ESTIMATION OF SUBSURFACE INFLOW, OUTFLOW AND STORAGE

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1. Subsurface Horizontal Flows

Subsurface horizontal flows form an important component of groundwater balance. Thus, a rational estimation of such flows is essential for a reliable quantification of groundwater resource. The contribution of subsurface flows towards the groundwater balance can be defined in terms of i) subsurface horizontal inflows and outflows across non-hydraulic boundaries (i.e., administrative or political boundaries and ii) stream aquifer interaction i.e., influent or effluent seepage). The present discussion is confined to the former only.

The flows across the non-hydraulic boundaries are quite sensitive to the status of groundwater development and/or recharge in the study area as well as across the boundary. Thus, such flows can be estimated for historical period only and can not be projected for modified pumping/recharge patterns unless the boundaries are extended to hydraulic boundaries (i.e., constant head or impervious) and distributed modelling resorted to. On the other hand, the stream-aquifer interaction can be computed for historical periods and as well be projected, provided the river stages are almost unaffected by the projected changes in the pumping/recharge patterns. This may generally hold good since for most of the rivers the order of magnitude of stream aquifer interflows is quite small as compared to the total discharge.

2. Dupuit Forchheimer's Assumptions and the Solution

Strictly speaking, there can not be an exclusively horizontal flow in unconfined aquifers. Flow in such aquifers occurs on account of vertical inclination of watertable which inevitably introduces vertical component to the flow. However, for most of the groundwater flow conditions, the vertical component may be small enough to render the flow 'near horizontal'. Another associated question is about the distribution of horizontal velocity in a vertical plane. The question is of great concern because the velocity computed in accordance with the watertable gradient represents the velocity at the watertable surface and may differ from the velocity occuring down below. Dupuit-Forchheimer's assumptions resolve these issues as follows.

i) it is assumed that the watertable is near horizontal,

ii) the horizontal velocity is assumed to be uniformly distributed over a vertical section, and

iii) the uniform horizontal velocity across any vertical section is assumed to be equal to the horizontal velocity at the watertable.

Thus, the horizontal flow per unit width per unit time (q) in accordance with the D-F assumptions is K i h or Ti (refer Fig.1). This formula is quite popular amongst practising professionals. i, the gradient of watertable is estimated by sampling the positions of watertable at two adjascent points in the direction of flow (refer Fig.2). However, it must be recognised that this formula is applicable only if the D-F assumptions are satisfied. These assumptions may be severely violated close a partially penetrating pumping well on river. In such locations, the formula (q = Ti) may lead to gross over estimation of horizontal flows since the free surface velocity may be much higher than the velocities down below (refer Fig.3). It may be inferred that the observation wells for monitoring of watertable gradients should be sufficiently away from any partially penetrating production well (or wells)or partially penetrating river.

The other uncertainty is about the choice of spacing of wells for calculation of watertable gradients. If the wells are too closely spaced, the computed gradient may contain large roundoff errors. On the other hand, if the distance is too long, the estimate will represent only average gradient (the resulting errors are known as truncation errors) Thus the wells should be located at such a minimum distance that the difference in their water elevations is large enough to be measured accurately.

3. System of Wells

It can be concluded that the subsurface horizontal flows across a boundary can be estimated by monitoring water levels in a system of wells, uniformly distributed across the boundary. It may be emphasized that it is important to have wells on both sides of the boundary. Ideally speaking there should be a series of sets of two wells across the boundary (figure 4). In the absence of such a systematical well field, contours of equal water table may be drawn and the gradients may be estimated through interpolated values.The boundary can be divided into stretches of near-uniform water table gradients. The horizontal flows over each stretch can be calculated and subsequently summed up.

4. Groundwater Storage

Fluctuation of groundwater storage is an important component of the water balance studies. Estimation of groundwater storage essentially involves integration of watertable elevation (above the base rock) over the horizontal space. Such a storage can be computed by sampling the water table elevation at discrete points well - spread over and across the study area. The fluctuation of groundwater storage in a given period can be computed in the following two ways

i) Calculation of groundwater storages at the beginning and at the end of the period. This essentially involves drawing of watertable contours for the beginning and end of the stipulated period. The water table contours can provide estimates of groundwater storage as follows

Superpose the watertable contours over the study area. Entire area must be covered by the contour system. For such a coverage there must be adequate number of observation wells, not only within the study area, but also outside it. The groundwater storage can be calculated as follows (refer figure 5)

$$S = S_y \sum_{i=1}^{n-1} h_i A_i$$

$$h_1 = \frac{h_i + h_{i+1}}{2}, \quad i = 2, \ldots\ldots, n-2$$

$$h_i = \frac{W_1 h_1 + W_2 h_2}{W_1 + W_2}, \quad i = 1$$

$$h_{n-1} = \frac{W_{n-1} h_{n-1} + W_n h_n}{W_{n-1} + W_n}, \quad i = n-1$$

Where W_1, W_2, W_{n-1} and W_n are the weight to be assigned to contours of (h_1, h_2) and (h_{n-1}, h_n) for estimation of groundwater storage in A_i and A_{n-1} respectively. These weights may be assigned subjectively depending upon location of the areas within the successive contours (i.e., closer the area A_1 to contour of h_2, higher will be the weight W_2 as compared to W_1). The areas $(A_1, A_2, \ldots\ldots, A_{n-1})$ can be measured by any appropriate technique, say by a planimeter.

ii) The procedure outlined in the preceding para can lead to large errors especially when there is a wide variation in watertable elevations within the area. In the presence of high watertable elevations, even small errors in drawing of contours can cause large errors in storage estimates. This may completely distant the estimates of storage fluctuations. This problem can be overcome by plotting contours of watertable fluctuations (during the stipulated period). The areas between adjacent contours of watertable fluctuations are planimetered and the resulting storage fluctuation is calculated in the same way as described earlier. However, unlike the watertable contours, the watertable fluctuation contours are generally quite irregular and multiple in nature (refer figure 6).

5. Computer Assisted Calculations

It can be verified from the preceding discussion that the procedures for estimation of subsurface horizontal flows and storages essentially involve respectively the differentiation and integration of the watertable elevations. In the suggested manual procedures, these operations are performed by drawing of watertable contours. However, such procedures may lead to large errors because of the inherent subjectivity and errors in the raw data. The problem can be overcome to some extent by approximating the true (but unknown) functional relation between the watertable elevation and coordinates along any two arbitrarily decided orthogonal directions (e.g., latitude and longitude), by an appropriate polynomial. Thus,

$$h(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 y^2 + \alpha_6 xy + \ldots\ldots (\alpha_o, \alpha_1, \alpha_2 \ldots)$$

The coefficient $(\alpha_o, \alpha_1, \alpha_2, \ldots)$ appearing in the assumed polynomial relation can be estimated by regressional analysis of the raw data as follows.

Let h_i^* be the observed data at the i^{th} observation well (coordinates: x_i, y_i). Thus, the sum of the squares of differences between the observed and the computed watertable elevations (Z) can be written as follows

$$Z = \sum_{i=1}^{n} [h_i^* - h(x_i, y_i)]^2$$

$$Z = Z(\alpha_1, \alpha_2, \ldots\ldots, \alpha_m) \quad [n >> m]$$

(α) can be estimated by minimizing Z i.e.,

$$\frac{dZ}{d\alpha_i} = 0 \quad i = i, \ldots. m$$

These m linear equations can be solved for n unknown coefficients. Thus, the polynomial is completely solved.

The optimal form of the polynomial (i.e., number of the terms) can be determined by minimizing the standard error (s) defined as follows

$$s^2 = \frac{\Sigma(h_i^* - h(x_i, y_i))^2}{n - m}$$

The adequacy of the adopted polynomial can be determined by checking the acceptability of s (the probable errors in computed h may be $\pm s$). Further, it can also be checked by R^2, to be computed as follows

$$R^2 = 1 - \frac{\Sigma[h_i^* - h(x_i, y_i)]^2}{\Sigma(h_i^* - \bar{h}^*)^2} \; ; \; \bar{h}^* = \frac{\Sigma h_i^*}{n}$$

A good fit is characterized by R^2 close enough to 1. The contribution of the each term can be quantified by computed t as follows

$$t = \frac{\alpha_i}{s.e.(\alpha_i)} \; ; \; s.e.(\alpha_i) = s\sqrt{C_{ii}}$$

where, C_{ii} is the i^{th} diagonal element of the corrected sum of squares and product matrix. The contribution of the i^{th} term can be considered to be insignificant if the computed t falls below the tabulated t for (n-m) degree of freedom at some chosen level of confidence (generally 95%). Such terms can be dropped. The 'goodness' of the truncated polynomial (TP) as compared to the full polynomial (FP) can be quantified by F test as follows.

$$F = \frac{SSR(TP) - SSR(FP)-Kd}{SSR(FP)/(n-m)}$$

where SSR stands for the minimized sum of the squares of the residues, Kd is the number of terms droped. If computed F is smaller than the tabulated F with Kd and (n-m) degrees of freedom at α percentile level of confidence, the partial polynomial can be considered to be satisfactory at the chosen level of confidence. Otherwise, the terms initially dropped may be reintroduced.

The consistency of the data points can be checked by computing standard residues as follows

$$es_i = \frac{h_i^* - h(x_i, y_i)}{s}$$

The data points displaying $|es_i|$ larger than 2 can be treated as 'Outliers'. One possible reason for such 'outliers' could be large observational errors. Thus, if no other explaination ocan be found, such point may be dropped. This is bound to improve the fit significantly (i.e. lower and higher R^2).

Thus, the optimal polynomial can be determined following the procefure outlined in the preceding paragraphs. Subsequently the two operations can be performed as follows.

i) Subsurface horizontal flows across the boundary (q_x)

$$q_x = - T \frac{\partial}{\partial x} h(x.y)$$

ii) Subsurface storage (S)

$$S = \int_A h(x,y) S \, dx \, dy$$

A Computer program in FORTRAN IV has been developed by the author to perform all the operations discussed earlier. The programme has been thoroughly tested for a large number of real life data sets. Extension services are available on request.

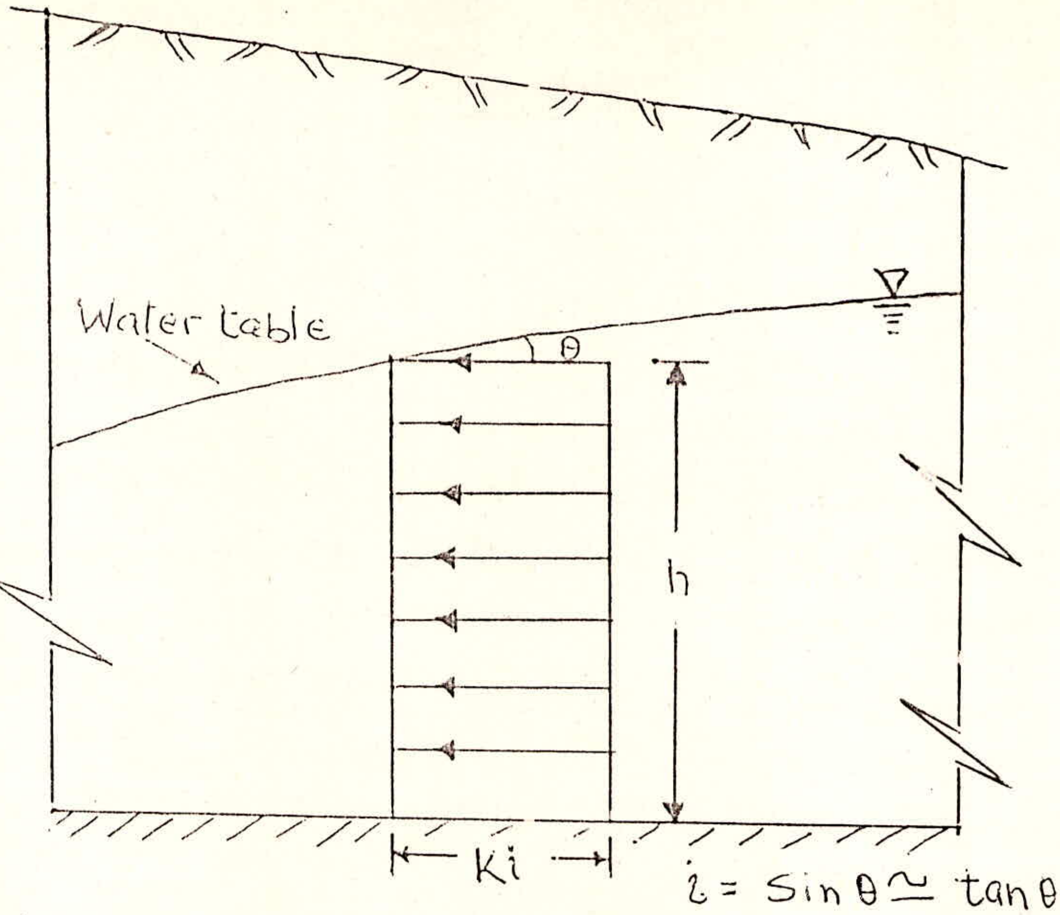


FIG - 1 Distribution of horizontal velocity in accordance with D-F assumptions.

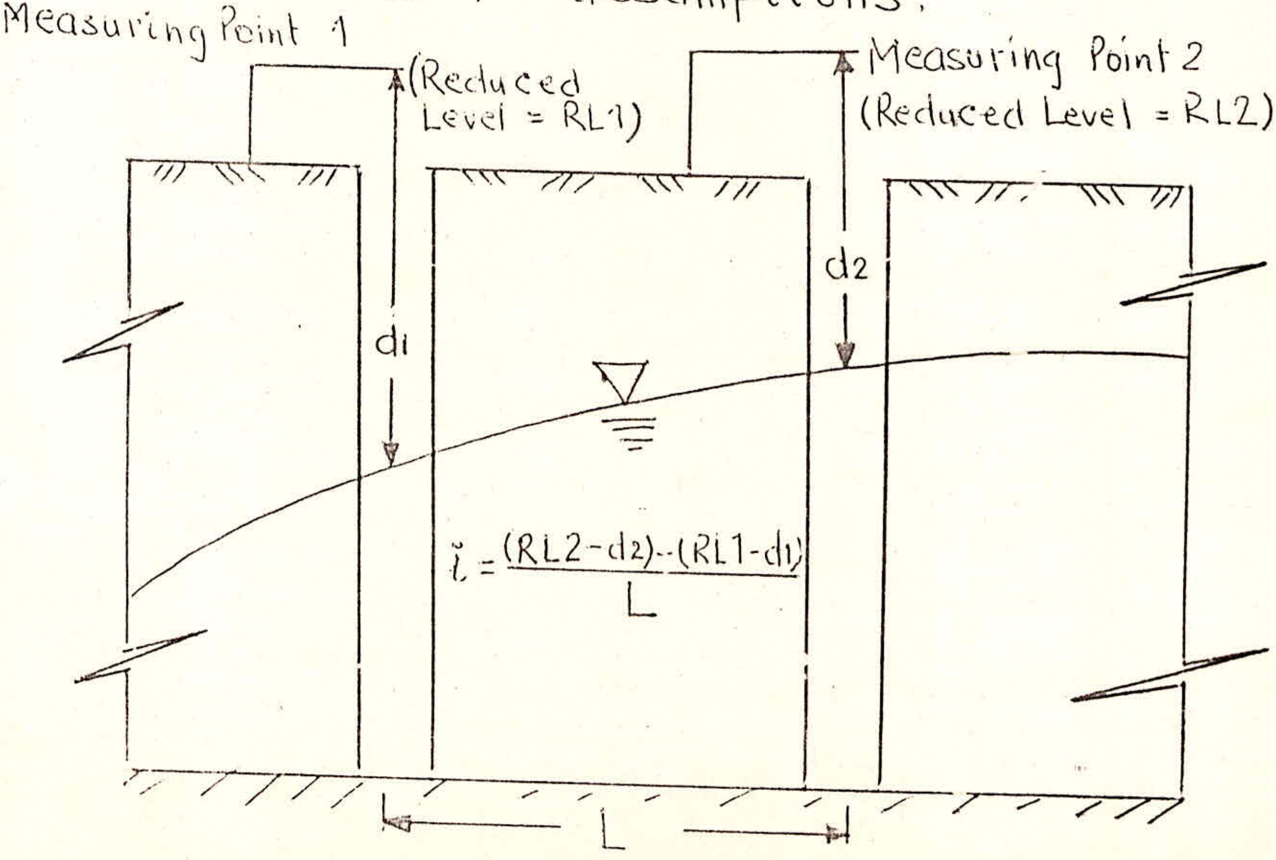


FIG-2 Estimation of Watertable gradient

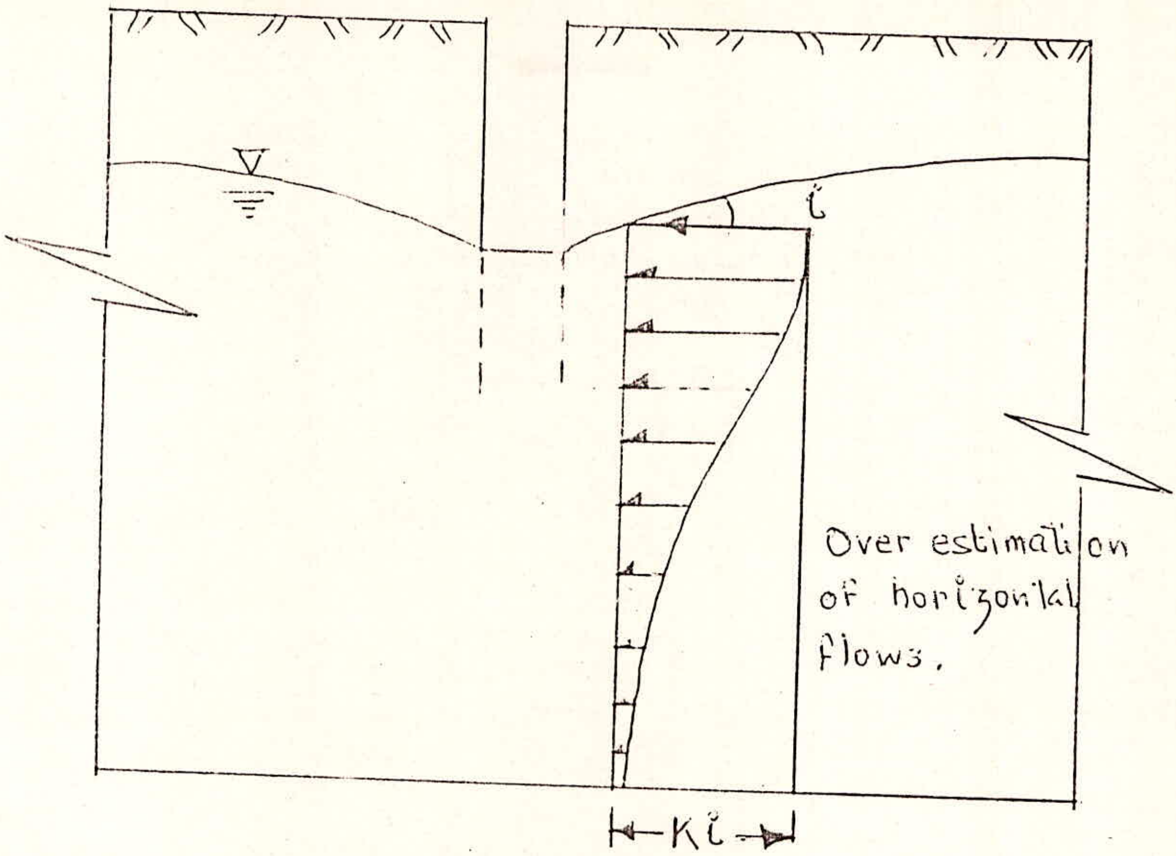
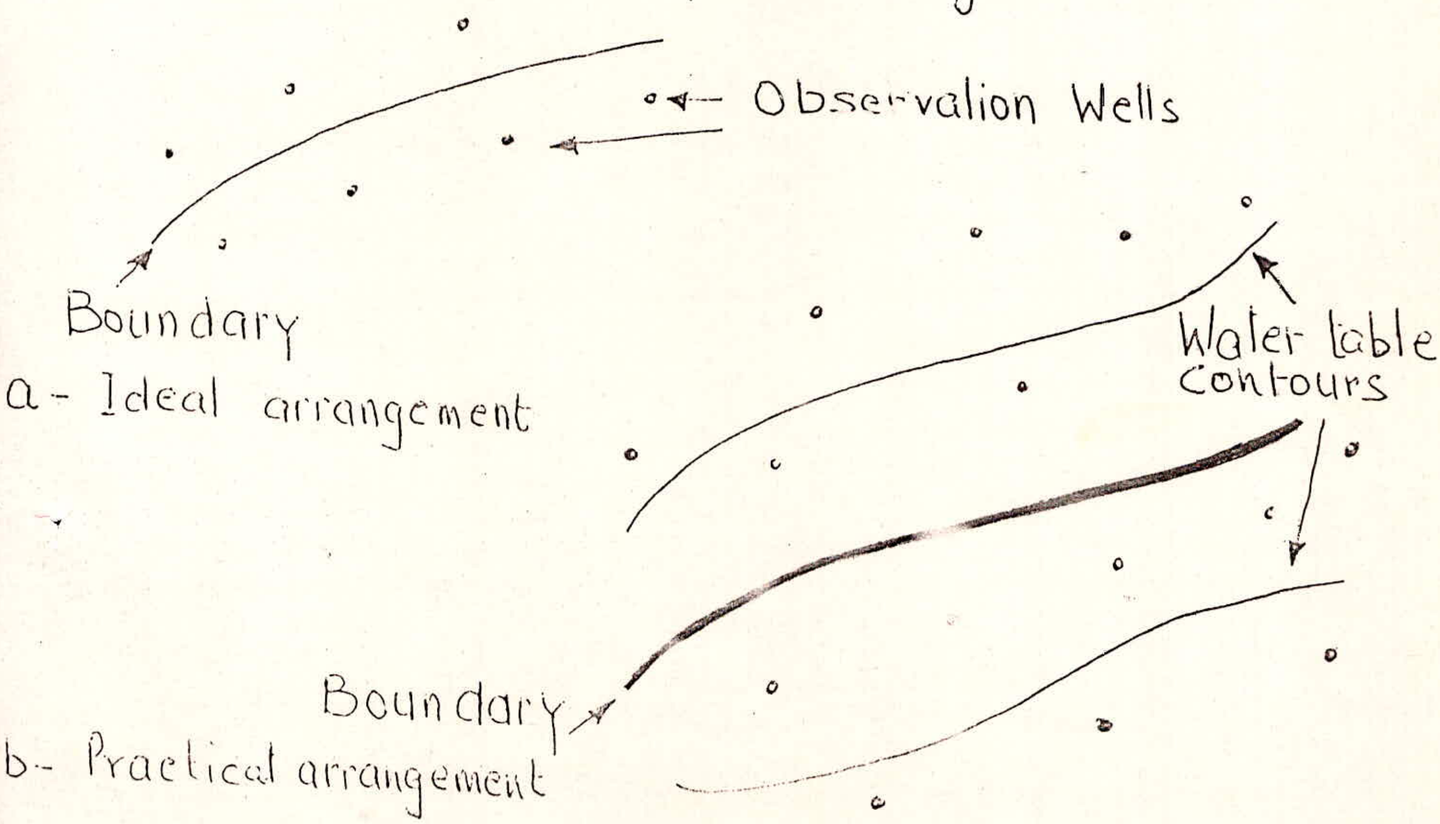


FIG - 3 Typical velocity distribution close to a partially penetrating well.



G - 4 System of wells for monitoring boundary flows.

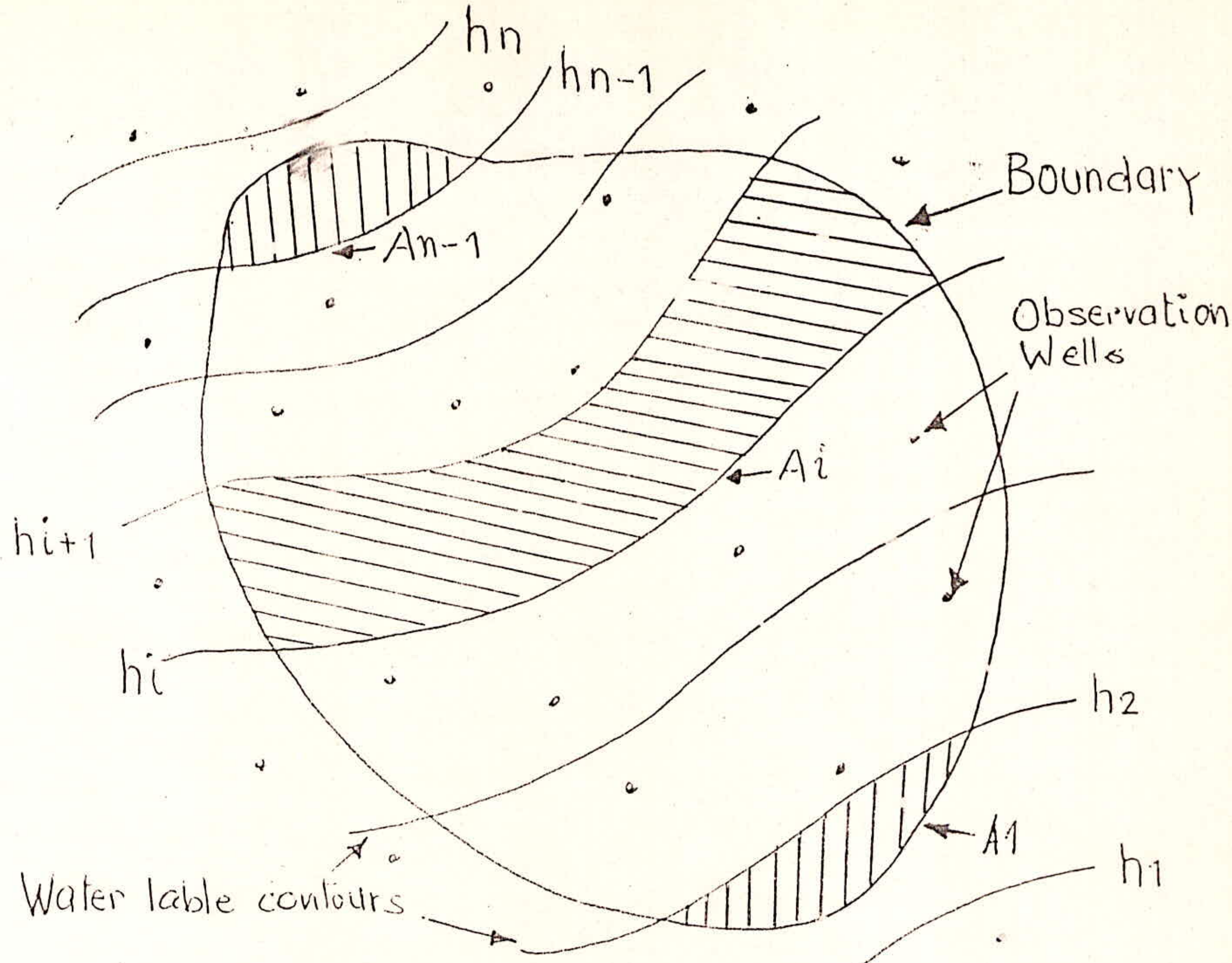


FIG-5 Estimation of ground water storage.

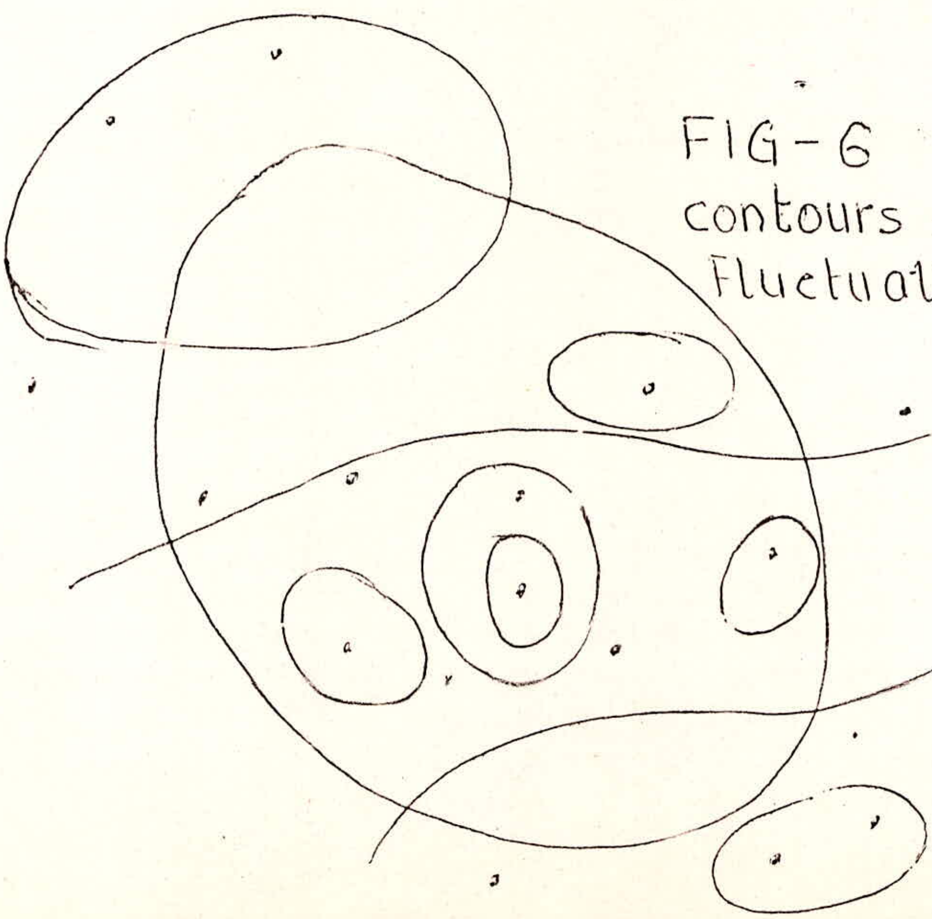


FIG-6 Typical contours of water table Fluctuations.