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Estimation of Hydraulic Conductivity by Neural Network-Ridge Function

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ABSTRACT: Aquifer properties exhibit significant spatial variation and assigning hydraulic conductivity values to a distributed parameter model based on the available scanty field data is an important problem in modeling of groundwater systems. An estimate of hydraulic conductivity for a two-dimension inverse model based on ridge functions and neural network for a phreatic leaky-aquifer is developed in this study. An objective function is minimized by combining a forward transient groundwater flow model with a proper optimization algorithm to obtain the best set of hydraulic conductivity values. The forward transient groundwater flow model is developed using the finite element method to obtain values of hydraulic conductivity from hydraulic head measurements. An artificial neural network that ensures correspondence between the integral representation of ridge function and neural network algorithm is then incorporated. To account for the high frequency fluctuations of the estimated hydraulic conductivity values in the model, the input weights are related to the spatial frequency. Later, using an inverse modeling the hydraulic conductivity values are estimated so that the mean square error between the measurements and the model prediction in terms of piezometric head is minimized. The results indicate that complex hydraulic conductivity values can be estimated from the piezometric head measurements taking only few parameters. This is sound to be suitable when hydraulic conductivity field map exhibits heterogeneity with large anisotropy in the aquifer. The procedure also helps to dampen erratic high frequency terms in the estimated parameters and hence is stable and attains fast converge.

INTRODUCTION

A ridge function $G(x) : R^n \rightarrow R$, $(n \geq 2)$ is defined on the basis of a univariate function $F : R \rightarrow R$ as $G(x) = F(a.x - b)$ where $a \in R^n$ is a fixed vector, $b \in R$ is an areal number, $x = (x_1, x_2, x_3, \ldots, x_n)$ $a \in R^n$ and $a.x = \sum_{i=1}^{n} a_i x_i$. Function $G(x)$ takes the same value on certain hyper-planes in R^n (whose normal vectors are parallel to a) and is not integrable in R^n even when the one-dimensional function is integrable on R.

Ridge functions and their generalization as linear combination take place in problems of neural networks applied for approximating multi-dimensional spatial functions (Chui and Li, 1992; Sun and Cheney (1992); Leshno *et al.*, 1993; Murata, 1996; Cande's 1999). Coppola *et al.* (2003), in general pointed out the advantages of Artificial Neural Networks (ANNs) models over conventional simulation methods. Chui and Li (1992) claimed that neural networks with one hidden layer that represent approximation by ridge functions can be designed to estimate any continuous functions. Murata (1996) proved that the inverse transform that is developed by using ridge functions provides a reasonable interpretation of the structure of three-layered networks since there is correspondence between transformation coefficients and parameters of networks. Mantoglou (2003) used ridge functions and neural networks to represent a two-dimensional transmissivity map of a synthetic confined aquifer with steady flow.

In this study, a two-dimensional parameterization of hydraulic conductivity is determined by ridge functions and neural networks. An inversion of aquifer model is then obtained such that an objective function is minimized in terms of the hydraulic head by combining a forward-transient groundwater flow model with Levenberg-Marquardt optimization algorithm to obtain stable estimates of hydraulic conductivity values.

THE DISCRETIZED TRANSFORM OF RIDGE FUNCTIONS AND ANNS RESENTATION

Murata (1996) introduced continuous integral representations using a linear combination of ridge functions to approximate an n-dimensional function with certain boundness and admissibility conditions, as,

$$g(x) = \int_{R^{n+1}} \tau(a,b)\rho(a.x-b)dadb \qquad \ldots (1)$$

in which $\tau(a,b)$ indicates the values of ridge transform (ridge coefficients) and $\rho: R \to R$ is one dimensional identical function. Admissibility conditions reveal that the function approximated has a bound on the first moment of the magnitude distribution of the Fourier transform, i.e. $\int_R \rho(t)dt = 0$. The value of the this integral can be approximated by summation of N discrete cell of integral, thus,

$$g(x) = \sum_{i=1}^{N} \tau_i \rho(a_i.x - \beta_i) \qquad \ldots (2)$$

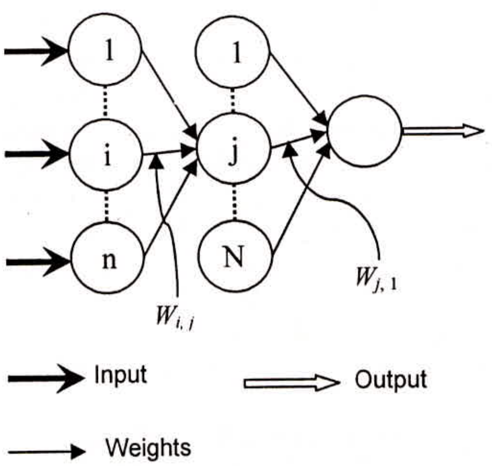

Fig. 1: Schematic hydrologic cross-section of study area

An architecture of a feedforward artificial neural network consisting of an input layer with n input-units, an output layer with only one neuron and a hidden layer of N neurons, depicted in Figure 1 is applied to compute the value of an n-dimensional $f(x): R^n \to R$. Using a linear activation (transfer) function for output layer, such architecture can be expressed as,

$$f(x) = \sum_{j=1}^{N} W_{j,1} Fn\left(\sum_{i=1}^{n} W_{i,j} x_i + b_j\right) \qquad \ldots (3)$$

where Fn is the transfer function of hidden layer and $W_{i,j}$ and $W_{j,1}$ are weights of input neurons and hidden neurons, respectively. A comparison of equation (2) with equation (3) shows that there is a correspondence between the discrete representations and neural network algorithm as depicted in Table 1.

Representing the feedforward ANN with the ridge functions provides attractive convergence properties which make them useful for estimating multidimensional functions (Mantoglou, 2003). Also, the mean square error of the approximation is inversely proportional to the number of neurons in hidden layer, N (Barron, 1993; Murata, 1996). Furthermore, to approximate the function with desirable accuracy the activation function of the networks should be no a polynomial (Leshno *et al.*, 1993).

Table 1: Corresponding Parameters between Ridge Functions Representation and ANN Algorithm

	Parameters			
Ridge functions representation	a	β	ρ	τ
ANN algorithm	$W_{i,j}$	b_j	Fn	$W_{j,1}$

THE INVERSE MODEL

The groundwater system of the Salmas Plain, in the north-west region of Iran consists of an unconfined and a confined aquifer separated by an aquitard. Quaternary alluvial, silt, sand, gravel and clay deposited unevenly throughout the aquifers cause a great heterogeneity of the hydro-geological features. The Zola River which is an unseasonal stream that runs from southwest to northeast of the plain and constitutes a prescribed potential boundary condition.

In this study, the hydraulic conductivity distribution of the phreatic aquifer is estimated to indicate the capability of the methodology. Figure 2 shows the schematic cross section of the aquifer system. The flow in the aquifers is assumed to be horizontal while it is vertical in the aquitard. The governing equation of flow in the unconfined leaky-aquifer takes the form,

$$\frac{\partial}{\partial x}\left[K(h-\eta)\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[K(h-\eta)\frac{\partial h}{\partial y}\right] + R + q_v - P = S_y \frac{\partial h}{\partial t} \qquad \ldots (4)$$

In the equation, $h(x,y,t)$ is the hydraulic head in the phreatic aquifer (L), K is the hydraulic conductivity assumed to be an isotropic two-dimensional function, $R = N + R_1 + RW$ where $N(x,y,t)$, $R_1(x,y,t)$, and $RW(x,y,t)$ represent the distributed rates (L/T) of natural replenishment artificial recharge, and return water through irrigation respectively, and, $P = P_1 + ET$ where $P_1(x,y,t)$ and $ET(x,y,t)$ denote the rates (L/T) of pumping, and evapotranspiration, respectively. The vertical leakage rate (L/T) through the semipervious layer, assumed to be linear is represented by,

$$q_v = K_{aqd}\frac{\phi - h}{B_{aqd}} = \frac{\phi - h}{c} \qquad \ldots (5)$$

where $\phi(x, y, t)$ is the piezometric pressure in the confined aquifer (L), and $c = B_{aqd}/K_{aqd}$ in which K_{aqd} and B_{aqd} are the permeability and the thickness of aquitard, respectively.

The study area is discretized into 1278 rectangular cells (grid) and the transient groundwater flow model is simulated subject to boundary and initial conditions using the finite element model referred to as FEGM. The nodes along the permanent stream flow are also considered to be a boundary with constant head.

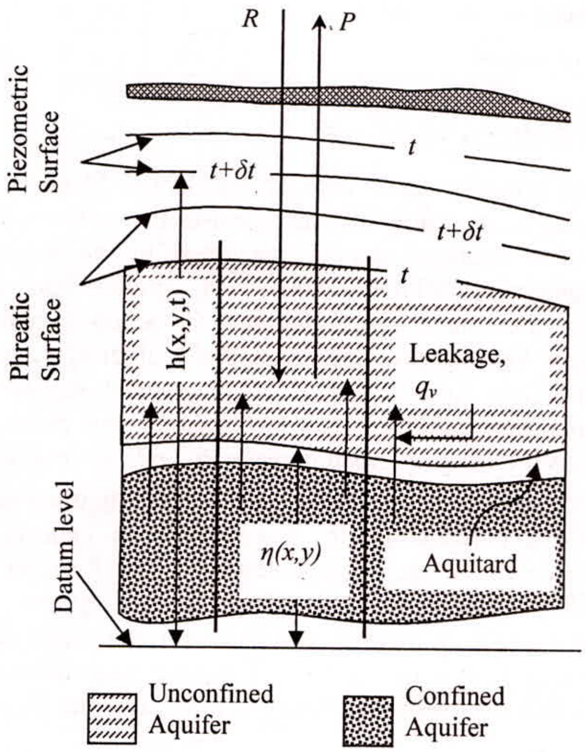

Fig. 2: Schematic hydrologic cross-section of study area

Using piezometric measurements that are available at only 29 locations throughout the aquifer, an inverse problem (parameter estimation problem) is conducted to estimate a set of particular values of $W_{i,j}$, $W_{i,1}$ and b_j that represent the hydraulic conductivity values (Eqn. 3) and simultaneously minimize some measure of misfit between the head measurements, h_m^t and the model prediction, h_m^e. One commonly used measure of misfit is the least square error, $E = \sum_{l=1}^{M}\left(h_l^t - h_l^e\right)^2$ where M is the number of head measurements. Substituting Eqn. (3) the least square error takes the form,

$$E = \sum_{l=1}^{M}\left(h_l^m - \left\langle \sum_{j=1}^{N} W_{j,1} Fn\left(\sum_{i=1}^{2} W_{i,j}x_i + b_j\right)\right\rangle_m\right)^2, \quad \ldots (6)$$

$i = 1, 2$

METHODOLOGY AND DISCUSSION

Taking the true values of hydraulic conductivity (Figure 3a), the values of head are determined by applying FEGM. The obtained values of head at only 29 locations (M=29) for which the measured head values were available are selected as true values. The optimized values of the parameters ($W_{i,j}^*$, $W_{j,1}^*$ and b_j^*) represent the hydraulic conductivity values are obtained through an optimization process. The error function (Eqn. 6) is minimized by the Levenberg-Marquardt algorithm to ensure convergence. The number of neurons, N in hidden layer is taken to be equal to 25. The geographical coordinates of the points of measured hydraulic head locations are used through minimization of error function to take into account the high frequency fluctuations of the hydraulic conductivity values in the model.

In addition to this since using a one-dimensional sigmoidal functions in ANNs do not satisfy the necessary boundness conditions in Ridge functions representation of the form (1), (Murata, 1996; Mantoglou, 2003) one-dimensional bell shaped functions with a maximum of one constructed by the first derivative of the tangential-sigmoidal function is used as transfer function, Fn of hidden neurons. The first derivative of the tangential sigmoidal function satisfies the admissibility condition and exhibits a behavior of oscillation (Mantoglou, 2003).

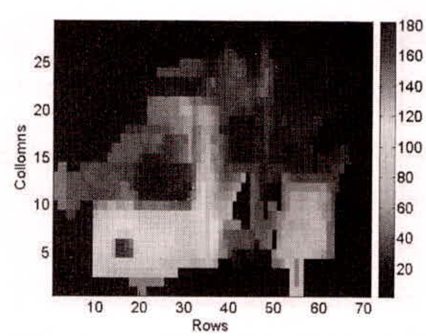

Fig. 3a: The spatial distribution of the true hydraulic conductivity, Abrishami *et al.* (1992)

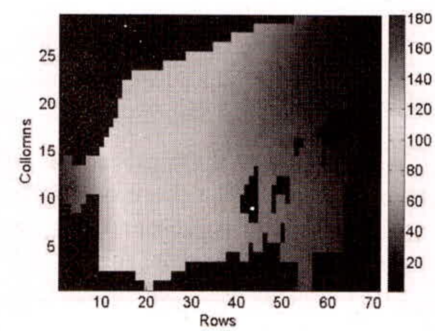

Fig. 3b: The spatial distribution of the hydraulic conductivity, estimated

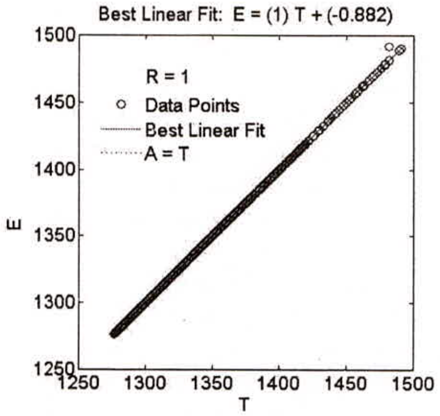

Fig. 4: The correlation between hydraulic head values due to estimated and real permeability

The optimal values of $W^*_{i,j}$, $W^*_{j,1}$ and b^*_j are used in Equation (3) to determine the spatial hydraulic conductivity values (Figure 3b). Figure 3 approximately showed a resemblance between the assumed two-dimensional hydraulic conductivity maps (true distribution) of Abrishami *et al.* (1992) and estimated hydraulic conductivity.

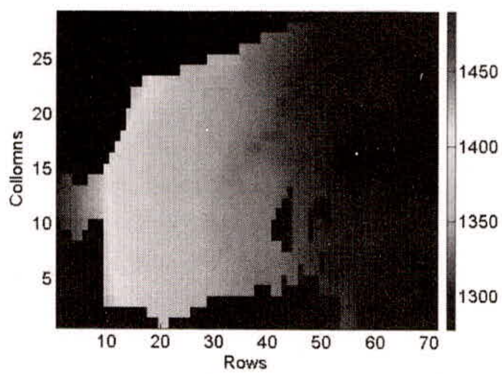

Fig. 5(a): The spatial distribution of hydraulic head corresponding to the true distribution of hydraulic conductivity

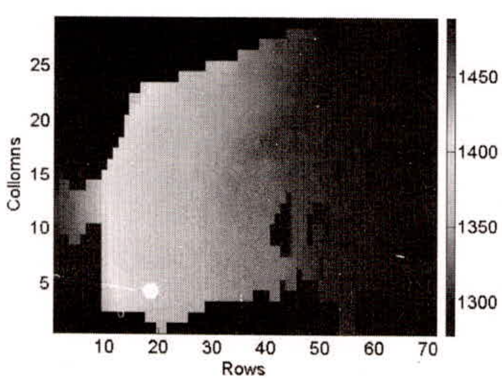

Fig. 5(b): The spatial distribution of hydraulic head corresponding to the estimated distribution of hydraulic conductivity

The hydraulic conductivity distribution so obtained and the spatial-true hydraulic conductivity were used in FEGM to obtain the corresponding hydraulic head values. The post regression analysis reveals that there exists an extremely high correlation (R = 1) between hydraulic head values computed using estimated true hydraulic conductivity values, Figure 4. Furthermore, it can be seen that the trend of hydraulic conductivity is dominated by the trend followed by the hydraulic head (Figures 4 and 5).

SUMMARY

Values of hydraulic conductivity by Ridge functions and Neural Network are determined through inverse problem by minimizing the error between the observed and computed hydraulic head values. The high frequency fluctuation of the predictable hydraulic conductivity values is simulated by the frequency of spatial co-ordinates. The results indicate that even though the target (true data) values of hydraulic are quite difference from the hydraulic conductivity values obtained numerically (Figure 3), the subsequent hydraulic head values obtained taking both target and estimated shows an extremely good match as depicted in Figure 5. Also, on comparing Figure 3 and Figure 5, one can see that the trends of hydraulic head have an impact on the tendency of estimated permeability values.

So it can be pointed out that this approach is competent to represent an essential two-dimensional map of hydraulic conductivity to simulate the dynamical groundwater flow in the phreatic leaky-aquifer even when the study area is essentially empty of field measurements. Furthermore, the procedure is not time consuming and converges so fast.

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