SEEPAGE LOSSES FROM CANALS

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Steady seepage from canals

Steady state seepage from a canal, when the water table is at large depth has been analysed by a number of investigators for various boundary conditions. Kozeny (1931) has shown that, if the shape of a ditch or canal conforms to the equation $[x - \frac{Q}{\pi K} \cdot \cos^{-1}(y/H)]^2 + y^2 = H^2$, where Q is the seepage rate, x and y are cartesian coordinates with origin at the centre of water surface and H is the maximum depth of water in the canal, the maximum width of sheet of water seeping down into the porous medium is equal to (B+2H); B being the width of canal at water surface. According to Kozeny, the seepage quantity from such a canal is K(B+2H), where, K is the coefficient of permeability. The result holds good if the porous medium is of very large thickness so that the seeping water can maintain its vertical downward movement indefinitely. This requirement prohibits the applicability of the solution to cases where the ground water table is at shallow depth. Muskat (1946) has compared the values of seepage discharge for three different shapes of canals and quantified that the extreme variation in seepage, due to the eff effect of shape of canal or ditch, is about 10 percent. Wedernikov (1937) obtained an exact solution for seepage from channels of triangular and trapezoidal shape with ground water table at infinite depth.

Unsteady seepage from canals when water table is at large depth

When the water table is at large depth, the canal is not hydraulically connected with the aquifer, and the seepage from canal is independent of the location of water table. In such a situation, there are three main aspects of the flow process, viz., the movement of water through the unsaturated zone till it reaches the deep water table, recharge to the aquifer after the wetting front reaches the water table, and evolution of water table after the onset of recharge.

According to Morel-Seytoux and Khanji (1974), until the wetting front reaches the water table, the infiltration rate can be represented adequately by a modified Green Ampt equation given below:

$$I_{(t)} = \frac{K\ (\bar{\theta}-\theta_1)[H(t)+H_c]+w(t)}{w(t)}$$

where,

$H(t)$ = depth of water above soil,

$w(t)$ = cumulative volume of infiltration expressed as depth,

K = hydraulic conductivity at normal saturation,

H_c = effective capillary drive,

$\bar{\theta}$ = water content at natural saturation, and

θ_1 = initial water content.

According to Abdul-razzak and Morel-Seytoux(1983), the seepage rate from a canal is not the recharge rate at the water table at all time. With the wetting front position between the canal bed and initial water table position, and for initially dry soil, the seepage rate varies in time, but the recharge rate is constant and zero. Even after hydraulic connection is established, unless the soil column between canal bed and initial water table position is saturated, the two rates will be different. When the depth of water table from canal bed is large, by the time the wetting front reaches the saturated zone it travels at a velocity approximately equal to the saturated hydraulic conductivity, K. Bouwer (1969) states that for the case of seepage from a canal for the shape of channel given by Kozeny, the vertical downward flow and maximum width of the flow system are essentially reached at a depth of 1.5 (B+2H) below the bed of the canal. The rate of recharge at the time the wetting front meets the ground water table is equal to the seepage rate and remains constant equal to K(B+2H).

An efficient yet accurate hydrologic model on the interaction between river (canal) and the alluvial aquifer has been developed by Morel-Seytoux and Daly(1975).The flow from the canal to the aquifer can be assumed to be linearly dependent on the difference of potentials at the periphery of the canal and in the aquifer near the canal. The following relation has been used by Morel-Seytoux:

$$Q_r(n) = - \Gamma_r[\sigma_r(n) - S_r(n)]$$

in which,

Γ_r = the constant of proportionality known as reach transmissivity,

$\sigma_r(n)$ = draw down of the water level in the canal r... during n^{th} time period measured from a high datum, and

$S_r(n)$ = drawdown of the water table in the aquifer measured from the same datum in the vicinity of the canal.

Using a simple potential theory Morel-Seytoux et al (1979) have derived the following expression of reach transmissivity for a canal embedded in a porous medium underlain by an impervious layer [Fig. 2]:

$$\Gamma_r = \frac{TL_r}{e} \cdot \frac{0.5W_p + e}{5\ W_p + 0.5\ e}$$

in which,

L_r = length of canal reach,

T = transmissivity of the aquifer,

W_p = wetted perimeter of the canal, and

e = saturated thickness below the canal bed.

Herbert (1970) has related the flow from a partially penetrating river, having semicircular cross section [Fig. 3], to the potential difference between the river and in the aquifer below the river bed. The expression is given by:

$$Q_r = \pi L_r K (h - h_o)/\log_e(0.5\, m/r_r)$$

in which,

L_r = length of river reach,

h_r = potential at the river boundary,

h_o = potential in the aquifer below the river bed,

m = saturated thickness of the aquifer, and

r_r = radius of the semicircular river cross section.

The reach transmissivity which could be obtained from from above equation is:

$$\Gamma_r = \pi L_r K/\log_e(0.5\, m/r_r)$$

Estimation of seepage from a canal when watertable is at shallow depth below the canal bed:

Let the time span be discretised by time steps of uniform size. According to the linear relationship, the seepage rate during time step 'n' could be expressed as

$$Q_r(n) = - \Gamma_r[\sigma_r(n) - S_r(n)] \qquad ..(1)$$

in which, $\sigma_r(n)$ is the drawdown to the water level in the canal measured from a high datum during n^{th} unit time period, and $S_r(0,n)$ is the drawdown in the aquifer under the canal measured from the same datum during n^{th} unit time step. According to Herbert (1970), the reach transmissivity, Γ_r, per unit length of canal is given by

$$\Gamma_r = \pi K/\log_e(0.5\ \pi e/W_p) \qquad ..(2)$$

in which, e is the saturated thickness below the canal bed, W_p is the wetted perimeter of the canal, K is the coefficient of permeability. According to Poluborinova-Kochina (1962) the water table rise at time t after the onset of recharge at a distance x from the centre of the strip due to continuous recharge taking place at a uniform rate of R m^3 per unit time per unit area from a strip source of width B is given by

$$S_1(x,t) = \frac{R\alpha t}{2T}\left[\mathrm{Erf}\left(\frac{x+0.5B}{\sqrt{4\alpha t}}\right)-\mathrm{Erf}\left(\frac{x-0.5B}{\sqrt{4\alpha t}}\right)\right]$$

$$+ \frac{R}{4T}\left[(x+0.5B)^2\mathrm{Erf}\left(\frac{x+0.5B}{\sqrt{4\alpha t}}\right)-(x-0.5B)^2\mathrm{Erf}\left(\frac{x-0.5B}{\sqrt{4\alpha t}}\right)\right]$$

$$+ \frac{R\sqrt{\alpha t}}{2T\sqrt{\pi}}\left[(x+0.5B)e^{-\frac{(x+0.5B)^2}{4\alpha t}} - (x-0.5B)e^{-\frac{(x-0.5B)^2}{4\alpha t}}\right]$$

$$- \frac{RB\sqrt{(x^2)}}{2T} \qquad \text{for } x \leq -B/2 \text{ and } x \geq B/2 \qquad ..(3)$$

$$= F(x,B,R,t) - \frac{RB\sqrt{(x^2)}}{2T} \qquad ..(4)$$

For $-B/2 \leq x \leq B/2$, the rise is given by

$$S_1(x,t) = F(x,B,R,t) - \frac{R}{2T}\left(x^2 + \frac{B^2}{4}\right) \qquad ..(5)$$

If unit recharge takes place from unit length of the strip, the recharge rate R will be equal to 1/B. If recharge continues at this rate, the rise in watertable height can be computed with the help of equation (3) and (5) depending on the value of x.

Let unit recharge take place from unit length of the strip during the first unit time step and let no recharge take place after that. Let the corresponding rise in water table be designated as $\partial(x,B,n)$. $\partial(x,B,n)$ coefficient can be computed from the following relationship:

$$\partial(x,B,n) = F(x,B,\tfrac{1}{B},n) - F(x,B,\tfrac{1}{B},n-1) \qquad ..(6)$$

$$\partial(x,B,1) = F(x,B,\tfrac{1}{B},1) - \frac{1}{2T}V(x^2) \quad \text{for } x \leq -B/2,\; x \geq B/2 \qquad ..(7)$$

$$= F(x,B,\tfrac{1}{B},1) - \frac{1}{2BT}\left(x^2 + \frac{B^2}{4}\right) \text{ for } -B/2 \leq x \leq B/2 \qquad ..(8)$$

Let it be assumed that the seepage rate and the recharge rate are equal. When a canal conveys the water, the recharge takes place at a varying rate. Let $Q_R(\gamma)$, $[\gamma=1,2,....n]$ be the seepage rate during time step γ. The rise in water table height at the end of time step n for a variable recharge rate is given by

$$S_1(x,n) = \sum_{\gamma=1}^{n} \partial(x,B,n-\gamma+1)\, Q_R(\gamma) \qquad ..(9)$$

Referring to Fig.4 the depth to water table height below the canal bed can be expressed as

$$S_r(0,n) = D_i - \bar{H} - \sum_{\gamma=1}^{n} \partial(0,B,n-\gamma+1)\, Q_R(\gamma) \qquad ..(10)$$

Replacing $S_r(0,n)$ in equation (1) with the help of equation (10), and rearranging

$$-\frac{Q_r(n)}{\Gamma_r} = \sigma_r(n) - D_i + \bar{H} + \sum_{\gamma=1}^{n} \partial(0,B,n-\gamma+1)\, Q_R(\gamma) \qquad ..(11)$$

Splitting the temporal summation into two parts, one part comprising the summation upto $(n-1)^{th}$ time step, and the second part consisting of the contribution of the n^{th} time step, and solving for $Q_r(n)$,

$$Q_r(n) = [D_i - \bar{\bar{H}} - \sigma_r(n) - \sum_{\gamma=1}^{n-1} \delta(C,B,n-\gamma+1), Q_r(\gamma)]$$

$$[\frac{1}{\Gamma_r} + \delta(O,B,1)] \qquad ..(12)$$

$Q_r(n)$ can be solved in succession starting from time step 1. If the depth of water in the canal remains constant, the term $D_i - \bar{\bar{H}} - \sigma_r(n)$ will be constant and equal to the depth to initial water table position below the canal bed. The decreasing seepage rate can be computed for a straight canal reach from equation (12).

A typical variation of unsteady seepage with time is shown in Fig.4. The seepage rate is more in the beginning and it decreases with time. In the very beginning the seepage rate is equal to the product of reach transmissivity and initial potential difference.

REFERENCES

Abdulrazzak, M.J., and H.J. Morel-Seytoux, (1983), 'Recharge from an Ephemeral Stream Following Wetting Front Arrival to Water Table', Water Resources Research, Vol.19, No.1, pp.194-200.

Bouwer, H., (1969), 'Theory of Seepage from Open Channels' Advances in Hydroscience, edited by V.T. Chow, Vol.5, Academic Press, New York, pp.121-172.

Harr, M.E., (1962),'Ground Water and Seepage', Mc. Graw Hill Book Company, New York.

Herbert, R., (1970),'Modelling Partially Penetrating Rivers on Aquifers Model', Ground Water.Vol.8,pp.29-36.

Morel-Seytoux,H.J., and J.Khanji,(1974), 'Derivation of an Equation of Infiltration', Water Resources Research, Vol.10,No.4,pp.795-800.

Morel-Seytoux, H.J., T.Illangsekare, and G. Peters, (1979), 'Field Verification of the Concept of Reach Transmissivity' I.A.H.S.-A.I.S.H.,Publication No.128,pp.335-359.

Muskat, M., (1946), 'The Flow of Homogeneous Fluids Through Porous Media', J.W. Edwards Publisher Inc., Ann Arbour, Michingan, USA.

Polubarinova - Kochina, P.Ya, (1962),'Theory of Ground Water Movement', Princeton University Press, USA.

Wedernikov, V.V.,(1937), 'Seepage from Triangular and Trapezoidal Channels', angewandete Math. Mech.,Vol.,pp.155-168, vide Harr (1962).