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Real-Time Flood Forecasting Using Muskingum Stage—Hydrograph Routing Method

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ABSTRACT: The application of a variable parameter Muskingum stage–hydrograph routing method for real–time flood forecasting at a river gauging site is demonstrated in this study. The estimation of forecast error is made using a two–parameter linear autoregressive model with its parameters updated at every time interval of 30 minutes at which the stage observations are made. This hydrometric data–based forecast model is applied for forecasting floods at the downstream end of a 15 km reach of the Tiber River in Central Italy. The study reveals that reliable forecast of flood estimate can be made for different lead times subject to a maximum lead time nearly equal to the travel time of the flood wave within the selected routing reach.

INTRODUCTION

Many communities owe much of their prosperity to advantages offered by adjacent and nearby streams, the more important being adequate commercial and municipal water supplies, navigation, power development and recreation. Adverse effects, however, are experienced when high flows occur in the form of floods causing loss of lives and damages to properties. Then, economically feasible measures must be taken to eliminate the loss of lives and damage to properties by providing structural measures such as levees and flood walls, channel improvement etc. However, structural measures cannot eliminate completely the risk, given the impossibility of building larger and larger structures to cope up with extremely low probability events. Therefore, an important role is left to non-structural measures to be compared, evaluated and actuated in real–time, which implies the need for accurate flood forecasts with a sufficient lead time to allow for their implementation. This is why flood forecasting with sufficient lead time has become an important non-structural measure for flood damage reduction and for minimizing flood related deaths. Therefore, it is essential that flood forecasting methods should be physically based, less data intensive and, over and above, should be easily understood by the field engineers.

Every flood forecasting model operates on two modes, viz., 1) the simulation mode, and 2) the operation mode (on-line forecasting). A flood forecasting model in the simulation mode attempts to produce the response of the system for the past recorded precipitation or upstream flow input. The response of the model is compared with the recorded response at the point of forecasting interest, and if both do not match, either the model structure is changed or the parameters

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are modified till the matching is done satisfactorily. Once the structure of the model and its parameters are identified during the calibration phase, the model can be used for forecasting purposes and it is said to be used in operational mode. While the basic structure of the model is not changed in the operational mode, the parameters need to be changed considering the current catchment conditions due to the variation of the input and subsequent change in other components of the rainfall-runoff process.

Flood forecasting models are typically made up of two components: 1) the deterministic flow component and 2) the stochastic flow component. While the former is determined by the hydrologic/hydraulic model, the latter is determined based on the residual (error) series of the difference between the forecasted flow for a specified lead time and the corresponding observed one. The residual series reflects both the model error, due to the inability of the model used for forecasting to correctly reproduce the flow process, and the observational error while measuring the flow. It is imperative, therefore, to use an appropriate approach to reduce the model error. The analysis focuses on this specific aspect by studying the use of a variable parameter Muskingum stage routing method as a component model of a hydrometric data based deterministic forecasting model. It would be shown later in the paper that the use of a physically based component model in a forecasting model enables the use of a simple stochastic error updating model to estimate the forecast error. The proposed forecasting model is tested considering several flood events which occurred in a 15 km river reach selected along the Tiber River, in Central Italy, bounded by Pierantonio and Ponte Felcino gauging stations.

VARIABLE PARAMETER MUSKINGUM STAGE–HYDROGRAPH ROUTING METHOD

The variable parameter Muskingum stage–hydrograph routing method, henceforth, referred to as the VPMS method, was developed by Perumal and Ranga Raju (1998a, 1998b) directly from the Saint Venant equations. The form of the routing equation developed is same as that of the Muskingum method, by replacing the discharge variable by the stage variable and, hence, the reason for adherence of the term "Muskingum". Further, the parameters vary at every routing time interval and they are related to the channel and flow characteristics by the same relationships as established for the physically based Muskingum method (Apollov *et al.*, 1964; Cunge, 1969; Dooge *et al.*, 1982; Perumal 1994a, 1994b). The detail development of the method can be found in Perumal and Ranga Raju (1998a, 1998b) and Perumal *et al.* (2007). Only the equations relevant to this study are presented herein.

Using the Approximate Convention–Diffusion (ACD) equation of the following flow depth formulation (Perumal and Ranga Raju, 1999),

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0 \qquad \ldots (1)$$

the Muskingum type routing equation can be arrived at as (Perumal 1998a),

$$y_u - y_d = K\frac{d}{dt}\left[y_d + \theta(y_u - y_d)\right] \qquad \ldots (2)$$

where y_u and y_d denote the flow depths at the upstream and downstream of the Muskingum reach, respectively. The travel time K may be expressed as,

$$K = \frac{\Delta x}{c_3} \qquad \ldots (3)$$

where Δx is the reach length of the Muskingum reach and c_3 is the wave celerity.

The weighting parameter θ, after neglecting the inertial terms, may be expressed as,

$$\theta = \frac{1}{2} - \frac{Q_3}{2S_0(\partial A/\partial y)_3 c_3 \Delta x} \qquad \ldots (4)$$

The subscript 3 attached with different variables denotes the evaluation of these variables at section 3, as shown in the definition sketch of the Muskingum reach of length Δ*x*, at which the normal discharge corresponding to the flow depth at the middle of the Muskingum reach passes during unsteady flow (see Figure 1); Q denotes the discharge; S_0 is the bed slope and $\partial A/\partial y$ is the top width of the water surface.

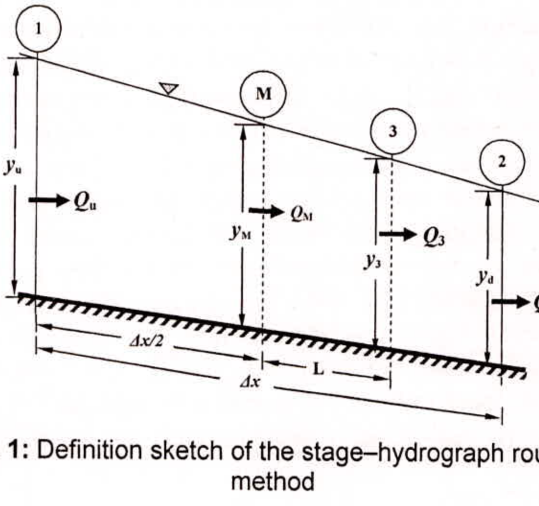

Fig. 1: Definition sketch of the stage–hydrograph routing method

Using equations (3) and (4) in equation (2) and expressing it in difference equation leads to a form similar to that of the Muskingum routing equation, but using flow depth as the operating variable and it is expressed as,

$$y_{d,j\Delta t} = C_1 y_{u,j\Delta t} + C_2 y_{u,(j-1)\Delta t} + C_3 y_{d,(j-1)\Delta t} \quad \dots (5)$$

where $y_{u,j\Delta t}$ and $y_{d,j\Delta t}$ denote the observed upstream and the estimated downstream flow depths at time $j\Delta t$, respectively, and $y_{u,(j-1)\Delta t}$ and $y_{d,(j-1)\Delta t}$ denote the observed upstream and downstream flow depths at time $(j-1)\Delta t$, respectively. The notation Δt is the routing time interval, and the coefficients C_1, C_2 and C_3 are expressed as,

$$C_1 = \frac{-K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \quad \dots (6a)$$

$$C_2 = \frac{K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \quad \dots (6b)$$

$$C_3 = \frac{K(1-\theta) - 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \quad \dots (6c)$$

It has been shown by Perumal *et al.* (2007) that the VPMS method can be applied for routing in a compound cross-section channel reach of trapezoidal section consisting of a main channel and a floodplain channel as shown in Figure 2. It has been shown therein that the wave celerity corresponding to flow in the main channel, c_{main}, ($y < y_m$) is expressed as,

$$c_{main} = \left[\frac{5}{3} - \frac{2}{3}\frac{R_{main}(\partial P_{main}/\partial y)}{(\partial A_{main}/\partial y)}\right]\left(\frac{Q_{main}}{A_{main}}\right) \quad \dots (7)$$

where A_{main}, P_{main}, and R_{main} represent the flow area, the wetted perimeter and the hydraulic radius for the main channel, while Q_{main} is the discharge of the main channel section.

The wave celerity for flow in the compound channel ($y > y_m$) is computed as (Perumal *et al.*, 2007):

$$c_{compound} = \left[\left(\frac{5}{3}\frac{\partial A_{main}}{\partial y}\right)v_{main}\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] + \left[\left(\frac{5}{3}\frac{\partial A_1}{\partial y} - \frac{2}{3}\frac{A_1}{P_1}\frac{\partial P_1}{\partial y}\right)v_1\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] + \left[\left(\frac{5}{3}\frac{\partial A_2}{\partial y} - \frac{2}{3}\frac{A_2}{P_2}\frac{\partial P_2}{\partial y}\right)v_2\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] \quad \dots (8)$$

where v_{main} denotes the velocity of the main channel, while v_1 and v_2 are the velocities in the floodplains (1) and (2) shown in Figure 2; A_1, P_1, A_2 and P_2 denote the flow area and wetted perimeter of the two floodplains and $A_{compound}$ is the total flow area.

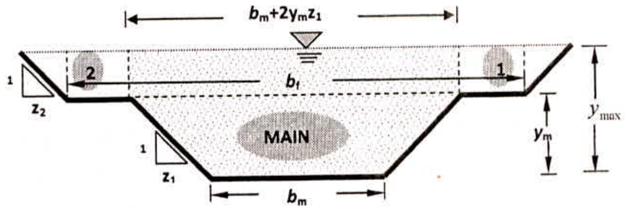

Fig. 2: Compartmentalization of the compound channel section into a main channel (shade) and two floodplains (1 and 2) for celerity computation

The velocity in the main channel and in floodplains (1) and (2) of the compound channel is evaluated as:

$$v_{main} = \frac{\sqrt{S_0}}{n}(R_{main})^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \quad \dots (9a)$$

$$v_1 = \frac{\sqrt{S_0}}{n}(R_1)^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \quad \dots (9b)$$

$$v_2 = \frac{\sqrt{S_0}}{n}(R_2)^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \quad \dots (9c)$$

where R_{main}, R_1 and R_2 denote the hydraulic radius of the main channel section and of the floodplains (1) and (2) of the compound channel section, respectively, and n is the Manning roughness coefficient.

VPMS Model for Real-Time Application

The variable parameter Muskingum stage routing model applicable for forecasting purposes is written by modifying equation (5) as,

$$\hat{y}_{d,(j\Delta t+T_L)} = C_1 y_{u,j\Delta t} + C_2 y_{u,j\Delta t} + C_3 \hat{y}_{d,((j-1)\Delta t+T_L)} \quad \dots (10)$$

where $j\Delta t$ is the time of forecast, $\hat{y}$ denotes the forecasted stage values and T_L is the forecasting lead time. The minimum T_L is Δt, the routing time interval at which the stage measurements are made, and this corresponds to one time interval ahead forecast. The maximum lead time interval that can be adopted depends on the accuracy of the obtained forecast. The larger the T_L, the poorer would be the accuracy of the forecast.

In order to apply the VPMS routing method in a river reach for real–time flood forecasting purposes, an error updating model also need to be developed for estimating the forecast error, which when added to the model estimated forecast for a given lead time would yield the final forecasted flow at the site of interest. It is proposed to use a second–order linear autoregressive error updating model of the following form for forecasting the error at time $(j\Delta t + T_L)$,

$$e_{est,(j\Delta t+T_L)} = a_1\, e_{obs,j\Delta t} + a_2\, e_{obs,(j-1)\Delta t} + \varepsilon_{(j\Delta t+T_L)} \quad \dots (11)$$

where $e_{obs,j\Delta t}$ and $e_{obs,(j-1)\Delta t}$ are the forecasting errors estimated at time $j\Delta t$ and $(j-1)\Delta t$, respectively, and $\varepsilon_{(j\Delta t+T_L)}$ is the random error (white noise). However, the flow depth forecasting can be made only after the lapse of certain initial period of the forecasting event, known as the warm up period. The difference between the observed stage and the VPMS routed stage in the warm up period is considered as the actual error and its series is assumed to be stochastic in nature. The initial parameters a_1 and a_2 of the error update model are assessed using this error series estimated in the warm up period. The duration of initial warm up period considered for developing the error update model should not be long to render the forecasting exercise to be of no practical use for forecasting the given event, and, at the same time, it should not be too short resulting in numerical problem while estimating the parameters a_1 and a_2 using the least squares approach. It may be noted that no attempt was made to study the sensitivity of the order of the stochastic error model and the initial warm up period on the estimates of the forecast. The parameters a_1 and a_2 are updated in real-time on the basis of the last available observations.

CASE STUDY AND RESULTS

The proposed forecasting model consisting of the VPMS routing method, as the basic model, and the second order linear autoregressive model, as the error updating model, is applied for forecasting the flow in a 15 km reach along the Tiber River, in Central Italy. The selected reach is bounded by Pierantonio and Ponte Felcino gauging stations and has an average bed slope S_0 of 0.0016.

Note that the approximation of the VPMS method for routing a given stage–hydrograph in a river reach requires the use of an equivalent prismatic channel reach; this involves the approximation of the actual river reach sections at the two ends to an equivalent prismatic section with a one-to-one relationship established between the flow depth of the actual section of a given flow area with the corresponding flow depth of the prismatic channel section of the same flow area. Based on the surveyed cross-sections at the ends of the actual river reach, it was considered appropriate to approximate the actual reach by a compound trapezoidal section reach. Accordingly, the surveyed cross-sections of the actual reach were overlapped and a two stage trapezoidal compound section geometry with $b_m = 25$ m, $y_m = 5$ m, $b_f = 59.5$ m and $z_1 = z_2 = 2.5$ as required for the prismatic channel reach conceptualization of the VPMS routing method was finalized by a trial and error approach by fitting the best relationships between the actual flow depths and the equivalent trapezoidal section ones as: ($y_{u\text{-}trap} = 0.8887\, y_{u\text{-}actual} + 0.11$) for Pierantonio section and ($y_{d\text{-}trap} = 1.0582\, y_{d\text{-}actual} - 0.1308$) for Ponte Felcino site. $y_{u\text{-}trap}$ and $y_{d\text{-}trap}$ are the equivalent upstream and downstream flow depths in the trapezoidal channel section corresponding to the flow depths $y_{u\text{-}actual}$ and $y_{d\text{-}actual}$ in the actual river section. Using the upstream section relationship, the observed stage hydrograph of any event was converted to equivalent trapezoidal section stage hydrograph to enable the routing using the VPMS method and, using the relationship ($y_{d\text{-}actual} = 0.945\, y_{d\text{-}trap} + 0.1236$), developed on the basis of the downstream relationship, the routed hydrographs of the equivalent trapezoidal section was converted to the actual end section estimated hydrograph.

For studying the applicability of the proposed forecasting model, 12 flood events recorded concurrently at Pierantonio and Ponte Felcino stations were used. The details of these events, each recorded at half-an-hour intervals, are shown in Table 1, where also the details of wave travel time, percentage of lateral flow and actual and equivalent trapezoidal peak flow depths at both stations are reported. The accuracy of the proposed forecasting model was studied using a warm up period of 5 hours and considering five forecast lead times (1.0, 1.5, 2.0, 2.5 and 3.0 hours). The efficiency of the forecast was evaluated using two criteria: 1) Nash–Sutcliffe (NS) criterion (Nash and Sutcliffe, 1970); and 2) Persistence Criterion (PC). As the NS criterion is well known in hydrological literature, only the Persistence criterion is explained herein. It compares the prediction of the proposed model against that obtained by the no-model, which assumes the steady state over the forecasting lead time, and is evaluated as,

$$PC = \left(1 - \frac{\sum_i \left(y_{i\Delta t} - \hat{y}_{i\Delta t}\right)^2}{\sum_i \left(y_{i\Delta t} - y_{(i\Delta t - T_L)}\right)^2}\right) \times 100 \quad \dots (12)$$

where y and $\hat{y}$ denote the observed and the forecasted flow depth values, respectively.

Table 1: Pertinent Characteristics of the Flood Events Studied

Event	Wave Travel Time (h)	Lateral Inflow (%)	Pierantonio Section		Ponte Felcino Section	
			Actual Peak Stage (m)	Equivalent Trapezoidal Peak Stage (m)	Actual Peak Stage (m)	Equivalent Trapezoidal Peak Stage (m)
December '96	1.5	1.9	4.74	4.32	4.22	4.33
April '97	1.5	6.5	5.07	4.62	4.57	4.70
November '97	1.0	5.4	4.22	3.86	3.81	3.90
February '99	2.0	4.4	5.06	4.61	4.52	4.65
December '99	0.0	24.7	2.71	2.52	2.79	2.82
December '00	2.0	flooding	5.92	5.37	5.25	5.42
April '01	2.0	0.2	3.68	3.38	3.23	3.29
November '05	2.5	flooding	7.10	6.42	6.92	7.19
3rd December '05	1.0	3.6	5.10	4.64	4.42	4.55
5th December '05	1.0	5.7	5.49	4.99	4.76	4.91
30th December '05	2.0	1.9	4.99	4.54	4.34	4.46
February '06	1.5	28.4	2.28	2.14	2.64	2.66

Tables 2, 3, 4, 5 and 6 show the forecasting results for peak flow depth forecast at Ponte Felcino station for all the selected flood events and for all the investigated lead times. The results also include the accuracy of peak reproduction, error in time to peak, Nash–Sutcliffe (NS) efficiency and Persistence criterion (PC) efficiency. Some of the floods studied herein are characterized by flooding events (December 2000 and November 2005) with flow spilled over the main channel almost in the entire stretch of the reach and also received unaccounted lateral flow.

Table 2: Forecasting Model Results for a Lead Time of 1 hour (err_y_{peak} = percentage error in peak stage; err_t_{peak} = percentage error in time to peak stage)

Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December '96	0.08	–1.5	99.82	93.80
April '97	–0.20	–0.5	99.95	97.80
November '97	0.97	–3.0	99.87	96.15
February '99	–0.77	–0.5	99.90	96.59
December '99	1.95	1.0	99.79	78.68
December '00	–0.64	0.5	99.80	90.11
April '01	–0.61	0.5	99.67	95.66
November '05	0.06	0.0	99.87	90.54
3rd December '05	–1.29	1.0	99.74	95.26
5th December '05	–0.17	0.5	99.80	93.66
30th December '05	0.02	0.0	99.91	92.60
February '06	1.50	1.0	99.62	81.56
Mean absolute value	0.69	0.83	99.81	91.87

Figures 3, 4, 5 and 6 show some typical forecasted events for various lead times. The given inflow hydrograph and the corresponding observed outflow hydrograph are also shown in these figures. It is inferred from the results given in Tables 2–6 that up to a lead time of 3.0 h, only two flood events (December 1999 and February 2006) could not be successfully forecasted as reflected by their PC estimates (<50%). These two events are characterized by significant lateral flows (>25% of inflow hydrograph volume). As the proposed forecasting model has been developed using the assumption of no lateral flow in the considered reach, it is expected that the efficiency of the model would be poorer in forecasting the flow depth when that event is associated with significant lateral flow. Though the error update model can, to some extent, improve the forecasts in the event of lateral flow, it may not give reliable forecasts when there is significant lateral flow in the reach.

Table 3: As for Table 2, but for a Lead Time of 1.5 hours

Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December '96	0.53	–1.00	99.70	95.33
April '97	–0.77	0.00	99.85	97.01
November '97	1.86	–2.50	99.79	97.13
February '99	–0.27	0.50	99.93	98.94
December '99	2.53	1.50	99.49	75.67
December '00	–0.82	–1.00	99.66	92.10
April '01	1.06	–0.50	99.57	97.44
November '05	–0.38	0.00	99.66	88.97
3rd December '05	–0.48	–0.50	98.89	90.59
5th December '05	0.39	0.00	99.59	94.12
30th December '05	0.96	0.00	99.87	94.95
February '06	3.40	0.00	98.86	74.01
Mean absolute value	1.12	0.63	99.57	91.36

Table 4: As for Table 2, but for a Lead Time of 2.0 hours

Event	*err_y_{peak}* (%)	*err_t_{peak}* (h)	*NS* (%)	*PC* (%)
December '96	0.96	–0.50	99.33	94.06
April '97	–0.31	–2.00	97.38	92.83
November '97	2.60	–3.50	99.40	95.33
February '99	0.43	0.50	99.62	96.54
December '99	2.94	2.00	98.79	66.30
December '00	–0.12	–8.50	99.30	90.53
April '01	3.72	0.00	97.79	92.29
November '05	–0.65	1.00	99.36	88.06
3rd December '05	1.62	–8.00	95.54	77.98
5th December '05	1.51	–3.00	98.58	88.34
30th December '05	1.17	1.50	99.67	92.65
February '06	5.70	0.50	97.20	62.68
Mean absolute value	1.81	2.58	98.50	86.47

Table 5: As for Table 2, but for a Lead Time of 2.5 hours

Event	*err_y_{peak}* (%)	*err_t_{peak}* (h)	*NS* (%)	*PC* (%)
December '96	3.82	–4.50	97.81	87.22
April '97	0.77	–1.50	97.93	84.36
November '97	5.78	–4.00	98.17	90.65
February '99	3.16	–4.00	98.18	89.21
December '99	3.63	2.50	97.04	45.34
December '00	3.95	–8.50	98.17	83.65
April '01	7.92	–1.00	90.73	78.61
November '05	–0.94	2.00	98.86	86.05
3rd December '05	7.74	–7.50	86.25	54.76
5th December '05	5.87	–4.00	94.66	71.37
30th December '05	1.75	0.50	98.88	83.90
February '06	7.81	1.50	94.07	47.23
Mean absolute value	4.43	3.46	95.90	75.20

Table 6: As for Table 2, but for a Lead Time of 3.0 hours

Event	*err_y_{peak}* (%)	*err_t_{peak}* (h)	NS (%)	PC (%)
December '96	7.79	–5.00	94.74	78.10
April '97	2.65	–7.50	95.48	75.72
November '97	10.09	–3.50	96.26	86.55
February '99	11.26	–3.50	95.27	80.04
December '99	4.02	4.50	95.87	45.41
December '00	8.79	–8.50	96.23	75.88
April '01	13.58	–0.50	79.09	65.16
November '05	–1.22	2.50	98.15	84.07
3rd December '05	13.06	–6.50	74.75	39.59
5th December '05	10.42	–3.50	90.50	63.89
30th December '05	2.46	0.00	97.80	77.68
February '06	9.85	2.00	90.88	41.20
Mean absolute value	7.93	3.96	92.09	67.77

It may be seen from Figures 3, 4, 5 and 6 and from the forecast results of other events (not shown herein) that almost for all the events studied the update error model overestimates the forecast error when the rate of increase of rising limb suddenly decreases resulting in increased forecast error around this time zone. The minimum *PC* estimated for the forecasted events is greater than 60%, except for three events (December 1999, 3rd December 2005 and February 2006) out of which two events are characterized by significant lateral flow.

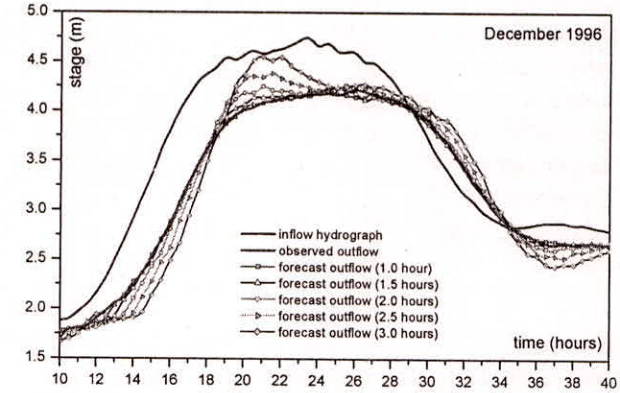

Fig. 3: December 1996 event: comparison between observed and forecast stage hydrographs for different lead times at Ponte Felcino section. The input stage hydrograph at Pierantonio site is also shown

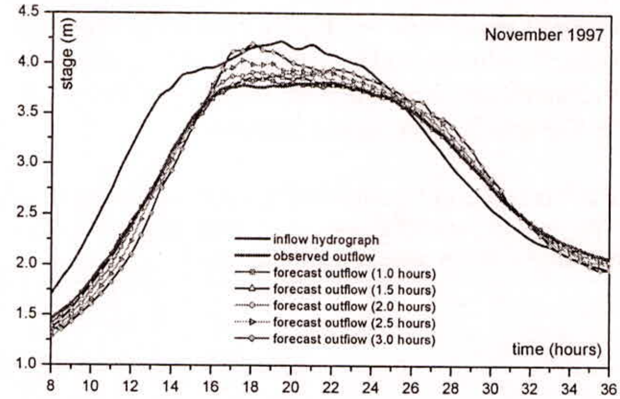

Fig. 4: As for Figure 3, but for the event of November 1997

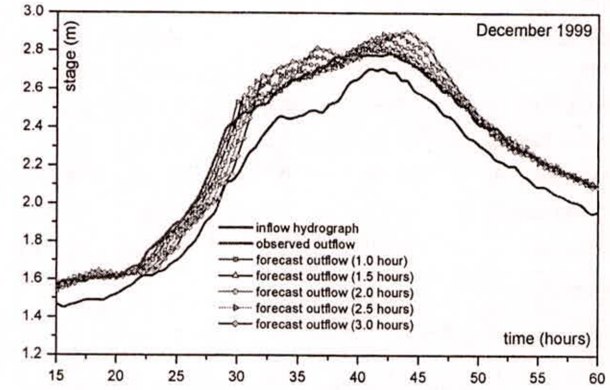

Fig. 5: As for Figure 3, but for the event of December 1999

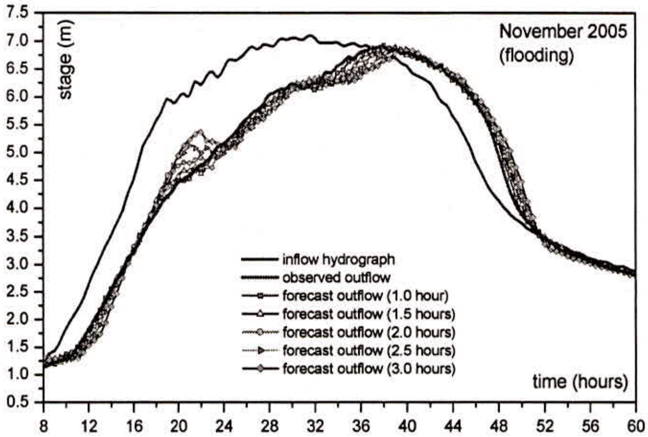

Fig. 6: As for Figure 3, but for the event of November 2005

CONCLUSIONS

The application of a variable parameter Muskingum stage–hydrograph routing (VPMS) method for real-time flood forecasting at a river gauging site is demonstrated in this study. Based on the forecasting performance for different investigated events, one can infer that the proposed model has the potential for practical forecasting applications in hydrometric data based modelling provided that the adopted forecasting lead time is not longer than the mean wave travel time of the selected river reach. Further investigations on different case studies have to be carried out in order to verify the proposed forecasting model accuracy and, furthermore, it should be advisable to extend the model formulation for taking into account not negligible lateral contribution entering along the selected river reach.

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