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The Methodology of Long Range Forecasting of Levels of Large Lakes

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ABSTRACT: The methodology of revealing of periodicities in time series of the lake level and water balance components is elaborated. It combines the development of the original physically based model of oscillations of mutually related lake characteristics and approximation of lake level and water balance components by sine functions using the method of least squares.

The periodicities were revealed respectively for the level of the Caspian Sea and the Ladoga Lake. The respective sums of the periodicities show a high correlation with the considered time series and were applied for their long range forecast estimations. The validation of the forecast results by the new independent data shows that the forecasted and real values of the level of the Caspian Sea and the Ladoga Lake are in a good accordance.

INTRODUCTION

The level and water balance components of large inland seas and lakes are significantly variable. Their increase or reduction causes the variation in coastal line, areas of water surface and islands, salt balance components and natural habitat of land and water plants and animals. This variability affects the development of different branches of economy such as fishery, transport, health resorting and tourism.

Research of variation of lakes' level is therefore important for the economy, natural management and ecology of inland regions. The important problem of modern hydrology and environmental studies concerns the revealing of periodicities in the dynamics of lake's characteristics.

Traditionally the methods of Fourier transform, correlation and spectral analysis are used for analysis of time series of lakes levels (Shlaymin, 1962). Present study develops a physically based model of dynamics of lake level and water balance components under oscillations of water inflow (Babkin, 2002) and applies it for the estimation of periodicities in their time series and their forecast computations with the lead time of 5–7 years. The case study was for the Caspian Sea and the Ladoga Lake data.

The Caspian Sea is the largest endorheic lake of the Earth with surface area more than 380,000 km^2. The catchment area of the sea is vast, about 3,000,000 km^2 (Ratcovich, 1993). The areas of the water surface and the catchment of Ladoga Lake are equal to 17,800 km^2 and 276,000 km^2 respectively (Sokolov, 1952). Ladoga Lake is a flowing reservoir, with many rivers of different sizes discharging into it. The lake drains into river Neva, which ultimately discharges to the Finnish Gulf.

THE MODEL OF DYNAMICS OF LAKE LEVEL AND WATER BALANCE COMPONENTS UNDER OSCILLATIONS OF WATER INFLOW

The Morphometric and Water Balance Characteristics of the Lakes

Let us evaluate the dependencies between the morphometric and the water balance characteristics of the lakes. Figure 1 shows (a) the relationships between evaporation (E) from the Caspian Sea and its surface area (S), (b) volume of outflow to Kara-Bogaz-Gol Gulf (R) and the sea level (H) and (c) sea level and area. The empirical dependencies can be approximated by straight lines of the general form,

$$H = kS + k', \qquad \dots (1)$$

$$E = eS + e', \qquad \dots (2)$$

$$R = rH + r', \qquad \dots (3)$$

where k, k', e, e', r and r' are empirical constant.

The combination of these equations of linear trends of these empirical dependencies into the system allows the calculations of the relation between the total water

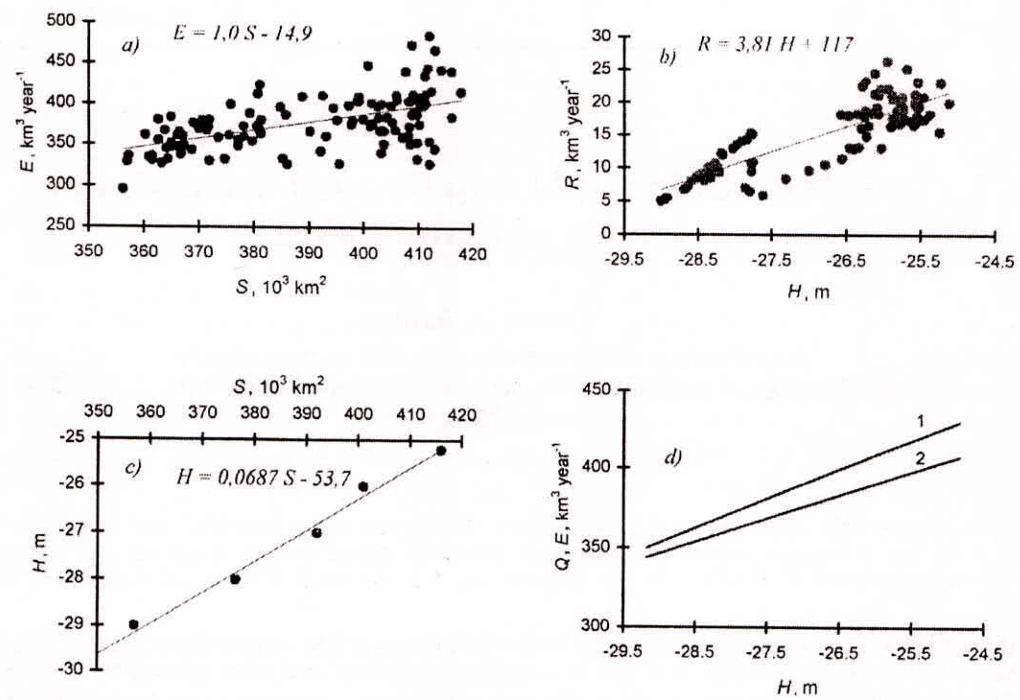

Fig. 1: The empirical dependencies between evaporation and water surface area of the Caspian Sea (*a*), volume of outflow to the Kara-Bogaz-Gol Gulf and sea level (*b*), sea level and area (*c*); components of the sea water balance under different height of its level (*d*)

Table 1: Coefficients of Relationships between the Morphometrical Characteristics and Water Balance Components of the Caspian Sea and the Ladoga Lake

Lake	*D, m*	*e, km/year*	*r, km^2/year*	*k, m/km^2*	*e', km^3/year*	*r' km^3/year*	*k' km*
Caspian Sea	from –29,2 to –24,8	$1 \cdot 10^{-3}$	$3{,}81 \cdot 10^{3}$	$6{,}87 \cdot 10^{-5}$	–14,9	117	$-53{,}7 \cdot 10^{-3}$
Ladoga Lake	from 3,7 to 5,7	$0{,}539 \cdot 10^{-3}$	$28{,}1 \cdot 10^{3}$	$3{,}33 \cdot 10^{-3}$	–2,65	–55,9	$-54{,}4 \cdot 10^{-3}$

output (Q) (sum of volumes of evaporation and water outflow to the gulf) and the sea level by the formula,

$$\frac{H}{1000} = \frac{Q}{\frac{e}{k}+r} - \frac{r'+e'+rk'}{\frac{e}{k}+r} + k'. \qquad \ldots (4)$$

If the output of water is balanced by the inflow to the sea (P) (the sum of rivers discharge to the sea, groundwater inflow and precipitation to the sea surface), the sea is in some equilibrium state. The replacement of the water output by inflow in the formula (4) permits to calculate the equilibrium levels of the Caspian Sea for different values of inflow. The relating to them magnitudes of water surface area, evaporation and outflow to the gulf are calculated by Eqns. 1–3.

The Caspian Sea water balance components under different levels are shown in Figure 1 (d). Curve 1 describes the output of water Q, and respectively, the dependence between the equilibrium level height and inflow, curve 2—the evaporation from the water surface area. The ordinate difference of magnitudes of curves 1 and 2 is equal to the volume of outflow to the Kara-Bogaz-Gol Gulf.

The dependencies between the evaporation from the Ladoga Lake and its surface area, volume of Neva River runoff and the lake level and lake level and area were analyzed analogically. The values of the coefficients e, r, k, e', r' and k' of the Caspian Sea and the Ladoga Lake are shown in the Table 1. The column D reflects the interval of values of levels of these lakes where the considered dependencies are reliable.

Parameterization of Dynamics of Lakes Characteristics

Let us propose that the variation of inflow is periodic with some frequency ω and period T. The maximal and minimal inflow P_2 and P_1 and its value $P_0 = 0,5(P_2 + P_1)$ presuppose equilibrium states with magnitudes of sea characteristics which may be calculated from the Eqns. 1–4,

$$E = E_1, R = R_1, H = H_1, S = S_1, P_1 = E_1 + R_1, \quad \dots (5)$$

$$E = E_2, R = R_2, H = H_2, S = S_2, P_2 = E_2 + R_2, \quad \dots (6)$$

$$E = E_0, R = R_0, H = H_0, S = S_0, P_0 = E_0 + R_0. \quad \dots (7)$$

With account of dependencies (1)–(4) the approach solution of lake water balance differential equation was found and the model, which presents the dynamics of sea characteristics as a response to the oscillation of water inflow was developed (Babkin, 2002). The analysis of the response of the Caspian Sea characteristics to the inflow variation was executed by the comparison of impact curve, reflecting the inflow dynamics,

$$\lambda = \frac{P - P_0}{P_2 - P_1} = 0{,}5 \cdot \mathrm{Sin}\omega t \quad \dots (8)$$

with the curve of response, reflecting the variation of water output and other linearly related sea characteristics,

$$\beta = \frac{Q - Q_0}{P_2 - P_1} = \frac{E - E_0}{E_2 - E_1} = \frac{R - R_0}{R_2 - R_1} = \frac{H - H_0}{H_2 - H_1} = \frac{S - S_0}{S_2 - S_1} \quad \dots (9)$$

The values of parameters λ and β may vary from –0,5 to 0,5.

The mean level of the Caspian Sea is approximately equal to –27 m (Khublaryan and Naidenov, 2000). According to Eqns. 1–4 this level is related with the following values of the Sea characteristics: S_0 = 389,000 km^2, E_0 = 376 km^3 year^{-1}, R_0 = 14 km^3 year^{-1} and Q_0 = 390 km^3 year^{-1}. Let us analyze the variation of the Caspian Sea characteristics when P_0 = 390 km^3 year^{-1}, $(P_2–P_1)$ = 64 km^3 year^{-1} and T = 140 years.

Figure 2(a) shows the dynamics of the Caspian Sea characteristics under variation of the water inflow. Curve 1 reflects the oscillation of water inflow, curve 2 describes by parameter β the variation of sea characteristics. The curve of dynamics of the sea characteristics under periodic inflow variation is also periodic. The periods of variation of level, water surface area, evaporation and outflow to the gulf are equal to the period of inflow oscillation.

The maxima and minima of parameter β are points of intersection of the curve of the sea characteristics variation and the curve of inflow variation. Comparing the Eqns. 8 and 9 it is possible to evaluate that at extreme points of β variation curve the inflow should be equal to water output. The increase of area, water level, evaporation from the sea surface and flow of water to the gulf is appeared if $P > Q$, when $P < Q$, these characteristics tend to decrease. The extremes of parameter β have a time lag τ if compared with parameter λ extremes. The lag time of extremes of the sea characteristics to the inflow is within 0–0, 5π.

Let us consider the amplitude of the response curve,

$$\delta\beta = \beta_{max} - \beta_{min} = \frac{H_{max} - H_{min}}{H_2 - H_1} = \frac{S_{max} - S_{min}}{S_2 - S_1} = \frac{E_{max} - E_{min}}{E_2 - E_1} = \frac{R_{max} - R_{min}}{R_2 - R_1}, \quad \dots (10)$$

where β_{max} and β_{min} are the adjacent maximum and minimum of the response curve. $\delta\beta$ may vary from 0 to 1. The response curve amplitude shows the portion of the variation amplitudes of the sea level, water surface area, evaporation, volume of outflow to the gulf from a difference of levels, areas, evaporation and volumes of outflow of two equilibrium states, determined by extreme values of inflow P_2 and P_1 respectively.

The amplitude of response curve $\delta\beta$ and the time lag τ were calculated consecutively for vast volume of values of inflow, period and amplitude of its variation. Computation shows that dependence of response curve amplitude $\delta\beta$ and time lag τ upon the amplitude of inflow variation $(P_2–P_1)$ is practically missing for typical values of inflow into the Caspian Sea, all possible periods and amplitudes of its variation.

Figure 2b, c shows the dependencies of response curve amplitude and the time delay of maximal and minimal values of the Caspian Sea characteristics to corresponding inflow extremes from the period of variation. These dependencies were computed for the following inflow values:

P_0 = 360 km^3 year^{-1}, P_0 = 390 km^3 year^{-1}, P_0 = 420 km^3 year^{-1}.

According to Eqn. 4 the inflow of 360 km^3 year^{-1} causes the sea level of –28,6 m. Such low height of the Caspian Sea level was observed at the 70th of the 20 century. The inflow of 420 km^3 year^{-1} presupposes the sea level about –25,4 m. Such increased values of sea level was at the 80th of 19 century.

The dynamics of parameters λ and β (curves 1 and 2) (*a*), dependencies of parameter β amplitude (*b*) and the time lag of the sea characteristics to inflow (*c*) from the period of variation (numbers near the curves show the values of inflow P_0).

Figure 2 shows that the parameter β amplitude grows with increase of variation period. For the same period the response amplitude $\delta\beta$ is the larger the water inflow to the sea is the lower. The time lag of the sea characteristics to inflow also grows with increase of variation period. For the same period the time lag τ is larger under high inflow to the sea than under low inflow.

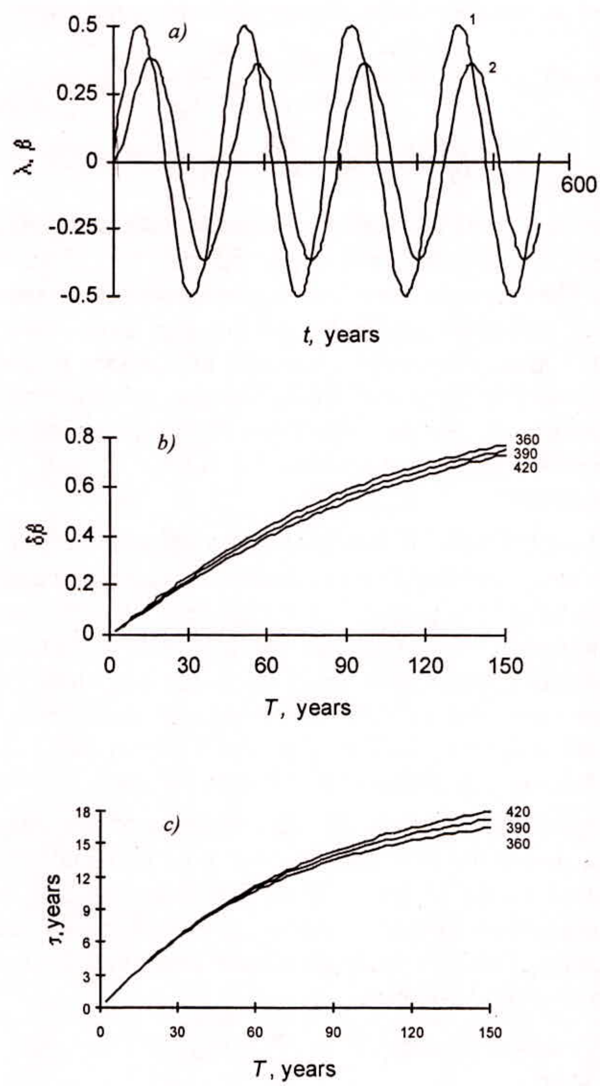

Fig. 2: Variation of the Caspian Sea characteristics under oscillation of water inflow

The time lag of the Ladoga Lake characteristics to the water inflow is much more smaller than τ of the Caspian Sea. The upper limit of τ of the Ladoga Lake is no more than 1 year, τ of the Ladoga Lake is almost equal to its upper limit for the periods of oscillations larger than 12 years.

THE ANALYSIS OF PERIODICITIES IN TIME SERIES OF LAKE'S CHARACTERISTICS

The time series of lakes level and water inflow were approximated by sine functions with using the method of the least squares (Linnik, 1962; Babkin, 2005),

$$S_{H,P} = \sum_{1}^{n} (a_i - a_0 - b\mathrm{Sin}\omega t - c\mathrm{Cos}\omega t)^2 = \sum_{1}^{n} ((a_i - a_0 - \frac{\delta a}{2}\mathrm{Sin}(\omega t + \varphi_a))^2 \quad \dots (11)$$

There a_i is a value of inflow or lake level variable from time series in the year t_i, i—number of the year in the observation range of n years; a_0—additional item of approximation, b and c—constant parameters which reflect the amplitude δa and phase φ_a of approximation sine.

The approximation sine is the best when the sum (11) is the smallest. To determine the best sine for any fixed period the particular derivatives of Eqn. 11 by parameters a_0, b and c were found. These derivatives were equated with 0 and combined into the system. Its solution shows the values of approximation parameters a_0, b and c.

The estimates of additional items H_0 and P_0, amplitudes $\delta H/2$ and $(P_2–P_1)/2$, phases φ_H and φ_P of the sine functions and least sum of squares of differences S_H and S_P respectively between time series of Caspian Sea level and inflow and the approximation functions are shown in the Table 2.

Let us consider the relation of sum of squires of differences between time series and approximation sine to the sum of squire deviations of time series from its mean value. This relation may vary from 0 to 1 and the relations of time series of different nature may be presented in the same graph. The sums of squire deviations of time series of Caspian Sea water inflow S_{0P} and level S_{0H} from their mean values are equal to $338{,}0 \cdot 10^3$ (km^3 year^{-1})^2 and 157,47 m^2 respectively.

Figure 3(a) illustrates the relations of sums of squares of differences S_H and S_P to the sums of square deviations of time series of water inflow and level of the Caspian Sea from their mean values S_{0H} and S_{0P}. The minimum of sums of square differences of inflow and level and approximation functions are marked near the same periods of 4, 7, 9, 11, 13, 16, 21–22 and 34–36 years.

Figure 3(b) presents the amplitudes of response $\delta\beta$ calculated in dependence from the variation period. The points reflect the relation of amplitude of level approximation sine to the difference of levels of two equilibrium states, caused by maximal and minimal inflow approximation sine values. Amplitudes of the level approximation sine and maximal and minimal values of inflow approximation sine are taken from the Table 2. The equilibrium levels of extreme inflow were calculated according to Eqn. 4. The curve reflects the model calculations under inflow of 390 km^3 year^{-1} (Figure 2(b)).

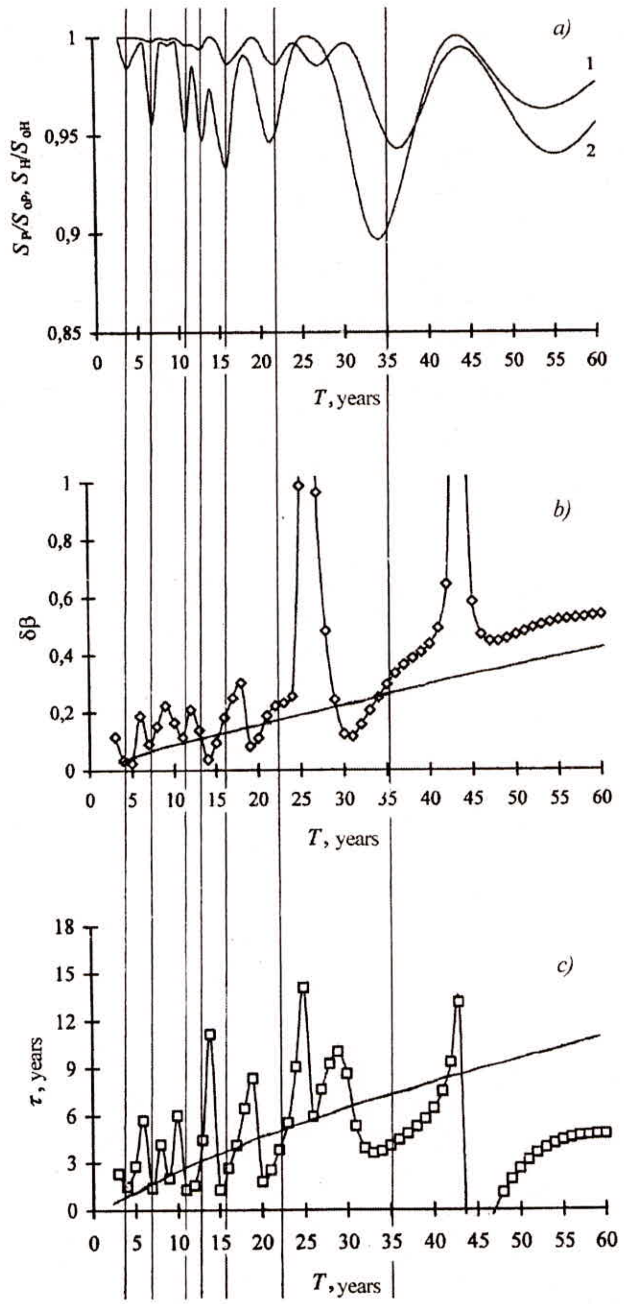

Fig. 3: The relations of sums of square differences respectively of time series of water inflow (1) and level (2) of the Caspian Sea and their approximation functions to sums of squire deviations of these ranges from their mean values (*a*); relation of amplitude of level approximation sine to the difference of levels of two equilibrium states caused by the extreme values of inflow approximation sine (*b*); time delay of sea level approximation sine to inflow approximation sine (*c*). The lines on the grapes (*b*) and (*c*) show the model results with water inflow equal to 390 km^3 year^{-1}

Figure 3(c) illustrates the time delay of sea level to inflow. Points reflect the delay of sine approximated sea level to the sine which approximates inflow. The time delay was estimated as multiplication of variation period and difference of phases of functions, which approximate the level and inflow. The multiplication was divided to 2π. When difference of phases of approximation functions was negative, 2π were added to its magnitude. The curve reflects the model calculations (Figure 2(c)).

The values of parameters $\delta\beta$ and τ of approximation functions and model results are often close near the periods where there is minimum of sum of squire differences of time series and approximation functions of level and inflow: 4, 7, 11, 13, 16, 21–22 and 34–36 years. So we may conclude these are some indications of periodicities.

The relation of amplitudes of level approximation sine to the difference of levels of two equilibrium states caused by extreme values of inflow approximation sine significantly exceeds model calculations near the periods of 26–28 years and 42–44 years and even exceeds 1 (its maximal value admitted by the model). These exceeding may be an indirect indication of periodicities of evaporation, another important water balance element.

The time series of the Ladoga Lake level and water inflow were analogically analyzed and the periods in their dynamics of 4, 6, 8, 11–12 and 31 years were revealed.

The Methodology of Forecast Estimations of Lake Level

The revealed periodicities in time series of the lakes level and water inflow were successively summed. The sum of periodicities is equal to the sum of the sum of respective sinusoids with the mean value of their additional items. When the periodicities were summed the correlation of their sums with respective time series were successively increased.

Correlation of respective sums of all periodicities with the time series of the level and total water inflow of the Caspian Sea and Ladoga Lake is larger than 70%. It is enough to apply them for the long range forecast estimations (Apollov *et al*., 1974).

Figure 4 illustrates the time series of the Caspian Sea and Ladoga Lake level and water inflow approximated by the sinusoids with the periods of 140 and 31 years respectively, and the sums of revealed periodicities.

Table 2: Sine Approximation of Time Series of Caspian Sea Level and Water Inflow

T, year	*H*₀, m	*δH*/2, m	*φ*_H, radian	*S*_H, m²	*P*₀, km³ year⁻¹	(*P*₂–*P*₁)/2, km³ year⁻¹	*φ*_P, radian	*S*_P, (km³ year⁻¹)²
3,0	–27,001	,016	3,222	157,469	385,833	2,567	1,787	337572,300
4,0	–27,001	,018	4,664	157,465	386,003	9,524	,732	332669,700
5,0	–27,001	,009	4,558	157,479	385,828	6,773	1,738	335239,200
6,0	–27,002	,045	–1,42	157,366	385,731	4,432	4,556	336810,000
7,0	–27,002	,077	1,914	157,141	385,838	15,882	3,161	323044,800
8,0	–27,002	,036	1,943	157,407	385,885	4,382	–1,084	336838,100
9,0	–27,001	,061	3,226	157,264	385,833	5,049	4,631	336466,700
10,0	–27,002	,037	–,799	157,404	385,807	4,167	2,985	336945,600
11,0	–27,004	,104	1,458	156,854	385,329	16,840	2,194	321726,800
12,0	–27,003	,106	4,632	156,842	385,732	9,418	–835	332860,300
13,0	–27,001	,129	4,084	156,504	385,969	17,392	–061	320177,900
14,0	–27,002	,026	1,195	157,444	385,902	12,466	–106	329004,400

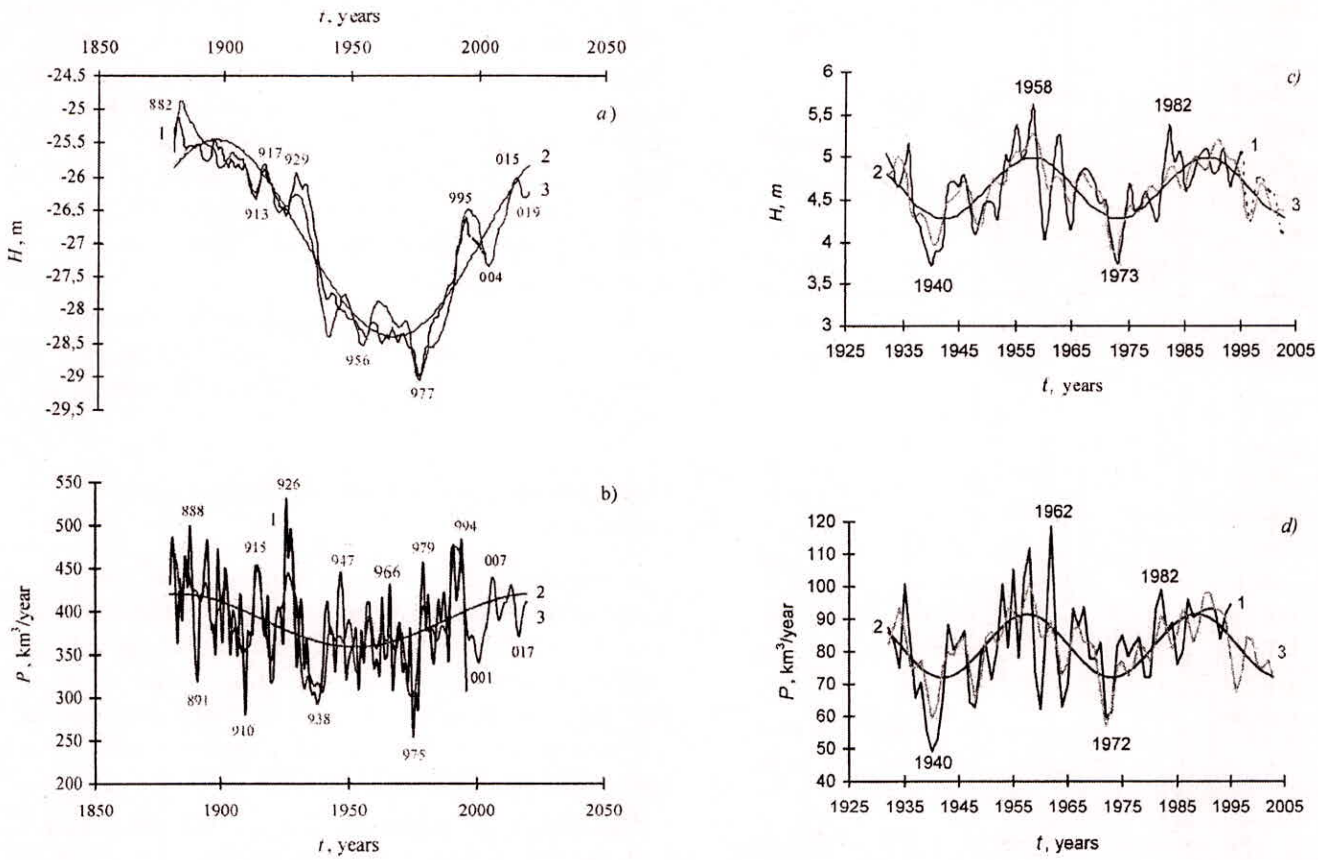

Fig. 4: The observation data for the level and water inflow of the Caspian Sea (a) and (b) and the Ladoga Lake (c) and (d). 1 – time series, 2 – sinusoids of approximation with a periods of 140 years and 31 years respectively, 3 – sums of periodicities

We may see that the extreme of the long wave sine approximated Caspian Sea level has a time lag to the extreme of the sine approximated water inflow and this time lag is almost the same as the τ calculated by the model (Figure 2(a)). The time difference between the extremes of the approximation sinusoids of the Ladoga Lake level and water inflow is equal to 1 year, which also is in a good agreement with the model results.

The forecast estimations of the Caspian Sea and Ladoga Lake level and water inflow were received.

The forecasts of the levels of these water bodies were tested by the new data (dotted lines on the Figure 4(a), (c)). The forecast is estimated as justified if the difference between the real and forecasted values of hydrological characteristics is no more than 67,4% from its mean standard deviation (Apollov *et al.*, 1974).

Mean standard deviation of the Caspian Sea level for the period 1880–1996 is equal to 1,16 m. So the accepted mistake of the forecast is no more than 0,782 m. Mean standard deviation and the accepted mistake of forecast of the Ladoga Lake level for the period of 1932–1995 are equal to 0,416 and 0,280 m respectively.

So, we may see (Figure 4(a), (c)), that the forecasts of level of the Caspian Sea and the Ladoga Lake are justified almost for all 7 years forward.

SUMMARY

The mathematical model of dynamics of the lake level and water balance components under oscillations of water inflow was elaborated. For the data of the Great Russian lakes—Caspian Sea and Ladoga Lake the nomographs of the time delay of lake's characteristics to the water inflow and relation of their amplitudes were calculated for different values of water inflow and oscillation periods. These nomographs are useful for revealing the periodicities in time series of lakes characteristics.

The time series of the Caspian Sea and Ladoga Lake level and water inflow were approximated by periodic functions. The periodicities in the changes of these characteristics were revealed by the comparison of the results of modelling and approximation. The application of the model permits to reveal the periodicities more reliable comparing with the traditional methods of the correlation and spectral analysis. We may conclude that most part of periodicities of the level of the Caspian Sea and the Ladoga Lake are not occasional but resulted from the respective periods of their total water inflow.

The periodicities in the time series of water bodies were successively summed and their sums were developed into the forecast equations. The forecast estimations of the levels of the Caspian Sea and the Ladoga Lake were received and tested by the new data. These forecasts with the lead time of 5–7 years may be considered as successful.

Analogical computations may be conducted for the other large lakes and reservoirs all over the world.

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