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Comparison of Bed Load Formulae in Hydraulic Geometry Using Similarity Principle

L. Yilmaz
Technical University of Istanbul
Civil Engineering Department, Hydraulics Division
80626, Maslak, Istanbul, TURKEY

ABSTRACT: Employing the similarity principle, this paper derives the bed load formulae. The formulae include relations for stream width, meander wave length, and bed slope, which are related to friction factor, bed load discharge, bed load diameter, and the water depth. On comparison using the data on the hydraulic geometry of several alluvial laboratory streams, the derived formulae are found to match well with those by Einstein (1950), Shields (1936), and Meyer-Peter and Muller model (1948).

INTRODUCTION

A rational understanding of the hydraulic behavior of alluvial streams is of paramount importance in the planning and design of hydraulic structures in alluvial settings. The sediment and water discharge of a river are primarily determined by the nature of the drainage area. Independent of the river itself, these are determined exclusively by the hydrology, geology, and topography of the drainage area. According to the water and sediment discharge, the river creates its own geometry, viz. slope, depth, width, and meandering pattern. Since the slope and meander pattern do not respond rapidly enough to follow seasonal variations of discharge, it is natural to invoke some kind of dominant or formative values of the discharge for these variables (Engelund and Hansen, 1966; Hansen, 1967; Kennedy and Alam, 1967).

In a given river, the water and sediment discharges normally increase in the downstream direction, and so do the depth and the width of the stream. The slope and the grain size usually decrease gradually from the source to estuary. According to Leviavsky (1955), the grain size decreases approximately exponentially in the downstream direction. These observations naturally tempt to look for empirical relations for the variation of the depth, width, and slope as functions of the water discharge, which has been turned out to be the most important quantitative independent parameter and a number of "regime" relations have been suggested using it. A recent account to such relations has been given by Blench (1957; 1966), as follows,

$$B \sim Q^{1/2} \qquad \text{... (1)}$$

$$D \sim Q^{1/3} \qquad \text{... (2)}$$

$$I \sim Q^{-1/6} \qquad \text{... (3)}$$

where B denotes the width, D the depth, I the slope and Q the water discharge. An empirical relation expressing the observations of meander "wave-lengths" has been suggested by Inglis (1947) as,

$$L \sim Q^{1/2} \qquad \text{... (4)}$$

Eqs. 1 and 4 describe a direct proportionality between the width of the stream and the meander length. Using the data of a large number of American and Indian rivers as well as for several small scale model tests, Leopold and Wolman (1957) derived the following popular relation,

$$L = 10B \qquad \text{... (5)}$$

Due to complex mechanics of the bed load and water transport, a number of variations of the above formulae have been proposed in literature. The objective of this paper is to derive bed load formulae using the principle of similarity and to compare them with the available popular ones using the laboratory data of several experimental alluvial streams.

SIMILARITY PRINCIPLE

According to the principle of similarity, a system of non-dimensional parameters is obtained to characterize the flow system. Three types of similarity exist between a prototype and its model in order to ensure that the model is completely similar to, and thus accurately represents the prototype. The main types of similarity are geometric, kinematic and dynamic.

A model should be similar in shape to the prototype. Any unspecialized person can recognize that the model is just another version of the prototype, but with a different scale, generally smaller. All dimensions in the prototype are reduced by a fixed factor called the scale ratio. The use of similarity considerations is feasible when the effect of viscous shear is negligible, i.e. for dunes and the upper flow regime. In case of ripples, a generally pronounced scale effect is expected.

Consider two flow systems, 1 and 2, that are supposed to be geometrical similar, with the scale ratio $\lambda = D_1/D_2$, D_i = flow depth for system i. The conditions for complete similarity, assuming the bed configuration to be dunes. Geometric and dynamic similarity demands,

$$(1)\ \frac{D_1}{d_1} = \frac{D_2}{d_2},\ \text{or}\ \lambda = \frac{D_1}{D_2} = \frac{d_1}{d_2} \qquad \dots (6)$$

(2) Equal Slopes $I_1 = I_2$

(3) The dimensionless effective bed shear stress must be the same,

$$\theta_1' = \theta_2'$$

$$\theta' = \frac{D'I}{(S-1)d} \qquad \dots (7)$$

where $S = \dfrac{\gamma_s}{\gamma}$ = relative density of sediment grains, d = mean fall diameter. The dune patterns will be similar when these conditions are similar. That is, $\theta_1 = \theta_2$ and $\theta_1'' = \theta_2''$

$$\theta'' = \frac{1}{2}F^2 \frac{\alpha h^2}{(S-1)dI} \qquad \dots (8)$$

F = Froude number, h = dune height, α = coefficient velocity. This says that the expansion loss is the same fraction of the total loss of mechanical energy in both streams. Since the slopes are the same for both the streams, the friction factor is the same because slopes are common, Froude numbers are identical. Thus, for a given sediment size, the flow system is completely defined if D, I, and θ' are specified. Then the scale ratios of sediment rates can be calculated,

$$\frac{f_1}{f_2} = \frac{f_1'}{f_2'} = \frac{\lambda_H}{\lambda_L} \qquad \dots (9)$$

λ_L = horizontal length scale ratio and λ_H = vertical length scale ratio.

Expressing hydraulic resistance and transport capacity of alluvial streams,

$$V = 10.9d^{-\frac{3}{4}}D^{\frac{5}{4}}I^{\frac{9}{8}}\ (\theta > 0.15 \text{ and for dunes only}) \qquad \dots (10)$$

$$f\Phi = 0.1\theta^{\frac{5}{2}} \qquad \dots (11)$$

Taking L for the wave length, B for width,

$$f\frac{L}{D} = c_1,\ f\frac{B}{D} = c_2 \begin{pmatrix} c_1 = 14 \\ c_2 = 1.4 \end{pmatrix} \qquad \dots (12)$$

The principle of similarity predicts,

$$\frac{L}{B} = 10$$

$$f\frac{L}{D} = 14 \qquad \dots (13)$$

Recalling definitions of θ,

$$\theta = \frac{DI}{(S-1)d},$$

$$I = f\frac{V^2}{2g}\frac{1}{D} = \frac{1}{2}fF^2,$$

$$F = \frac{V}{\sqrt{gD}} \qquad \dots (14)$$

$$\Phi = \frac{q_T}{\sqrt{(S-1)qd^3}},$$

$$q_T = \frac{Q_T}{B}, \qquad B = \text{water surface width}$$

Utilizing these relations, one gets,

$$B = 0.78d^{-0.316}Q^{0.525}$$

$$D = 0.108\left(\frac{Q_T}{Q}\right)^{-\frac{2}{7}} d^{0.21}Q^{0.317}$$

$$I = 12.8\left(\frac{Q_T}{Q}\right)^{\frac{4}{7}} d^{0.527}Q^{-0.212} \qquad \dots (15)$$

$$L = 7.8d^{-0.316}Q^{0.525}$$

Comparison of the Similarity Principle Results with Shields (1936) Approach

On the basis of his experimental results, Shields (1936) proposed a dimensionally homogeneous transport function,

$$Q_b\gamma_s / Q\gamma S = 10(\tau_0 - \tau_c)/(\gamma_s - \gamma)d_m \qquad \dots (16)$$

where

Q_b is bed load transport rate (tons/hour);

d_m is the effective diameter of sediment (mm).

Shields (1936) hypothesized that the rate of transport was a function of the dimensionless resistance coefficient,

$$\tau_o\left[(\gamma_S-\gamma)d_m\right]^{-1}$$

the critical value controlling the incipient motion of the bed load. The computation of the shear stress uses the hydraulic radius R and the slope S of the flume,

$$\tau_o=\gamma RS$$

whereas the critical shear stress is obtained from Straub's graph (1935) for various sediment sizes. The bed load transport rate obtained from the Shields formula is the mass of the solid particles. This rate value should be multiplied by a factor 1.60, in accordance with the density of sand, to obtain the volumetric value,

$$I=f\frac{V^2}{2g}\frac{1}{D}=\frac{1}{2}fF^2;\ F=\frac{V}{\sqrt{gD}}$$

$$\frac{\gamma_sQ_b}{\gamma QI}=\frac{10(\tau_0-\tau_c)}{(\gamma_s-\gamma)d_m};\ q_B=\frac{Q_B}{B}\ \ B=\text{water surface width}$$

$$\frac{\gamma_sQ_B}{\gamma QI}=\frac{10DI}{(s-1)d\gamma}$$

$$\frac{\gamma_sQ_B}{QI^2}=\frac{10D}{(s-1)d}$$

$$\frac{\gamma_sq_BB}{Qf^2\dfrac{V^4}{(2g)^2}\dfrac{1}{D^2}}=\frac{10D}{(s-1)d}$$

$$\frac{\gamma_sq_BB}{Qf^2\dfrac{V^4}{384.94}\dfrac{10D}{D^2}}=\frac{1}{(1.65)d}$$

$$\frac{\gamma_sq_BB}{Qf^2\dfrac{V^4 0.02597}{D}}=\frac{1}{(1.65)d}$$

$$\gamma_sq_BB=\frac{0.0157Qf^2V^4}{dD}$$

Using the equations like in similarity principle, the parameters B, L, D and I is computed and gives not the same values of Engelund and Hansen (1967) as can be seen below.

q_B is the rate of bed load transport in volume of material per unit time width of section. Thus the dimensionless form of the bed load discharge is given also by Einstein (1950). Einstein's dimensionless bed load discharge and dimensionless form of the bed shear is a rather complicated expression. However, for small sediment discharges, it follows rather closely the simple relation as given by Engelund and Hansen (1967),

$$\Phi_B=8(\theta-0.047)^{3/2}$$

Shields Approach,

$$\frac{\gamma_sq_BB}{\gamma Qf^2\dfrac{V^4}{4g^2}\dfrac{1}{D^2}}=\frac{10D}{\left(\dfrac{\gamma_s}{\gamma}-1\right)d_m}$$

$$\frac{q_BBD^2 4g^2}{Qf^2V^4}=\frac{10D}{\dfrac{\gamma_s}{\gamma}\left(\dfrac{\gamma_s}{\gamma}-1\right)d_m}$$

Using also other formulas,

$$\frac{q_BB^5D^5 4g^2}{Q^5f^2}=\frac{10}{\dfrac{\gamma_s}{\gamma}\left(\dfrac{\gamma_s}{\gamma}-1\right)d_m}\quad \text{using } q_B=\frac{Q_B}{B}$$

$$Q_B=\frac{10Q^5f^2}{\dfrac{\gamma_s}{\gamma}\left(\dfrac{\gamma_s}{\gamma}-1\right)d_mB^4D^5 4g^2}$$

$$Q_B=\left(\frac{10}{4g^2}\right)Q^5f^2\left[\frac{\gamma_s}{\gamma}\left(\frac{\gamma_s}{\gamma}-1\right)\right]^{-1}d_m^{-1}B^{-4}D^{-5}$$

$$Q_B=0.02598Q^5f^2\left[\frac{\gamma_s}{\gamma}\left(\frac{\gamma_s}{\gamma}-1\right)\right]^{-1}d_m^{-1}B^{-4}D^{-5}$$

$$B^4=\frac{Q^5}{Q_B}0.005941f^2d_m^{-1}D^{-5}$$

$$B=(0.2776)\frac{Q^{1.25}}{Q_B^{0.25}}f^{0.5}d_m^{-0.25}D^{-1.25}$$

$$D=0.3587\frac{Q}{Q_B^{0.20}}f^{0.4}d_m^{-0.2}B^{-0.8}$$

$$L=3.587\frac{Q}{Q_B^{0.20}}f^{0.4}d_m^{-0.2}B^{-0.8}$$

$$I=0.6612\left(\frac{Q_B}{Q}\right)^{0.5}d^{0.5}D^{-0.5}$$

COMPARISON THE SIMILARITY PRINCIPLE RESULTS WITH EINSTEIN'S (1950) APPROACH

The bedload distribution due to Einstein (1950) is given in below formulae where the intensity of bed-load transport can be expressed as,

$$\Phi_*=(i_{Bw}.q_{bw}/i_{bw}.g.\ \rho_s)\ \sqrt{\rho/(\rho_s-\rho)}.\ (\sqrt{1/gD^3})\ q_{bw}=q_s$$

$$\Psi_*=(\gamma_s-\gamma)\ D/\gamma_s\ R\ S$$

where

i_{BW} is the fraction of the total sediment load for a given sediment size D;
i_{bw} is the fraction of the bed sediment of a given sediment size D;
q_{bw} is the bed load discharge by weight per unit width per unit time;
g is the gravitational acceleration (m/sec^2);
ρ_s is the density of sediment (kg-sec^2/m^4);
ρ is the density (kg-sec^2/m^4);
D is the particle size (m);
q_s is the sediment discharge in gr. per second per length on a dry weight basis;
Ψ_* is the shear intensity parameter;
γ_s is the specific weight of solid particles (T/ m^3);
γ is the specific weight of water (T/m^3); and
S is the slope.

The similarity principle results can be compared with the Einstein (1950) bedload function. The flow intensity parameter, ψ_*, is calculated from flow hydraulics and the characteristics of the sediment on the stream-bed; and the bedload intensity, Φ_{*1} is read from the bed load function. The unit bedload, q_{sbi}, and particle availability, P_i, are the only unknown parameters. Where the particle size appears in the flow intensity calculation, the particle availability, P_i, is not a parameter.

Setting the particle availability parameter, P_i, equal to 1, one obtains,

$$q_{sbi} = P_i \Phi_* \gamma_s / \{[\gamma/\gamma_s - \gamma\}]/g D_{si}^3\}^{1/2}$$

where D_{si} is the grain diameter for which si per cent is finer;

q_{sbi} is the intensity of bed load movement of size class i;
Φ_* is the bed load intensity;
γ is the unit weight of water;
γ_s is the unit weight of sediment particles;
g is the gravitational acceleration;
P_i is the particle availability parameter~ G_{di}/G_d (bed surface gradation);
G_d is the total weight of sediment in the bed surface layer, and
G_{di} is the weight of the i^{th} size class in the bed surface layer.

The bed surface layer in this equation is a zone near the bed surface called the "active layer". The surface elevation is changed as sediment is deposited into or scoured out of this layer.

The transport capacity of the meandering flume was expressed as,

$$q_s = \Sigma q_{pbi} P_i$$

where

q_{pbi} is the transport potential of bed material load for size class i;
P_i is the fraction of bed surface particles in size class i, and
i is the particle size class interval.

Using the equations,

$$I = f\frac{V^2}{2g}\frac{1}{D} = \frac{1}{2}fF^2$$

$$F = \frac{V}{\sqrt{gD}}$$

$$\frac{Q_B}{B} = \frac{\Phi_*\gamma_s}{\left[(\gamma/\gamma_s - \gamma)/gD_{si}^3\right]^{1/2}}$$

$$q_B = \frac{Q_B}{B} \qquad B = \text{water surface width}$$

Einstein's (1950) Approach,

$$B = \frac{Q_B\left[\left(\gamma^2/(s-1)\gamma\right)/gD_{si}^3\right]^{1/2}}{\Phi_*\gamma_s}$$

$$B = Q_B\Phi_*^{-1}\gamma_s^{-1}\left[\left(\gamma^2/(s-1)\gamma_s\right)/gD_{si}^3\right]^{1/2}$$

$$L = 10Q_B\Phi_*^{-1}\gamma_s^{-1}[(\gamma^2/(s-1)\gamma_s)/gD_{si}^3]^{1/2}$$

$$I = f\frac{Q^2/A^2}{2g}\frac{1}{D} = f\frac{Q^2/B^2D^2}{2g}\frac{1}{D} = f\frac{Q^2}{B^2D^3}\frac{1}{2g}$$

$$D^3 = fQ^2B^{-2}0.050968(\text{I})^{-1}$$

$$D^3 = fQ^2B^{-2}0.050968\frac{1}{2}fV/\sqrt{gD}$$

$$D^3 = fQ^2B^{-2}0.025484fV(gD)^{-1/2}$$

$$D^3 = fQ^2B^{-2}0.008141fVD^{-1/2}$$

$$D^{3.5} = f^2Q^2B^{-2}0.008141V$$

$$D = f^{0.571}Q^{0.571}B^{-0.571}0.25291V^{0.28571}$$

$$D = 0.2529f^{0.571}Q^{0.571}B^{-0.571}V^{0.28571}$$

$$V = 122.93f^{-1.998}Q^{-1.998}B^{1.998}D^{3.5}$$

$$I = fQ^2B^20.0509D^{-3}$$

Comparison of the Similarity Principle Results with Meyer-Peter and Muller (MPM) Model (1948)

Meyer-Peter and Muller (1948) carried out flume experiments in the Laboratory for Hydraulic Research and Soil Mechanics at the Swiss Federal Institute of Technology, Zurich, and derived a formula for bed load discharge with the aim to develop a more practical formula. The U.S. Bureau Reclamation (U.S.B.R.) modified it, converted from metric units to English units, and obtained the following form,

$$q_b = (\gamma_s/\gamma_s - \gamma)\,(\sqrt{g/\gamma})\,(M - N)^{3/2}$$

where

$$M = 0.0667\,\gamma\,(Q_b/Q)\,[(d_{90})^{1/6}/n_b]^{1.5}\,y\,S$$

$$N = 0.0076\,(\gamma_s - \gamma)\,d_m$$

where

- q_b is the bed load discharge (tons/ ft day);
- g is the gravitational acceleration (m/s^2);
- Q_b is the effective water discharge that determines the bed load transport (m^3/s);
- Q is the total water discharge (m^3/s);
- y is the average water depth (ft);
- d_{90} is the diameter of particles 90% finer obtained from sieving test (mm); and
- n_b is the coefficient of roughness.

This formula was derived from experiments using a laboratory flume with a maximum width of 2 m, very different from the conditions encountered in large channels. The formula depends primarily on the grain diameter and water discharge. Bogardi (1978) indicated several difficulties that are encountered in application of this formula.

According to the Meyer-Peter and Muller (1948) (MPM) Model,

$$\gamma DS = 0.047(\gamma_s - \gamma)D_s + 0.25\rho^{1/3}q_s^{2/3}$$

when $q_s = 0$ this equation reduces to,

$$\frac{\tau_c}{\gamma_s^i D_s} = 0.047$$

which defines the beginning of motion,

$$S = (\gamma d)^{-1}\left[0.047(\gamma_s - \gamma)D_s + 0.25\rho^{1/3}q_s^{2/3}\right]$$

$$\frac{Q_B}{B} = (\gamma_s/\gamma_s - \gamma)(\sqrt{g/\gamma})(M - N)^{3/2}$$

$$B = Q_B[(\gamma_s/\gamma_s - \gamma)(\sqrt{g/\gamma})(M - N)^{3/2}]^{-1}$$

$$L = 10Q_B[(\gamma_s/\gamma_s - \gamma)(\sqrt{g/\gamma})(M - N)^{3/2}]^{-1}$$

$$D = \left[0.047(\gamma_s - \gamma)D_s + 0.25\rho^{1/3}q_s^{2/3}\right]\gamma^{-1}S^{-1}$$

DISCUSSIONS

Engelund and Hansen (1967) applied Bagnold's stream power concept and the similarity principle to obtain their sediment transport equation,

$$f\phi = 0.1\theta^{5/2}$$

with

$$f' = \frac{2gSD}{V^2}$$

$$\phi = \frac{q_t}{\gamma_s\sqrt{\frac{(\gamma_s - \gamma)}{\gamma}gd^3}}$$

$$\theta = \frac{\tau}{(\gamma_s - \gamma)d}$$

in which g = gravitational acceleration; S = energy slope; V = average flow velocity; q_t = total sediment discharge by weight per unit channel width; γ_s and γ = specific weight of sediment and water, respectively; d = median particle diameter; and τ = shear stress along the bed. Strictly speaking, Engelund and Hansen's equation should be applied to flows with dune beds in accordance with the similarity principle. However, Eqn. 6.6 can be applied to flows with dune beds and upper regime bed forms with particle sizes greater than 0.15 mm, without serious error. According to Yang (1987), a river can adjust its roughness, geometry, profile and pattern through the processes of sediment transport. Qualitative descriptions of these dynamic adjustments of natural streams have been made mainly by geologists and geomorphologists. Empirical regime types of equation have been developed by engineers to solve design problems. Attempts have been made in recent years to explain these adjustments based on different extremal theories and hypotheses.

REFERENCES

Bagnold, R.A. (1954). "Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear." Proc. Roy. Soc., Vol. 225, A.

Bagnold, R.A. (1956). "The Flow of Cohesionless Grains in Fluids." *Philosophical Trans. of Roy. Soc. of London, A*, Vol. 249.

Blench, T. (1957). "Regime Behavious of Canals and Rivers. " *Butterworths Scientific Publications, London.*

Blench, T. (1966). "Mobile-Bed Fluviology." Dept. Tech. Serv., Tech III. Div., Univ. of Alberta, Edmonto, Alberta, Canada.

Brooks, N.H. (1958). "Mechanics of Streams with Movable Beds of Fine Sand." Trans. Am. Soc. Civ. Engrs., Vol. 123.

Chao-Lin, Chiu and Mc Sparrow, J.E. (1966). "Effect of Secondary Flow on Sediment Transport." Journ. Hydr. Div. HY 5, *Proc. Am. Soc. Civ. Engrs., September.*

Du Boys, P. (1879). "Le Rhone et les rivieres a lit affouillable." *Annales des ponts et chaussees*. Series 5, Vol. 18.

Einstein, H.A. (1950). "The Bed-Load Function for Sediment Transportation in Open Channel Flows." *Technical Bulletin No. 1026.* U.S. Dept. of Agriculture.

Einstein, H.A. and Barbarossa, N.L. (1952). *River Channel Roughness.* Trans. Am. Soc. Civ. Engrs. Vol. 117.

Engelund, F. and Hansen, E. (1966). "Investigations of Flow in Alluvial Streams." *Acta Polytechnical Scandinavica.* Civil Engineering and Building Construction Series No. 35, Copenhagen.

Engelund, F. (1966). "Hydraulic Resistance of Alluvial Streams." *Journ. Hydr. Div. Vol. 92 HY 2, Proc. Am. Soc. Civ. Engrs.*, March.

Engelund, F. and Hansen, E. (1967). "Comparison between Similarity Theory and Regime Formulae." *Basic Research-Progress Report No. 13, January,* Hydraulic Laboratory, Techn. Un. of Denmark.

Engelund, F. (1967). "A Sediment Transport Theory Based on Similarity." *Basic Research-Progress Report No. 13,* Jan, Hydraulic Laboratory, Tech. Un. of Denmark.

Guy, H.P., Simons, D.B. and Richardson, E.V. (1966). "Summary of Alluvial Channel Data from Flume Experiments, 1956–1961." *U.S. Geological. Survey Professional Paper 462–I.*

Hansen, E. (1967). "On the formation of Meanders as a Stability problem." *Basic Research-Progress Report No. 13,* Jan., Hydraulic Laboratory, Tech. Uni. of Denmark.

Inglis, C.C. (1947). "Meanders and their Bearing on River Training." *Proc. Instn. Civ. Engrs.,* Maritime and Waterways Pap. No. 7, January.

Kennedy, J.F. (1963). "The Mechanics of Dunes and Antidunes in Erodible-Bed Channels." *Journal of Fluid Mech.,* Vol. 16, Part 4.

Kennedy, J.F. and Alam, A.M.Z. (1967). Discussion of "Hydraulic Resistance of Alluvial Streams" *in Journ. Hydr. Div.,* Vol. 93, HY 1, January.

Leviavsky, S. (1955). "An Introduction to Fluvial Hydraulics." Constable and Company Ltd., London.

Meyer-Peter, E. and Muller, R. (1948). "Formulas for Bed-Load Transport." *Proc. 3rd Meet. Int. Ass. Hydr. Res.*, Stockholm.

Nikuradse, J. (1933). "Stromungsgesetze in rauhen Rohren." Forsch.-Arbeiten Ing.-Wesen, No. 361.

Nordin, C.F. and Algert, J.H. (1966). "Spectral Analysis of Sand Waves." *Journ. Hydr. Div. HY 5, Proc. Am. Soc. Civ. Engrs.*, September.

Prandtl, L. (1927). "Uber die ausgebildete Turbulenz." Verh. D.H. Intern. Kongr. F. Techn. Mech., Zurich.

Raudkivi, A.J. (1963). "Study of Sediment Ripple Formation." *Journ. Hydr. Div. Hy 6, Proc. Am. Soc. Civ. Engrs.,* November.

Reynolds, A.J. (1965). "Waves on the Erodible Bed of an Open Channel." *Journ. of Fluid Mech.,* Vol. 22, Part 1.

Richardson, J.F. and Zaki, W.N. (1954). "Sedimentation and Fluidisation." Trans. Instn. Chem. Engrs. London, Vol. 32.

Rouse, H. (1939). "Experiments on the Mechanics of Sediment Suspension." *Proc. 5th Intern. Congr. Appl. Mech.,* Cambridge, Mass.

Simons, D.B., Richardson, E.V. and Albertson, M.L. (1961). "Flume Studies Using Medium Sand (0.45 mm)." *Geological Survey Water-Supply Paper 1498-A.*

Shields, I.A. (1936). "Anwendung der Ahnlichkeitmechanik und der Turbulenzforschung auf die Geschiebebewegung." Mitt. Preuss. Ver. –Anst., Berlin, No. 26.

Vanoni, V.A. (1946). "Transportation of Suspended Sediment by Water." Trans. Am. Soc. Civ. Engrs., Vol. 111.