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Modeling Suspended Sediment Movement during Floods in Surface Flows

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ABSTRACT: A stochastic diffusion jump model in response to extreme flows is proposed herein to describe the movement of sediment particles in surface waters. The proposed particle tracking model classifies the movement of particles into three categories—a drift motion, a Brownian type motion due to turbulence in the flow field for example, and jumps due to the occurrence of extreme events. In the proposed model, a random term mainly caused by the fluid eddy motions is modeled as a Wiener process. In addition, the occurrence of the extreme flow event is modeled as a Poisson process. The magnitude of particle movement in response to extreme flow events, characterized as the Poisson jump, depends on the characteristics of the extreme events and the properties of sediment particles. The frequency of occurrence of the extreme events in the proposed model can be explicitly accounted for when evaluating the movement of sediment particles. As such, the proposed particle tracking model, when coupled with an appropriate hydrodynamic model, can assist in developing a forecast model to predict the movement of particles in the presence of extreme flows. The mean and variance of particle trajectory can be obtained from the proposed stochastic model via simulations.

INTRODUCTION

Stochastic approaches have been widely implemented in biological, chemical and environmental engineering fields. McNair *et al.* (1997) developed a stochastic diffusion model for organic particle transport by considering both the molecular diffusion and turbulence eddies. A Lagrangian Stochastic (LS) model for the dispersion and deposition of submicron-size particles in turbulent air flows was presented by Reynolds (1999). A Stochastic Differential Equation (SDE) of particle displacement in a spatially homoeneous porous medium was developed by adding a stochastic term to Darcy's equation (Verwoerd and Kulasiri, 2003). Recently, Dean and Russell (2004) developed a numerical framework approximating the dispersivity of solute transport in porous media. Another widely used category of stochastic transport models is known as Random-walk Particle Tracking Models (PTMs), in which mass is transported as discrete particles. For instance, Dimou and Adams (1993) applied the Fokker-Plank equation to develop the PTMs for well mixed estuaries and coastal waters. Particle tracking algorithms are popular for modeling transport in various fields (Reimus and James, 2002; Pederson *et al.*, 2003; Perianez, 2004). Here we will develop a conceptual stochastic diffusion jump model based on similar stochastic principles of the particle tracking models. However, in addition to keeping the intrinsic advantages of particle tracking models, the proposed model has a potential to account for the frequency of the extreme events.

GOVERNING EQUATION

A stochastic process is defined as a spatial or temporal process involving probability (Yen, 2002). Movement of sediment particles in a water system can be characterized as a stochastic process. The displacement of a suspended particle in natural rivers is considered to follow a stochastic jump diffusion process consisting of a drift term $\overline{u}$, a diffusion coefficient σ, a driving Wiener process B_t, and a Poisson process P_t, as shown in Equation (1) (e.g. Hanson, 2005),

$$dX_t = \underbrace{\overline{u}(t, X_t)dt}_{drift\ term} + \underbrace{\sigma(t, X_t)dB_t}_{random\ term} + \underbrace{h(t, X_t)dP_t}_{jump\ term} \quad \ldots (1)$$

where $X(t)$ is the trajectory of a particle, a three-dimensional vector, expressed as $X_t = [x(t)\ y(t)\ z(t)]^T$; h

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is the jump amplification factor; B_t is a three-dimensional vector of the Wiener process. $B_t - B_s$ has a normal distribution with a zero mean and a variance of $(\sigma\sigma^T)$ $(t - s)$ for $s \le t$, , which is independent of X_t. Parameters $\bar{u}$, σ and h are all continuous functions related to the characteristics of the transport process. The drift term $\bar{u}$ and the diffusion coefficient σ can be obtained by the heuristic method (e.g. Hunter *et al.*, 1993; Dimou and Adams, 1993; Heemink, 1995; McNair *et al.*, 1997; Man and Tsai, 2007) by comparing the Fokker-Planck equation with the advection-diffusion equation,

$$\bar{u}(t, X_t) = \begin{Bmatrix} \bar{U}(t,x,y,z) + \partial D_x / \partial x \\ \bar{V}(t,x,y,z) + \partial D_y / \partial y \\ \bar{W}(t,x,y,z) - w_s + \partial D_z / \partial z \end{Bmatrix} \quad \dots (2)$$

where w_s is the particle settling velocity; $\bar{U}$ is the mean streamwise fluid velocity; $\bar{V}$ is the mean transverse fluid velocity; and $\bar{W}$ is the mean normal fluid velocity. D_x, D_y and D_Z are the turbulent diffusivity in the streamwise, transverse and vertical direction, respectively. If the coordinate system is aligned with the flow, then $\sigma(t, X_t)$ is a 3×3 diagonal matrix. Herein, the diffusion coefficient tensor is treated as a diagonal matrix as,

$$\sigma(t, X_t) = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} \quad \dots (3)$$

The relationship between the diffusion coefficients and the turbulent diffusivities can be expressed as,

$$\frac{1}{2}\left(\sigma\sigma^T\right)_{i,i} = D_i \quad \dots (4)$$

The jump amplification factor h is related to the physical characteristics of extreme flow events such as tsunamis, floods and large flow perturbations in the environmental engineering field, whereas the Poisson process P_t can be characterized to reflect the frequency (or the return period) of the extreme event occurrence.

MODELING EXTREME FLOW EVENTS AS A POISSON PROCESS

The frequency of the occurrence of the extreme event can be modeled as a Poisson process, $dP(t)$. Incorporating the frequency of an extreme event in the modeling of particle movement offers a more comprehensive evaluation of an effective risk of sediment concentrations exceeding a designated level of protection. Furthermore, the proposed approach classifies the movement of particles into three categories—drift motion, a Brownian type motion due to turbulence in the flow field for example, and jumps due to the occurrence of extreme flow events, as shown in Equation (1). The frequency of the extreme event, $dP(t)$, behaves as an indicator function of the number of extreme event occurrences with a negligible error $O^2(\lambda\, dt)$. For instance, $dP(t) = 0$ with an asymptotic probability $(1 - \lambda dt)$ if there is no jump; and $dP(t) = 1$ with an asymptotic probability $(\lambda\, dt)$ if there is one jump, while the probability of multiple simultaneous jumps are likely to be negligible. The Poisson process $dP(t)$ can then be simulated by the Monte Carlo simulation. Based on the stationary property of the incremental Poisson process, we have,

$$dP(t) = P(t + \Delta t) - P(t) \quad \dots (5)$$

$$\Pr[dP(t) = k] = e^{-\lambda dt} \frac{(\lambda dt)^k}{k!} \quad \dots (6)$$

where Pr denotes the probability and λ represents the average number of extreme event occurrences. Let T_j represent the j^{th} jump time, then the distribution of the inter-jump time $\Delta T_j = T_{j+1} - T_j$ conditioned on T_j is exponentially distributed,

$$\Pr[\Delta T_j \le \Delta t | T_j] = 1 - e^{-\lambda \Delta t} \quad \dots (7)$$

where $T_0 = 0$. The exponential random variable can be generated from a uniform random distribution. Let U be a uniform (0, 1) random variable,

$$X_e = -\mu \ln(U) \quad \dots (8)$$

in which X_e is exponentially distributed with the mean μ where $\mu = 1/\lambda$. Equation (8) can be used to generate an exponential distribution from a uniformly distributed pseudo-random number generator to simulate the Poisson process. The small time increment process can be numerically simulated by a standard uniform number generator and the method of acceptance-rejection (Fleming and Rishel, 1975; Hanson, 2005) such that the open interval (0, 1) is partitioned into a centered interval of length λdt and the complement of (0, 1).

The jump amplification factor h is related to the particle property and its ambient environment, reflecting the effect of the extreme event on particle movement. The magnitude of the jump amplification factor can be obtained beginning from the SDE of particle trajectory. Equation (1) can be rewritten as,

$$dX_t = \left[\bar{u}(t, X_t) + \sigma(t, X_t) dB_t / dt + h(t, X_t) dP_t / dt\right] dt \quad \dots (9)$$

The particle velocity V_p can be approximately expressed as (Oksendal, 1998),

$$V_p = \frac{dX_t}{dt} = \bar{u}(t, X_t) + \sigma(t, X_t)dB_t / dt + h(t, X_t)dP_t / dt \quad \dots (10)$$

The particle velocity herein is categorized into two parts; the first part is the particle velocity in regular flow environments including a mean drift term and a random term, and the second part is the difference of the particle velocity gained or lost in the presence of the extreme event. According to Newton's second law of motion,

$$m_p \frac{dV_p}{dt} = F_r + F_e \quad \dots (11)$$

where m_p is the mass of the particle and F denotes the summation of all the forces acting on the particle. F_r is the external force acting on the particle in the absence of the extreme event, while F_e is the transitional force to move the particle from a regular flow state to an extreme flow state. The additional force F_e needed for the particle to move from one state to the other state can be expressed as,

$$F_e = m_p \frac{u - V_p}{\tau_p} \quad \dots (12)$$

where u is the mean flow velocity in the presence of extreme events. τ_p is the relaxation time, defined as the time needed for a particle to move from the regular flow state to the extreme flow state. In analogy to Zaichik *et al.* (1997), the relaxation time for a sediment particle to move from its original state to the other in response to an extreme flow event can be obtained based on the force balance equation.

In regular flows, the particle follows the fluids exactly except for the settling velocity. Note that T is the duration of the extreme event. Integrating Equation (11) with the help of Equations (12) and (13) over the entire extreme event duration T gives,

$$V_p = V_{p0} + \int_0^T (F_r)/m_p d\tau + \int_0^T \frac{u - V_p}{\tau_p} d\tau \quad \dots (13)$$

Where V_{p0} is the particle velocity immediately before the occurrence of the extreme event. In other words, it is the particle velocity in regular flow conditions; the second term on the right hand side is the change of particle velocity due to the regular flow force during extreme events; and the third term is the change of particle velocity due to the occurrence of extreme flow events. The first two terms are the particle velocity in regular flow environments, which are related to the mean drift and randomness terms. The third term is the particle velocity change occurring mainly due to the extreme flow event within time τ_p. Now, integrating Equation (13) over the entire extreme event duration gives the particle movement during the extreme event,

$$dX_t = \int_t^{t+T} V_p dt = \int_t^{t+T} \left[V_{p0} + \int_0^T (F_r)/m_p d\tau + \int_0^T \frac{u - V_p}{\tau_p} d\tau \right] dt \quad \dots (14)$$

When the event duration is relatively small compared to the overall simulation time, it is assumed that the impact of the extreme flows is reflected mainly on the jump term during the extreme event. By comparing Equations (10) and (13), we can obtain,

$$\bar{u}(t, X_t) + \sigma(t, X_t)dB_t / dt = V_{p0} + \int_0^T (F_r)/m_p d\tau \quad \dots (15)$$

If exactly one extreme event occurs within a time interval $[t, t + dt]$, $dP_t = 1$. One can compare Equations (1) and (15) during the extreme event period from t to $t + T$ and obtain,

$$h(t, X_t) = \int_t^{t+T} \left(\int_0^T \frac{u - V_p}{\tau_p} d\tau \right) dt \quad \dots (16)$$

For a very fine particle (i.e., $d_p \to 0$) to move from one state to the other in response to an extreme flow event, the relaxation time τ_p approaches zero. We assume that such an immediate response time be modeled by the reciprocal of the Dirac-delta function, i.e.

$$\frac{1}{\tau_p} = \delta(T/2) \quad \dots (17)$$

Following the fundamental property of the delta function, we can obtain,

$$\int_0^T \frac{u - V_p}{\tau_p} d\tau = \int_0^T (u - \dot{V}_p)\delta(T/2)dt = u - V_p \quad \dots (18)$$

As a result, the stochastic jump model for fine particles can be described as,

$$dX_t = \bar{u}(t, X_t)dt + \sigma(t, X_t)dB_t + \left[\int_t^{t+T} (u - V_p)dt\right]dP_t \quad \dots (19)$$

where dP_t is the number of extreme event occurrences within a time interval $[t, t + dt]$. $\int_t^{t+T} (u - V_p)dt$ is the average magnitude of an extreme event effect within the time interval $[t, t + dt]$. For finer particles, Equation (19) describes the particle trajectory composed of two parts: one related to regular flow (the first two terms on the right hand side), and the other due to the occurrence of extreme flow events (the last term on the right hand side).

ILLUSTRATIVE EXAMPLE

Here we present one example to illustrate the impact of the Poisson jumps in the presence of extreme flow events. We assume that the Poisson process and Wiener process are two independent stochastic processes for these examples. For simplicity, the duration of extreme event is set as the computational time step in the following example.

The example shows modeling of the particle trajectory in a one-dimensional flow field. Two different cases are presented here for a prismatic rectangular channel with a bed slope of 0.2% and the cross-section width of 3 m, and the Manning roughness coefficient of 0.03. One case shows a particle in a stationary flow $\bar{u}$ = 0 m/s. The other illustrates that the particle in a regular flow with a water depth of 0.05 m, corresponding to a constant mean drift velocity, $\bar{u}$ = 0.2 m/s. In order to test the jump term effect and for simplicity, the diffusivity in both cases is assumed to be σ = 0.1 m^2/s. Both cases are subject to extreme flow events with an average number of occurrences, λ = 10. Assuming the occurrence of a flood event in this example, has increased the flow rate to 8.2 m^3/s. As such, the corresponding water depth is 1.97 m and flow velocity is 1.39 m/s. The jump term in Equation (19) is set as $1.2dt$ in this example, i.e. the mean drift velocity in the extreme event has increased to u = 1.39 m/s if the duration of the extreme event is dt. The time step in this example is 0.2 s and the simulation time is 100 seconds.

Figure 1 illustrates sample realizations of particle trajectory as well as their ensemble mean (based on 5000 simulations) in the aforementioned two scenarios, respectively. It is noted that in the absence of extreme events, the particle moves randomly but stays in average in its original position in stationary flow, as shown in Figure 1. Note that the relative time herein is defined as the time normalized by the total simulation time. It is shown from this example that the impact of extreme flow event occurrences on the particle trajectory can be quantified given the frequency of extreme flow event occurrences and the mean drift flow characteristics of extreme flows. As can be observed from Figure 1, the particle movement in response to an extreme flow event is subject to a translocation jump that deviates significantly from the movement of a particle in stationary flow in the absence of the extreme event.

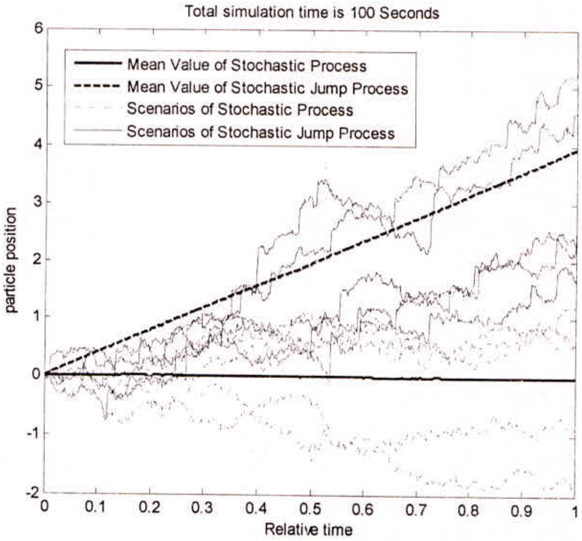

Fig. 1: Sample realizations and ensemble of particle trajectory inflow with a zero drift term, diffusivity = 0.1 m^2/s and the flow drift = 1.4 m/s in extreme flows

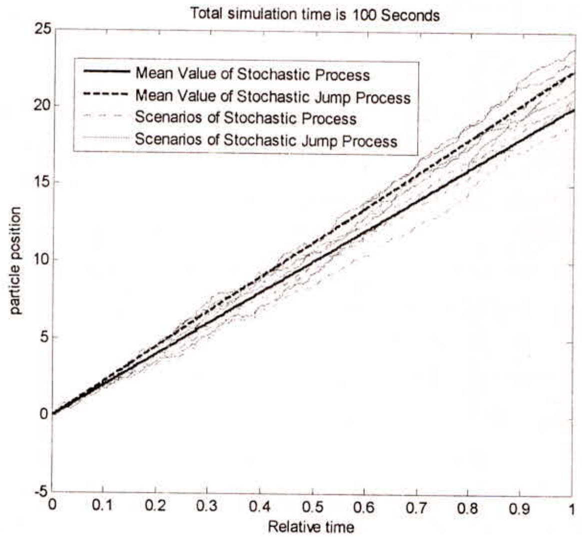

Fig. 2: Sample realizations and ensemble of particle trajectory with flow mean drift = 0.2 m/s, diffusivity = 0.1 m^2/s and the mean flow drift = 1.4 m/s in extreme flows

Figure 2 shows simulation of scenario 2 that the particle initially locates in a one-dimensional flow with a mean drift velocity of 0.2 m/s. The sample realizations of particle trajectory inflow without jumps (solid line) and with jumps (dashed line) are illustrated in Figure 2. It is shown in Figure 2 that the particle subject to jumps moves further downstream compared to that without jumps, as the occurrence of the extreme flow events in this example results in an increased mean flow drift velocity. The variance of particle trajectory for the first scenario in the example is illustrated in Figure 3. It can be seen from Figure 3 that the difference of particle trajectory between the regular flow and the extreme flow can be quantified. Such variances are increasing with respect to time. The source of such variations can be attributed to both the randomness of particle movement and the uncertainty of the extreme event occurrence. Figure 4 presents the variance of particle trajectory for the second scenario. Comparing Figure 3 and 4, it can be concluded that the variance in both scenarios is not affected by the mean drift.

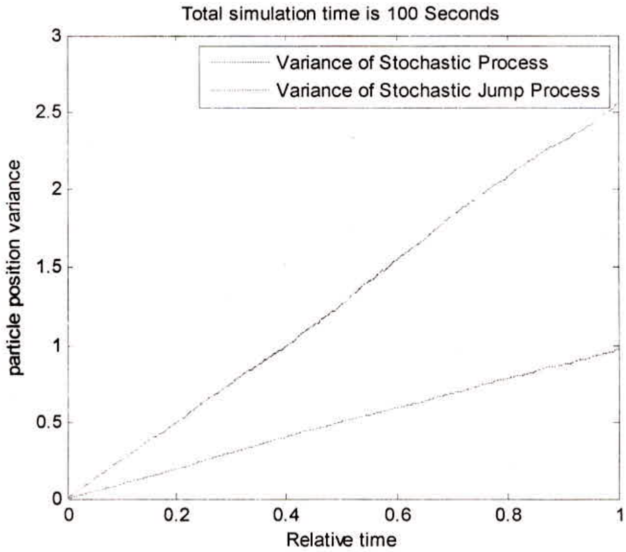

Fig. 3: Variance of particle trajectory in flow with a zero drift term, diffusivity = 0.1 m^2/s and the flow drift = 1.4 m/s in extreme flows

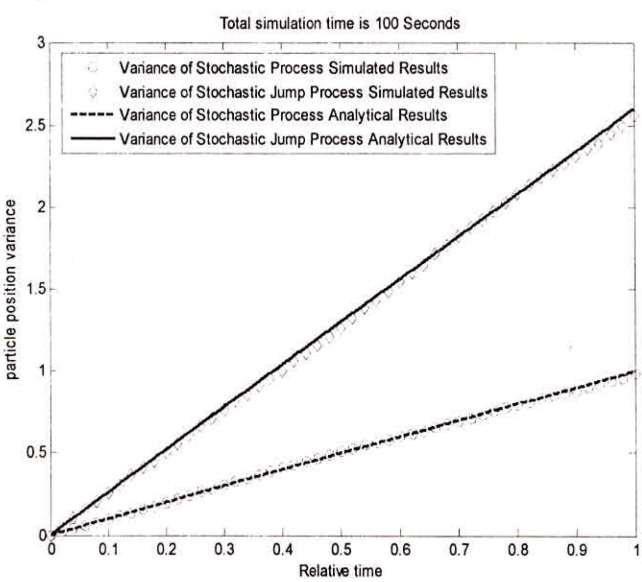

Fig. 4: Variance of particle trajectory with flow mean drift = 0.2 m/s, diffusivity = 0.1 m^2/s and the mean flow drift = 1.4 m/s in extreme flows

CONCLUSIONS

A stochastic particle tracking model simulating suspended sediment movement in surface water flows is proposed herein. The proposed model takes into account both the randomness of particle movement caused by turbulence and the stochastic jumps in the presence of extreme flow events. In the proposed model, a random term mainly caused by the fluid eddy motions is represented by a Wiener process and the occurrence of the extreme event is modeled as a Poisson process. The magnitude of the Poisson jump can be quantified using the flow characteristics of the extreme events and the property of sediment particles. It is demonstrated from the examples that the particle, when subject to positive jumps due to extreme flow events, travels further and faster downstream and also is subject to more frequent resuspensions due to flow accelerations incurred in extreme flow events. Both the ensemble mean and variance of particle trajectory can be quantified using the proposed stochastic diffusion jump model. Compared to the particle tracking models, the proposed model here can be capable of incorporating the frequency of the extreme flow event occurrence in modeling the particle trajectory, which is a step forward in building a forecast model to analyze the movement of particles in flows when there are random extreme flow events.

This paper formulates and presents an alternative approach as opposed to the traditional sediment modeling of movement of sediment particles in both regular flows and during extreme flows. However, more work is needed to refine the proposed method such as incorporation of the large eddy simulation model to better quantify the flow randomness and collection of detailed flow and particle movement data to further validate the proposed particle tracking model.

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