# DYNAMIC PROGRAMMING MODEL FOR OPTIMISATION OF STORMWATER RETENTION PONDS IN MULTIPLE CATCHMENT SYSTEM

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# INTRODUCTION

Suitably designed detention facilities for stormwater can be used to reduce the negative effects of urban development, especially with regard to drainage options. Such facilities have to satisfy environmental regulations, both in quality and quantity aspects. Since land values are generally higher in urban areas and, at the same time one cannot afford to relax the runoff and pollution control standards, it is essential that a suitable approach is adopted to design a stormwater detention pond which can take various factors into account to obtain a least cost solution. The methodology proposed by Papa and Adams (1997), for optimising the pond geometry of multiple parallel catchment system, has been chosen and modified suitably. In this system, each catchment may have a pond with variable performance but the collective pond performances are to meet a specified pollution control level at a common discharge point (Behera et al., 1999). The objectives of the present study are: (i) to develop an optimisation model for stormwater detention ponds in multiple parallel catchments using dynamic programming, and (ii) to apply the developed model to the Hyderabad urban catchment in India.

## **STUDYAAREA**

The area chosen for the study is a part of Hyderabad city in South India. Hyderabad city and its surroundings extend from 17 10'N to 17 30'N latitudes and 78 10'E to 78 50'E longitudes. Located at an average altitude of 500 m above mean sea level, the city comprises single and multi-family residential, industrial, commercial and public zones. Due to rapid urbanisation,

about 50 to 60% of the land area in the city is now impervious. The average annual rainfall in the city is about 1100 mm and temperature varies from 10 e during winter to as high as 44 e during summer. The major part of the rainfall occurs during the monsoon months of June to October. The city is developing into a city with modern information technology related infrastructure. The area taken for the present study is 852 hectares in extent and consists of Rajendra Nagar area and both sides of Usa river, a tributary of the Musi river. The area is traced from the Survey of India (SOI) topo-sheet No: 56KI7ISE (1:25000). The study area extends from 78 22' 30"E to 78 25'00"E longitudes and from 17 20'00"N to 17 22'30"N latitudes (see Figure 1). The area is divided into five sub-catchments. The statistical parameters of the rainfall series of the watershed are: Inter Event Time = 2 hours; Inverse Mean Rainfall Event Duration = 0.198 h<sup>-1</sup>; Inverse Mean Rainfall Event Volume= 0.31 mm<sup>-1</sup>; Inverse Mean Rainfall Inter Event Time = 0.025 h<sup>-1</sup>. The total suspended solids in the stormwater of the study area ranges from 92-296 mg 1<sup>-1</sup>. Figure 2 shows the schematic representation of part of the Hyderabad urban watershed used in the optimisation.

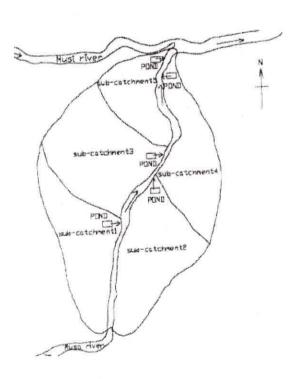
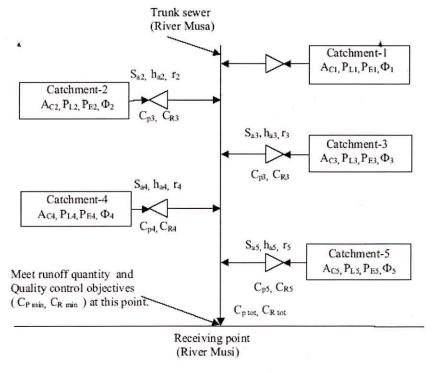


Fig. 1 Study area map

#### DYNAMICAPROGRAMMING

Dynamic programming (DP) is used for the optimisation and the procedure adopted is explained below. In the approach adopted, the multiple catchment problem is transformed into series of individual catchment problems (stages). The solutions of the smaller problems are then combined to obtain the solution or policy of the overall (multiple catchment) problem. However, the interdependence among the individual stages is preserved, i.e. the overall pollution control and runoff control levels are not violated by decomposition (Papa and Adams, 1997).



Legend: Represents a sub-catchment SWM pond with storage volume Release rate, depth

Fig. 2 Schematic representation of parallel catchment system

Typically, a large urban catchment/watershed is composed of a number of small to mediumsized sub-catchments, each having a dominant land use pattern (residential/ industrial/ commercial/open). Hence, they can be characterised by different parameters affecting runoff quantity and quality. Consider the system of parallel catchments illustrated in Figure 2, each with a stormwater management (SWM) pond responsible for controlling quantity and quality of runoff from its catchment. These ponds are located upstream of its outlet draining into a common collection system (e.g. trunk sewer) which subsequently carries the runoff into a receiving water. Each catchment is defined spatially, based on its local topography and runoff coefficient, as well as its land and construction costs. In the present work, it is assumed that the meteorological characteristics for each of the multiple sub-catchments are the same as the entire catchment.

# **OBJECTIVEAFUNCTION**

The objective is to minimise the total cost of implementing the SWM ponds in each of the catchments. The costs considered include the cost of the land on which the stormwater detention facility is built, costs of construction, and operation, maintenance and repairs (OMR) of the pond. The cost of local conveyance (from the pond to its collector) is not included, as SWM ponds are usually located very close to the trunk sewer. (Adams and Papa, 2000). In the present work, the SWM pond is used as the extended detention dry pond type, in which the catchment runoff is stored until the entire volume is used and the release rate is exceeded by the inflow rate, at which point a spill or bypass results. The cost of a single SWM pond can be estimated by

$$P_{vond} = P_L A_P + P_E V_P \tag{1}$$

where  $A_p$  and  $V_p$ = pond surface area (m<sup>2</sup>) and volume of the pond (m<sup>3</sup>) respectively,  $P_{pond}$  = total cost of pond (Indian Rupees);  $P_1$  = present land value (Indian Rupees/ m<sup>2</sup>); and  $P_E$  = present value of construction and OMR costs (Indian Rupees/ m<sup>3</sup>).

The decision variables in this optimal design procedure are the active storage volume of the pond ( $S_A$  expressed in millimetres uniformly distributed over the catchment), the controlled release rate from the pond (r, expressed in mm uniformly distributed over the catchment per hour) and pond depth ( $h_A$  in metres). The cost function is, hence, expressed in terms of these

decision variables as:

$$V_{p} = A_{p}h_{A} = A_{c}S_{A}10 \tag{2}$$

where  $A_C$  is the catchment area in ha;  $S_A$  the active storage volume in mm (as explained in previous section); and the multiplier 10 is the dimensionless conversion unit for obtaining pond volume,  $V_P$  in  $m^3$ .

Substituting eqn.(2) in eqn.(1) and solving for P<sub>pond</sub>, the following relation is obtained

$$P_{pond} = P_{L} \frac{A_{C} S_{A} 10}{h_{A}} + P_{E} A_{C} S_{A} 10 \tag{3}$$

The least-cost combination of SWM ponds in the multiple catchments scenario may be obtained through a DP optimisation procedure. In this formulation, the main objective is to obtain the least total cost associated with the SWM ponds in all catchments, while satisfying the desired levels of overall system performances in terms of runoff quantity and quality control. The objective function of the DP for the problem taken up for the present study may be expressed as

Minimize 
$$\{TC = \sum_{i=1}^{5} (P_{pond})_i = \sum_{i=1}^{5} (P_{Li} \frac{A_{ci} S_{Ai} 10}{h_{Ai}} + P_{Ei} A_{Ci} S_{Ai} 10)\}$$
 (4)

where TC is the total cost associated with implementation of ponds in Indian Rupees; Index, i denotes the number of sub-catchments in the study area (in the present case, i = 1 to 5, and also it denotes stage of the computation in DP);  $P_{pond}$  is the cost associated with implementation;  $P_L$  is the cost of land occupied by pond in Rupees per  $m^2$ ;  $P_E$  is the unit cost of excavation in Rupees/ $m^3$  This is subjected to the following system constraints

#### Pollution control constraint:

Pollution control means that the amount (in percentage) of total suspended solids (TSS) reductionIremoval by the sedimentation process in the stormwater detention pond, when the

stormwater is routed through the detention pond. For more details of the sedimentation process, TSS removal, pollution control isoquants, the reader can refer to Adams and Papa (2000). The pollution control restraint ensures that the overall pollution control obtained from the system, prior to the discharge into the receiving water, must achieve at least the minimum specified level of control. This constraint is represented as

$$C_{Ptot} = \frac{\sum_{i=1}^{5} C_{Pi} A_{Ci}}{\sum_{i=1}^{5} A_{Ci}} \ge C_{Pmin}$$
(5)

where  $C_{plot}$  is the area-weighted average level of pollution control provided by parallel catchments;  $C_{pmin}$  the minimum pollution control objective to be satisfied at the outfall (expressed as fraction); and  $C_{pi}$  is the pollution control performance expressed as fraction for the i-th catchment.

# Runoff control constraint:

Runoff control is the amount of percentage of stormwater controlled through optimum releases of stormwater from detention ponds, prior to discharging directly into downstream receiving waters. For more details of runoff control isoquants, the reader can refer Behera et al. (1999). This constraint ensures that the overall runoff control from the system must achieve at least the minimum specified level of control, which can be represented as

$$C_{Rtot} = \frac{\sum_{i=1}^{3} C_{Ri} A_{Ci}}{\sum_{i=1}^{3} A_{Ci}} \ge C_{R \min}$$

$$(6)$$

where  $C_{Rtot}$  is the area-weighted average level of runoff control provided by parallel catchments;  $C_{R min}$  is the minimum runoff control objective to be satisfied at the outfall (expressed as fraction); and  $C_{Ri}$  is the runoff control performance expressed as a fraction for the i-th catchment.

The DP is characterised by a forward recursive equation, which is a partial objective function expressing the cost for each combination of states at any stage in terms of the possible decisions that can be made during that stage and states in the next stage. The forward

recursive equation for the problem is

$$f_m^*(S_m) = Min_{Dm} \left\{ r_m(S_m, D_m) + f_{m-1}^*(S_m - 1) \right\}$$
 (7)

where  $S_m$  is the state variable describing the level of pollution control provided by the system ( $C_{Ptot}$ ) at stage (catchment) m,  $D_m$  the design (decision variable values) of the pond in catchment m ( $S_a$ , r, h).  $r_m$  is the cost of the pond at stage (catchment) m, and  $f_m$  ( $S_m$ ) is the total cost of all ponds that can be provided by a SWM pond in the catchment (Adams and Papa, 2000).

# RESULTSAANDADISCUSSION

The computation procedure begins with the arrangement of a catchment area. The catchments are numbered from 1 to 5. The input data for the DP model are shown in Table 1. The results presented herein are for the set target of 60% ( $C_{ptot}$ = 60) of TSS reduction and runoff control of 90% ( $C_{Rtot}$ = 90) at the downstream receiving waters, i.e. the state variables for DP model are set at 60% and 90% respectively for pollution reduction and runoff control. The state variables are discretised into 10 values at increments of 5% representing different states of reduction total pollution and runoff control. The pond depth is discretised at increments of 0.10 m between the pond depths of 1 m and 3 m for optimising the pond cost at every stage. The feasible set of state variables and the optimal values of decision variables are computed for stage 1 (sub-catchment 1), stage 2 (sub-catchment 2) and so on. Due to space restrictions, the results of the stage-wise computation, figures for pollution control isoquants and runoff control isoquants are not presented in the paper. The optimal design policy for the five parallel catchment system for 60% pollution reduction and 90% runoff control is shown in the Table 2. The DP model allows the user to set the various targets (state variables) of pollution reduction and runoff control.

Table 1. Input data for DP Model

Design inputs	Catchment 1	Catchment 2	Catchment 3	Catchment 4	Catchment 5
Area, A <sub>c</sub> (hectares)	221	173	206	150	102
Co-efficient of runoff,(F)	0.6	0.55	0.5	0.55	0.5
Land cost P, (R <sub>s</sub> \m²)	370	450	500	450	500
Construction &maintenance cost, P <sub>E</sub> (R <sub>2</sub> /m <sup>3</sup> )	175	250	300	250	300
Minimum pond depth ,h,(m)	1.0	1.0	1.0	1.0	1.0
Maximum pond depth, h, (m)	3.0	3.0	3.0	3.0	3.0

Table 2. Optimum storage- release pattern (for 60% pollution control level and 90% runoff control)

Optimum design outputs					
	Catchment1	Catchment2	Catchment3	Catchment4	Catchment
Catchments area,(ha)	221	173	206	150	102
Pollution control,(%)	0.6	0.55	0.5	0.55	0.5
Runoff control,(%)	0.58	0.74	0.62	0.85	0.42
Pond depth.(m)	1.9	1.0	1.2	3.0	3.0
Storage volume(mm)	11.4	27.6	9.8	23.4	4.8
Storage volume(m³)	25194	47748	20188	35100	4896
Control release rate(mm/h)	0.3 ·	0.38	0.27	0.25	0.15
Control release rate(m³/s)	0.184	0.182	0.155	0.104	0.043
Cost of the pond(R <sub>s</sub> )	93,15,150	3,34,23,600	1,44,68,066	1,40,40,000	22,84,800

# **CONCLUSIONS**

A dynamic programming model has been developed for optimal design of detention facilities in a multiple parallel catchment system. The DP model needs information on the contributing area, the coefficient of runoff, the land cost, the construction cost and the runoff and pollution control criteria. Investigations were carried out adopting various levels of runoff control, pollution control and pond depth and the total cost for detention facilities. For the set target of 60% TSS reduction (pollution removal) and 90% stormwaterI runoff control at the downstream receiving waters (the present case is the River Musi), the optimum storage volumes required for the proposed detention facilities at each sub-catchment (prior to joining the trunk sewer, are 25 194 m<sup>3</sup>, 47 748m<sup>3</sup>, 20 188m<sup>3</sup>, 35 100m<sup>3</sup> and 4896m<sup>3</sup> respectively, for catchments 1, 2, 3, 4 and 5. The optimum pond depths are 1.9 m, 1 m, 1.2 m, 3 m. The total cost of the stormwater management plan, through detention facilities, would be around 73.6 million Indian rupees. The study shows that better control levels could be achieved at the upstream detention ponds as compared to control levels at the receiving water system, though the optimal pond configuration may be different.

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