Training Course

On

Hydrological Processes in an Ungauged Catchment

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CHAPTER-2

Methods of Developing Rating Curves

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CHPATER-2

METHODS OF DEVELOPING RATING CURVES

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2.0 INTRODUCTION

Stream flow is the variable usually required for hydrological analysis but continuous measurement of flow past a river section is usually impractical, inconvenient, or prohibitively expensive. However, river stage (or the depth of flow with respect to a datum) can be observed continuously or at regular (short) time intervals with comparative ease and economy. Fortunately, a relation exists between stage and the corresponding discharge at a river section. This relation is termed a stage-discharge relationship or stage-discharge rating curve or simply, a rating curve.

A rating curve is established by making a number of concurrent observations of stage and discharge over a period of time covering the expected range of stages and discharges at the river gauging section. At many locations, the discharge is not a unique function of stage; variables such as surface slope or rate of change of stage with respect to time must also be known to obtain the complete relationship in such circumstances. The rating relationship thus established is used to transform the observed stages into the corresponding discharges.

The shape, reliability and stability of the stage-discharge relation are controlled by a section or reach of channel at or downstream from the gauging station which is known as the station control. The establishment and interpretation of stage discharge relationships requires an understanding of the nature of controls and the type(s) of control at a particular station. The channel characteristics forming the control include the cross-section area, shape of the stream channel, expansions and restrictions in the channel, channel sinuosity, the stability and roughness of the streambed, and the vegetation cover. Collectively, these constitute the factors determining the channel conveyance.

2.1 TYPES OF STATION CONTROL

The character of the rating curve depends on the type of control which, in turn, is governed by the geometry of the cross section and physical features of the river downstream of the section. Station controls are classified in four categories:

- section and channel controls
- natural and artificial controls
- complete, compound and partial controls
- permanent and shifting controls

When any change in the physical characteristics of the channel downstream to the control has no effect on the flow at the gauging section, such control is termed as section control. In other words, any disturbance downstream to the control will not be able to pass the control in the upstream direction. Natural or artificial local narrowing of the cross-section (waterfalls, rock bar, gravel bar) creating a zone of acceleration are some examples of section controls. The section control necessarily has a critical flow section at a short distance downstream.

A cross section where no acceleration of flow occurs or where the acceleration is not sufficient to prevent passage of disturbances from the downstream to the upstream direction is called as a channel control. The rating curve in such a case depends upon the geometry and the roughness of the river downstream of the control. The length of the downstream reach of the river affecting the rating curve depends on the normal or equilibrium depth h_e and on the energy slope S [$L \infty h_e/S$, where h_e follows from Manning's $Q = K_m B h_e^{5/3} S^{0.5}$ (wide rectangular channel) so $h_e = [(Q/K_m S^{0.5})^{3/5}]$. The length of channel effective as a control increases with discharge. Generally, the flatter the stream gradient, the longer the reach of channel control (DHV 1999).

An artificial section control or structure control is one which has been specifically constructed to stabilise the relationship between stage and discharge and for which a theoretical relationship is available. These include weirs and flumes, discharging under free flow conditions. Natural section controls include a ledge of rock across a channel, the brink of a waterfall, or a local constriction in width (including bridge openings).

Natural controls vary widely in geometry and stability. Some consist of a single topographical feature such as a rock ledge across the channel at the crest of a rapid or waterfall so forming a complete control. Such a complete control governs the stage-discharge relation throughout the entire range of stage experienced. However, in many cases, station controls are a combination of section control at low stages and a channel control at high stages and are thus called compound or complex controls. The section control begins to drown out with rising tailwater levels so that over a transitional range of stage the flow is dependent both on the elevation and shape of the control and on the tailwater level.

Where the geometry of a section and the resulting stage-discharge relationship does not change with time, it is described as a stable or permanent control. Shifting controls change with time these may be section controls such as boulder, gravel or sand riffles which undergo periodic or near continuous scour and deposition, or they may be channel controls with erodible bed and banks. Shifting controls thus typically result from:

- scour and fill in an unstable channel,
- growth and decay of aquatic weeds, and
- overspilling and ponding in areas adjoining the stream channel.

The amount of gauging effort and maintenance cost to obtain a record of adequate quality is much greater for shifting controls than for permanent controls. Since rating curves for the unstable controls must be updated and/or validated at frequent intervals, regular and frequent measurements are required. In contrast, for stable controls, the rating curve can be established once and needs validation only occasionally. Since stage discharge observations require significant effort and money, it is always preferred to select a gauging site with a section or structure control. However, this is not practicable in many cases and one has to be content with either channel control or a compound control.

2.2 ESTABLISHING THE STAGE DISCHARGE RATING CURVE

If Q and h are discharge (m^3/s) and river stage or elevation of water surface (m), then the relationship can be expressed as:

$$Q = f(h) \tag{2.1}$$

Where *f(h)* is an algebraic function of water level. A graphical presentation of stage discharge curve helps in visualizing the relationship and to transform stages manually to discharges whereas an algebraic relationship can be advantageously used for analytical or computerized transformation.

A simple stage discharge relation is one where discharge depends upon stage only. A complex rating curve occurs where additional variables such as the slope of the energy line or the rate of change of stage with respect to time are required to define the relationship. The need for a particular type of rating curve can be ascertained by first plotting the observed stage and discharge data on a simple orthogonal plot. The scatter in the plot gives a fairly good assessment of the type of stage-discharge relationship required for the cross section. Examples of the scatter plots obtained for various conditions are illustrated below. If there is negligible scatter in the plotted points and it is possible to draw a smooth single valued curve through the plotted points then a simple rating curve is required. This is shown in Fig 2.1.

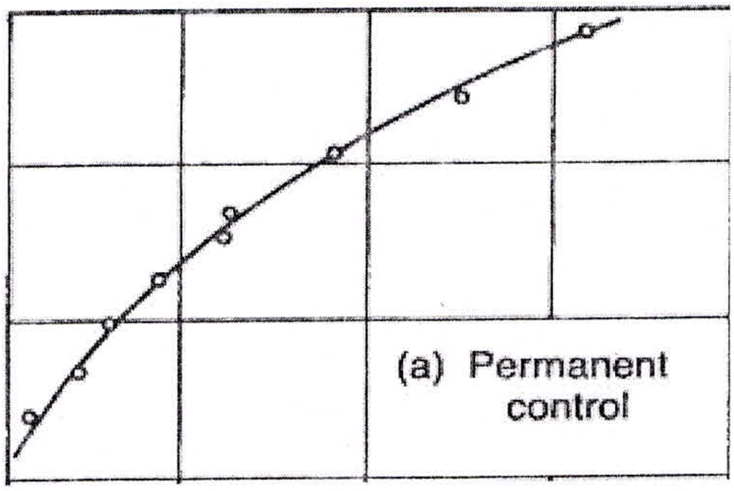

Fig. 2.1 Permanent control

However, if scatter is not negligible then it requires further probing to determine the cause of such higher scatter. There are four distinct possibilities:

a) **Backwater Effects:** If the station is affected by the variable backwater conditions arising due for example to tidal influences or to high flows in a tributary joining downstream, then , in such cases, if the plotted points are annotated with the corresponding slope of energy line (≈surface slope for uniform flows) then a definite pattern can be observed. A smooth curve e passing through those points having normal slopes at various depths is drawn first. It can then be seen that the points with greater variation in slopes from the corresponding normal slopes are located farther from the curve. This is as shown in Fig. 2.2a and b.

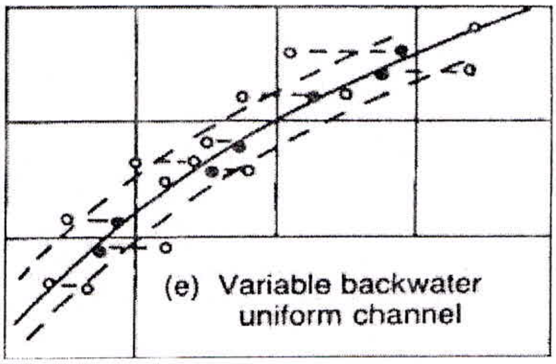

Fig. 2.2a Rating curve affected by variable backwater(uniform channel)

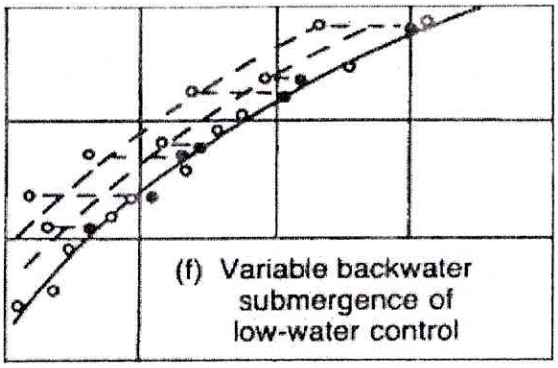

Fig. 2.2b Rating curve affected by variable backwater (submergence of low water control)

b) **Local acceleration effect:** If the stage discharge rating is affected by the variation in the local acceleration due to unsteady flow, in such case, the plotted points can be annotated with the corresponding rate of change of slope with respect to time. A smooth curve (steady state curve) passing through those points having the least values of rate of change of stage is drawn first. It can then be seen that all those points having positive values of rate of change of stage are towards the right side of the curve and those with negative values are towards the left of it. Also, the distance from the steady curve increases with the increase in the magnitude of the rate of change of stage. This is as shown in Fig. 2.3.

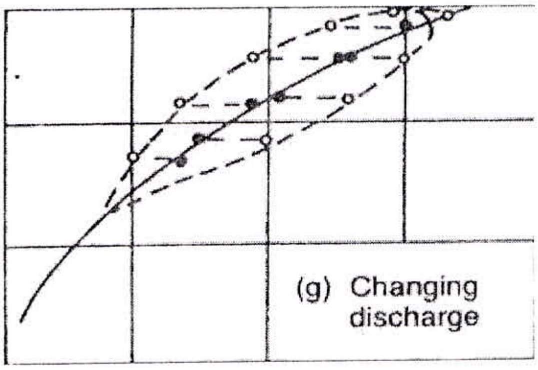

Fig. 2.3 Rating curve affected by unsteady flow

c) **Scouring effect:** The stage discharge rating is affected by scouring of the bed or changes in vegetation characteristics. A shifting bed results in a wide scatter of points on the graph. The changes are erratic and may be progressive or may fluctuate from scour in one event and deposition in another. Examples are shown in Fig. 2.4.

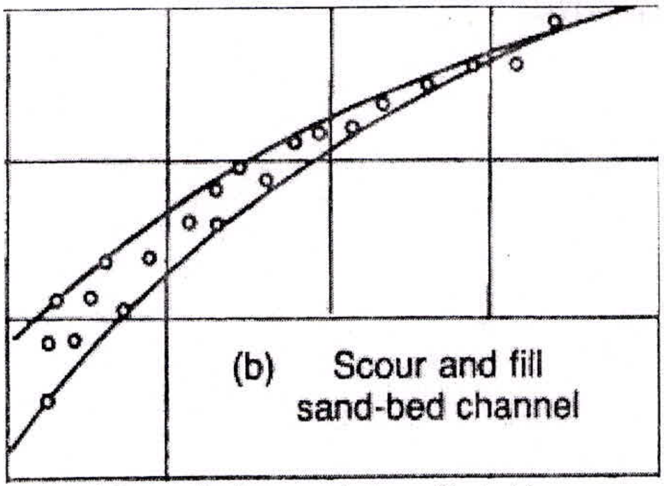

Fig. 2.4 Stage-discharge relation affected by scour and fill

If no suitable explanation can be given for the amount of scatter present in the plot, then it can perhaps be attributed to the observational errors. Such errors can occur due to non-standard procedures for stage discharge observations.

2.3 TYPES OF RATING CURVES

Based on the interpretation of scatter of the stage discharge data, the appropriate type of rating curve is fitted, and there are four main cases as follows:

a) **Simple rating curve**: If simple stage discharge rating is warranted then either single channel or compound channel rating curve is fitted according to whether the flow occurs essentially in the main channel or also extends to the flood plains.

b) **Rating curve with backwater corrections**: If the stage discharge data is affected by the backwater effect then the rating curve incorporating the backwater effects is to be established. This requires additional information on the fall of stage with respect to an auxiliary stage gauging station.

c) **Rating curve with unsteady flow correction**: If the flows are affected by the unsteadiness then the rating curve incorporating the unsteady flow effects is established. This requires information on the rate of change of stage with respect to time corresponding to each stage discharge data.

d) **Rating curve with shift adjustment**: A rating curve with shift adjustment is warranted in case the flows are affected by scouring and variable vegetation effects.

2.3.1 Fitting of single channel simple rating curve

Single channel simple rating curve is fitted in those circumstances when the flow is contained the main channel section and can be assumed to be fairly steady. There is no indication of any backwater affecting the relationship. The bed of the river also does not significantly change so as create any shifts in the stage discharge relationship. A typical scatter plot of the stage and discharge data shows a very little scatter if the observational errors are not significant. The scatter plot of stage discharge data in such situations is shown in Fig. 2.1. The fitting of simple rating curves can conveniently be described under the following headings:

a) equations used and their physical basis,
b) determination of datum correction(s),
c) number and range of rating curve segments,
d) determination of rating curve coefficients,
e) estimation of uncertainty in the stage discharge relationship.

2.3.2 Equations Used and their Physical Basis

Two types of algebraic equations are commonly fitted to stage discharge data:

(1) *Power type equation* which is most commonly used

$$Q = c\ (h + a)^b \qquad (2.1)$$

(2) *Parabolic type of equation*

$$Q = \quad c_2\ (h_w + a)^2 + c_1\ (h_w + \ a) + c_0 \qquad (2.2a)$$

where Q = discharge (m^3/sec), h = measured water level (m), a = water level (m) corresponding to Q = 0, and c_i = coefficients derived for the relationship corresponding to the station characteristics.

The power type equation is most frequently used in India and is recommended. Taking logarithms of equation (2.1) results in a straight line relationship of the form:

$$\log\ (Q) = \log\ (c) + b\ \log\ (h + a) \qquad (2.2b)$$

or $Y = A + B\ X$

That is, if sets of discharge (Q) and the effective stage (h + a) are plotted on the double log scale, they will represent a straight line. Coefficients A and B of the straight line fit are functions of a and b. Since values of a and b can vary at different depths owing to changes in physical characteristics (effective roughness and geometry) at different depths, one or more straight lines will fit the data on double log plot. This is illustrated in Fig. 2.5, which shows a distinct break in the nature of fit in two water level ranges. A plot of the cross section at the gauging section is also often helpful to interpret the changes in the characteristics at different levels.

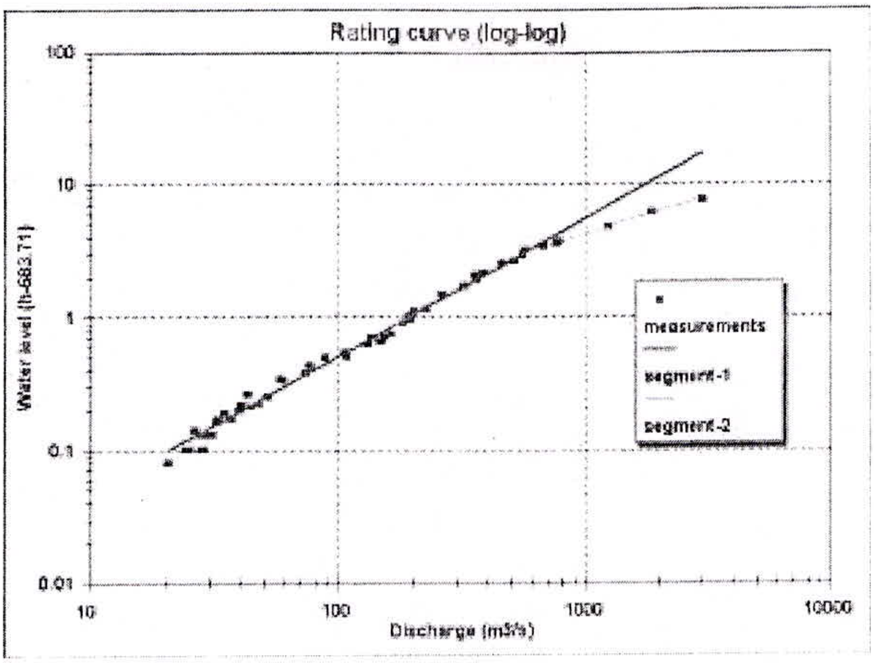

Fig. 2.5 Double logarithmic plot of rating curve showing a distinct break

The relationship between rating curve parameters and physical conditions is also evident if the power and parabolic equations are compared with Manning's equation for determining discharges in steady flow situations. The Manning's equation can be given as:

$$Q = (1/n)A(R)^{2/3}\sqrt{S} \tag{2.3}$$

$Q =$ *function of (roughness and slope) (depth and geometry)*

Hence, the coefficients a, c and d are some measures of roughness and geometry of the control and b is a measure of the geometry of the section at various depths. Changes in the channel resistance and slope with stage, however, will affect the exponent b. The net result of these factors is that the exponent for relatively wide rivers with channel control will vary from about 1.3 to 1.8. For relatively deep narrow rivers with section control, the exponent will commonly be greater than 2 and sometimes exceed a value of 3. Note that for compound channels with flow over the floodplain or braided channels over a limited range of level, very high values of the exponent are sometimes obtained (>5).

2.3.3 Determination of datum correction (a)

The datum correction (a) corresponds to that value of water level for which the flow is zero. From eq. (2.1), it can be seen that for Q = 0, (h + a) = 0 which means: a = -h.

Physically, this level corresponds to the zero flow condition at the control effective at the measuring section. The exact location of the effective control is easily determined for artificial controls or where the control is well defined by a rock ledge forming a section control. For the channel controlled gauging station, the level of deepest point opposite the gauge may give a reasonable indication of datum correction. In some cases identification of the datum correction may be impractical especially where the control is compound and channel control shifts progressively downstream at higher flows. Note that the datum correction may change between different controls and different segments of the rating curve.

The commonly used methods for estimating the datum correction are:

a) trial and error procedure,
b) arithmetic procedure,
c) computer-based optimization.

Wherever possible, the estimates should be verified during field visits and inspection of longitudinal and cross sectional profiles at the measuring section.

Trial and error procedure was the method most commonly used before the advent of computer-based methods. The stage discharge observations are plotted on double log plot and a median line is fitted through them. However, as explained earlier, if the stages are adjusted for zero flow condition, i.e. datum correction a, then this line should be a straight line. This is achieved by taking a trial value of "a" and plotting (h + a), the adjusted stage, and discharge data on the same double log plot. It can be seen that if the unadjusted stage discharge plot is concave downwards then a positive trial value of "a" is needed to make it a straight line. Conversely, a negative trial value is needed to make the line straight if the curve is concave upwards. A few values of "a" can be tried to attain a straight line fit for the plotted points of adjusted stage discharge data. The procedure is illustrated in Fig. 2.6. This procedure is slow but quite effective when done manually. Use of general spreadsheet software for such trial and error procedure can be very convenient and faster.

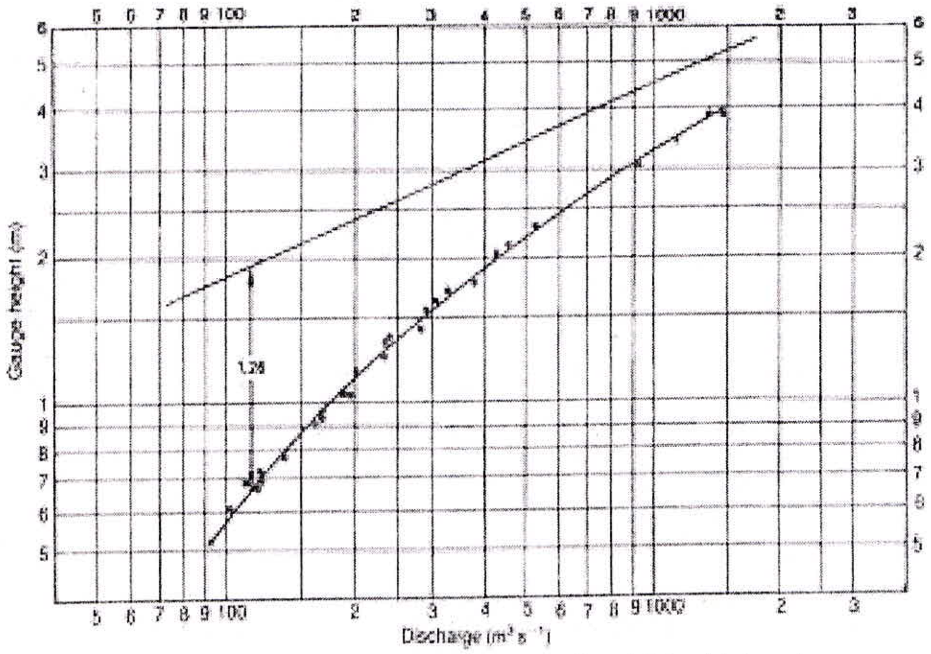

Fig 2.6 Determination of datum correction (a) by trial and error

Arithmetic procedure:

This procedure is based on expressing the datum correction "a" in terms of observed water levels. This is possible by elimination of coefficients b and c from the power type equation between gauge and discharge using simple mathematical manipulation. From the median curve fitting the stage discharge observations, two points are selected in the lower and upper range (Q_1 and Q_3) and the third point Q_2 is computed from $Q_2^2 = Q_1 * Q_3$, such that:

$$\frac{Q_1}{Q_2} = \frac{Q_2}{Q_3} \tag{2.4}$$

If the corresponding gauge heights for these discharges read from the plot are h_1, h_2 and h_3 then using the power type, we obtain:

$$\frac{c(h_1 + a)}{c(h_2 + a)} = \frac{c(h_2 + a)}{c(h_3 + a)} \tag{2.5}$$

which yields

$$a = \frac{h_2^2 - h_1 h_3}{h_1 + h_3 - 2h_2} \tag{2.6}$$

From this equation the value of "a" can be obtained directly. This procedure is known as Johnson method and is described in the WMO Operational Hydrology manual on stream gauging (Report No. 13, 1980).

Optimization procedure:

This procedure is suitable for automatic data processing using computer and "a" is obtained by optimization. The first trial value of the datum correction "a" is either input by the user based on the field survey or from the computerized Johnson method described above. Next, this first estimate of "a" is varied within 2 m so as to obtain a minimum mean square error in the fit. This is a purely mathematical procedure and probably gives the best results on the basis of observed stage discharge data but it is important to make sure that the result is confirmed where possible by physical explanation of the control at the gauging location. The procedure is repeated for each segment of the rating curve.

2.3.4 Number and ranges of rating curve segments:

After the datum correction "a" has been established, the next step is to determine if the rating curve is composed of one or more segments. This is normally selected by the user rather than done automatically by computer. It is done by plotting the adjusted stage, (h-a) or simply "h" where there are multiple segments, and discharge data on the double log scale. This scatter plot can be drawn manually or by computer and the plot is inspected for breaking points. Since for (h-a), on double log scale the plotted points will align as straight lines, breaks are readily identified. The value of "h" at the breaking points give the first estimate of the water levels at which changes in the nature of the rating curve are expected. The number and water level ranges for which different rating curves are to be established is thus noted. For example, Fig. 2.5 shows that two separate rating curves are required for the two ranges of water level – one up to level "h1" and second from "h1" onwards. The rating equation for each of these segments is then established and the breaking points between segments are checked by computer analysis (See below).

2.3.5 Determination of rating curve coefficients:

A least square method is normally employed for estimating the rating curve coefficients. For example, for the power type equation, taking a and b as the estimates of the constants of the straight line fitted to the scatter of points in double log scale, the estimated value of the logarithm of the discharge can be obtained as:

$$Y = \alpha + \beta X \tag{2.7}$$

The least square method minimizes the sum of square of deviations between the logarithms of measured discharges and the estimated discharges obtained from the fitted rating curve. Considering the sum of square the error as E, we can write:

$$E = \sum_{i=1}^{N} (Y_i - Y_i)^2 = \sum_{i=1}^{N} (Y_i - \alpha - \beta X_i)^2 \tag{2.8}$$

Here i denote the individual observed point and N is the total number of observed stage discharge data. Since this error is to be minimum, the slope of partial derivatives of this error with respect to the constants must be zero. In other words:

$$\frac{\partial E}{\partial \alpha} = \frac{\partial \{\sum_{i=1}^{N} (Y_i - \alpha - \beta X_i)^2}{\partial \alpha} = 0 \tag{2.9}$$

and

$$\frac{\partial E}{\partial \beta} = \frac{\partial \{\sum_{i=1}^{N} (Y_i - \alpha - \beta X_i)^2}{\partial \beta} = 0 \tag{2.10}$$

This results in two algebraic equations of the form:

$$\sum_{i=1}^{N} Y_i - \alpha N - \beta \sum_{i=1}^{N} X_i = 0 \tag{2.11}$$

and

$$\sum_{i=1}^{N} (X_i Y_i) - \alpha \sum_{i=1}^{N} X_i - \beta \sum_{i=1}^{N} (X_i)^2 = 0 \tag{2.12}$$

All the quantities in the above equations are known except a and b. Solution of the equations yields:

$$\beta = \frac{N\sum_{i=1}^{N}(X_iY_i) - (\sum_{i=1}^{N}X_i)(\sum_{i=1}^{N}(Y_i)}{N\sum_{i=1}^{N}(X_i)^2 - (\sum_{i=1}^{N}X_i)^2} = 0 \qquad (2.13)$$

and

$$\alpha = \frac{\sum_{i=1}^{N}Y_i - \beta\sum_{i=1}^{N}X_i}{N} = 0 \qquad (2.14)$$

$$b = \beta \qquad \& \qquad c = 10^{\alpha} \qquad (2.15)$$

2.4 COMPOUND CHANNEL RATING CURVE

If the flood plains carry flow over the full cross section, the discharge (for very wide channels) consists of two parts:

$$Q_{river} = (hB_r)(K_{mr}h^{2/3}S^{1/2}) \qquad (2.16)$$

and

$$Q_{floodplain} = (h-h_1)(B-B_r)[K_{mf}(h-h_i)^{2/3}S^{1/2}] \qquad (2.17)$$

assuming that the floodplain has the same slope as the river bed, the total discharge becomes:

$$Q_{total} = hB_r(Kmrh^{2/3}S^{1/2}) + (h-h_1)(B-B_r)[K_{mf}(h-h_i)^{2/3}S^{1/2}] \qquad (2.18)$$

This is illustrated in Fig. 2.9. The rating curve changes significantly as soon as the flood plain at level (h-h_1) is flooded, especially if the ratio of the storage width B to the width of the river bed Br is large. The rating curve for this situation of a compound channel is determined by considering the flow through the floodplain portion separately. This is done to avoid large values of the exponent b and extremely low values for the parameter c in the power equation for the rating curve in the main channel portion.

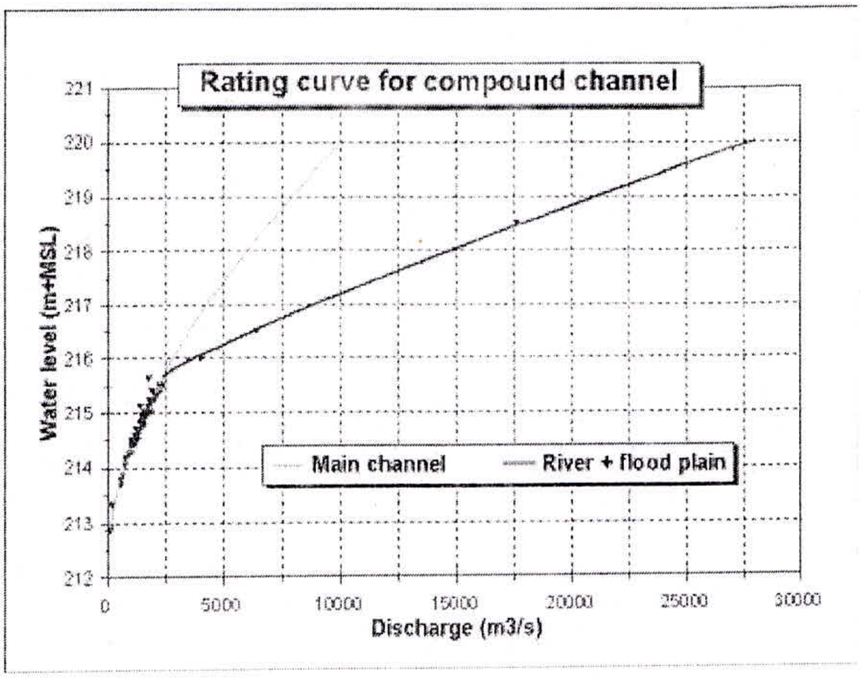

Fig 2.9 Example of rating curve for compound cross-section

The last water level range considered for fitting rating curve is treated for the flood plain water levels. First, the river discharge Q_r will be computed for this last interval by using the parameters computed for the one but last interval. Then a temporary flood plain discharge Q_f is computed by subtracting Q_r from the observed discharge (Q_{obs}) for the last water level interval, i.e.

$$Q_f = Q_{obs} - Q_r \tag{2.19}$$

This discharge Q_f will then be separately used to fit a rating curve for the water levels corresponding to the flood plains. The total discharge in the flood plain is then calculated as the sum of discharges given by the rating curve of the last but one segment applied for water levels in the flood plains and the rating curve established separately for the flood plains.

The rating curve presented in Fig 2.9 for Jhelum river at Rasul reads:

For $h < 215.67$ m + MSL: $Q = 315.2(h-212.38)^{1.706}$
For $h > 215.67$ m + MSL: $Q = 315.2(h-212.38)^{1.706} + 3337.4(h-215.67)^{1.145}$

Hence the last part in the second equation is the contribution of the flood plain to the total river flow.

2.5 RATING CURVE WITH BACKWATER CORRECTION

When the control at the gauging station is influenced by other controls downstream, then the unique relationship between stage and discharge at the gauging station is not maintained. Backwater is an important consideration in streamflow site selection and sites having backwater effects should be avoided if possible. However, many existing stations in India are subject to variable backwater effects and require special methods of discharge determination. Typical examples of backwater effects on gauging stations and the rating curve are as follows:

a) by regulation of water course downstream,
b) level of water in the main river at the confluence downstream,
c) level of water in a reservoir downstream,
d) variable tidal effect occurring downstream of a gauging station,
e) downstream constriction with a variable capacity at any level due to weed growth etc.
f) rivers with return of overbank flow

Backwater from variable controls downstream from the station influences the water surface slope at the station for given stage. When the backwater from the downstream control results in lowering the water surface slope, a smaller discharge passes through the gauging station for the same stage. On the other hand, if the surface slope increases, as in the case of sudden drawdown through a regulator downstream, a greater discharge passes for the same stage.

The presence of backwater does not allow the use of a simple unique rating curve. Variable backwater causes a variable energy slope for the same stage. Discharge is thus a function of both stage and slope and the relation is termed as slope-stage-discharge relation. The stage is measured continuously at the main gauging station. The slope is estimated by continuously observing the stage at an additional gauge station, called the auxiliary gauge station. The auxiliary gauge station is established some distance downstream of the main station. Time synchronization in the observations at the gauges is necessary for precise estimation of slope. The distance between these gauges is kept such that it gives an adequate representation of the slope at the main station and at the same time the uncertainty in the estimation is also smaller. When both main and auxiliary gauges are set to the same datum, the difference between the two stages directly gives the fall in the water surface. Thus, the fall between the main and the auxiliary stations is taken as the measure of surface slope. This fall is taken as the third parameter in the relationship and the rating is therefore also called *stage-fall-discharge relation*.

Discharge using Manning's equation can be expressed as:

$$Q = K_m R^{2/3} S^{1/2} A \tag{2.20}$$

Energy slope represented by the surface water slope can be represented by the fall in level between the main gauge and the auxiliary gauge. The slope-stage-discharge or stage fall- discharge method is represented by:

$$\frac{Q_m}{Q_r} = (\frac{S_m}{S_r})^p = (\frac{F_m}{F_r})^p \tag{2.21}$$

where Qm is the measured (backwater affected) discharge, Qr is a reference discharge, Fm is the measured fall, Fr is a reference fall, and p is a power parameter between 0.4 and 0.6.

From the Manning's equation given above, the exponent "p" would be expected to be ½. The fall (F) or the slope (S = F/L) is obtained by the observing the water levels at the main and auxiliary gauge. Since, there is no assurance that the water surface profile between these gauges is a straight line, the effective value of the exponent can be different from ½ and must be determined empirically. An initial plot of the stage discharge relationship (either manually or by computer) with values of fall against each observation, will show whether the relationship is affected by variable slope, and whether this occurs at all stages or is affected only when the fall reduces below a particular value. In the absence of any channel control, the discharge would be affected by variable fall at all times and the correction is applied by the constant fall method. When the discharge is affected only when the fall reduces below a given value the normal (or limiting) fall method is used.

2.6 CONSTANT FALL METHOD

The constant fall method is applied when the stage-discharge relation is affected by variable fall at all times and for all stages. The fall applicable to each discharge measurement is determined and plotted with each stage discharge observation on the plot. If the observed falls do not vary greatly, an average value (reference fall or constant fall) Fr is selected. For manual computation an iterative graphical procedure is used. Two curves are used (Figs. 2.10 and 2.11):

1. All measurements with fall of about *Fr* are fitted with a curve as a simple stage discharge relation (Fig. 2.10). This gives a relation between the measured stage *h* and the reference discharge Q_r.
2. A second relation, called the adjustment curve, either between the measured fall, *Fm*, or the ratio of the measured fall for each gauging and the constant *fall (Fm / Fr)*, and the discharge ratio (*Qm / Qr*) (Fig. 2.11)

This second curve is then used to refine the stage discharge relationship by calculating Q_r from known values of *Qm* and *Fm/Fr* and then re-plotting h against Q_r. A few iterations may be done to refine the two curves.

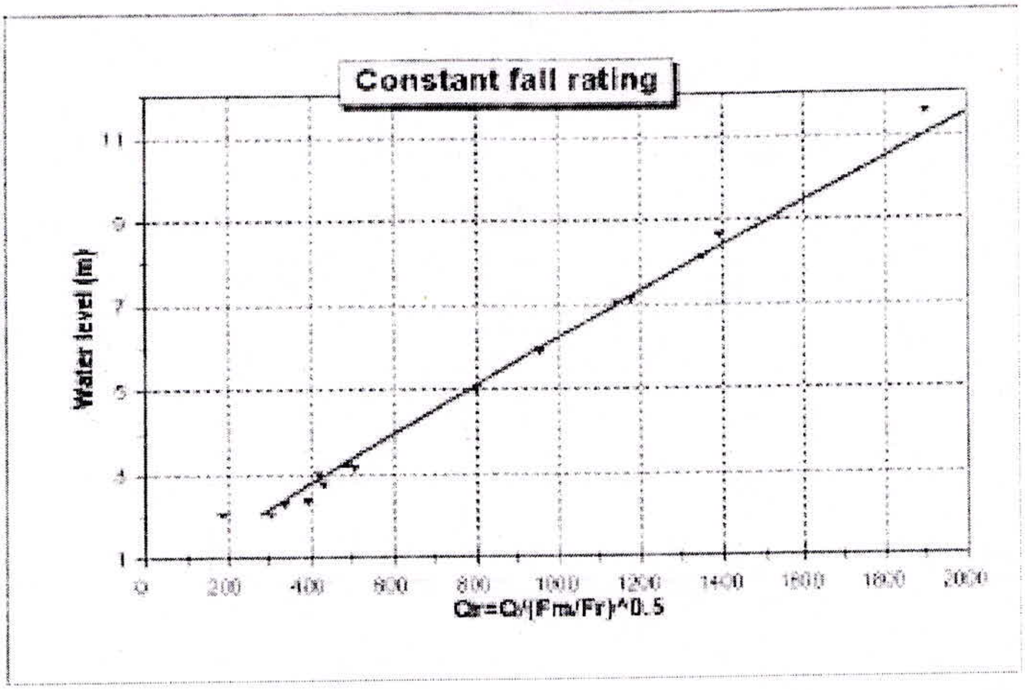

Fig 2.10 Qr=f(h) in constant fall rating

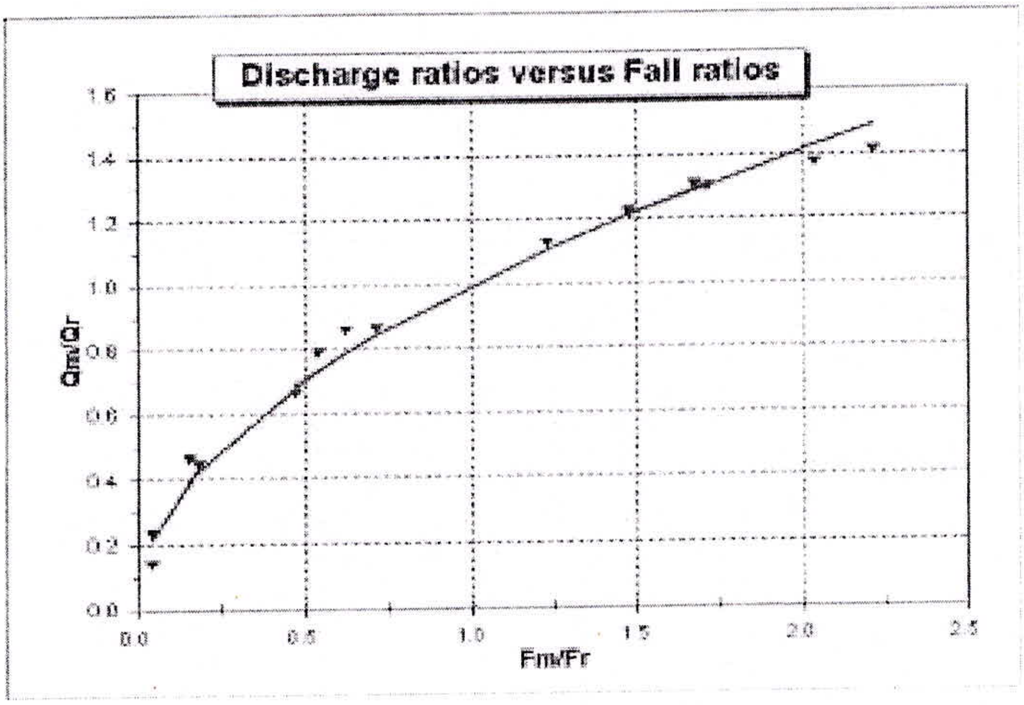

Fig 2.11 Qm/Qr = f(Fm/Fr)

The discharge at any time can be then be computed as follows:

1. For the observed fall (*Fm*), calculate the ratio (*Fm/Fr*),
2. Read the ratio (*Qm / Qr*) from the adjustment curve against the calculated value of (*Fm/Fr*)

3. Multiply the ratio (Qm / Qr) with the reference discharge Qr obtained for the measured stage h from the curve between stage h and reference discharge Qr.

2.7 NORMAL FALL METHOD

The normal or limiting fall method is used when there are times when backwater is not present at the station. Examples are when a downstream reservoir is drawn down or where there is low water in a downstream tributary or main river. The simplified procedure for such cases is given below step by step.

a) Compute the backwater-free rating curve using selected current meter gauging (the Q_r-h relationship).
b) Using values of Qr derived from (1) and Fr derived from the following relation

$$F_r = F_m\left(\frac{Q_r}{Q_m}\right)^{1/p} \tag{2.22}$$

A parabola is fitted to the reference fall in relation to stage *(h)* as:

$$F_r = a + bh + ch^2 \tag{2.23}$$

The parameter p is optimized between 0.4 and 0.6.

The discharge at any time, corresponding to the measured stage h and fall Fm, is then calculated by:

a) Obtaining Fr for the observed h from the parabolic relation between h and Fr
b) Obtaining Qr from the backwater free relationship established between h and Qr
c) Then calculate discharge corresponding to measured stage h as:

$$Q = Q_r\left(\frac{F_m}{F_r}\right)^p \tag{2.24}$$

2.8 RATING CURVE WITH UNSTEADY FLOW CORRECTION

Gauging stations not subjected to variable slope because of backwater may still be affected by variations in the water surface slope due to high rates of change in stage. This occurs when the flow is highly unsteady and the water level is changing rapidly. At stream gauging stations located in a reach where the slope is very flat, the stage-discharge relation is frequently affected by the superimposed slope of the rising and falling limb of the passing flood wave. During the rising stage, the velocity and discharge are normally greater than they would be for the same stage under steady flow conditions. Conversely, during the falling stage the discharge is normally less for any given gauge height than it is when the stage is constant. This is due to the fact that the approaching velocities in the advancing portion of the wave are larger than in a steady uniform flow at the corresponding stages. In the receding phase of the flood wave the converse situation occurs with reduced approach velocities giving lower discharges than in equivalent steady state case.

Thus, the stage discharge relationship for an unsteady flow will not be a single-valued relationship as in steady flow but it will be a looped curve as shown in the example below. The looping in the stage discharge curve is also called hysteresis in the stage-discharge relationship. From the curve it can be easily seen that at the same stage, more discharge passes through the river during rising stages than in the falling ones.

2.8.1 Application

For practical purposes the discharge rating must be developed by the application of adjustment factors that relate unsteady flow to steady flow. Omitting the acceleration terms in the dynamic flow equation, the relation between the unsteady and steady discharge is expressed in the following form which is known as the Jones formula:

$$Q_m = Q_r \sqrt{\left(1 + \frac{h}{cS_0}\frac{dh}{dt}\right)} \qquad (2.25)$$

where Qm is measured discharge, Q_r is estimated steady state discharge from the rating curve, c is wave velocity (celerity), S_0 is energy slope for steady state flow, *dh/dt* is rate of change of stage derived from the difference in gauge height at the beginning and end of a gauging (+ for rising; - for falling). Here, Q_r is the steady state discharge and is obtained by establishing a rating curve as a median curve through the uncorrected stage discharge observations or using those observations for which the rate of change of stage had been negligible. Care is taken to see that there are sufficient number of gauging on rising and falling limbs if the unsteady state observations are considered while establishing the steady state rating curve.

Rearranging the above equation gives:

$$\frac{1}{cS_0} = \frac{(Q_m/Q_r)^2 - 1}{dh/dt} \qquad (2.26)$$

The quantity *(dh/dt)* is obtained by knowing the stage at the beginning and end of the stage discharge observation or from the continuous stage record. Thus the value of factor ($1/cS_0$) can be obtained by the above relationship for every observed stage. The factor ($1/cS_0$) varies with stage and a parabola is fitted to its estimated values and stage as:

$$\frac{1}{cS_0} = a + bh + ch^2 \qquad (2.27)$$

A minimum stage h_{min} is specified beyond which the above relation is valid. A maximum value of factor ($1/cS_0$) is also specified so that unacceptably high value can be avoided from taking part in the fitting of the parabola. Thus unsteady flow corrections can be estimated by the following steps:

a) Measured discharge is plotted against stage and beside each plotted point is noted the value of *dh/dt* for the measurement (+ or -)

b) A trial *Qs* rating curve representing the steady flow condition where *dh/dt* equals zero is fitted to the plotted discharge measurements.

c) A steady state discharge Q_r is then estimated from the curve for each discharge measurement and Q_m, Q_r and *dh/dt* are together used in the Equation (2.26) to compute corresponding values of the adjustment factor ($1 / cS_0$)

d) Computed values of $1 / (cS_0)$ *are* then plotted against stage and a smooth (parabolic) curve is fitted to the plotted points. For obtaining unsteady flow discharge from the steady rating curve the following steps are followed:

i. obtain the steady state flow *Qr* for the measured stage *h*
ii. obtain factor ($1/cS_0$) by substituting stage *h* in the parabolic relation between the two
iii. obtain (*dh/dt*) from stage discharge observation timings or continuous stage records
iv. substitute the above three quantities in the Equation (2.26) to obtain the true unsteady flow.

It is apparent from the above discussions and relationships that the effects of unsteady flow on the rating are mainly observed in larger rivers with very flat bed slopes (with channel control extending far downstream) together with significant rate change in the flow rates. For rivers with steep slopes, the looping effect is rarely of practical consequence. Although there will be variations depending on the catchment climate and topography, the potential effects of rapidly changing discharge on the rating should be investigated in rivers with a slope of 1 metre/ km or less. Possibility of a significant unsteady effect (say more than 8–10%) can be judged easily by making a rough estimate of ratio of unsteady flow value with that of the steady flow value.

2.8.2 Shift in Rating Curve

The properties of a permanent control do not change with time. In this case, it is not necessary to update the rating curve frequently. However, very few channels are stable over all stages and at all times. If the geometry and properties of the cross-section change with time due to scour or deposition, growth of vegetation, etc., this is known as shifting control. Such a situation requires frequent updating of the rating curve. The changes in the properties of cross-section may be slow due to processes, such as erosion and deposition or these may be sudden due to alterations in channel. The growth and decay of vegetation in the cross-section and downstream channel may also result in seasonal changes in the properties of the control.

The changes in channel and flow properties may be either gradual or abrupt and this change is also reflected in shift in the rating curve. It is important to note that the stage-discharge relation is subject to small random fluctuations and the rating curve usually represents an average condition. A shift in the rating curve is said to have occurred if a group of consecutive observations of discharge plots a certain percentage (say, about 5%) either to the left or the right of the established rating curve. If, however, only a few points depart from the curve, this could be due to random errors that are associated with streamflow measurements. If the change in the rating curve lasts for a shorter duration, the original curve is used with adjustments but if the duration is a month or longer, it signals that a new curve may have to be developed.

The vertical movement or shift of the rating curve is accounted for through a shift adjustment. In the simplest case, for a discharge measurement, the shift adjustment is obtained by subtracting the observed gauge value from the gauge value which corresponds to that discharge on the rating curve. To obtain discharge from the rating curve under shifting conditions, the shift adjustment is added to the gauge height and then the rating curve is used to read discharge. For example, consider that the growth of vegetation in the channel, which increases roughness, has resulted in a shift of the rating curve to the left and now, for a given discharge, stages are 0.1m higher than the original condition. In the new situation, if the measured stage is 1.5m, the discharge is read from the rating curve corresponding to the stage value 1.5 – 0.1 = 1.4m. Of course, several discharge measurements are not likely to give the same shift adjustment and an average value may be used in such cases.

2.8.3 Extrapolation of Rating Curve

Rating curves are required to be extrapolated in situations where the discharge measurements are not available over the entire range of observed river stages. In simple cases, the rating curve may be smoothly extended but this will not be correct if, in the extended range, there is a change in the channel geometry, there is flow over flood plain, and the roughness coefficient changes significantly.

For extrapolation of the rating curve in low flow ranges, it is better to plot the curve on a simple graph paper since zero discharge cannot be plotted on a log scale. Now, a smooth curve is drawn to join the gage of zero flow with the lowest end of the rating curve (see Fig. 2.12).

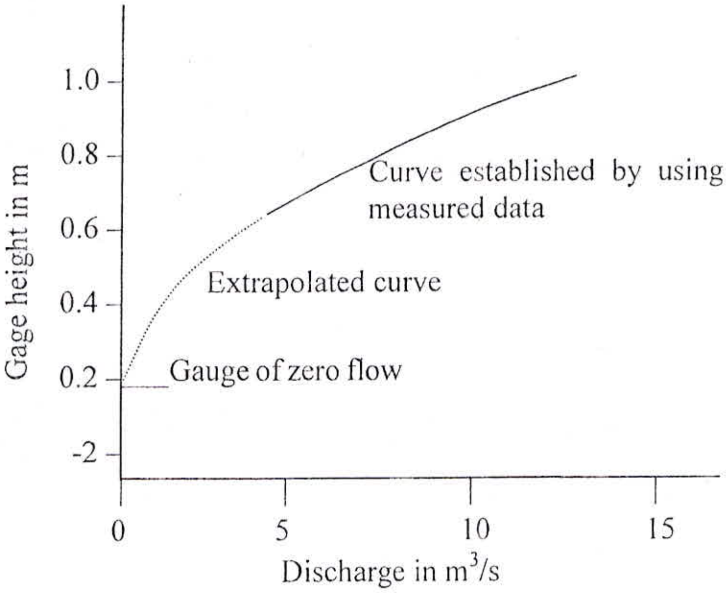

Fig. 2.12 Extrapolation of rating curve in lower ranges.

The extrapolation of the rating curve in high flow ranges should be done with utmost caution and should be attempted if indirect methods of discharge estimation such as the slope-area method cannot be used. Among the methods that can be used to extrapolate a rating curve, Rantz (1982) recommends the use of the conveyance-slope method as it is superior to the velocity-area method. The concept of conveyance can be understood from the Manning's equation:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} = K S^{1/2} \tag{9}$$

Here K is known as the conveyance and it includes all the elements of cross-section that can be measured or estimated. A curve of K vs gauge height, covering the entire range of gauge height, is plotted on an ordinary graph paper. Now, for the measured discharges, S can be obtained by dividing the discharge by the corresponding K and squaring the result. Using these data, a curve of gauge height vs S is plotted and is extrapolated to the required gauge height (see Fig. 2.13). It is helpful to remember that slope tends to attain a constant value at higher river stages under certain conditions. The discharge at the desired gage can be obtained by reading the values of K and S from the respective curves, taking the square root of S and multiplying it by K. Rantz (1982) recommends that the extrapolation of discharge beyond a value exceeding twice the measured discharge should be attempted only as a last resort.

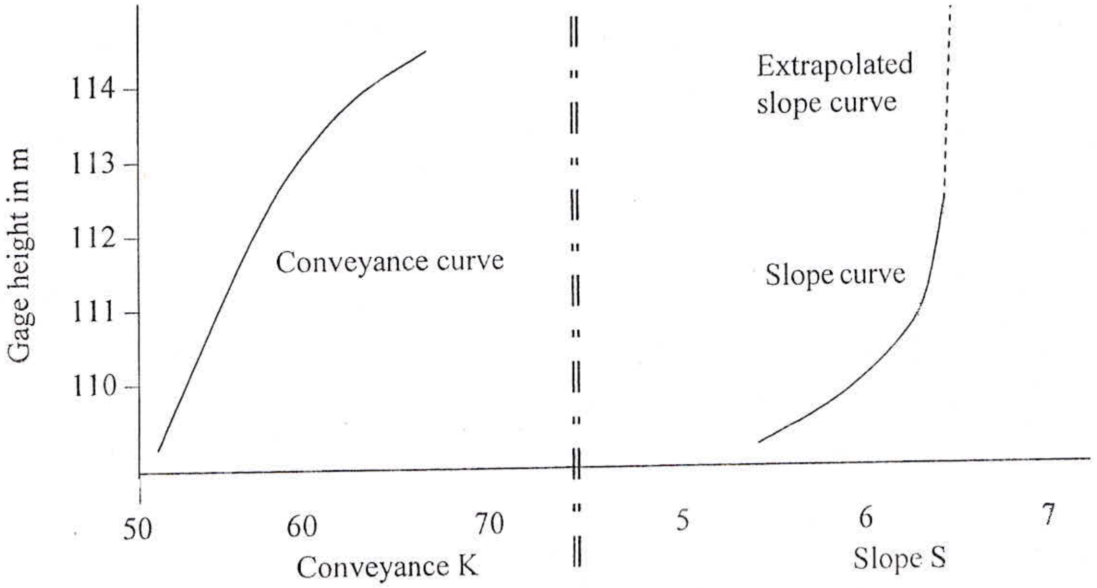

Fig. 2.13 Conveyance slope method of extrapolation of rating curve.

2.9 NEW TECHNIQUES FOR DEVELOPMENT OF STAGE-DISCHARGE RELATION

Establishment of a rating curve is a mapping problem where stage is the input variable and discharge is the output. Among the emerging data analysis techniques, the Artificial Neural Networks (ANN) are powerful procedures for non-linear function mapping. Jain and Chalisgaonkar (2000) applied this technique to establish stage-discharge relations. The inputs to the ANN were river stages at current and previous periods and discharges in previous periods. A three-layer ANN was trained using error back-propagation algorithm. The results of the ANN method were compared with the conventional approach and it was found that the ANN estimates were much closer to the observed values. Another important finding of the study was that an ANN whose inputs included current and previous stage as well as previous discharges could successfully model a looped rating curve.

Moramarco and Singh (2001) developed a method to estimate discharge at a site where only stage measurements are available, by using the discharge data at an upstream station. The method is applicable in situations where the lateral inflows in the intervening channel reach are negligible. The stage at a section can be related to remotely measured discharge by using flood routing. But this requires that a number of parameters be estimated and the results are not always satisfactory. The proposed method relates local river stage with the upstream discharge using a 2-parameter model without involving flood routing. The method is suitable when the downstream boundary is unknown and gives better results than the conventional flood routing methods.

Fitting of stage discharge relationships should not be considered simply a mathematical exercise in curve fitting. Personnel involved in this exercise should be familiar with the field conditions. Standards have been developed by many countries for establishing stage-discharge relation. The international standard ISO 1100/2 (see ISO, 1982) deals with determination of stage-discharge relations. Rantz (1982) has given a detailed discussion of the topic.

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