Proceedings WARMICE-2012
Sudhir Kumar, et al. (eds)
I.A.H., 2012

Runoff Estimation from a Small Watershed using GIUH Approach in a GIS Environment

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Abstract : A geomorphological instantaneous unit hydrograph (GIUH) is derived from the geomorphological characteristics of Kothuwatari (29.67 km^2) watershed, a sub-watershed of upper Damodar Valley, Hazaribagh, Jharkhand, India and it is related to the parameters of the Clark instantaneous unit hydrograph (IUH) model for deriving its complete shape.

The model parameters of the Clark model were determined by Newton-Rapson non-linear optimization technique, using the Geomorphological characteristics of the watershed. Various geomorphologic characteristics of the study watershed have been determined from the toposheets 72H/7 and 72H/8 by using the GIS software, Integrated Land and Water Information System ILWIS 3.0.

The developed GIUH based Clark model has been applied for simulation of the direct surface run-off (DSRO) hydrographs for nine rainfall-runoff events of the Kothuwatari watershed. The performance of the model has been evaluated by employing performance indices viz., coefficient of efficiency, absolute average error, root mean square error, average error in volume and absolute percentage deviation in peak flow rates. The results indicates that the coefficient of efficiency varies from 63.13 to 97.78, absolute average error varies from 0.32 to 2.56 m^3/s, root mean square error varies from 0.54 to 4.724 m^3/s, average error in volume varies from -0.037 to 0.005 m^3 and absolute percentage deviation in peak flow rates varies from 0.422 to 8.80.The DSRO hydrographs are computed with reasonable accuracy by the GIUH based Clark model, which simulate the DSRO hydrographs of the catchment considering the Kothuwatari watershed to be ungauged.

Key Words: GIS, GIUH Approach, Runoff, Ungauged Watershed.

INTRODUCTION

Most watershed modeling studies require the handling of a vast amount of spatial data for computation of parameters associated with the models. Whenever catchments are gauged, mathematical models can be developed, which may be calibrated and validated for available historic data. In developing countries like ours, most of the small catchments are ungauged due to the involvement of high cost in setting up gauging stations. For such ungauged catchments, the options available are, either to go for the regionalization of the parameters based on the data available for the gauged catchments in nearby hydro-meteorologically similar regions or to use its geomorphological details for modelling their hydrological response. Regionalization of the parameters is, however a very tedious task to accomplish since the hydrological behavior of many nearby catchment have to be ascertained before being confident about the values of the parameters. On the other hand, the geomorphological approach has many advantages over the regionalization techniques as it avoids the requirement and interpretation of hydrologic data from the neighboring gauged catchments in the region.

The GIUH theory was introduced by Rodriguez-Iturbe and Valdes (1979). Valdes et al. (1979) compared the GIUHs for some real world basins with the IUHs derived from the discharge hydrograph. Corradini et al.(1995) established the variation of the GIUH with basin order reduction. Rinaldo and Rodriguez-Iturbe (1996), Rodriguez-Iturbe and Rinaldo (1997) expressed the pdf of travel times as a function of the basin forms. Karvonen et al. (1999) used the GIUH model for different land-use

areas and calibrated the GIUH routing parameters using an optimization technique. Jain et al. (2000) derived a GIS supported GIUH for flood estimation in an ungauged basin and found that the peak characteristics of the design flood are more sensitive to the storm pattern. Kumar et al. (2002) studied the sensitivity analysis of the GIUH based Clark model. Chandramohan et al. (2002) applied GUIH to a small catchment, Barchi, which lies in the North Kanara district, Karnataka.. Kumar et al. (2007) applied GIUH based Clark and Nash models for simulation of the direct surface run-off (DSRO) hydrographs for ten rainfall-runoff events of the Ajay catchment up to the Sarath gauging site of eastern India. Cao(2010) adopted a kinematic-wave-based geomorphologic instantaneous unit hydrograph (KW-GIUH) model to estimate runoff in two ungauged mountain watersheds, the Yingjing River watershed and Tianquan River watershed in Sichuan.

In the present study, GIUH based Clark model has been developed by integrating the conceptual modelling of instantaneous unit hydrograph (IUH) given by Clark (1945) with the GIUH equations developed by Rodriguez-Iturbe and Valdes (1979), for the Kothuwatari watershed, which is the sub-watershed of Tilaiya dam catchment of upper Damoder Valley, Hazaribagh, Jharkhand, India. The geomorphological parameters of the study area were derived from toposheets with the help of GIS software ILWIS. The developed model was used for deriving the storm wise unit hydrographs as well as direct surface runoff hydrographs. For performance evaluation of the model, various statistical indices viz., coefficient of efficiency, absolute average error, root mean square error, average error in volume and absolute percentage deviation in peak flow rates were employed.

STUDYAREA

The Kothuwatari watershed lies between 24^0 12'18"- 24^0 16'49"N latitude and 85^0 24'18" - 85^0 28'08"E longitude and it has an area of 29.67 km^2 with relief varying from 289 to 479 m above the mean sea level. It has a heterogeneous land physiography as it is composed of hill rocks, rolling uplands, depressional lands and ravines occurring side by side. The climate of this watershed varies from sub-tropical to sub-temperate. Soil textures are coarse or coarse loamy in upper surfaces, becoming finer with depth, which indicates well drained conditions and maturity of the soils. Rain starts from middle of June and continue till October as a result of South-West monsoon. Rain in abundance occurs during the months of July and August. About 90 percent of rain occurs from mid June to September; about 4 percent from October to February, and about 6 percent from March to May. The humidity is very high from June to September. Nearly 53.7 percent of the total watershed area consists of agricultural land, 26.38 percent as forest land, 18.6 percent as waste land and the remaining 1.32 percent may be categorized under miscellaneous uses. The drainage network map of the watershed is shown in Figure 1.

The required rainfall and runoff data for the study area were collected from Damodar Valley Corporation (DVC), Hazaribagh, Jharkhand, while the geomorphological data were generated with the help of toposheets procured from Survey of India, Dehradun. In this study, nine storm events for the years of 1992-1996 recorded at Karso gauging station that resulted in single peaked runoff hydrographs were selected.

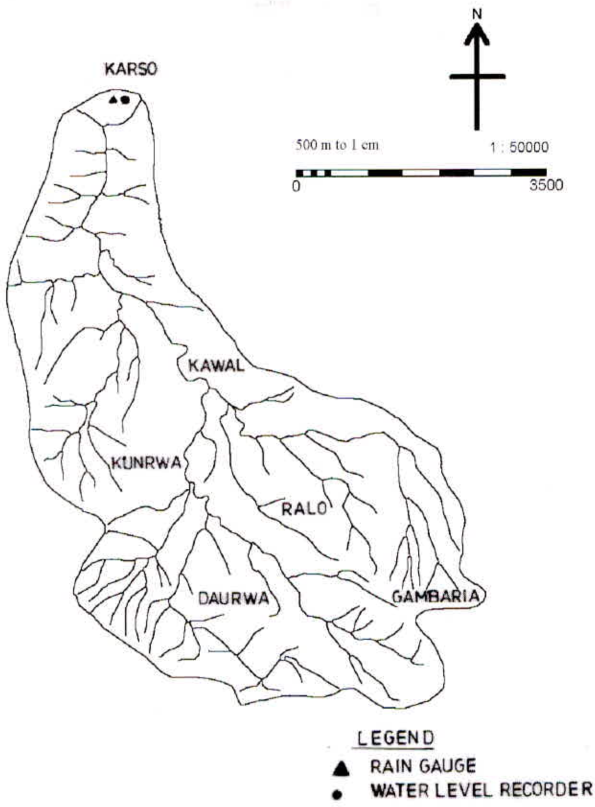

Fig. 1. Drainage network map of Kothuwatari watershed

MATERIALS AND METHODS

Preparation of Geomorphologic Database

Geomorphological Database of the study area was developed with the help of GIS software, ILWIS 3.0. The boundary of the watershed and all the streams have been traced at a scale of 1:50,000 from Survey of India toposheets 72H/7 and 72H/8. Three maps, i.e., boundary map, drainage map and contour map of the same scale were prepared. Then these three maps were scanned and converted into appropriate file formats and were opened in ILWIS environment for the on screen digitization. The area and perimeter of the basin can be computed after converting boundary (segment) map to polygon map. After converting the contour map into digital form, it was rasterised. Then interpolation from isolines was carried out on this map. This interpolated map gives the elevation at each point (pixel) in the basin. For ordering of streams of the drainage network Strahler's method (Strahler, 1964) of stream ordering was followed. In the system, the length of each stream was stored in a table. The various geomorphologic characteristics such as bifurcation ratio, lengh ratio, area ratio, length of main stream etc. were determined by using the Horton's law of stream number, stream length and stream area (Horton 1945). The geomorphologic details of streams of different order for the study area are presented in the Table 1.

Table 1. Geomorphologic detail of different stream orders for Kothuwatari watershed

Stream order	Total number of streams	Total length of streams (km)	Mean stream length (km)	Mean stream area (km^2)	Other computed geomorphologic characteristics
1	87	50.1403	0.5763	0.1649	Bifurcation ratio (R_B) = 4.547
2	23	18.1349	0.7885	1.1352	Length ratio (R_L) = 2.337
3	4	11.3489	2.8372	6.0627	
4	1	6.3697	6.3697	29.676	Area ratio (R_A) = 5.614

Preparation of Time-Area Diagram Using GIS

The time-area diagram illustrates the spatial distribution of travel time of overland flow in a catchment. The time-area methods were developed in recognition of the importance of the time distribution of rainfall on runoff in the hydrologic design of storage and regulation of water works. Application of GIS makes preparation of the time-area diagram of a catchment less time consuming and quite easier. The distance from the most upstream point in the basin upto the gauging site, along the main stream was measured from the line attributes of digitized drainage map. It was assumed that the time of travel between two points is proportional to the distance and inversely proportional to square root of the slope between these points, and given by:

$$t = CL / \sqrt{S} \qquad \ldots(1)$$

where, t is time of travel, L is the length of stream, S is the slope of the stream between two points and C is a proportionality constant. An initial estimate of time of concentration may be made by the Kirpich's formula as:

$$T_c = 0.06628 \ L^{0.77} \ H^{-0.385} \qquad \ldots(2)$$

where, T_C is the time of concentration in h, L is the length of stream in km and H is the average slope of the stream. Substituting the values of L and H in equation (3), the value of t_c can be calculated. This value of t_c may be substituted in the equation (2) and then the final formula for C obtained as:

$$C = t_c \sqrt{S_A / L} \qquad \ldots(3)$$

where, S_A is mean slope of the main stream. Now the computed values of constant of proportionality C may be used in the equation (1) for computing the time of travel between the two points of the catchment. The time of travel between various locations over the catchment is progressively computed, beginning from the gauging site of the catchment. All the values of the time of travels for each stream were then marked on the map of the catchment. Then, these points are transferred in the digital form. Using interpolation technique a map of time distribution is drawn through these points, from the time distribution map values, a map of isochrones at a desired interval, e. g. 0.25 h is prepared. The inter-isochronal areas were estimated in the ILWIS environment to prepare the time-area diagram of the catchment as shown in Figure 2.

Model Development

Rodriguez Iturbe *et al.* (1979) first introduced the concept of geomorphologic instantaneous unit hydrograph (GIUH), which led to the renewal of

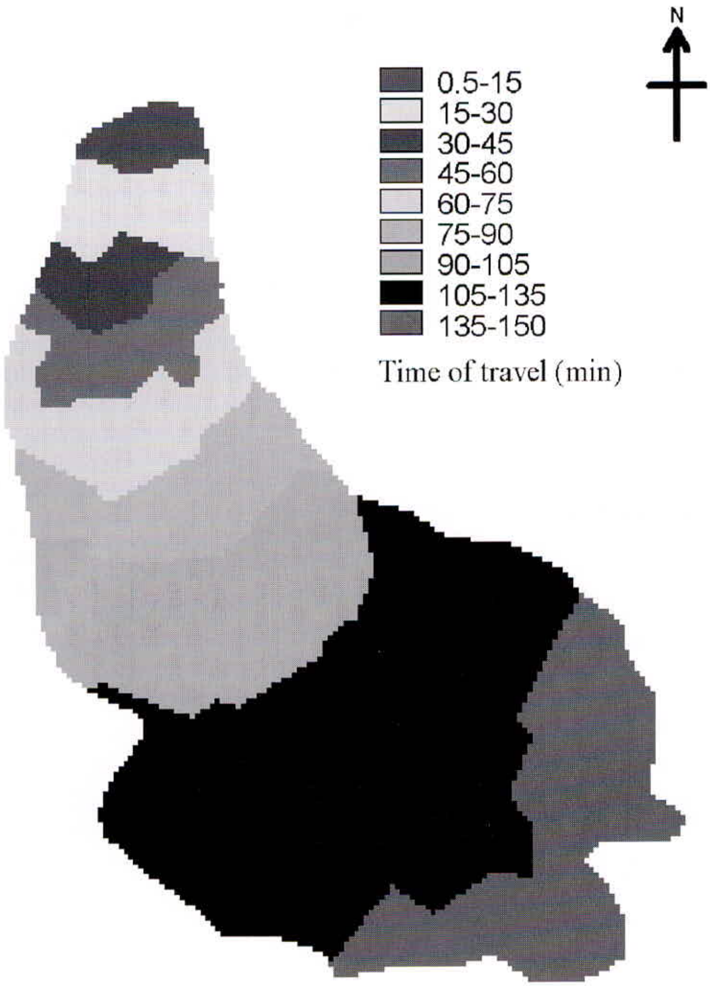

Fig. 2. Time-area diagram of Kothuwatari watershed

research in hydrogeomorphology. The expressions for peak discharge and time to peak of IUH were determined by the following formulae as given by (Rodriguez Iturbe and Valdes, 1979):

$$q_p = 1.31 R_L^{0.43} \left(\frac{V}{L_\Omega} \right) \quad \ldots(4)$$

$$t_p = 0.44 \left(\frac{L_\Omega}{V} \right) \left(\frac{R_B}{R_A} \right)^{0.55} (R_L)^{-0.38} \quad ..(5)$$

where, L_W is the length of the stream of order W in km, V is the expected peak velocity in m/s, q_p is the peak flow in h^{-1}, t_p is the time to peak in h and R_B, R_L and R_A are the bifurcation ratio, length ratio and area ratio given by the Horton's laws of stream numbers, lengths and areas, respectively.

Empirical results indicated that for natural basins the values for R_B normally range from 3 to 5, for R_L from 1.5 to 3.5 and for R_A from 3 to 6 (Smart, 1968).

On multiplying Eq. (5) and (6), we get a non dimensional term $q_p * t_p$ as given below:

$$IR = q_p * t_p = 0.5764 \left(\frac{R_B}{R_A} \right)^{0.55} (R_L)^{0.05} \qquad \ldots (6)$$

This dimensionless ratio IR is not dependent upon the flow velocity and thereby, on the storm characteristics and hence is only a function of the catchment characteristics.

Derivation of Direct Surface Runoff Hydrograph Using the GIUH Based Clark Model Approach

The Clark model (Clark, 1945) suggests that the instantaneous unit hydrograph (IUH) can be derived by routing the unit inflow in the form of time area diagram, which is prepared from the isochronal map, through a single reservoir. For the derivation of IUH, the Clark model uses two parameters, time of concentration (T_C) in h, which is the base length of the time area diagram, and storage coefficient (K) in h of a single linear reservoir in addition to the time area diagram. The governing equation of IUH for this model is given as,

$$u_i = C\, I_i + (1 - C)\, u_{i-1} \qquad \ldots (7)$$

where, u_i is the i^{th} ordinate of the IUH, u_{i-1} is the $(i-1)^{th}$ ordinate of the IUH, C & (1 C) is the routing coefficients

$$C = Dt / (K + 0.5\, Dt) \qquad \ldots (8)$$

where, Dt is the computational interval, h, I_i is the i^{th} ordinate of the time area diagram. A UH of desired duration (D) may be derived using the following equation,

$$U_i = \frac{1}{n}(0.5u_{i-n} + u_{i-n} + u_{i-n+1} \ldots\ldots\ldots + u_{i-1} + 0.5u_i) \qquad (9)$$

where, U_i is the i^{th} ordinate of unit hydrograph of duration D hour and at computational interval of Dt h, n is number of computational intervals and u_i is the i^{th} ordinate of the IUH. The steps involved in the derivation of direct surface runoff hydrographs of a specific duration from the rainfall-runoff data of historical events using the GIUH based Clark model approach are: (i) Geomorphological characteristics (R_A, R_B and R_L) of the study area are evaluated using the GIS software ILWIS. (ii) A non-dimensional time-area diagram of the catchment is developed from the developed time-area diagram considering t/T_c on x-axis and cumulative contributing area on y-axis. (iii) The direct surface runoff (DSRO) hydrograph ordinates are computed by separating the baseflow from the observed hydrograph ordinates. (iv) The f-index method is used for the estimation of the excess rainfall hyetographs. (v) A suitable peak velocity (V) for a given storm is selected in order to compute peak discharge values of the hydrograph. (vi) The time of concentration (T_c) is computed using the equation,

$$T_c = 0.2778\, L / V \qquad \ldots (10)$$

where, L is the length of main stream channel in m and V is the peak velocity in m/s. (vii) using this values of T_c, time-area diagram of the catchment is computed at each computational time interval employing the non-dimensional time-area diagram. (viii) Peak discharge (q_p) of GIUH is computed by equation (4). (ix) Values of storage coefficient (K) is computed by Newton-Rapson optimization procedure, so that the absolute difference between the GIUH peak and IUH peak is minimized. (x) IUH is computed by equation (7) using the time of concentration (T_c), time-area diagram and optimized values of storage coefficient (K) as computed in step (ix). (xi) D-hour unit hydrograph is computed using the equation (9). (xii) DSRO hydrograph is computed by convoluting the excess rainfall hyetograph with unit hydrograph obtained in step (xi). (xii) DSRO hydrograph is compared with the observed DSRO hydrograph. In case the reproduction of the DSRO is not found to be satisfactory, another suitable value of velocity is chosen and the above methodology is repeated for estimating the D-hour unit hydrograph and computed DSRO. Thus a trial and error method is adopted to arrive at the most suitable value of the velocity and hence the D-hour unit hydrograph.

RESULTS AND DISCUSSION

Determination of Parameters for the GIUH Based Clark Model

As the geometric properties of gauging section and the Manning's roughness coefficient for the basin under study as well as the flow velocities corresponding to discharge passing through the gauging section at different depths of water flow were not known for the watershed, the model was calibrated by adopting the peak velocity as a variable for different storm events under the study. The parameters of GIUH based Clark model for various rainfall-runoff events are given in Table 2. It can be observed from the table that the values of Clark model parameters (T_C & K) vary from 0.440 to 1.045 h and 0.409 to 1.003 h respectively. The ratio between storage coefficient (K) and the sum of storage coefficient and time of concentration (T_c), i.e., $K/(K+T_c)$ has a unique value for a particular catchment (NIH, 1993). Its average value for nine storm events has been computed as 0.487 for the GIUH based Clark model for the Kothuwatari watershed.

Comparison of Observed and Computed DSRO Hydrographs

The direct surface runoff (DSRO) hydrographs computed by GIUH based Clark model were compared with the observed DSRO hydrographs. The accuracy of models was also tested by comparison of computed and observed DSRO hydrographs shown in Figure 3 through Figure 5 for events E1, E5 and E8. The values of peak discharge of the DSRO hydrographs for the various storm events are given in Table 3.

Performance Evaluation of GIUH Based Clark Model

The statistical indices employed in the present study for evaluation of the direct surface runoff (DSRO) hydrographs derived by the GIUH based Clark model with respect to the observed DSRO hydrographs are as follows:

Table 2. Parameters of GIUH based Clark model for different storm events

Event	Date	GIUH based Clark Model		
		T_c (h)	K (h)	$K/(K+T_c)$
E1	June 24-25,1992	0.826	0.770	0.48246
E2	October 12-13,1993	0.788	0.750	0.48765
E3	November 2-3,1993	0.986	0.944	0.48912
E4	June 14,1994	1.045	1.003	0.48975
E5	June 18,1994	0.440	0.409	0.48174
E6	July 4,1994	0.661	0.622	0.4848
E7	August 13,1996	0.826	0.786	0.48759
E8	September 15,1996	1.008	0.963	0.48858
E9	October 15-16,1996	1.009	0.964	0.4886

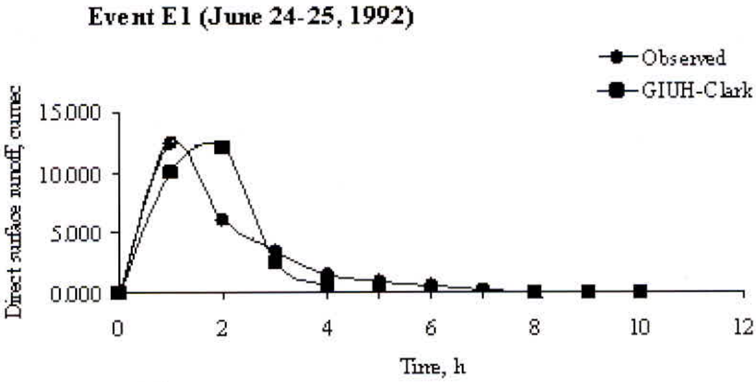

Fig. 3. Comparison of observed and predicted direct surface runoff hydrographs of event E1 for Kothuwatari watershed

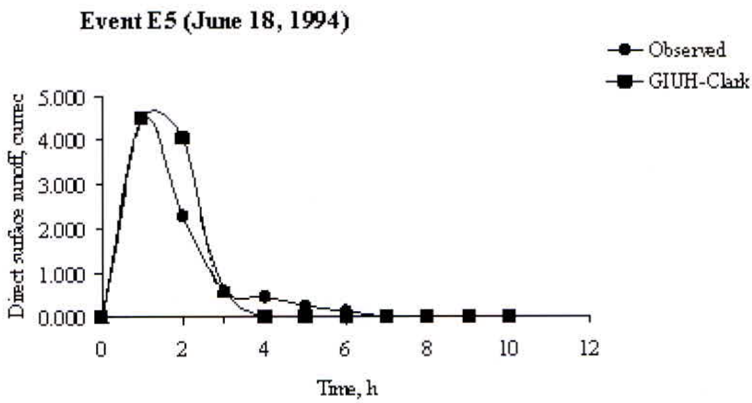

Fig. 4. Comparison of observed and predicted direct surface runoff hydrographs of event E5 for Kothuwatari watershed

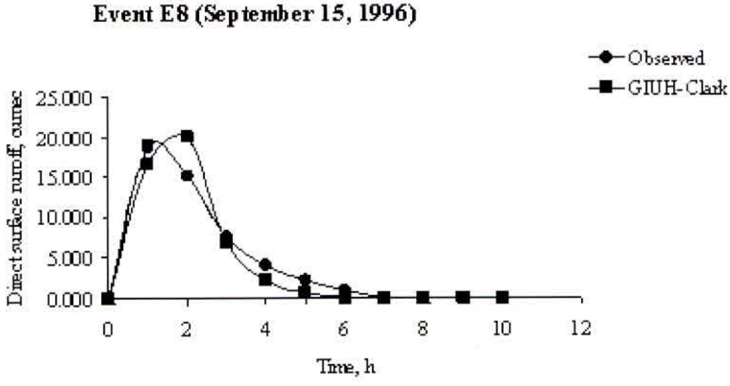

Fig. 5. Comparison of observed and predicted direct surface runoff hydrographs of event E8 for Kothuwatari watershed

Table 3 . Comparison of peak discharge obtained by GIUH based Clark and observed discharge

Events	Peak discharge, Q_p (m^3/s)	
	Observed	GIUH-Clark
E1	12.417	12.052
E2	30.552	27.862
E3	7.183	7.225
E4	2.893	2.990
E5	4.500	4.481
E6	2.985	3.189
E7	6.621	6.688
E8	18.582	20.037
E9	7.297	7.784

Coefficient of efficiency (CE)

The goodness of fit between observed and predicted values of DSRO ordinates was examined using following equation of coefficient of efficiency (Yu *et al.*, 1994),

$$CE = \frac{\sum_{i=1}^{n}\left(Q_{oi} - \overline{Q}\right)^2 - \sum_{i=1}^{n}\left(Q_{oi} - Q_{ci}\right)^2}{\sum_{i=1}^{n}\left(Q_{oi} - \overline{Q}\right)^2} * 100 \qquad (16)$$

where, Q_{oi} is the i^{th} ordinate of the observed discharge, $\overline{Q}$ is the average of the ordinates of observed discharge, Q_{ci} is the computed discharge and n is the number of ordinates. It can be observed from the Table 4 that the values of CE vary from 63.13 % to 97.78 % for GIUH based Clark Model. In the present study the model, yielding coefficient of efficiency (CE) values 60 % or more, is considered as applicable for the study area.

Absolute average error (AAE)

Sarma *et al.* (1973) gave the equation of absolute average error (AAE) between observed and predicted values as follows,

$$AAE = \frac{\sum_{i=1}^{n}\left|\left(Q_{oi} - Q_{ci}\right)\right|}{n} \qquad \ldots(17)$$

The AAE values lie in the range of 0.320 to 2.560 for the GIUH based Clark Model, which are well within the acceptable limits of less than five, the criteria adopted in the study.

Root mean square error (RMSE)

The root mean square error (RMSE) between the observed and the predicted values are determined by following equation (Yu *et al.*, 1994),

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_{oi} - Q_{ci}\right)^2}{n}} \qquad \ldots(18)$$

Table 4. Estimated values of performance evaluation indices for GIUH based Clark model and for different storm

Events	Date	Models	Performance evaluation indices				
			CE	AAE (m^3/s)	RMSE (m^3/s)	AEV (m^3)	PEP
E1	June 24-25,1992	GIUH-Clark	68.80	1.330	2.115	0.000	2.940
E2	October 12-13,1993	GIUH-Clark	75.34	2.560	4.724	0.005	8.800
E3	November 2-3,1993	GIUH-Clark	73.93	1.110	1.966	-0.003	0.580
E4	June 14,1994	GIUH-Clark	97.78	0.820	1.220	0.001	3.350
E5	June 18,1994	GIUH-Clark	80.98	0.350	0.640	-0.037	0.422
E6	July 4,1994	GIUH-Clark	69.44	0.320	0.540	0.000	6.835
E7	August 13,1996	GIUH-Clark	81.56	0.510	0.920	0.001	1.011
E8	September 15,1996	GIUH-Clark	87.54	1.370	2.350	0.003	7.830
E9	October 15-16,1996	GIUH-Clark	63.13	0.800	1.350	0.001	6.673

It is found that the values of RMSE ranges from 0.540 to 4.724 for the GIUH based Clark model.

Average error in volume (AEV)

The average error in volume (AEV) for the prediction models can be estimated by using the following equation (Sarma *et al.*, 1973)

$$AEV = \frac{(Vol_O - Vol_c)}{n} \quad \ldots(19)$$

where, Vol_o is the observed runoff volume and Vol_c is the computed runoff volume. The values of AEV vary from 0.000 to 0.037 for GIUH based Clark model.

Percentage absolute deviation in peak flow rates (PEP)

As suggested by Wang *et al.*, (1992) the percentage absolute deviations between observed and predicted values are determined by the following equation,

$$PEP = \frac{(Q_{OP} - Q_{PP})}{Q_{OP}} \quad \ldots(20)$$

where, PEP is percentage absolute deviation in peak flow rates, Q_{OP} is the observed peak flow rate and Q_{PP} is the predicted peak flow rate. The values of PEP vary from 0.422 % to 8.800 % for GIUH based Clark model.

CONCLUSIONS

The results of the present study reveal that the Kothuwatari watershed was found to be fourth ordered hydrologic unit. The various geomorphological parameters like bifurcation ratio (R_B), length ratio (R_L), area ratio (R_A), length of main stream (L_W) and length of highest order stream (L) of Kothuwatari watershed were determined as 4.548, 2.337, 5.614, 11.893 km and 6.36 km, respectively. Based on the percentage absolute deviation in peak flow rates (0.422 to 8.80), coefficient of efficiency (63.13 to 97.78) and average error in volume values (-0.037 to 0.005 m^3), the GIUH based Clark model estimate direct surface runoff hydrographs in agreement with the observed hydrographs for the study watershed.

Comparison of direct surface runoff hydrographs estimated using GIUH based Clark model with the observed surface runoff hydrograph indicates that the geomorphologic instantaneous unit hydrograph (GIUH) can be used as a transfer function in ungauged watersheds for modeling the transformation of excess rainfall into surface runoff.

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