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Velocity Profiles Assessment in Natural Channels During High Floods

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ABSTRACT: The accuracy of three different approaches for velocity profiles assessment during high floods has been investigated. For high floods is here intended the flow condition when the velocity points sampling is carried out only in the upper portion of the flow area. The first two methods assume the classical logarithmic law with additional terms, to take account of the dip-phenomenon in the curvature of the velocity profile. In this case the two methods need the estimation of three and four parameters, respectively. The third one is based on the entropy theory and uses a velocity profile distribution depending on the maximum flow velocity occurring in the flow area. This approach requires the assessment of one parameter. A sample of velocity measurements carried out during a period of 10 years at Pontelagoscuro gauged section, located along the Po River, northern Italy, has been considered for the analysis. Four flood events have been selected and for each of them the high flood condition has been surmised in terms of sampling. The accuracy of the investigated methods has been evaluated in terms of mean errors in estimating both the mean velocity along each sampled vertical and the mean flow velocity. Results showed that for high floods, the first two methods were not able to accurately reproduce the velocity profiles, showing a mean error on mean flow velocity greater than 20%; whereas the entropic approach was found more accurate showing a mean error which did not exceed 5%. Based on the obtained results, a procedure for addressing the velocity measurements during high floods is also proposed. The procedure can be conveniently adopted for practical applications allowing both to short remarkably the time of the velocity measurements sampling and to estimate quickly the discharge.

INTRODUCTION

The hydrologic/hydraulic physical processes have been often addressed through deterministic approach. However many gaps are still involved in the analysis and the probabilistic approach can be considered suitable to face them and to find a better response in the processes analysis. A fundamental probabilistic approach is the entropy theory which was introduced almost sixty years ago by Claude Shannon (1948) in his historical paper which represents the basis of the actual Information Theory. The Shannon concept was later extended by Jaynes in 1957, who introducing the Maximum Entropy Principle completely modified the approach followed for solving the statistic inference issues. It is well known that the information entropy represents a measure of the uncertainty linked to a probability distribution (Chapman, 1986) and it is fundamental for solving several problems based on statistical models, where the absence of data requires general assumptions for parameter estimating (Singh *et al.*, 1986). This is the case of the flow velocity distribution at river cross-sections. The velocity distribution has been investigated using deterministic as well as probabilistic approaches. An important probabilistic formulation was developed by Chiu (1987) introducing the formulation of the velocity distribution in the probability domain by considering the random sampling of flow velocity in a channel section. However, as such data are usually not available, Chiu proposed a link between the probability domain and the physical one. He derived possible expressions of the cumulative probability distribution function in terms of the coordinates in the physical space. However, estimation of two-dimensional velocity distribution is not always simple and may require as many as six parameters (Chiu and Chiou, 1986). The probability density function of the velocity was then obtained by applying the maximum entropy principle (Chiu, 1987, 1988, 1989; Barbé *et al.*, 1991). Using this probabilistic formulation, the mean velocity, u_m, can be expressed as a linear function of the maximum velocity, u_{max}, through a dimensionless entropy parameter M (Chiu, 1991). Xia (1997) investigated this correlation for some equipped sections along the Mississippi river and he found a perfect linear relationship between mean and maximum velocity. These results have been also confirmed by Moramarco *et al.* (2004), who analyzed the velocity measurements carried out during a period of 20 years in different gauged river sites of the Upper Tiber basin in Central Italy. They also modified the two-dimensional velocity distribution approach introduced by Chiu and Chiou (1986), so drastically reducing the number of parameters involved. Therefore, the possibility to assess the velocity distribution only considering the maximum velocity and the entropic parameter M can be of fundamental interest in the context of discharge monitoring by traditional technique and, in particular, during high floods. Likewise, there exists a multitude of methods to estimate the velocity distribution in a cross-sectional flow area. Traditional logarithmic approaches describe velocity profiles by using equations with a limited number of parameters which can be determined on the basis of velocity points sampled along each vertical. In particular, these approaches need a number of velocity measurements equal or greater than of the parameters involved, along with the position of the velocity points sampled. Fenton (2002) introduced a modified procedure of the traditional three-points or four-points method to estimate mean velocity along a vertical. In fact, for the proposed procedure, velocity sampling has not to be performed at fixed heights in the vertical from the bottom. Other interesting approaches were developed as that proposed by Ardiclioglu *et al.* (2005), who introduced a dip-correction factor to account the velocity dip phenomenon that always exists close to sidewalls. Although there are a large number of studies on velocity profiles in natural channel, few studies have been addressed for estimating the spatial velocity distribution during high flood conditions when it is not possible to sample the whole velocity field and in particular in the lower portion of the flow area. The sampling procedure of velocity measurements in a river cross section, in this case, could be difficult and dangerous for cableway operators. On the other hand, the value of maximum flow velocity could be more easily sampled since its position is located in the upper portion of the flow area where velocity measurements can be carried out also during high flow conditions. Considering that the rating curve accuracy is strictly connected to experimental data availability which have to be referred both to low and high flow depths, we well known how a quick and accurate determination of flow passing through a river section is fundamental in rating curve assessment. Therefore, a model able to assess the velocity profiles, also when velocity data are not available in any portion of the flow area should be welcome.

The objective of the paper is then to test the reliability of the aforementioned approaches to estimate the mean flow velocity in a natural river section during high flood, when the sampling of velocity points is made only in the upper portion of the flow area. The velocity data collected during four flood events at Pontelagoscuro site, along the Po River, Northern Italy, are used for the analysis.

VELOCITY PROFILE DISTRIBUTION MODELS

The classical logarithmic law describing the velocity distribution, u, along a vertical of a cross-sectional flow area, for turbulent flow over a rough bed, can be expressed as,

$$u(y) = \frac{u_*}{k} \ln \frac{y}{y_0} \qquad \ldots (1)$$

where,

- y is the distance from the bottom;
- u_* is the shear velocity, $u_* = (gRS)^{0.5}$ (g is the gravitation acceleration, R is the hydraulic radius and S is the energy slope);
- k is the Karman constant;
- y_0 is the location where the velocity hypothetically equals zero.

To introduce the possibility that the velocity profile is deviating from a logarithmic form and that it may present a maximum value at a point under the water surface, an additional term can be added to Eqn. (1). Fenton (2002) proposed the following expression,

$$u(y) = \frac{u_*}{k} \ln \frac{y}{y_0} + a_1 \frac{y}{D} \qquad \ldots (2)$$

where a_1 is a further unknown coefficient, having the same units of velocity, and D is the vertical depth. If three velocity points, u_1, u_2 and u_3, are sampled at different positions y_1, y_2 and y_3, all three unknown quantities included in Eqn. (2), u_*/k, y_0 and a_1, can be estimated by calibration procedure.

Eqn. (2) can be integrated thus obtaining the mean flow velocity value along the vertical,

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} \quad \ldots (3)$$

Fenton (2002) introduced an additional quadratic term in Eqn. (2) to better reproduce the curvature of velocity profile, yielding,

$$u(y) = \frac{u_*}{k}\ln\frac{y}{y_0} + a_1\frac{y}{D} + a_2\left(\frac{y}{D}\right)^2 \quad \ldots (4)$$

where a_2 is the additional unknown coefficient to be found by measurements. In this latter case four velocity points sampled at different positions along each vertical are needed. Therefore, the mean flow velocity can be derived as,

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} + \frac{a_2}{3} \quad \ldots (5)$$

Three velocity points, sampled at different distances from the bed, are needed to describe by Eqn. (5) the entire velocity profile along the vertical.

The entropic model proposed by Moramarco *et al.* (2004) allows the estimation of the velocity profile using a simplification of the analytical formulation introduced by Chiu (1987, 1988, 1989),

$$u(y) = \frac{u_{\max_v}}{M}\ln\left(1 + \left(e^M - 1\right)\frac{y}{D-h}\exp\left(1 - \frac{y}{D-h}\right)\right) \quad \ldots (6)$$

where $u_{\max_v}$ is the maximum velocity sampled along the investigated vertical and h is the location of the maximum velocity in terms of distance from water surface.

M is the entropic parameter, which is a characteristics of the river cross section and can be estimated by using the linear relationship (Chiu and Murray, 1992),

$$u_m = \Phi(M)u_{\max} \quad \ldots (7)$$

where u_m and u_{max} are the mean and the maximum flow velocity, respectively. $\Phi(M)$ can be expressed by (Chiu, 1989),

$$\Phi(M) = \frac{u_m}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad \ldots (8)$$

The entropic parameter M can be estimated, for the investigated cross section, on basis of pairs (u_m, u_{max}) of available data from measurements sampling (Moramarco *et al.*, 2004).

Once M has estimated and $u_{\max_v}$ sampled, Eqn. (6) gives the velocity profile along the vertical.

DATA COLLECTION

To address the velocity distribution analysis during high flood, the velocity measurements data sampled at Pontelagoscuro hydrometric site on Po river, in northern Italy, has been considered. Figure 1 shows the sketch of the gauged section. The sample is constituted by 48 velocity measurements carried out in the period 1984–1999. For each measurement, data refer to: 1) velocity points sampled along verticals in terms of elevation above the bed and observed value; 2) vertical location respect to left sidewall; 3) hydrometric level; 4) mean flow velocity; and 5) discharge. Measurements cover discharge values varying from 500 m^3s^{-1} up to 5000 m^3s^{-1}. The mean flow velocity and the maximum velocity vary in the range (0.5–2) m s^{-1} and (0.8–2.71) m s^{-1}, respectively.

The three velocity distribution equations, Eqns. (2), (4) and (6), were tested by using the velocity data collected during four flood events, for a total number of verticals and velocity points sampled equal to 52 and 400, respectively. Table 1 summarizes the main characteristics of the selected flood events.

The sampling configuration considers only the availability of the velocity measurements carried out in the upper portion of the flow area. In this way, the sampling during high flood can be represented.

Table 1: Stage, Flow Area (Area), Mean Velocity, u_m, Maximum Velocity, u_{max}, and Discharge, Q, for the Selected Events

Event	*Stage (m)*	*Area (m^2)*	*u_m (ms^{-1})*	*u_{max} (ms^{-1})*	*Q (m^3s^{-1})*
February 13, 1985	5.53	2052	1.13	1.80	2358
February 24, 1987	4.65	1853	0.94	1.43	1779
October 16, 1987	8.68	2448	2.04	2.71	5026
July 5, 1988	5.54	2105	1.07	1.59	2283

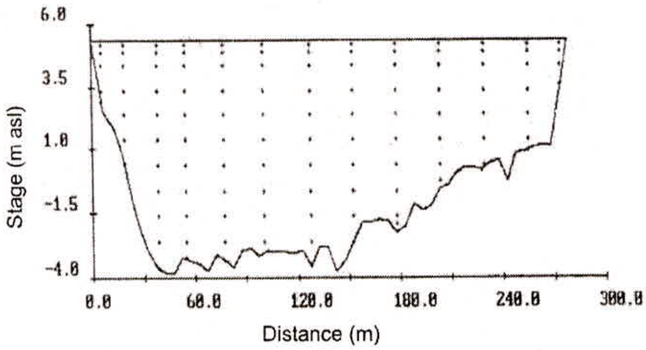


Fig. 1: Topographical survey of the Pontelagoscuro gauged river section

RESULTS

For the application of the Eqn. (6) the entropic parameter, M, was estimated, on basis of pairs (u_m, u_{max}) of 48 flow measurements performed during the period 1984–1997 In this case, the maximum velocity, u_{max}, has been assumed as the maximum value of sampled velocity points. By Eqn. (7), Φ(M) was found equal to 0.668 (see Figure 2) and, then, by Eqn. (8), M = 2.162. It is shown as the linear relationship underestimates the actual values of the mean flow velocity, mainly when the maximum velocity is greater than 2.0 ms^{-1}. This is coherent with results obtained by Moramarco *et al.* (2004) on different hydrometric sites located on the Upper Tiber basin in Central Italy.

As regards the application of the logarithmic distribution, Eqns. (2) and (4), unknown parameters have been estimated by sampling along each vertical, at equal distance, three and four velocity points, respectively. Table 2 shows the percentage errors for estimating the mean flow velocity of selected events. As it can be seen the approaches performance is quite similar.

The three velocity distributions have been also applied considering the velocity points only sampled in the upper portion of the flow area. As regards the application of Eqns. (2) and (4), the third and fourth velocity point, respectively, is represented from the bottom velocity which is surmised equal to zero; whereas in order to drastically reduce the sampling period during the measurement, we assume that Eqn. (6) is applied only considering the maximum velocity point in the flow area, u_{max}, and assuming the behaviour of the maximum velocity quantity in the cross-sectional flow area represented through an elliptical curve,

$$u_{\max_v}(x) = u_{\max}\sqrt{1-\left(\frac{x}{x_S}\right)^2} \quad \dots (9)$$

where $x_S = x_{SX}$ o $x_S = x_{DX}$ represents the distance from the right or left sidewall of the vertical, with reference x = 0, along which the maximum velocity, u_{max}, is sampled, respectively. Figures 3a and 3b show the comparison between the maximum velocity, $u_{\max_v}$, sampled along each vertical and the computed one by the elliptical approach, Eqn. (9), for the measurements carried out during the flood events occurred on 16th October 1987 and on 24th February 1987, respectively. As it can be seen, the elliptical trend reproduces with a fair accuracy the behaviour of the maximum velocities sampled in the flow area, for both events. Figure 3b shows, in the central portion of the section, an irregular distribution of the maximum velocities most probably due to secondary flows that, obviously, can not be modeled by Eqn. (9).

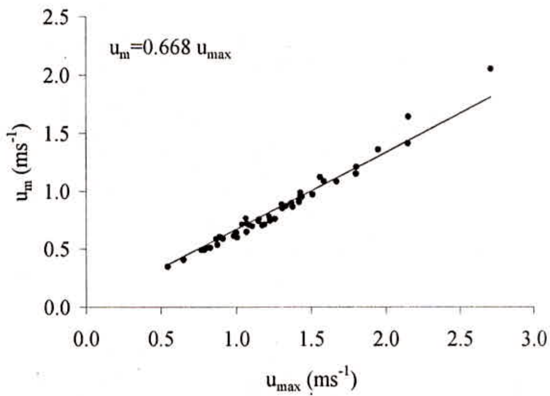


Fig. 2: Relation between mean and maximum velocities at the gauged river section of Pontelagoscuro

Table 2: Percentage Error in Estimating the Mean Flow Velocity, u_m, for the Selected Flood Events

Flood Event	Stage (m asl)	Discharge (m^3s^{-1})	u_m (ms^{-1})	Errors (%)		
				Eqn. (2)	Eqn. (4)	Eqn. (6)
February 13, 1985	5.53	2358	1.13	1.5	–0.3	4.0
February 24, 1987	4.65	1779	0.93	5.4	3.1	2.8
October 16, 1987	8.68	5026	2.04	5.0	–0.8	–0.3
July 05, 1988	5.54	2283	1.07	2.8	1.6	0.1

Applying Eqn. (6) coupled with Eqn. (9) for each vertical, the location, h, below the water surface where u_{maxv} is sampled, is assumed constant and corresponding to location of the maximum velocity, u_{max}.

This assumption could be inappropriate mainly in portions of flow area close sidewalls giving, however, errors not significant in the cross-sectional mean flow velocity assessment. As regards the verticals depth, D, it can be estimated on the basis of topographical surveys of the river section, which are generally available during the working period of the gauged site. Figure 4 shows, for the event on 24th February 1987, the comparison between the observed spatial distribution of the velocity in the flow area and the reconstructed one by using Eqn. (2) and (6). For Eqn. (6), results obtained by using the maximum velocity sampled along each vertical are also shown. An overall overview shows the field of velocity fairly represented by Eqn. (6) in terms of both direction and module; whereas Eqn. (2) provided a poor representation at the same way of Eqn. (4). These insights can be also inferred through Figure5 where the spatial distribution of percentage errors, in magnitude, is shown.

Figure 6 shows, for the three approaches, the percentage errors in estimating the mean velocity along each vertical respect to dimensionless distance from the location where the maximum velocity is sampled (x = 0). The mean error was found about 11% for Eqns. (2) and (6) and 23% for Eqn. (4).

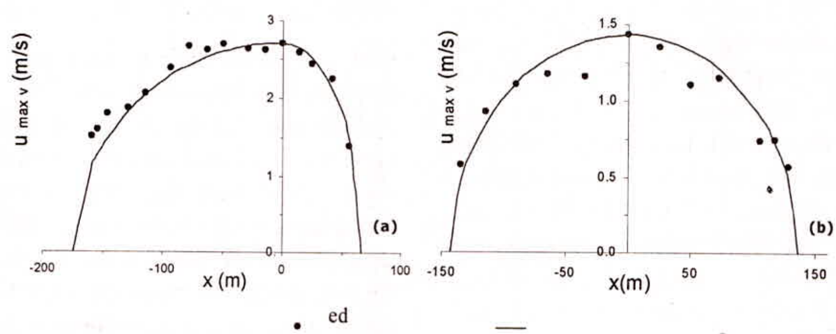


Fig. 3: Elliptical distribution, Eqn. (9), of the maximum velocities, u_{maxv}, along verticals plotted against the observed ones for the flood event October 16 1987 (a) and February 24 1987 (b)

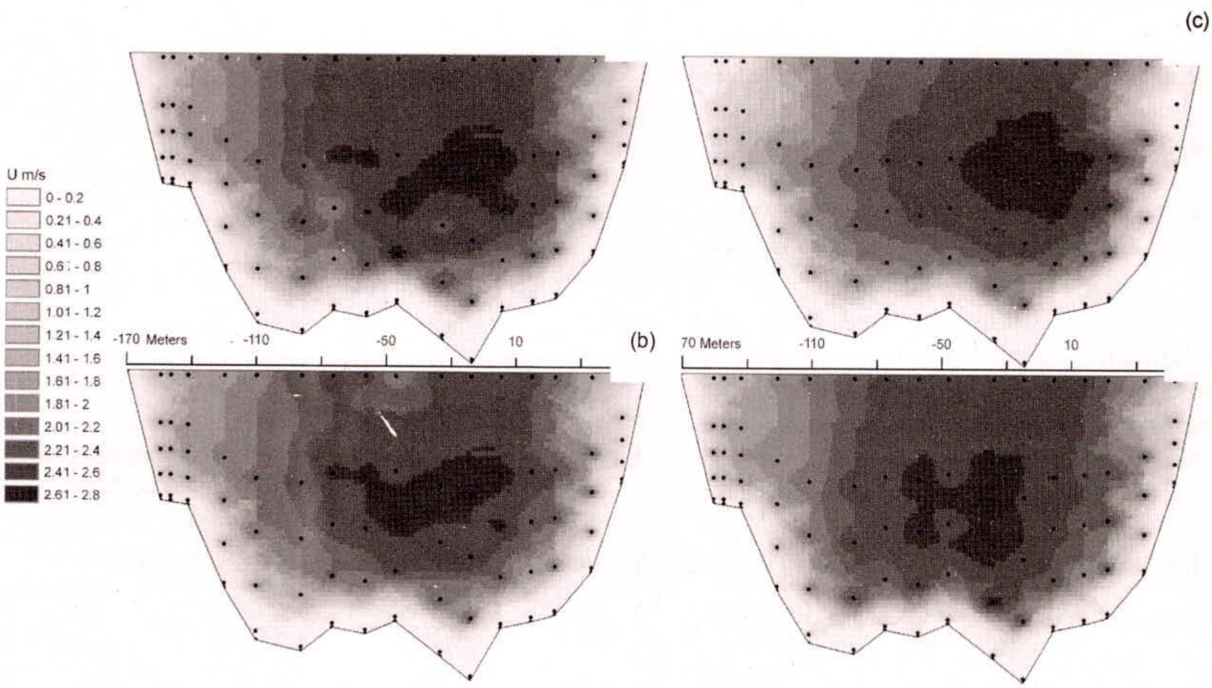
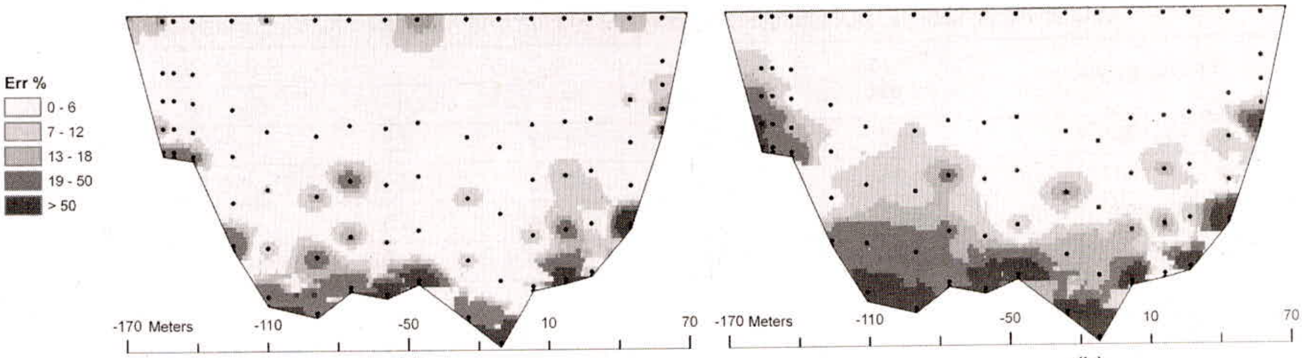


Fig. 4: Spatial distribution of flow velocity obtained by: sampled velocity points (a), Eqn. (6) using the velocity points sampled in the upper portion of the flow area (b); Eqn. (6) coupled to Eqn. (9) (c) and Eqn. (2) (d). Sampled velocity points are also shown



(a) (b)

Fig. 5: Percentage errors in estimating the flow velocity spatial distribution by using Eqn. (6) with maximum velocity sampled along each vertical, (a); and Eqn. (2), (b)

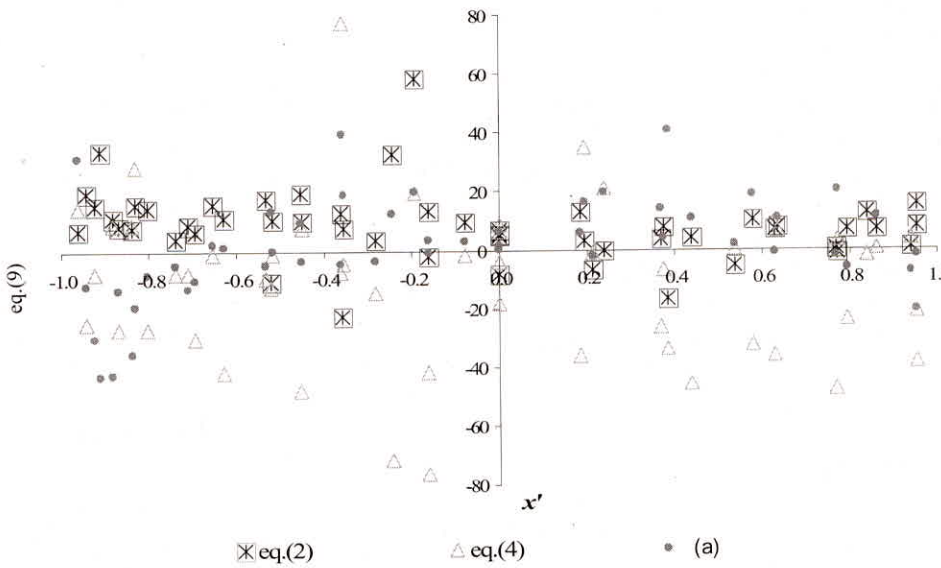


Fig. 6: Percentage error in estimating the mean velocity along the 52 sampled verticals. Distance *x*' represents the dimensionless horizontal distance of each vertical from that in which u_{max} was observed ($x = 0$)

Figure 7 shows the cumulated frequency of the percentage error in magnitude. As can be seen, Eqn. (6), coupled with Eqn. (9), and Eqn. (2) have a similar trend with an error lower than 20% for 92% and 86% of sampled verticals, respectively. The slightly lower accuracy of Eqn. (6) is due to Eqn. (9) which is unable to take account of secondary flows effects that for some verticals determined a reduction of maximum velocity along verticals, $u_{\max_v}$. Eqn. (4) was found poorly accurate with a percentage error exceeding 20% for 46% of verticals.

In terms of error in mean flow velocity estimation, from Table 3 can be inferred that Eqn. (6) provided a mean error lower than 5%; whereas for Eqns. (2) and (4) it increased, in magnitude, up to 9% and 14%, respectively.

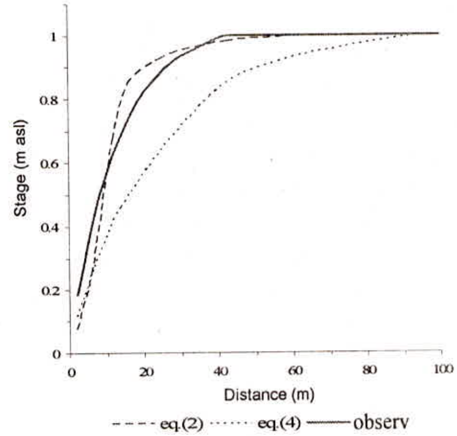


Fig. 7: Cumulated frequency of the percentage error, in magnitude, in estimating the mean velocity along the 52 investigated verticals

Table 3: As in Table 2, but Considering only the Maximum Flow Velocity, u_{max}, Sampling

Flood Event	*Stage (m asl)*	*Discharge (m^3s^{-1})*	*u_m (ms^{-1})*	*Errors (%)*		
				Eqn. (2)	*Eqn. (4)*	*Eqn. (6)*
February 13, 1985	5.53	2358	1.13	6.3	−2.7	13.8
February 24, 1987	4.65	1779	0.93	12.8	−28.8	7.8
October 16, 1987	8.68	5026	2.04	9.5	−10.9	−4.9
July 05, 1988	5.54	2283	1.07	5.9	−12.4	3.1

CONCLUSIONS

Based on the results here obtained, the following conclusions can be drawn:

(a) the logarithmic methods for high flood conditions produced, along each vertical, percentage errors comparable with the ones corresponding to the application of the entropic approach;

(b) the velocity profiles reconstructed by the modified entropic approach, Eqn. (6), were found well accurate using the velocity points sampled in the upper portion of the flow area, and fairly accurate through the only sampling of the maximum velocity;

(c) the procedure here addressed for velocity measurements during high flood, without losing the accuracy in estimating the mean flow velocity, it allows both to operate in safety conditions and to reduce the time of measurement which is fundamental for high floods.

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