STREAM AQUIFER INTERACTION

INTRODUCTION

The interaction between a river and an aquifer has been examined in some detail in recent years. There are two main aspects of this process: i) the flow from the aquifer to support river flow and the flow from river to the aquifer. Recharge may occur whenever the stage in a river is above that of the adjacent ground water table, provided that the bed comprises permeable or semi-permeable material. This type of ground water recharge may be temporary, seasonal or continuous. Also it may be a natural phenomenon or induced by man. Man can induce ground water recharge from rivers by lowering the water table adjacent to rivers through ground water abstraction.

A river in general penetrates fully or partially the upper aquifer. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. A single aquifer river interaction problem has been studied analytically by several investigators (Morel-Seytoux and Daly,1977,Todd,1955,Cooper and Rorabough, 1963) for finite and infinite aquifer. The expressions for aquifer recharge in the time of varying river stages have been derived by these investigators. The analysis made by Cooper and Rorabough, is for a fully penetrating river. Therefore the influence of the river width on river aquifer interaction cannot be ascertained from their analysis. Morel-Seytoux and Daly (1977) have analysed the river aquifer interaction problem for varying river stage in a partially penetrating river. In the analysis presented by Morel-Seytoux and Daly the river width has been assumed to remain in variant during the variation of stage. During passage of a flood, the stage as well as the river width change. It is therefore pertinent to analyse the river aquifer interaction during passage of a flood incorporating both the changes in river stage and width.

The following expression for exchange of flow between a partially penetrating river and an aquifer has been derived by Morel-Seytoux and Daly. In the derivation the width of a river reach does not change with change in river stage. Starting from the relation $Q_r(n) = \Gamma_r[\sigma_r(n)-S_r(n)]$, in which Γ_r is the reach transmissivity, $\sigma_r(n)$ is the river stage measured from a high datum during time n, $S_r(n)$ is the depth to piezometric surface below the river at time n measured from the same high datum, the following integral equation has been obtained by them:

$$Q_r(t)+\Gamma_r \int_0^t Q_r(\tau)k_{rr}(t-\tau)d\tau = \Gamma_r\,\sigma_r(t) \qquad (1)$$

where k_{rr} (.) is the reach kernel (Morel-Seytoux 1975). The above expression is valid for the case in which the interaction is taking place through a single reach. In case of several pervious reaches the generalized equation has been given as

$$Q_r(t)+\Gamma_r \sum_{\rho=1}^{R} \int_0^t Q_\rho(\tau)k_{r\rho}(t-\tau)d\tau = \Gamma_r\,\sigma_r(t) \qquad (2)$$

where R is the number of reaches. Equation(2) is a system of R integral equation to be solved simultaneously. Discretising the time parameter and assuming the river flow to be uniform within a time step the following solution to the integral equation(2) has been given by Morel-Seytoux and Daly:

$$Q_r(n)+\Gamma_r \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} \partial_{r\rho}(n-\gamma+1)Q_\rho(\gamma) = \Gamma_r\,\sigma_r(n) \qquad (3)$$

in which

$$\underset{r\neq\rho}{\partial_{r\rho}(n)} = \frac{1}{4\pi T}[E\{\frac{\phi d_{r\rho}^2}{4Tn}\}-E\{\frac{\phi d_{r\rho}^2}{4T(n-1)}\}] \qquad (4)$$

$$\partial_{rr}(n) = \frac{1}{\phi ab}\int_0^1 \operatorname{erf}\{\frac{a}{2}[\frac{\phi}{4T(n-\tau)}]^{\frac{1}{2}}\}.\operatorname{erf}\{\frac{b}{2}[\frac{\phi}{4T(n-\tau)}]^{\frac{1}{2}}\}\,d\tau \qquad (5)$$

ϕ = storage coefficient, T = transmissivity, a = length of the river reach, b = width of the river reach, $d_{r\rho}$ = distance from centre of the r^{th} reach to ρ^{th} reach, Γ_r = reach transmissivity of the r^{th} reach, and $\sigma_r(n)$ = river stage of the rth reach during time period n measured from a high datum.

INTERACTION OF AN AQUIFER AND A PARTIALLY PENETRATING STREAM WHOSE WIDTH AND STAGE CHANGE

A schematic section of a partially penetrating river in a homogeneous and isotropic aquifer of infinite areal extent is shown in Fig.1. The river and the aquifer are initially at rest condition. Due to passage of a flood, the river stages changes with time. The changes are identical over a long reach of the river. The width of the river changes with change in river stage. The change may be gradual or abrupt. It is required to find the recharge from the river to the aquifer and the flow from the aquifer to the river after the recession of the flood.

METHODOLOGY

The following assumptions are made for the analysis:

i) The flow in the aquifer is in horizontal direction and one dimensional Boussinesq's equation governs the flow in the aquifer.

ii) The time parameter is discretised. Within each time step, the river stage, width and the exchange flow rate between the river and the aquifer are seperate constant but they vary from step to step.

iii) The exchange of flow between the river and the aquifer is linearly proportional to the difference in the potentials at the river boundary and in the aquifer below the river bed.

ANALYSIS

The differential equation which governs the flow in the aquifer is

$$T \frac{\partial^2 S}{\partial x^2} = \phi \frac{\partial S}{\partial t}$$

in which

S = the water table rise in the aquifer, T = transmissivity of the aquifer, ϕ = storage coefficient of the aquifer.

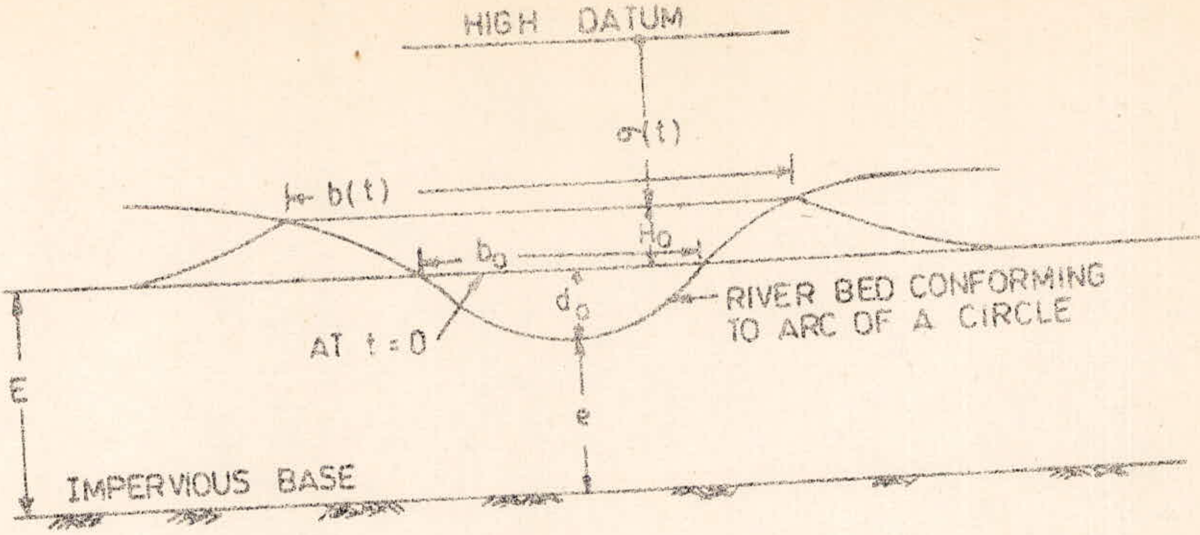

Fig.1(a) Schematic section of a river whose width changes gradually during the passage of a flood

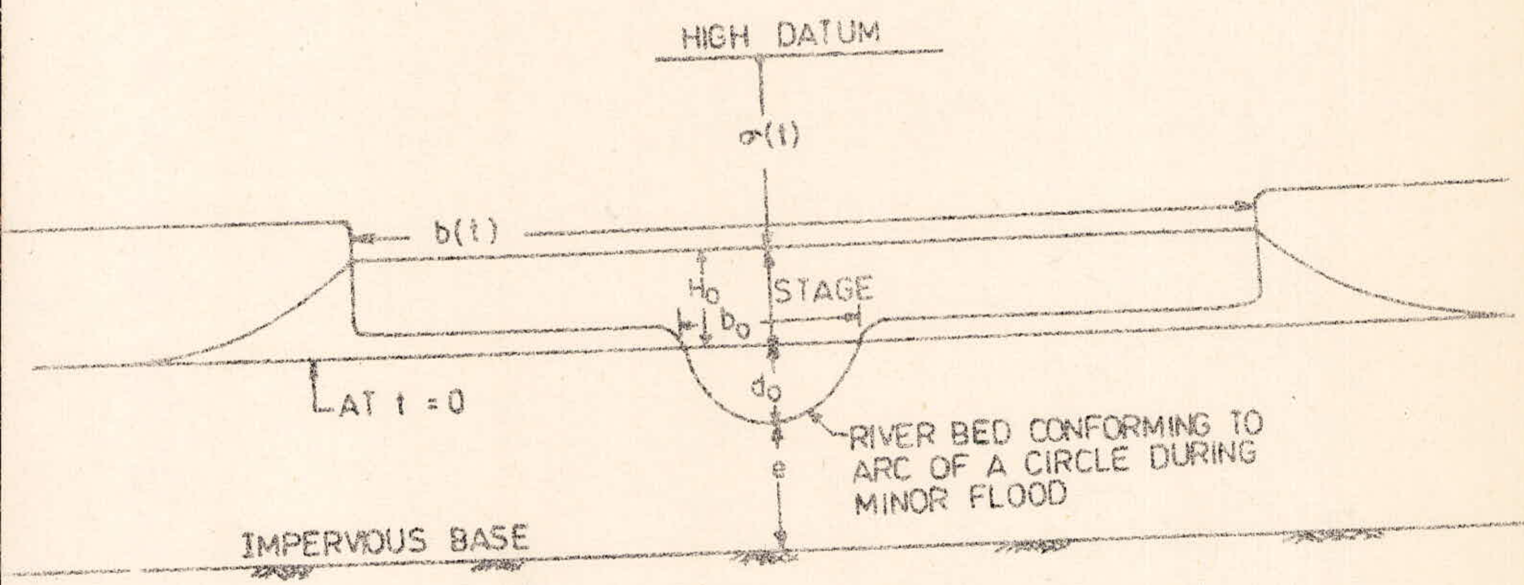

Fig.1(b) Schematic section of a river whose width changes abruptly during the passage of a high flood

If the aquifer and the river were initially at rest, the initial condition to be satisfied is:

$S(x,0) = 0$

The boundary conditions to be satisfied are:

$S(\pm\infty,t) = 0$

At the river and the aquifer interface recharge from the river to the aquifer takes place in a manner similar to that from an overlying bed source to an underlying aquifer through an intervening aquitard. The river resistance and the aquitard resistance are analogous. If the river fully penetrates the upper aquifer, then $S(0,n)=\sigma(n)$ i.e. the rise in water table height at the river is equal to rise in river stage, $\sigma(n)$. For a partially penetrating river $S(0,n)\neq\sigma(n)$ and $S(0,n)$ is to be determined as a part of the solution. The recharge which can be assumed to be linearly proportional to the potential difference, $[\sigma(n)-S(r,n)]$, is to be incorporated at the river boundary

The solution to the problem has been obtained following the principle of superposition.

If the recharge takes place at unit rate per unit length of the river and if the width of the river is 'W', the rise in piezometric surface at distance x from the centre of the river would be as given below:

$$S(x,t)=F(x,T,\phi,W,t)-\frac{[x^2+0.25W^2]}{2TW}$$

for $|x|<\frac{W}{2}$

$$=F(x,T,\phi.W,t)-\frac{\sqrt{x^2}}{2T}$$

for $|x|>\frac{W}{2}$

in which

$$F(x,T,\phi,W,t) = \frac{t}{2\phi W}[\mathrm{erf}\{\frac{x+0.5W}{\sqrt{4\alpha t}}\} - \mathrm{erf}\{\frac{x-0.5W}{\sqrt{4\alpha t}}\}]$$

$$+ \frac{1}{4TW}[\{x+0.5W\}^2 \mathrm{erf}\{\frac{x+0.5W}{\sqrt{4\alpha t}}\} - \{x-0.5W\}^2 \mathrm{erf}\{\frac{x-0.5W}{\sqrt{4\alpha t}}\}]$$

$$+ \frac{\sqrt{\alpha t}}{2TW\sqrt{\pi}}[\{x+0.5W\}\exp\{-\frac{(x+0.5W)^2}{4\alpha t}\} - \{x-0.5W\}\exp\{-\frac{(x-0.5W)^2}{4\alpha t}\}],$$

$\alpha = T/\phi$,

W = width of the river, and

x = distance measured from the centre of the river to the point of observation.

Let the rise in piezometric surface at a distance x from the centre of of the river at the end of n^{th} unit time step if unit recharge takes place from unit length of the river during the first unit time period, in which the river width is $W(1)$, be designated as $\partial[x,W(1),n]$. Hence

$$\partial[x,W(1),n] = F[x,T,\phi,W(1),n] - F[x,T,\phi,W(1),n-1]$$
for $n > 2$

$$\partial[x,W(1),1] = F[x,T,\phi,W(1),1] - \frac{\sqrt{(x)}^2}{2T}$$
for $|x| > W(1)/2$

$$\partial[x,W(1),1] = F[x,T,\phi,W(1),1] - \frac{1}{2TW(1)}[0.25W^2(1) + x^2]$$
for $|x| < W(1)/2$

Dividing the time span into discrete time steps, and assuming that, the recharge per unit length is constant within each time step but varies from step to step, the rise in piezometric surface below the centre of the river due to time variant recharge taking place through varying width can be written as

$$S(0,n) = \sum_{\gamma=1}^{n} q(\gamma)\partial[0,W(\gamma),n-\gamma+1]$$

in which $q(\gamma)$ is the recharge rate per unit length per unit time which is taking place through a width of $W(\gamma)$ during time step γ.

Substituting for $S(0,n)$ in the expression

$$q(n) = \Gamma_r(n)[\sigma(n)-S(0,n)]$$

and simplifying

$$\frac{q(n)}{\Gamma_r(n)} = \sigma(n) - \sum_{\gamma=1}^{n} q(\gamma)\partial[0,W(\gamma),n-\gamma+1]$$

in which $\Gamma_r(n)$ is the reach transmissivity.

Splitting the temporal summation into two parts and rearranging

$$q(n)\{\frac{1}{\Gamma_r(n)} + \partial[0,W(\gamma),1]\} = \sigma(n) - \sum_{\gamma=1}^{n-1} q(\gamma)\partial[0,W(\gamma),n-\gamma+1]$$

Hence,

$$q(n) = \frac{\sigma(n) - \sum_{\gamma=1}^{n-1}[q(\gamma)\partial[0,W(\gamma),n-\gamma+1]}{\frac{1}{\Gamma_r(n)} + \partial[0,W(n),1]}$$

$q(n)$ can be solved in succession starting from time step 1.

STREAM AQUIFER INTERACTION FOR A FULLY PENETRATING STREAM

Let the aquifer and the river be at initially rest condition. Let, there be a sudden drawdown of magnitude σ in the river stage and the river stage remains unchanged in the new position.

Solution to the one dimensional Boussinesq's is equation that satisfies the above initial and boundary condition is given by

$$S(x,t) = \sigma\left[1-\text{erf}\left(\frac{x}{\sqrt{4\frac{T}{\phi}t}}\right)\right]$$

in which $S(x,t)$ is the drawdown at a distance x from the river bank, t is the time measured since the sudden change in the river stage, σ, took place, and $\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-v^2} dv$. The return flow to the river from both sides of the river bank from a reach of length, L_r, is given by

$$q_r(t) = -2L_r T \left.\frac{\partial S}{\partial x}\right|_{x=0} = 2L_r T \left[\frac{2}{\sqrt{\pi}} e^{-\frac{x^2}{4\frac{T}{\phi}t}} \frac{1}{\sqrt{4\frac{T}{\phi}t}}\right]_{x=0}$$

$$= \frac{2 L_r T}{\sqrt{\pi \frac{T}{\phi} t}}$$

If the river stage changes with time the return flow can be computed as follows. Let the time span be discretised by uniform time step. The time step size may be a day or a week or a month. The return flow during n^{th} unit time step is given by

$$q_r(n) = 2L_r\sqrt{\frac{T\phi}{\pi}} \int_0^n \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{n-\tau}}$$

Discretising the time span

$$q_r(n) = 2L_r\sqrt{\frac{T\phi}{\pi}} \int_0^1 \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{n-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_1^2 \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{n-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_{\gamma-1}^{\gamma} \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{n-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_{n-1}^{n} \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{n-\tau}}$$

Let the change in river stage during a particular time step be uniform i.e. $\frac{\partial\sigma}{\partial\tau}$ changes from time step to step. Then

$$q_r(n) = 2L_r \sqrt{\frac{T\phi}{\pi}} \sum_{\gamma=1}^{\gamma} [\sigma(\gamma) - \sigma(\gamma-1)] \int_{\gamma-1}^{\gamma} \frac{d\tau}{\sqrt{n-\tau}}$$

Integrating

$$q_r(n) = 4L_r \sqrt{\frac{T\phi}{\pi}} \sum_{\gamma=1}^{n} [\sigma(\gamma) - \sigma(\gamma-1)](\sqrt{n-\gamma+1} - \sqrt{n-\gamma})$$

Let a coefficient $\delta(m)$ be defined as

$$\delta(m) = 2L_r\sqrt{(\frac{T\phi}{\pi})} \int_{o}^{1} \frac{d\tau}{\sqrt{m-\tau}} = 4L_r\sqrt{(\frac{T\phi}{\pi})}\ (\sqrt{m} - \sqrt{m-1})$$

The return flow $q_r(n)$ can be expressed as

$$q_r(n) = \sum_{\gamma=1}^{n} [\sigma(\gamma) - \sigma(\gamma-1)]\delta(n-\gamma+1)$$

Using this relation the return flow during time period n can be estimated for varying river stage for a fully penetrating river.

REFERENCES

Bouwer, H. 1969. Theory of seepage from open channels. Advances in Hydroscience, edited by V.T. Chow. Academic Press, V. 5, pp.121-172.

Cooper, H.H., Jr., and M.I. Rorabough, 1963. Ground water Movements and Bank Storage Due to Flood Stages in Surface Streams, Geol. Surv. Water Supply Paper 1536-J.

Morel-Seytoux, H.J. 1975. Optimal Legal Conjunctive Operation of Surface and Ground Waters. Proceedings Second World Congress, International Water Resources Association, New Delhi.

Morel-Seytoux, H.J. and C.J. Daly, 1977. A Discrete Kernel Generator for Stream Aquifer Studies. Water Resources Research Journal.

Todd, D.K., 1955. Ground Water flow in relation to a flooding stream, Proc. Amer. Soc. Civil Engineers, Vol.81, Sep. 628, pp.20.