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and for initial water table positions $H/\bar{H} = 0.005$, 0.007 and 0.009

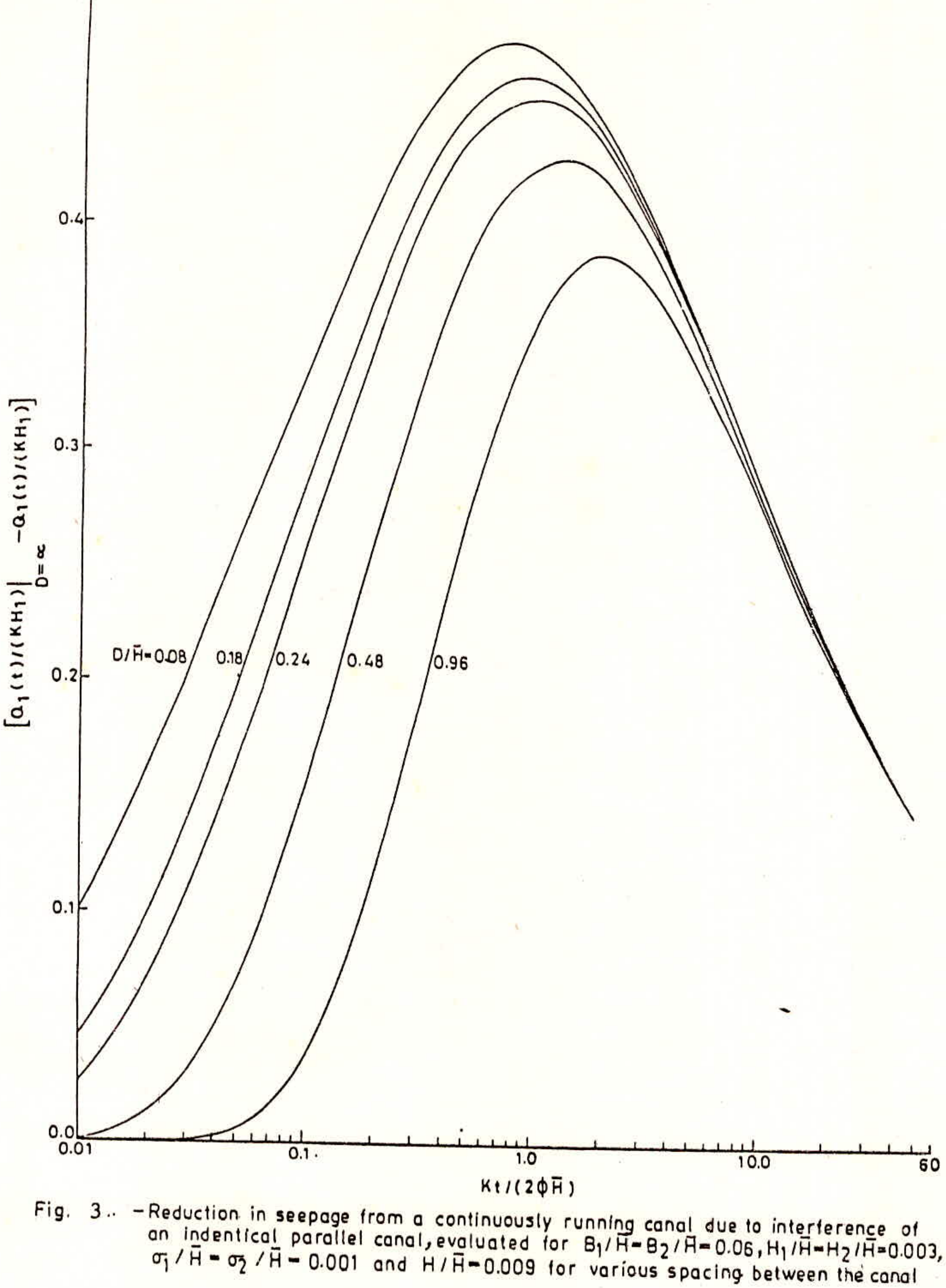

Fig. 3.. – Reduction in seepage from a continuously running canal due to interference of an indentical parallel canal, evaluated for $B_1/\bar{H}=B_2/\bar{H}=0.06$, $H_1/\bar{H}=H_2/\bar{H}=0.003$, $\sigma_1/\bar{H}=\sigma_2/\bar{H}=0.001$ and $H/\bar{H}=0.009$ for various spacing between the canal

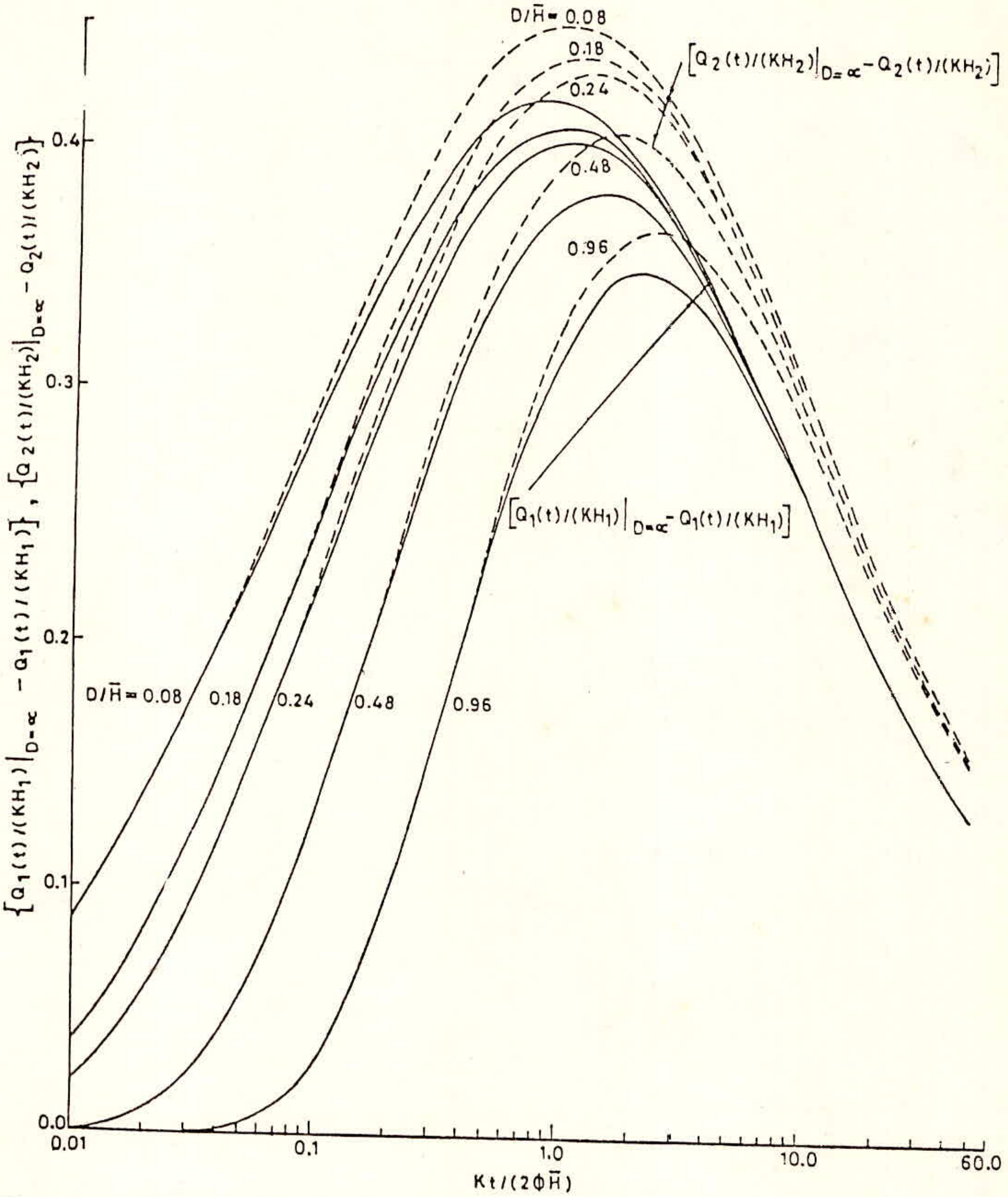

Fig. 4 – Reductions in seepage due to interference of two unequal continuously running parallel canals spaced at various distances, evaluated for $B_1/\bar{H} = 0.06$, $B_2/\bar{H} = 0.03$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$

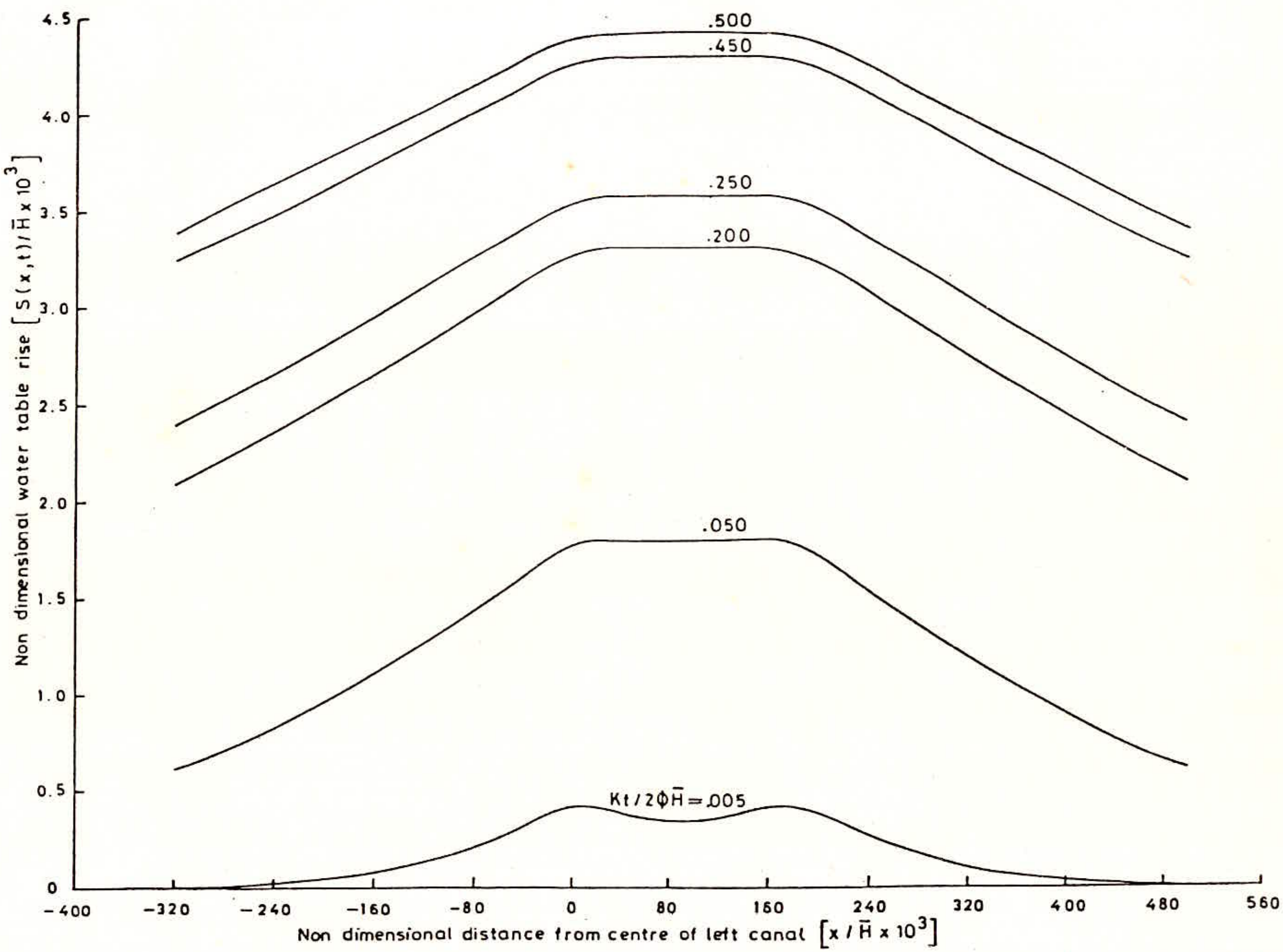

Fig.5 - Water table evolution due to seepage from two continuously running identical parallel canals predicted for $D/\bar{H} = 0.18$, $B_1/\bar{H} = B_2/\bar{H} = 0.06$, $H_1/\bar{H} = H_2/\bar{H} = 0.003$, $\sigma_1/\bar{H} = \sigma_2/\bar{H} = 0.001$ and $H/\bar{H} = 0.009$