Training Course

On

Hydrological Processes in an Ungauged Catchment

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CHAPTER-1

Use of Defined Rating Curve Methods

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CHPATER-1

USE OF DEFINED RATING CURVE METHODS

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1.0 INTRODUCTION

A rating table or curve is a relationship between stage (S) and discharge (Q) at a cross section of a river. In most cases, data from stream gages are collected as stage data. In order to model the streams and rivers, the data needs to be expressed as stream flow using rating tables. Conversely, the output from a hydrologic model is a flow, which can then be expressed as stage for dissemination to the public. The development of rating curve involves two steps. In the first step the relationship between stage and discharge is established by measuring the stage and corresponding discharge in the river. And in the second part, stage of river is measured and discharge is calculated by using the relationship established in the first part. For such cases, a series of streamflow measurements using a current meter are plotted versus the accompanying stage, and a smooth curve is drawn through the points. The measured value of discharges when plotted against the corresponding stages show a definite relationship between the two and it represents the integrated effect of a wide range of channel and flow parameters. The combined effect of these parameters is termed control. If the S-Q relationship for a gauging section is constant and does not change with time, the control is said to be permanent. If it changes with time, it is known as shifting control.

There can be significant scatter around this curve. Because of this, when using a rating curve, it is good to keep in mind that the discharge read from the curve is the most likely value, but it could be a little different from the measured value. Also, since rating curves are developed with few stage/discharge measurements, and measurements of high flows are rare, there can be significant errors in rating curves at high levels, especially around record level flows. Rating curves usually have a break point, which is around the stage at which the river spreads out of it's banks, or it could be at a lower stage if the river bed cross section changes dramatically. Above that stage, the river does not rise as fast, given that other conditions remain constant.

1.1 DEVELOPMENT OF RATING CURVE

A majority of streams and rivers, especially non-alluvial rivers exhibit permanent control. For such a case, the relationship between the stage and the discharge is a single-valued relation which is expressed as:

$$Q=a(H-H_0)^b \qquad (1.1)$$

in which Q = stream discharge, H = gauge height (stage), H_o = a constant which represent the gauge reading corresponding to zero discharge, a and b are rating curve constants. This relationship can be expressed graphically by plotting the observed stage against the corresponding discharge values in an arithmetic or logarithmic plot. A typical rating curve is shown in Fig. 1.1. Logarithmic plotting is advantageous as gauge-discharge relationship forms a straight line in logarithmic coordinates. The advantage of using the double logarithmic plot is two fold : (i) firstly, the plot would produce a straight line since the general form of rating curve is parabolic, and (ii) secondly, different straight lines allow to further grouping of data. This is shown in an example in Fig. 1.2. A part of the entire range of stage may form a straight line. It gives an indication about the stage at which the slope of the straight line changes if more than one lines are used to fit the data points.

While plotting the data on double log plot a prior knowledge about the value of H_o is necessary. As a first approximation the value of H_o is assumed to be the level of the bottom of the channel as determined from the cross section of the gauging station. Marginal adjustment in the values of H_o may be required in order to produce a straight line giving better fit to the plotted points. There is a possibility that more than one straight lines are fitted if so required to represent the changing conditions at different stages.

1.1.1 Least Square Method

The best values of a and b for a given range of stage can be obtained by the least-square-error method. Thus by taking logarithms, Eq.(1.1) may be represented as

$$\log Q = \log a + b \log (H - H_0) \qquad (1.2)$$

or $$Y = m X + c$$

in which the dependent variable $Y = \log Q$, independent variable $X = \log (H-H_o)$ and $c = \log a$. The values of the coefficients for the best-fit straight line using data of N observations of X and Y are:

$$m = \frac{N\sum_{i=1}^{N}(X_i Y_i) - (\sum_{i=1}^{N} X_i)(\sum_{i=1}^{N} Y_i)}{N\sum_{i=1}^{N}(X_i)^2 - (\sum_{i=1}^{N} X_i)^2}$$

$$c = \frac{\sum_{i=1}^{N} Y_i - m\sum_{i=1}^{N} X_i}{N} \qquad (1.3)$$

The Eqn. (1.3) is known as the rating equation of the stream and can be used for estimating the discharge Q of the stream for a given gauge reading H within the range of data used in its derivation.

The constant H_o representing the stage (gauge height) for zero discharge in the stream is a hypothetical parameter and can not be measured in the field. As such, its determination poses some difficulties. Different alternative methods are available for its determination. However generally it is found by extrapolating the rating curve by eye judgement to find H_0 as the value of H corresponding to Q=0. Using the value of H_o, plot log Q vs. log $(H - H_o)$ and verify whether the data plots as a straight line. If not, select another value in the neighbourhood of previously assumed value and by trial and error find an acceptable value of H_o which gives a straight line plot of log Q vs. log $(H-H_o)$.

Rating curve is established by concurrent measurements of stage (H) and discharge (Q) covering expected range of river stages at section over a period of time. If Q-H rating curve not unique, then additional information required on (i) Slope of water level (backwater), (ii) Hydrograph in unsteady flow condition.

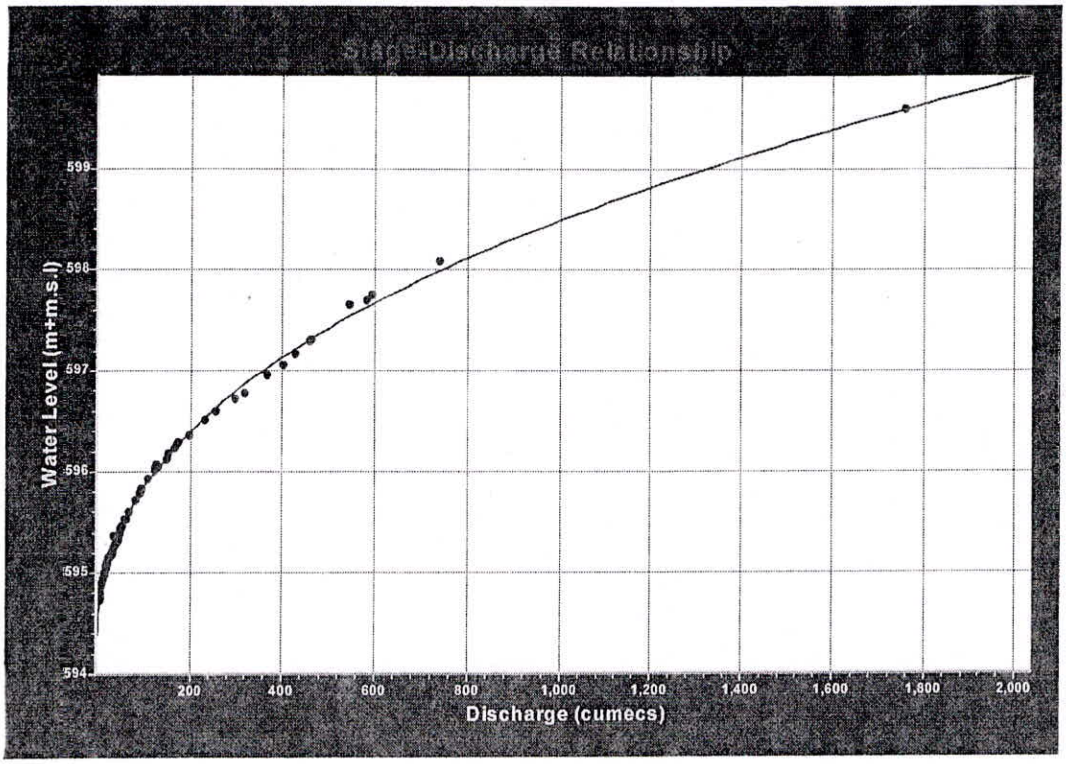

Fig. 1.1 A typical rating curve

Example 1.1

Develop a rating curve for the river Chasnman using the available stage-discharge data during the period January 1997 to December 1997. The rating curve using the power type of equation given in Eq. (1.1) may be used for this case.

No.	Water level (M)	Monthly mean Q (m^3/s)		Difference	Relative difference error (%)
		Observed	Computed		
1	594.800	9.530	8.541	0.989	10.38
2	595.370	36.480	46.661	-10.181	-27.91
3	596.060	127.820	136.679	-8.859	-6.93
4	596.510	231.400	226.659	4.741	2.05
5	598.080	738.850	783.019	-44.169	-5.98
6	597.700	583.340	610.35	-27.019	-4.63

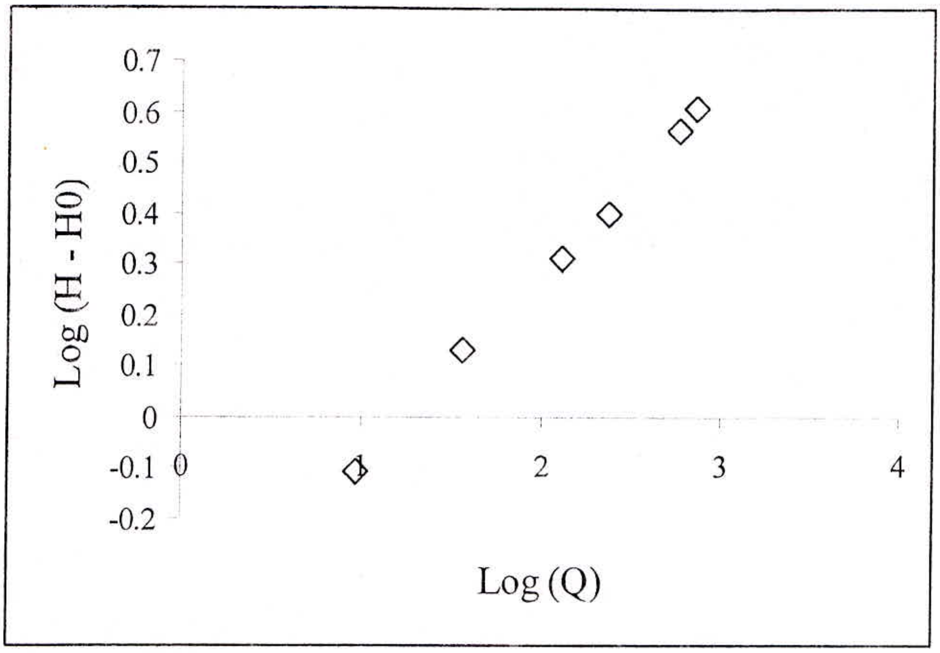

Fig. 1.2 A typical rating curve

a = -594.025 , b = 2.531, c = 22.63

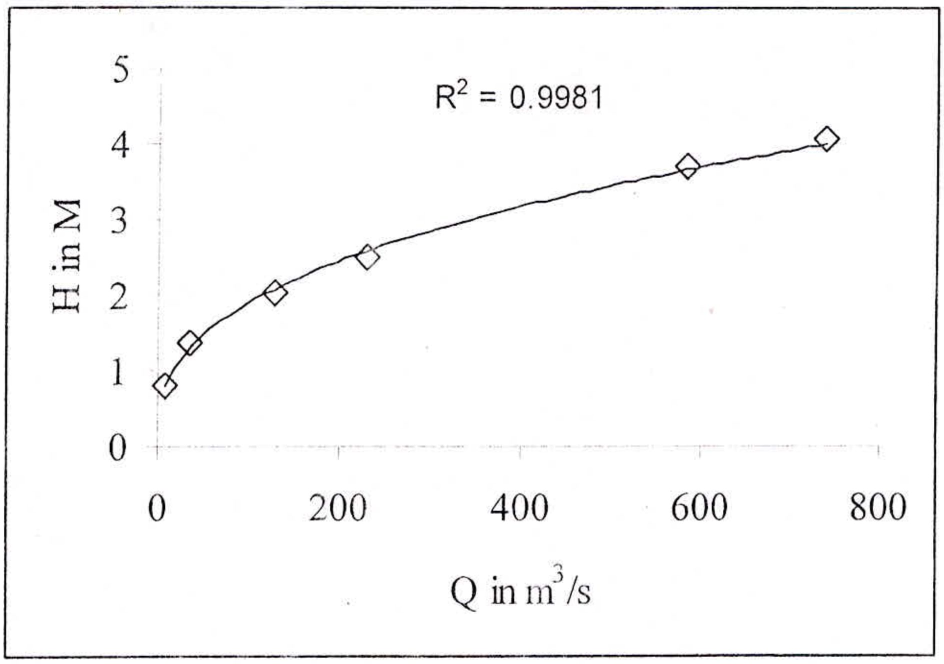

Fig. 1.3 A typical rating curve

A simple way of judging the results with a criteria of goodness-of-fit using the following
Relative difference error (%) = [Observed - Computed]/ Observed x 100
This is given in the last column of table. A minus sign indicates the over-estimated value and

positive the under-estimated value i.e. the estimated or computed monthly discharge value.

1.1.2 Confidence Limits of Rating Curve

Stage-discharge equation is a line of best fit to the measurements, the curve provides a better estimate than any of the individual measurements, however, position of the line is also subject to uncertainty. The measure of certainty for any kind of fitting is given by confidence bands where one can judge the percentage of certainty. This is generally given by standard error of the mean relationship S_{mr} expressed by the following relationship:

$$S_{mr} = S_e \sqrt{\frac{1}{n} + \frac{(P_i - \overline{P})^2}{S_p^{\,2}}}, \quad CL_{95\%} = \pm t S_{mr} \tag{1.4}$$

Where:
t = Student t-value at 95% probability,
$P_i = \ln(h_i + a)$,
S^2_P = variance of P
If n = 25, the $S_{mr} \approx 20\%$ Se indicating the advantage of using the curve over the individual measurements

Example 1.2:

For a typical catchment gauging site , the following monthly mean stage level in M above m.s.l and the corresponding runoff data were observed in the month of July

Year	Monthly mean stage level (H) in meters above msl.	Monthly mean Runoff (Q) in cumecs
1953	42.39	13.26
1954	33.48	3.31
1955	47.67	15.17
1956	50.24	15.50
1957	43.28	14.22
1958	52.60	21.20
1959	31.06	7.70
1960	50.02	17.64
1961	47.08	22.91
1962	47.08	18.89
1963	40.89	12.82
1964	37.31	11.58
1965	37.15	15.17
1966	40.38	10.40
1967	45.39	18.02
1968	41.03	16.25

Compute 95% confidence interval for regression coefficients

The 95% confidence interval for coefficients a and b is determined as follows:

(i) Compute the standard error of the rating curve equation as

$S^2 = \Sigma(y_i - \hat{y}_i)^2/(n - 2) = 123 \times 7/(1-2) = 8.83$

N = degree of freedom = 1 as the variable is one i.e. the runoff.

Therefore, S = 2.97.

(ii) Compute standard error of a (S_Q) as

$$S_Q = S\left(\frac{1}{n} + \frac{\bar{x}^2}{\Sigma(x_i - \bar{x})^2}\right)^{1/2}$$

$$= 2.97\left(\frac{1}{16} + \frac{42.94 \times 42.94}{570 \times 0.0559}\right)^{1/2} = 5.39$$

(ii) Compute $t_{(1-\alpha/2),(n-2)}$ from t-table (Chow, 1964) for $\alpha = 0.05$ and n = 16. It is equal to 2.14.

(iii) Compute 95% confidence interval for Q as given in Eq. 1.4:

$L_Q = a - t_{(1-\alpha/2),(n-2)} S_a = -13.1951 - 2.14 \times 5.39 = 24.73$

$U_Q = a + t_{(1-\alpha/2),(n-2)} S_a = -13.1951 + 2.14 \times 5.39 = -1.66$

1.2 TYPES OF STATION CONTROLS

The development of rating curve involves two steps. In the first step the relationship between stage and discharge is established by measuring the stage and corresponding discharge in the river. And in the second part, stage of river is measured and discharge is calculated by using the relationship established in the first part. Stage is measured by reading gauge installed in the river. If the stage discharge relationship doesn't change with time then it is called *permanent control* and if this relationship changes it is called *shifting control.* In a nutshell, the ccharacter of rating curve depends on type of control, governed by:

a) Geometry of the cross-section
b) Physical features of the river d/s

And, station controls are classified in the following ways:

a) Section and channel controls
b) Natural and artificial controls
c) Complete, compound and partial controls
d) Permanent and shifting controls

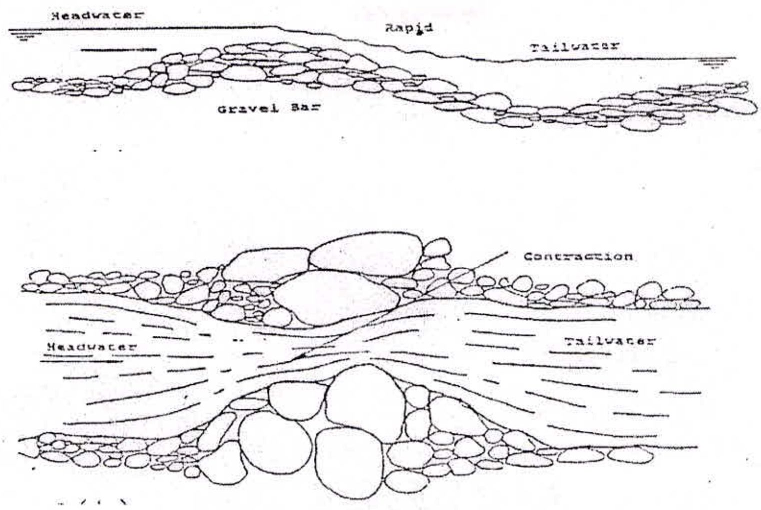

Fig. 1.4a Control configuration in natural channel

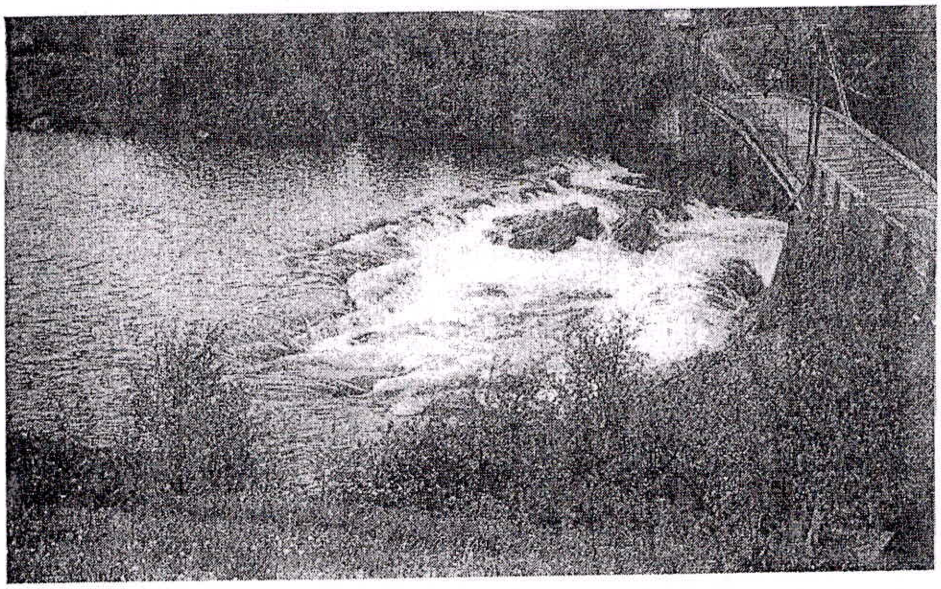

Fig. 1.4b Section control

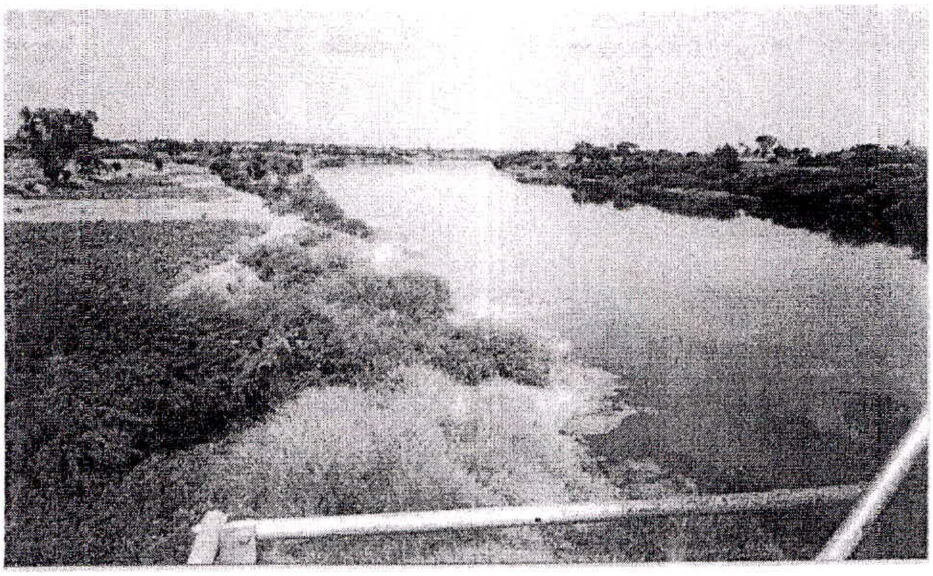

Fig. 1.4c Partial channel control

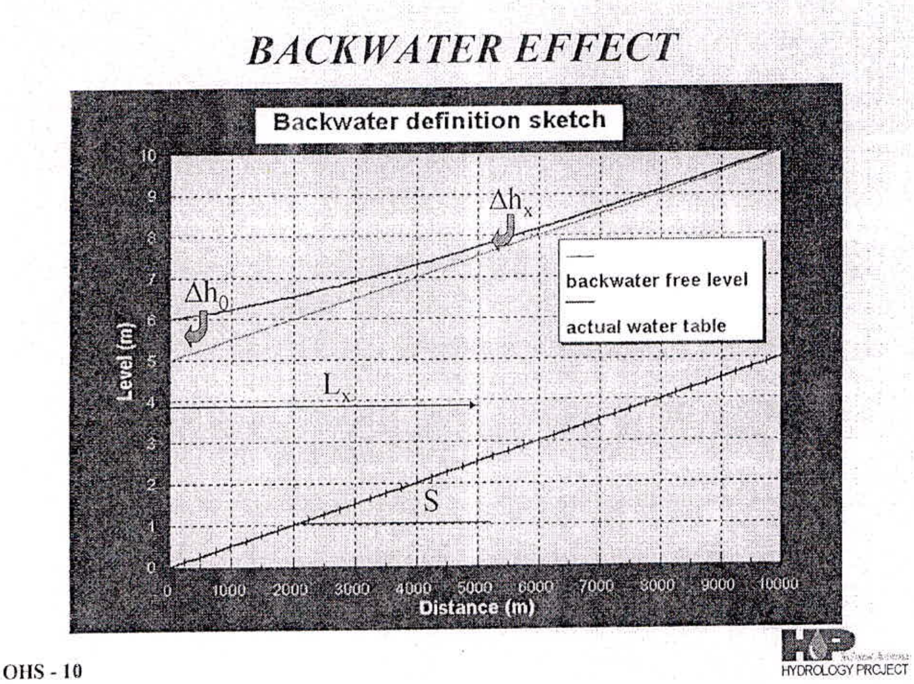

Fig. 1.5 Backwater effect

1.3 EXTENT OF CHANNEL CONTROL

Variable backwater given in Fig. 1.5, causes variable energy slope for the same stage. Hence, discharge is a function of both stage and of slope. The slope-stage-discharge relation is generally is the energy slope approximated by water level slope. The first order approximation of backwater effect in a rectangular channel can be given as follows:

at $x = 0$: $h_0 = he + \Delta h_0$

at $x = Lx$: $h_x = h_e + \Delta h_x$

Backwater: $\Delta h_x = \Delta h_0.\exp[(-3.S.L_x)/(h_e(1-Fr^2)]$

Where,

Froude: $Fr^2 = u^2/(gh)$ often << 1

Manning: $Q = K_m B h_e^{5/3} S^{1/2}$

So with $q = Q/B$: $he = \{q/(K_m S^{1/2})\}^{3/5}$

and $\ln(\Delta h_x/\Delta h_0) = -3.S.L_x/h_e$

At: $\Delta h_x/\Delta h_0 = 0.05$: $L_x = h_e/S$

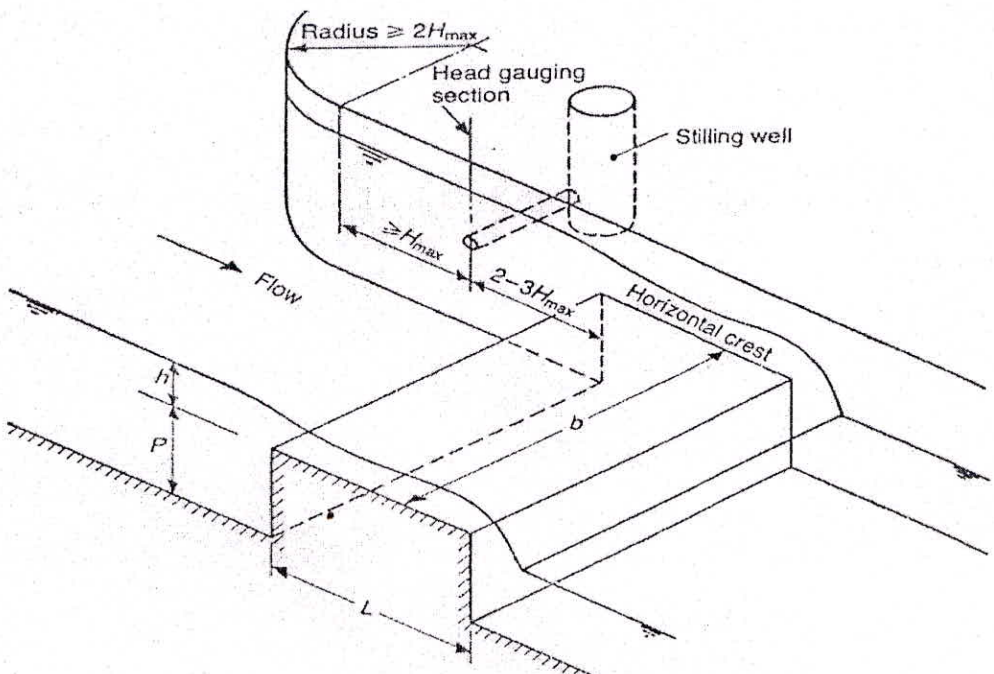

Fig. 1.6 ARTIFICIAL CONTROL

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