# Study on Inter-Plant Economical Operation of Cascade Hydropower

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ABSTRACT: Regional Operation of Electricity market has begun in China. With the development of China's market economy, hydropower will enter the arena of bidding for connection to the grid in the light of "fair, impartial and open" principles. Under electricity market, how to formulate a new model, which is accordant with not only the rule of operation of hydropower station (group) but also the regulation of operation of electricity power grid and loyal to the principle of electricity market at the same time, is the new task facing the talents engaging in hydroelectricity project to solve urgently. Solving Inter-plant Economical Operation of a cascade Hydropower System is fearfully important on the condition of electricity market. If the hydropower company is incapable of predicting the generation capability and the amount of generation of hydropower stations and risk-rate corresponding to various generations, there is no information available for Hydropower Company to refer to when they make futures agreements in a certain period. The paper sets up mathematical models of total minimum stored energy consumption and total minimum water consumption for the cascade hydropower system to provide a solution to the problem of rational distribution of next-day generating schedule in a cascade hydropower system awarded a contract in the bidding for connection to the grid. A case study is conducted to verify the rationality of the model, and its application scope is analyzed briefly.

#### INTRODUCTION

"Break monopoly and introduce competition mechanism to optimize the distribution of China's electric power resources" is the key to the sustainable development of China's power industry. It is inevitable that hydropower enters the arena of bidding for connection to the grid in accordance with "fair, impartial and open" principles (Zunlian, 2001; Yimin, 2002).

In a planned economy, perpendicular monopoly is practiced in the power industry. The operation of hydropower plants, subordinates of power administrative bureaus (power group corporations), is dispatched by the unified administrative directives from the grid dispatching departments (Yonghao, 2003). But, in a market economy, if power plants in a cascade hydropower system are under the administration of different power companies, the plants in the lower reaches would be unable to predict exactly their generating capacity during their biddings; because each plant takes part in the market competition, and therefore the bidding price will be kept as confidential information.

Besides, if the plant upstream wins the bidding whereas the one downstream fails, then the plant downstream may, even if it has normal storage, discharge its storage as waste not for power generation, which goes against the principle of making full use of water resources. Similarly, if the plant downstream wins the bidding whereas the one upstream fails, the plant upstream may not discharge for power generation, and consequently the plant downstream will operate in a non-economic manner, and even will not have sufficient water for power generation to meet the requirements of the contract, thus compensation liability. All the factors taken into account, the hydropower plants in a cascade system should be under the administration of one company. In this case, all the plants in a cascade system can be merged into a simulated generating set for bidding. After winning a bid for connection to the grid, then an optimal distribution of generation load can be made among the plants and within plants using a criterion such as the minimal total stored energy consumption in the cascade system. Such a practice can avoid the above-mentioned drawbacks.

# DAY GENERATION SCHEDULE AND INTER-PLANT ECONOMIC OPERATION OF HYDROPOWER PLANTS (CONEJO, 2002; GUANGYU, 2003; YULONG, 2003)

Under a given total loading process of hydropower plants, one short-term optimal dispatching objective of cascade hydropower plants is to obtain the optimal output process of each plant (short-term generation schedule of the plant) and the optimal output process of each generating set (set generating schedule) in accordance with the principle of the minimal total energy consumption of the plants. In a market economy, as an independent economic entity, a hydropower company with several cascade hydropower plants seeks to maximize profit as its main objective. But because the company has lost its previously enjoyed preferential treatment (that is, hydropower does not have the priority to be connected to the grid), it has to observe all the regulations of a market economy, and enter the bidding competition for connection to the grid. In this case, the company can merge its plants into a simulated generating set for bidding, and after its success in bidding, break down the total loading process into each plant and each generating set according to its optimal dispatching scheme.

This paper attempts to study inter-plant economic operation, that is, how to reasonably distribute total loading process to each plant when cascade plants are merged into a simulated generating set in bidding.

# MATHEMATICAL MODELS FOR INTER-PLANT ECONOMIC OPERATION

Consider a cascade hydropower system of reservoirs with power generation and flood control as their main purposes and water supply and navigation as other functions. In a market economy, the operation of the plants according to the bidding generating schedule may contradict with the comprehensive utility of reservoirs (Chuntian, 1997). Due to space limitations, this problem will be addressed in another article. In this paper, a model is established based on the principles of the minimum total stored energy consumption and minimum water consumption of the cascade hydropower plants. The model is suitable for plant operation in dry seasons whereas in wet seasons, hydropower load is normally determined by water volume (SenLin, 2001; Yushan, 2004). The mathematical model (Yinggui, 1994; Shawwash, 2000; Yadong, 2000) is as follows:

### **Objective Functions**

(a) Minimum Total Stored Energy Consumption

$$EC^* = \min \sum_{k=1}^{K} \sum_{i=1}^{I} (\sum_{j=i}^{I} \eta_j \cdot h_{j,k} \cdot Q_{i,k}) t_k \qquad \dots (1)$$

in which  $EC^*$  denotes sum (kwh) of total stored energy consumption of I plants in K periods;  $\eta_i$  is the output coefficient of Plant j;  $h_{j,k}$  (m) is the water head of Plant j in k period,  $Q_{i,k}$  (m<sup>3</sup>/s) is the discharge of Plant i in k period;  $t_k$  is hours in k period;  $t_k$  is period number;  $t_k$  is total number of periods;  $t_k$  is total number of plants; and  $t_k$  denote plant number,  $t_k$  is  $t_k$  is  $t_k$  denote plant number,  $t_k$  is  $t_k$  denote plant number,  $t_k$  is  $t_k$  denote plant number,  $t_k$  is  $t_k$  in  $t_k$  denote plant number.

(b) Minimum Total Water Consumption

$$W = \min \sum_{k=1}^{K} \sum_{i=1}^{I} Q_{ik} \cdot t_{k} \qquad ... (2)$$

where W (10<sup>6</sup> m<sup>3</sup>) denotes the sum of total water consumption of I plants in k periods.

#### Constraints

In accordance with characteristics of the research object, the following constraints should be taken into account.

(a) Output Equilibrium Equation

$$\sum_{i=1}^{I} P_{i,k} = P_{x,k} \ k = 1, 2, ..., K \qquad ... (3)$$

where  $P_{i,k}$  (MW) is the output of Plant i in k period and  $P_{x,k}$  is the load required for generation (MW) of the cascade system in k period.

(b) Plant Output Constraint

$$P_{i,\min} \le P_{i,k} \le P_{i,\max} \qquad \dots (4)$$

where  $P_{i,min}$  (MW) is the minimal output of Plant i;  $P_{i,k}$  is the output of Plant i in k period; and  $P_{i,max}$  (MW) is the capacity available or expected output of Plant i.

(c) Water Balance Equation

$$V_{i,k+1} = V_{i,k} + \delta(I_{i,k} + Q_{i,k})$$
 ... (5)

in which  $V_{i,k}$  (m<sup>3</sup>) denotes the storage of Plant i at the beginning of k period;  $V_{i,k+1}$  (m<sup>3</sup>) is the storage of Plant i at the end of k period;  $I_{i,k}$  (m<sup>3</sup>/s) is the inflow of Plant i in k period;  $Q_{i,k}$  (m<sup>3</sup>/s) is the discharge of Plant i in k period; and  $\delta$  is a coefficient for unit conversion.

# (d) Flow Connection among Cascade Plants

The impact of delayed flow exists but so far there are no effective measures to eliminate it. Usually, some simplified methods are adopted to deal with this problem with the prerequisite to satisfaction of requirements of the system operation. Jinwen (2003) proposed a periodic modal, which meets periodic constraint conditions, to determine the discharge of each reservoir in a flow-delayed section. This study deals only with the impact of the delayed flow (if there is a considerable variation of flow within a day, then the variation of delayed flow will be taken into account). In order to mitigate the impact of the discharge of the plant upstream in  $\tau$  duration, the last duration of the pervious day, on the daily optimal operation decision of the plant downstream before  $\tau$  duration, the calculation durations can be modified into  $(24 + \tau)$  hours. After the calculation, the results of the 24 hours excluding  $\tau$  will be taken,

$$I_{i,k} \le I_{qi,k} + Q_{i-1,k-\tau i}$$
 ... (6)

where  $I_{qi, k}$  stands for the sectional flow (m<sup>3</sup>/s) of Plant i in k duration and  $\tau_i$  is the delayed time of flow between Plant i+1 and Plant i.

# (e) Constraint of Reservoir Storage

$$V_{i,\min} \le V_{i,k} \le V_{i,\max} \qquad \dots (7)$$

where  $V_{i,min}$  (m<sup>3</sup>) stands for the dead storage of Reservoir i or the minimum storage for the comprehensive utility of Reservoir I and  $V_{i,max}$  (m<sup>3</sup>) is the maximum storage permissible of Reservoir I ( such as normal storage, or flood regulation).

# (f) Discharge Constraint

$$Q_{i,\min} \le Q_{i,k} \le Q_{i,\max} \qquad \dots (8)$$

where  $Q_{i,min}$  (m<sup>3</sup>/s) stands for the minimal discharge permissible or for comprehensive utility of the water turbine of Plant I and  $Q_{i,max}$  is the maximal flow capacity of the water turbine of Plant i or maximal discharge permissible.

# (g) End-of-day Reservoir Level Constraint

$$Z_{i,k+1} \ge Z_{i,\min} \qquad \dots (9)$$

where  $Z_{i, k+l}$  (m) denotes the reservoir level of Plant I at the end of day and  $Z_{i,min}$  (m) is the lowest reservoir level permissible of Plant i at the end of day. It should be noted that fewer lowest reservoir levels permissible are assumed as possible lest no solution should be obtained.

#### (h) Non-negativity Conditions

All the above-mentioned variables should be larger than or equal to zero.

#### CASE STUDY

A hydropower generating company, which has two cascade hydropower plants, Plants A and B, is awarded the contract in the bidding for connection to the grid the next day. The proposed models of economic operation are used to calculate the optimal generating process of each plant for the next day.

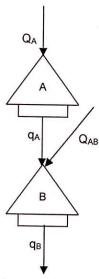


Fig. 1: cascade power plants

Plants A and B have yearly regulation capacity, their characteristics are shown in Table 1, and their locations are shown in Figure 1. The predicated reservoir inflow and sectional flow at each duration (24, 48 or 96 durations corresponding to the trading period), and the total load of the cascade system for the next day are shown in Column (2), (3) and (4) of Table 2. It is known that Plant A has 1100 MW capacity available in each duration whereas Plant B has 500 MW. Calculated with the preceding models, the generating process for each plant can be obtained, see Columns (5) and (6) of Table 2 [the objective functions is Eq. (1)] and Columns (7) and (8) [the objective function is Eq. (2)]. Their dynamic indexes are shown in Tables 3 and 4, respectively.

Table 2 shows that different models generate different output processes for each plant. Tables 3 and 4 indicate that for Plant A to generate the same amount of electricity, the model of minimum total stored energy consumption generates water consumption for generation of 11.48 million m<sup>3</sup>, daily electricity

generation of 3.637 MWH, and water level variation of 0.069 m down. In contrast, the model of minimum total water consumption generates water consumption for generation of 20.18 million m³, daily electricity generation of 5.960 MWH, and water level variation of 0.151 m down. However, for Plant B to generate the same amount of electricity, the model of minimum total stored energy consumption generates water consumption for generation of 28.12 million m³, daily electricity generation of 4.453 MWH, and water level variation of 0.039 m dropping; while the model of minimum total water consumption generates water consumption for generation of 14.42 million m³, daily electricity generation of 2.130 MWH, and water level variation of 0.028 m up.

The aforementioned indicates that priority should be given to the model of minimum total stored energy

consumption in the calculation of water for reservoirs downstream, while the model of minimum total water consumption should be first used in the calculation of water for reservoirs upstream. The reason is that the same volume of water has more potential in the reservoir upstream than downstream. Thus, with a view of minimum stored energy consumption, water in the reservoir upstream should be saved possibly and use of water in the reservoir downstream should be given first priority. On the other hand, to generate the same amount of electricity, a higher water head needs less water. So, from the point of view of minimum total water consumption, water of higher head should be used for generation as much as possible. In the case study, Plant A has higher water head (see Table 1) and accordingly, water in Plant A's reservoir should be first used.

Table 1: Characteristics of Reservoir Power Plants A and B

Item Plant	Normal Storage (m)	Dead Water Level (m)	Regulating Storage (10 <sup>6</sup> m <sup>3</sup> )	Designed Head (m)	Installed Capacity (mw)	Guarantee Output (mw)
Plant A	413	380	4966	112	1500	167
Plant B	261	242	5350	64.2	700	166

Table 2: Indexes of Dynamic Energy of the Hydropower Power Plant

Duration	Flow(m³/s)		Cascade	Minimum Total Stored Energy Consumption Model		Minimum Total Water Consumption Model	
	Reservoir A	Between A and B	Load (mw)	Load of Plant A (mw)	Load of Plant B (mw)	Load of Plant A (mw)	Load of Plant B (mw)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	50.0	46.0	0	0	0	0	0
2	50.0	46.0	0	0	0	0	0
3	50.0	46.0	0	0	0	0	0
4	50.0	46.0	0	0	0	0	0
5	50.0	46.0	0	0	0	0	0
6	49.0	44.0	0	0	0	0	0
7	48.0	43.0	514	200	314	400	114
8	47.0	42.0	502	222	280	370	132
9	46.0	41.0	480	200	280	290	190
10	46.0	41.0	850	375	475	560	290
11	46.0	40.0	206	0	206	206	0
12	44.0	39.0	0	0	0	0	0
13	43.0	37.0	0	0	0	0	0
14	43.0	37.0	435	0	435	330	105
15	44.0	38.0	442	200	242	350	92
16	44.0	38.0	1070	570	500	810	260
17	44.0	38.0	1016	516	500	742	274
18	45.0	39.0	1540	1040	500	1100	440
19	46.0	39.0	797	314	483	564	233
20	47.0	40.0	238	0	238	238	0
21	47.0	41.0	0	0	0	0	0
22	48.0	43.0	0	0	0	0	0
23	48.0	44.0	0	0	0	0	0
24	48.0	44.0	0	0	0	0	0

	Table 5. Allaly	313 OF ITTUCKES OF D	filanno Energy e		
Item Model	Inflow (10 <sup>6</sup> m³)	Generating Flow (10 <sup>6</sup> m <sup>3</sup> )	Storage Variation (10 <sup>6</sup> m <sup>3</sup> )	Water Level Variation (m)	Daily Power Generation (10 <sup>6</sup> kwh)
Min. total stored energy consumption	4.22	11.48	-7.26	-0.069	3.637
Min. total water consumption	4.22	20.18	-15.96	-0.151	5.960

Table 3: Analysis of Indexes of Dynamic Energy of Plant A

Table 4: Analysis of Indexes of Dynamic Energy of Plant B

Item Model	Inflow (10 <sup>6</sup> m³)	Generating Flow (10 <sup>6</sup> m <sup>3</sup> )	Storage Variation (10 <sup>6</sup> m³)	Water Level Variation (m)	Daily Power Generation (10 <sup>6</sup> kwh)
Min. total stored energy consumption	15.07	28.12	-13.05	-0.039	4.453
Min. total water consumption 23.77		14.42	9.35	0.028	2.130

The analysis shows that from the point of view of energy saving, the model of minimum total stored energy consumption should be used, whereas from the angle of water saving, the model of minimum total water consumption should be used. The model of minimum total stored energy consumption tends to overuse reservoirs downstream, which may lead to non-economic operation in the later periods. In contrast, the model of minimum total stored energy consumption tends to overuse reservoirs upstream. In general, in a cascade hydropower system, the first cascade plant has the highest water head. Too much discharge of storage at the first step may result in noneconomic operation of the whole system in later periods. Therefore, in actual dispatching operation, the application of the models should be tailored to the specific situations of different cascade systems. In the use of the model of minimum total stored energy consumption, a lower limit should be designated to storage level of reservoirs downstream at the end of a day in constraint condition (9) of the model (the value can be obtained by adjusting the-end-of-day reservoir level designated by interim dispatch). In the same manner, a lower limit should be designated to storage level of reservoirs upstream.

#### SUMMARY AND CONCLUSIONS

This paper presented mathematical models of minimum total stored energy consumption and minimum total water consumption for a cascade hydropower system to provide a solution to the problem of rational generation of load distribution of next-day generating schedule in a cascade hydropower system awarded a contract in the bidding for connection to the grid. The

case study indicated that the model of minimum total stored energy consumption is more suitable for the cascade hydropower system with power generation as its main task, but due attention should be paid to overuse of water in reservoirs downstream. In addition, it is hard to establish a general model for a cascade hydropower system due to considerable differences in adjustability, water head, storage, usable storage, output coefficient, and comprehensive utilization of cascade hydropower systems. In the application of the models, the constraint conditions should be adjusted to specific conditions.

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