

CONCEPTUAL MODELS FOR IUH DERIVATION

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OBJECTIVES

After attending this lecture, the participants would be able to understand the concept of some well known conceptual models such as Conventional Nash model, Integer Nash model and Clark Model for the derivation of Unit Hydrograph of a particular duration from a rainfall-runoff event.

INTRODUCTION

The direct surface runoff and effective rainfall are assumed to be linearly related with unit hydrograph ordinates. Thus, the ordinates of the unit hydrograph are found through a solution of the set of linear algebraic equations which involves the matrix operations. Even small errors in effective rainfall and direct surface runoff make the general linear solution numerically ill conditioned resulting in unrealistic shape of the unit hydrograph with fluctuating ordinates. The above approach becomes impracticable while relating the unit hydrograph parameters with catchment characteristics. Therefore, it becomes necessary to postulate a general linear hydrologic model (conceptual model) which can be represented by a limited number of parameters. This implies choosing a fixed form or equation for the instantaneous unit hydrograph (IUH) keeping in view the various constraints imposed by the nature of the hydrologic system. However, the number of parameters which is used to define the fixed form of IUH is limited by a number of independent significant relations which can be established with the catchment characteristics. If there are more parameters than this, the extra parameters can not be evaluated from the catchment characteristics for ungauged catchments and must either be given fixed values or related to the other parameters. In either case, the effective number of degrees of freedom is reduced to the number of independent relationships established.

Thus, the following requirements must be fulfilled while choosing a general IUH equation.

- (i) The IUH ordinates are all positive
- (ii) The shape of the IUH is preserved
- (iii) The errors in input data should not be amplified during the IUH derivation.
- (iv) The number of parameters of the chosen form is limited to the number of independent relationships established between the response and the catchment characteristics.
- (v) The form should reflect, as far as possible, the physical relationship between the input and output.
- (vi) Within the restrictions (iv) and (v), the form chosen should be as flexible as possible, particularly, in regard to fitting the observed events.

Now it is required to evaluate the parameter or parameters used in IUH equation to derive its shape. Moreover, it is desirable to evaluate those parameters from the records. In this lecture some well known conceptual models, namely conventional Nash Model, Integer Nash Model and Clark Model, for IUH derivation are discussed. Methodologies for unit hydrograph derivation are also described with the help of illustrative examples for each model.

NASH MODEL CONCEPT

Nash (1957) considered that the instantaneous unit hydrograph could be obtained by routing the instantaneous inflow through a cascade of linear reservoirs with equal storage coefficient. The outflow from the first reservoir is considered as inflow to the second reservoir and so on. Fig.1 illustrates the concept of Nash model.

Procedure to Determine Unit Hydrograph using Nash Model

The procedure to estimate unit hydrograph using Nash model from observed rainfall runoff event, is detailed below:

- (i) *Estimation of effective rainfall and direct surface runoff* : The effective rainfall and direct surface runoff may be computed separating the losses and base flow respectively.
- (ii) *Estimation of moments of effective rainfall and direct surface runoff* : The moments of effective rainfall and direct runoff for use in estimating the parameters of Nash model are to be computed by the following formulae :

$$r^{M'} y = \frac{i = \sum_1^N \frac{Y_i + Y_{i+1}}{2} \Delta t(t_i)'}{i = \sum_{i=1}^N \frac{Y_i + Y_{i+1}}{2} \Delta t}$$

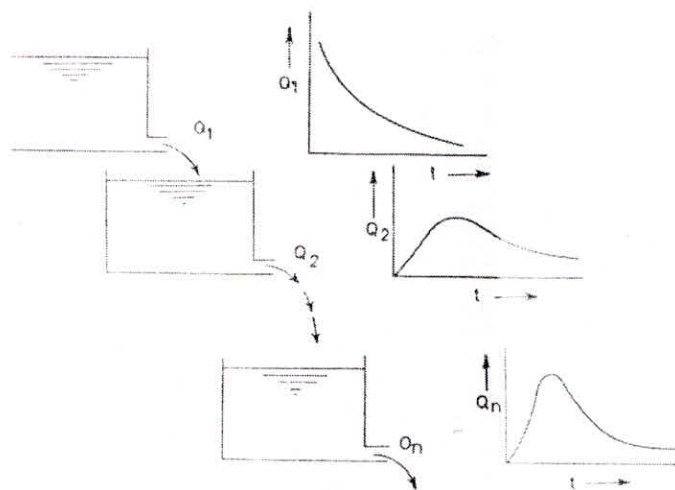


Fig. 1: Nash model concept

$$r^{M'} y = \frac{\sum_{i=1}^N \bar{Y}_i(t_i)^r}{\sum_{i=1}^N \bar{Y}_i} \quad (1)$$

$$R^{M'} x = \frac{\sum_{i=1}^M x_i \Delta t(t_i)^r}{\sum_{i=1}^M x_i \Delta t}$$

Or,

$$r^{M'} x = \frac{\sum_{i=1}^M X_i(t_i)^r}{\sum_{i=1}^M X_i} \quad (2)$$

where

- $r^{M'} y$ is the moment of the direct surface runoff about the origin
- $r^{M'} x$ is the rth moment of the excess rainfall hyetograph about the origin
- Y_i is the ith ordinate of direct surface runoff hydrograph
- X_i is the ith ordinate of excess rainfall hyetograph
- N is the no. of direct surface runoff hydrograph ordinates
- M is the no. of excess rainfall hyetograph ordinates
- Δt is the sampling interval (hrs), and
- t_i is the time to the mid point of the ith interval from the origin

(iii) *Estimation of Parameters of Nash Model* : Nash (1958) introduced the theorem of moments relating the moments of excess rainfall hyetograph (input) and direct surface runoff hydrograph (output) with the moments of instantaneous unit hydrograph.

Using the theorem of moments the first and second moment of effective rainfall and direct surface runoff are related with the corresponding moments of IUH as follows :

$$1^{M'} u = 1^{M'} y - 1^{M'} x \quad (3)$$

$$2^{M'} u = 2^{M'} y - 2_1^{M'} x - 1^{M'} u - 2^{M'} x \quad (4)$$

Where

- $1^{M'} u$ is the first moment of IUH about the origin
- $2^{M'} u$ is the second moment of IUH about the origin
- $1^{M'} y$ is the first moment of direct surface runoff about the origin
- $2^{M'} y$ is the second moment of direct surface runoff about the origin
- $1^{M'} x$ is the first moment of excess rainfall hyetograph about the origin, and
- $2^{M'} x$ is the second moment of excess rainfall hyetograph about the origin

The values of $1^M y$, $2^M u$, $1^M x$ and $2^M x$ can be obtained by using Eq. (1) and (2). However, $1^M u$ and $2^M u$ are related with the parameters of Nash Model as follows :

$$1^M u = nK^2 \quad (5)$$

$$2^M u = (nK)^2 + nK^2 \quad (6)$$

Substituting the values of $1^M u$ and $2^M u$ of the Eq. (5) and (6) into Eq.(3) and (4) the resulting equations may be written as :

$$nK = 1^M y - 1^M x \quad (7)$$

$$n(n+1) K^2 + 2nK 1^M x = 2^M y - 2^M x \quad (8)$$

In eq. (7) and (8) only unknowns are n and K. Therefore these two equations can be solved to estimate the parameters of Nash Model, n and K,

(iv) *Estimation of Instantaneous unit hydrograph (IUH)* : Nash's equation for the ordinate of IUH using the two parameters n and K may be given as :

$$u(t) = \frac{1}{K\Gamma n} (t/K)^{n-1} e^{-t/K} \quad (9)$$

It may be noted that Γn has been used in place of $(n-1)!$ to account for the non integer values of n as computed from the observed data.

(v) *Estimation of unit hydrograph of T hour duration* : The equation for T-hour unit hydrograph is given as follows :

$$U(T,t) = \frac{1}{T} [I(n, t/K) - I(n, (t-T)/K)] \quad (10)$$

Where $I(n,t/K)$ is the incomplete gamma function of order n at (t/K) .

To estimate the ordinates of $U(T,t)$, the tables of incomplete gamma functions are available which enable one to compute $I(n,t/K)$ and $I(n,(t-T)/K)$ for known values of n and K. The computer programme may also be used for this purpose.

(vi) The T-hour unit hydrograph obtained from (Eq.10) is converted to the unit of cumec using the following equation :

$$U'(T,t) = U(T,t) * 0.277 * VOL * CA \quad (11)$$

Where $U'(T,t)$ is T-hour unit hydrograph ordinates in cumec

VOL is the unit volume of T-hour UH in mm, and

CA is the catchment area (Sq.km).

Computational steps for deriving the unit hydrograph by Nash Model :

The following steps are involved for deriving the unit hydrograph by Nash Model using the rainfall-runoff data of a particular storm :

- (i) Obtain mean rainfall values at each computational interval taking the weighted mean of the observed values at different stations.
- (ii) Estimate direct surface runoff separating the baseflow from the discharge hydrograph using the base flow separation techniques
- (iii) Estimate the excess rainfall hyetograph separating the loss from total rainfall hyetograph.
- (iv) Estimate the first and second moment of effective rainfall hyetograph about the origin using the Eq (2)
- (v) Estimate the first and second moment of direct surface runoff hydrograph about the origin using Eq.(1)
- (vi) Solve Eq.(7) and (8) for the parameters n and K using the values of moments obtained from step (iv) and (v)
- (vii) Estimate the unit hydrograph of duration T hours using Eq.(10) and (11).

INTEGER NASH MODEL

To derive unit hydrograph for a specific duration using conventional Nash Model one has to evaluate the incomplete gamma function. When n is not an integer the incomplete gamma function can only be evaluated either from the incomplete gamma function table or the computer programmes. Even the evaluation of n also requires the use of complete gamma function tables or the computer programmes. Such type of computations make the field practitioners somewhat reluctant to use Nash Model in rainfall-runoff studies. Integer Nash Model, which is a simplified form of the conventional Nash Model, takes the parameter n approximated to the nearest integer and computes the incomplete gamma function using a simplified procedure where the use of pearson table is fully avoided or the use of computer programme is not essential. Thus the field practitioners are able to compute the unit hydrograph very easily by this method using simple scientific calculator. The rainfall-runoff data of a storm event is analysed by this method in the following steps :

- (i) Follow the steps (i) to (iv) described for conventional Nash model given in the previous section of this lecture.
- (ii) Consider an integer value of n and modify the values of K to preserve the first moment of IUH taking K equal to the ratio of the first moment of IUH about the origin to the integer value of n . However, one may face the problem of deciding the integer ' n ' value when the computed value of n , for example, lies somewhere nearer to 3.5. In such circumstances one may adopt a value of 3 and 4 and check for its closeness with the second moment of IUH about the centroid (nK^2). The integer n which leads to closer nK^2 is considered for integer Nash model.
- (iii) Derive the unit hydrograph of T -hour duration using the following equations :

$$U(T,t) = \frac{1}{T} [I(n,y) - I(n,y_1)] \tag{12}$$

Where $I(n,y) = 1 - e^{-y} \sum_{m=0}^{n-1} \frac{y^m}{m!}$ (13)

$$I(n,y_1) = 1 - e^{-y_1} \sum_{m=0}^{n-1} \frac{y_1^m}{m!} \tag{14}$$

$$y = t/K \tag{15}$$

$$y_1 = (t-T)/K \tag{16}$$

n = An integer value of n obtained from step (ii)

K = storage co-efficient obtained from step (ii)

(iv) Compute T -hour unit hydrograph ordinates in cumec using the following equation :

$$U'(T,t) = U(T,t) * 0.277 * VOL * CA \tag{17}$$

Where $U'(T,t)$ is T -hour unit hydrograph in cumec,

VOL is unit volume of UH (mm)

CA is catchment area (Sq.km)

CLARK MODEL CONCEPT

Clark (1945) suggested that the IUH can be derived by routing the unit inflow in the form of time area concentration curve, which is constructed from isochronal map, through a single linear reservoir. The isochronal map for a typical watershed is shown in Fig.2. However, Fig.3 shows the time area concentration curve constructed from the isochronal map. The concept of Clark Model for IUH derivation is illustrated in Fig. 4.

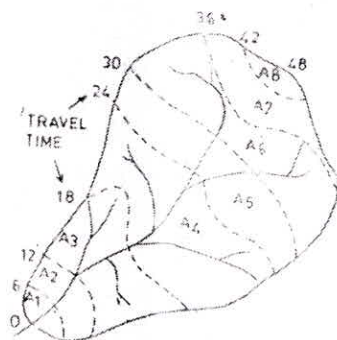


Fig. 2: Isochronal map of a watershed constructed

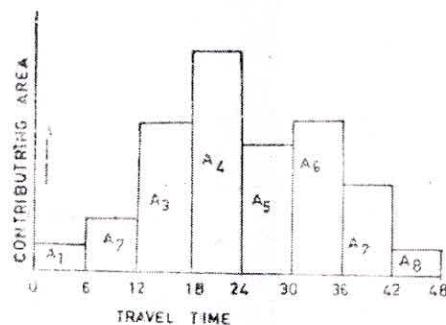


Fig. 3: Time area curve from isochronal map

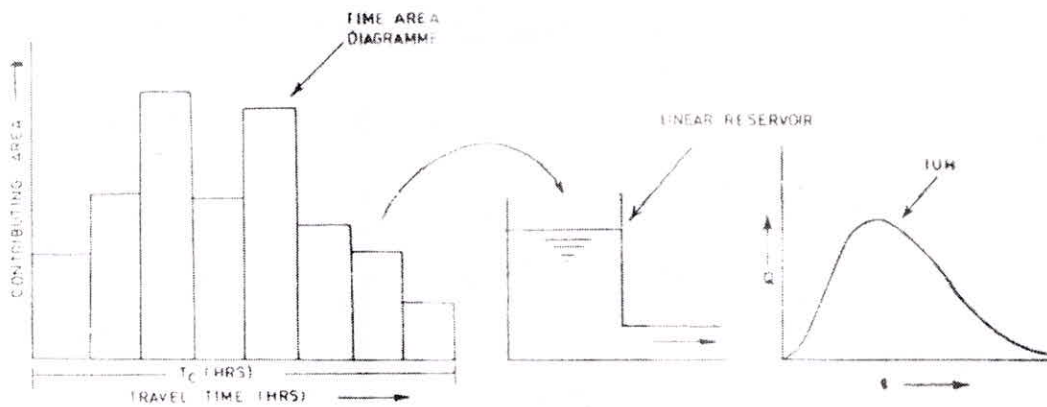


Fig. 4: Clark IUH model concept

Procedure to determine Unit Hydrograph

The procedure to determine unit hydrograph using Clark Model, is detailed below:

- (a) **Estimation of effective rainfall and direct surface runoff** : The effective rainfall and direct surface runoff may be estimated separating the losses from the rainfall hyetograph and base flow from the discharge hydrograph respectively.
- (b) **Estimation of parameters for Clark Model** : The Clark model uses two parameters and a time area relation to define the IUH.
 - (i) **Time of concentration (T_c)** : This represents the travel time of a water particle from the most upstream point in the basin to the out flow location. An initial estimate of this lag time is the time from the end of effective rainfall (plus snowmelt, if any) over the basin to the inflexion point on the recession limb of the surface runoff hydrograph. This time of concentration is used in developing the time area relation.
 - (ii) **Storage co-efficient (R)** : This is an attenuation constant which has the dimension of time. This parameter is used to account for the effect that storage in the river channel has on the hydrograph. This parameter can be estimated by dividing the flow at the point of inflexion of the surface runoff hydrograph by the rate of change of discharge (slope) at the same time. Another technique for estimating R is to compute the volume remaining under the recession limb of the surface runoff hydrograph following the point of inflexion and divide by the flow at the point of inflexion. In either case, R should be an average value determined by using several hydrographs.

Figure 5 illustrates the above definitions of T_c and R from a typical hydrograph. Since the shapes of hydrographs reflect many irregularities of rainfall and stream patterns, therefore the estimate obtained in this way are usually satisfactory only for the first approximation. Hence the best fit parameters should be determined for the best reproduction of hydrographs using some standard optimization techniques.

For better performance of the method :

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- The duration of unit rainfall excess and computation interval Δt must be shorter than T_c . Preferably shorter than one third of T_c
- The storms selected for study should be several times longer than the computation interval Δt in order to provide representative basin coverage of rainfall.

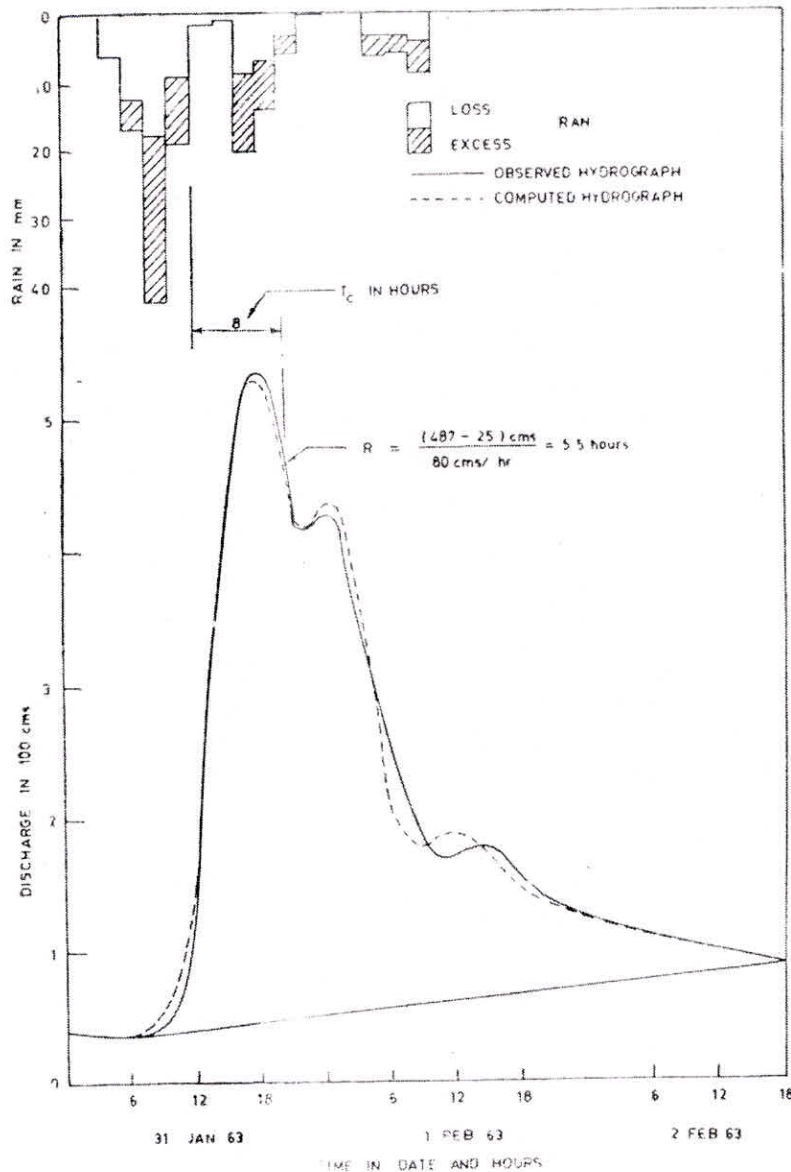


Fig. 5: Determination of Clark coefficients from a typical hydrograph

(iii) Time-area diagrams : The other necessary item to compute an IUH using Clark model is the time-area relation. When T_c has been determined the basin is divided into incremental runoff producing areas that have equal incremental travel times to the outflow location. The computational steps involved, in constructing the time-area curve, are:

- Measure the distance from the most upstream point in the basin to the outflow location along the principal water course.
- Estimate the time of travels as the ratio L/\sqrt{S} along the water course where L is the length of a segment and S is the slope of the segment.
- Laid out the isochrones representing equal times of travel to the outflow location after establishing the location of lines using the ratio L/\sqrt{S} of different segments
- Measure the area between the isochrones and tabulate them in upstream segments versus the corresponding incremental travel time for the each incremental area.

The increment of time used to sub-divide the basin need only be small enough to adequately define the areal distribution of runoff while the time period selected as the computation interval must be equal to or less than the unit duration of excess. Since the former is frequently larger than the later, a plot percent of time of concentration versus accumulative area is useful in determining time-area relationship (Fig.6). Such a curve facilitates rapid development of unit hydrographs for various computation intervals and unit duration of excess.

(c) Estimation of IUH : The resulting shape of time area diagram is routed through a linear reservoir to simulate the storage effects of the basin and the resulting outflow represents the IUH.

Before going for linear reservoir routing, the runoff from the contributing areas (between the isochrones) which would be translated to the outflow location, should be expressed in proper unit. The conversion to proper units of discharge can be made through the relationship.

$$I_i = Ka_i/\Delta t \quad (18)$$

Where I_i = Ordinate in proper units of discharge of the translation hydrograph at the end of period i.

a_i = Ordinates in units of area-depth of excess of the translation hydrograph at the end of period i.

K = Conversion factor to convert a_i to I_i

Δt = Time period of computation interval in hours.

Then the linear reservoir routing is accomplished using the general equation.

$$U_i = C I_i + (1-C)U_{i-1} \quad (19)$$

U_i is the IUH at the period i

U_{i-1} is the IUH at the period i-1

$$C = \frac{\Delta t}{R + 0.5\Delta t} \quad (20)$$

(d) **Estimation of Unit Hydrograph** : The hydrograph that results from routing these flows from the incremental areas is the IUH. The IUH can be converted to a unit hydrograph of unit rainfall duration Δt by simply averaging the two ordinates of IUH spaced at an interval Δt apart as follows :

$$U H_i = 0.5 (U_i + U_{i-1}) \quad (21)$$

The IUH can be converted to a unit hydrograph of some unit rainfall duration other than Δt , provided that it is an exact multiple of Δt , by the following equation :

$$U H_i = \frac{1}{n} [0.5U_{i-n} + U_{i-n+1} + \dots + U_{i-1} + \dots 0.5 U_i] \quad (22)$$

Where,

$U H_i$ = ordinate at time i of unit hydrograph of duration D -hour and computational interval Δt

$$n = \frac{D}{\Delta t}$$

D = Unit hydrograph duration (hours)

Δt = Computational interval (hours)

U_i = Ordinate at time of i of IUH

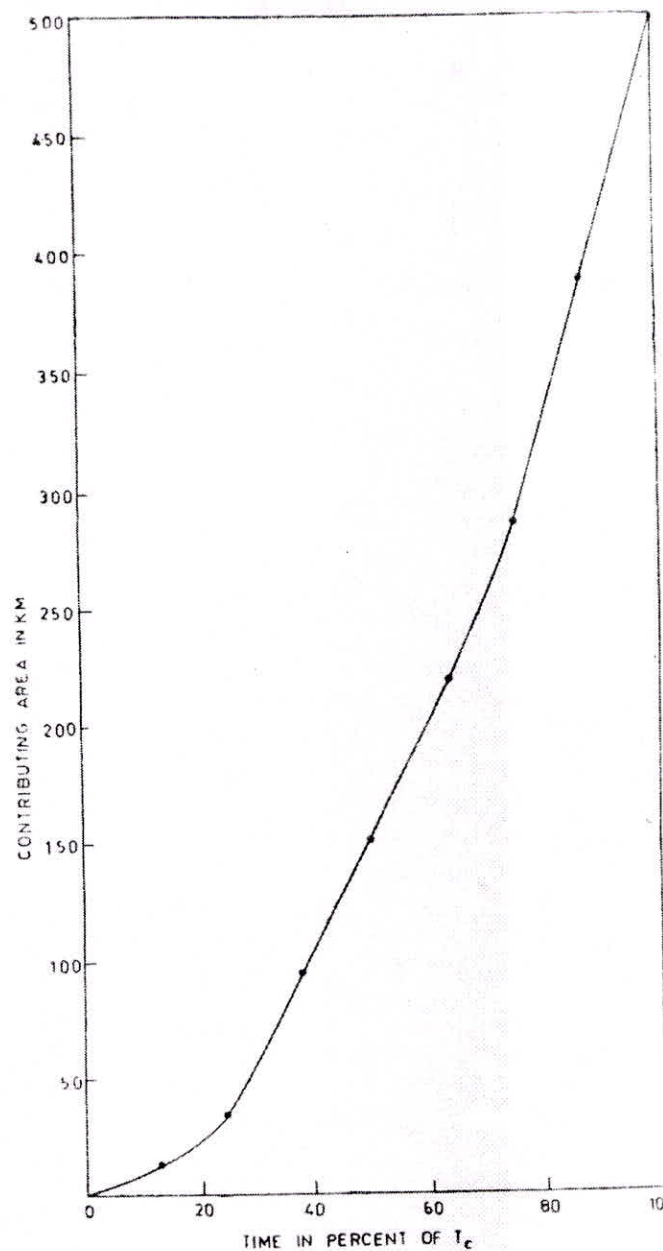


Fig. 6: Watershed time-area relationship

Computational Steps :

The computational steps involved in Clark Model are :

- (i) Make first estimate of Clark Model parameters, T_c and R , from the excess rainfall hyetograph and direct surface runoff hydrograph.
- (ii) Construct the time-area curve, taking the T_c value obtained from step (i), using the procedure described earlier in this lecture.

- (iii) Measure the area between each pair of isochrones by planimeter.
- (iv) Plot the curve of time versus cumulative area. Note that the abscissa is expressed in percent of T_c . Tabulate increments between points that are computational interval Δt apart.
- (v) Convert the units of inflow using the Eq. (18)
- (vi) Route the inflow obtained from step (v) using the Eq. (19) and (20) to get IUH ordinates
- (vii) Compute the unit hydrograph of the excess rainfall duration using Eq. (22) and (23)

BIBLIOGRAPHY

1. Chow, V.T. (1965), 'Runoff' Section 14, Hand Book of Applied Hydrology McGraw Hill book Co. Inc., New York.
2. Nash, J.E. (1959), 'Note on a unit hydrograph study with particular reference to British catchment', Manuscript'.
3. Nash, J.E. (1982-83), 'Lecture notes on Hydrologic System', International Post Graduate course on Hydrology, University College Galway, Ireland (Unpublished)
4. Pearson, K. (1965), 'Tables of the incomplete Gamma function', Published by Charles Criffin and Company ltd. for the Biometrika Trustees.
5. Singh, R.D. and S.M. Seth (1984-85), 'Comparative study of Unit Hydrograph Methods', Report no.CS-7, National Institute of Hydrology, Roorkee
6. Singh, R.D. and S.M. Seth (1984-85), 'Unit Hydrograph Derivation', Report No.UM-8, National Institute of Hydrology, Roorkee
7. Singh, R.D. M. Perumal and S.M. Seth (1986), 'Integer Nash Model for Runoff computation', Proceedings of the 53rd annual Research and Development Session of CBIP held at Bhubaneswar, 8-10 May.
8. U.S. Army Corps of Engineers (1973), 'Hydrograph Analysis; Vol.4, October
9. Wilson, EM (1969), 'Engineering Hydrology, Mac-Millan, London.