RAINFALL-RUNOFF MODELLING USING TANK MODEL

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INTRODUCTION

Various rainfall-runoff models with different levels of complexity have been developed for scientific and operational applications. However, depending on the specific purpose and study region a good model performance is not always assured. Usually, the application of sophisticated models in a different context from the one they were developed provides unsatisfactory results. Measurement based evidence indicating that rainfall alone could sustain river flow laid basis for searching mathematical relationships between precipitation and streamflow. Mathematical problem formulation is convenient in that it provides a documentable and repeatable way for problem definition and analysis. It can also be relatively objective, assuming that the structure of the model is based on well proven, widely accepted hydrological and/or system analytical principles. As simulation of rainfall-runoff relationships has been a prime focus of hydrological research for several decades, an abundance of mathematical models has been proposed to quantify transformation of precipitation into streamflow. Mathematical rainfall-runoff models can crudely be classified as metric, conceptual and physics-based. Metric models are strongly observation-oriented, and they are constructed with little or no consideration of the features and processes associated with the hydrological system. Conceptual models describe all of the component hydrological processes perceived to be of importance as simplified conceptualisations. This usually leads to a system of interconnected stores, which are recharged and depleted by appropriate component processes of the hydrological system. Finally, physicsbased models attempt to mimic the hydrological behaviour of a catchment by using the concepts of classical continuum mechanics.

THE TANK MODEL

The tank model is a very simple model, composed of four tanks laid vertically in series as shown in Figure 1. Precipitation is put into the top tank, and evaporation is subtracted sequentially from the top tank downwards. As each tank is emptied the evaporation shortfall is taken from the next tank down until all tanks are empty. The outputs from the side outlets are the calculated runoffs. The output from the top tank is considered as surface runoff, output from the second tank as intermediate runoff, from the third tank as sub-base runoff and output from the fourth tank as base flow.

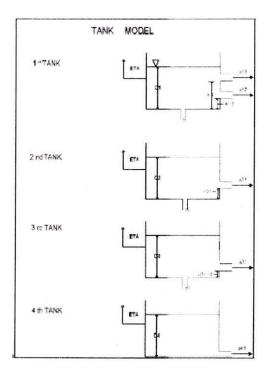


Figure 1: Structure of Tank Model

Despite this simple conceptualisation the behaviour of the tank model is not so simple. The behaviour of the model is strongly influenced by the content of each of the stores. Under the same rainfall and different storage volumes the runoff generated is significantly different. The tank model is applied to analyse daily discharge from daily precipitation and evaporation inputs. The concept of initial loss of precipitation is not necessary, because its effect is included in the non-linear structure of the tank model.

Runoff

The total runoff is calculated as the sum of the runoffs from each of the tanks. The runoff

from each tank is

calculated as

$$q = \sum_{x=1}^{4} \sum_{y=1}^{nx} (C_x - H_{xy}) a_{xy}$$
 (1)

Where q is the runoff depth in mm, C_x the water level of tank. X, H_{xy} the outlet height and a_{xy} is runoff coefficient for the respective tank outlet. Note if the water level is below the outlet no discharge occurs.

Evapotranspiration

The evapotranspiration is calculated using Beken's (1979) equation.

ETA=ETP(1-exp(
$$-\alpha \sum_{X=1}^{4} Cx$$
) (2)

Where ETA is the evapotranspiration in mm, α the evapotranspiration coefficient (0.1) and C_x the water level of tank.

Infiltration

The infiltration in each tank is calculated using:

$$I_{x} = C_{x} B_{x} \tag{3}$$

Where I_x is the infiltration in mm, C_x the water level of tank x and B_x the infiltration coefficient tank x.

Storage

The amount of water in each tank affects the amount of rainfall, infiltration, evaporation and runoff. The storages are calculated from the top to the bottom tank. The evaporation is initially deducted from the first storage up to a maximum of the potential rate. The remaining potential evapotranspiration is taken from each of the lower tanks until the potential rate is reached or all of the tanks have been evaporated. After evaporation has been taken from the tanks rainfall is added to the top tank and based on the revised level runoff and infiltration is estimated. This is subsequently deducted from the storage level. The next tank subsequently receives the infiltration from the tank above. The process continues down through the other tanks.

Default values

The RRL is configured with a set of default values for each model parameter. These default values specify the initial parameter value plus the upper and lower bounds for that parameter. **Table 1** lists the default values for the Tank model.

Table 1. Default parameter values for the Tank model

Parameter	Default value	Default minimum	Default maximum
H11	0	0	500
al 1	0.2	0.0	1.0
a12	0.2	0.0	1.0
a21	0.2	0.0	1.0

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a31	0.2	0.0	1.0
a41	0.2	0.0	1.0
alpha	0.1	0.0	1.0
b1	0.2	0.0	1.0
b2	0.2	0.0	1.0
b3	0.2	0.0	1.0
C1	20	0	100
C2	20	0	100
C3	20	0	100
C4	20	0	100
H12	0	0	300
H21	0	0	100
H31	0	0	100
H41	0	0	100

DESCRIPTION TANK MODEL:

Precipitation is put into the top tank (Figure 2), and evaporation is subtracted from the top tank. If there is no water in the top tank, evaporation is subtracted from the second tank; if there is no water in both the top and the second tank, evaporation is subtracted from the third tank; and so on. The outputs from the side outlets are the calculated runoffs. The output from the top tank is considered as surface runoff, output from the second tank as intermediate runoff, from the third tank as sub-base runoff and output from the fourth tank as baseflow. This may be considered to correspond to the zonal structure of underground water shown typically in Fig. 3. In spite of its simple outlook, the behavior of the tank model is not so simple. If there is no precipitation for a long time, the top and the second tanks will empty and the tank model will look like Fig. 4a or 4b. Under such conditions, runoff is stable. In the case of Fig. 4a the discharge will decrease very slowly, and in the case of Fig. 4b the discharge will be nearly constant. If there is a comparatively heavy rain of short duration under these conditions, the tank model will move to one of the states shown in Fig. 4c and 4d. In these cases, a high discharge of short duration will occur before the model returns to the stable state as before. In these cases, most of the discharge is surface runoff from the top tank and there is little or no runoff from the second tank.

If heavy precipitation occurs over a longer period then the tank model will take on the state shown in Fig. 4e. When the rain stops, the water in the top tank will run off quickly and the tank model will move to the state shown in Fig. 4f. Then, the output from the second tank will decrease slowly, forming the typical downward slope of the hydrograph following a large discharge.

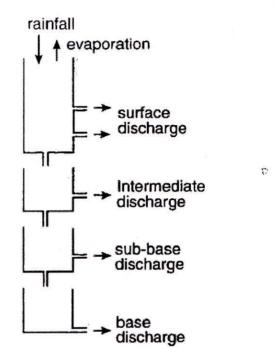


Figure 2: Various components considered in Tank Model

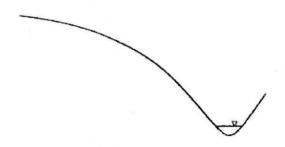


Figure 3: Zonal structure of underground water

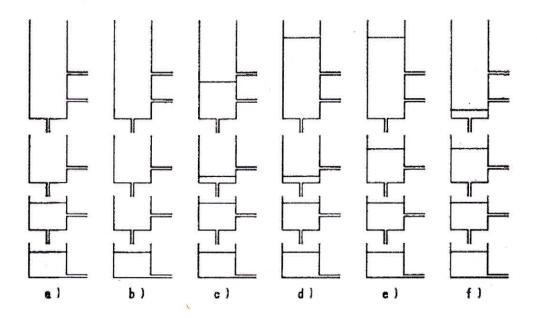


Figure 4: Tank Model behavior under different precipitation conditions

The Tank Model represents such many types of hydrographs because of its non-linear structure caused by setting the side outlets somewhat above the bottom of each tank (except for the lowest tank).

The tank model described above is applied to analyse daily discharge from daily precipitation and evaporation inputs. The concept of initial loss of precipitation is not necessary, because its effect is included in the non-linear structure of the tank model.

For flood analysis the tank model shown in Fig. 5 is applied, where the inputs are usually hourly precipitation and the outputs are hourly discharge. This model contains only two tanks; the third and the fourth tanks are replaced by a constant discharge because the flows from lower tanks form a negligible part of the large flood discharge.

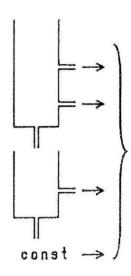


Figure 5: Tank model structure for flood analysis

REMARKS

More basically, the tank model is useful for modeling the surface runoff component. Sugawara presents piece-wise linear models. The parameters for Tank model may have physical meaning. A single tank yields a linear storage model. A series of tanks yields a Nash cascade. Therefore, tank models are very much in the spirit of linear systems analysis for IUH analysis. However tank models have a major advantage and a major disadvantage in terms of mathematical development. Interestingly, the advantage and the disadvantage are the same - the model can be physically visualized. For the empiricist and the engineer that is an advantage. For the theoretician and the mathematician that is a disadvantage.

REFERENCES

Sugawara, M., et al., 1974. Tank model and its application to Bird Creek, Wollombi Brook, Bikin River, Kitsu River, Sanaga River and Nam Mune. Research note of the National Research Center for Disaster Prevention, No.ll:1-64.

Sugawara, M., 1979. Automatic calibration of the tank modeL. Hydrol. Sci. Bull. 24(3):375-388

Rainfall-Runoff Modelling (March 11-15, 2013)

Sugawara, M., et al., 1984. Tank model with snow component. Research note of the National Research Center for Disaster Prevention, No.65: 1-293.

Sugawara, M., 1993. On the weights of precipitation stations. Advances in theoretical hydrology, European Geophys. Soc. series on hydrological sciences, 1, Elsevier.