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Determination of Unsaturated Soil Parameters from Infiltration Experiment Using Inverse Procedure

P.N. Chandramouli, K.S. Hariprasad[1], C.S.P. Ojha and Talari Venkata Gopal
Department of Civil Engineering
Indian Institute of Technology, Roorkee - 247 667, INDIA
E-mail: suryafce@iitr.ernet.in

ABSTRACT: The inverse problem of determining unsaturated soil hydraulic properties from one-dimensional, transient infiltration event is analyzed. Hydraulic properties are assumed to be described by Van Genuchëten's (1980) model. Unknown parameters in the model are estimated from observed infiltration rates during transient flow by numerical inversion of the unsaturated flow equation. The inverse problem is solved using Levenberg-Marquardt method. It is observed that, the inverse procedure estimates the unsaturated soil parameters quite accurately and the parameter estimates are comparable with the obtained by direct measurement. It is concluded that, inverse procedure can be effectively used to estimate unsaturated soil parameters from infiltration experiment.

Keywords: Unsaturated Zone, Parameter Estimation, Objective Function, Inverse Modeling, Richard's Equation, Levenberg-Marquardt Method.

INTRODUCTION

Solution of Richard's equation requires knowledge of soil hydraulic conductivity and water content verses water pressure head functions (referred to hereafter as soil hydraulic properties). Unsaturated soil parameters can be determined directly from the laboratory or in situ field-measurements. It has been observed that the laboratory methods of estimation of hydraulic properties are time consuming and may not represent the field situations (Russo *et al.,* 1991). Alternatively, these properties can be estimated using parameter estimation methods from transient flow events by numerical or analytical inversion of Richard's equation (Kool *et al.,* 1985; Parker *et al.,* 1985; Kool *et al.,* 1987; Kool and Parker, 1988; Russo *et al.,* 1991).

Inherent in this approach is the assumption that the soil hydraulic properties may be described by a relatively simple deterministic model (referred to hereafter as the hydraulic model) that contains a small number of unknown parameters. The unknown parameters are estimated by minimizing deviations between observed and predicted flow-controlled attributes (e.g., water contents, pressure heads, etc). Fundamental to this approach, is that model parameters are determined in such a way, that the ability of the flow model to reproduce the transient flow event is optimized. Kool *et al.* (1987) reviewed the parameter estimation methods, for unsaturated flow and transport models. Their analysis on hypothetical data sets showed that the van Genucheten's parameters α and n can be estimated uniquely using only the information on pressure head data, but simultaneous estimation of three or more parameters require additional information. Russo *et al.* (1991) analyzed the identifiability and uniqueness of different hydraulic models when infiltration rates are used as optimization criterion. They showed analytically that with infiltration data as boundary condition at the ground surface, two-parameter models (Brooks and Corey, 1964, Van Genucheten's, 1980) are unidentifiable.

Simunek and Genucheten's (1996) estimated the soil hydraulic properties by a parameter estimation procedure that combines the Levenberg Marqurdt non-linear optimization involving weighted least square, with quasi-three-dimensional numerical model, which solves the variably saturated flow equation. Ghidoui and Prasad (2000) discussed the a priori identifiability of the unsaturated soil parameters from the steady state and transient field experiments using inverse procedures. They concluded that a parametric model with two parameters can be identified uniquely from the pressure head/moisture content measurements. However, parametric models with more than two

[1]Conference speaker

parameters can't be identified uniquely using transient data. In the present study, the inverse procedure is used to estimate the unsaturated soil parameters from infiltration experiments.

GOVERNING EQUATION

The vertical transient unsaturated flow in a rigid porous medium is governed by the well-known Richards equation given as (1931),

$$\frac{\partial}{\partial z}\left\{K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right\}=C(\psi)\frac{\partial \psi}{\partial t} \quad \ldots (1)$$

Where, ψ is the pressure head, K is the hydraulic conductivity, $C(\psi)$ $(d\theta/d\psi)$ is the soil moisture capacity, θ is the volumetric moisture content, z is the vertical co-ordinate taken positive upwards and t is the time co-ordinate. Given, K-ψ and θ-ψ relations, solution of Eqn. (1) with appropriate initial and boundary conditions constitute the forward problem. Any conventional numerical technique such as finite difference, finite element methods can be used to solve Eqn. (1) to obtain the discrete pressure heads at successive time intervals. For the case of infiltration into a dry soil, the initial and boundary conditions are,

$$\begin{aligned} \psi &= \psi_0 \quad 0 \le z \le L, \quad t=0 \\ \psi &= \psi_t \quad z=L, \quad t \ge 0 \\ \psi &= \psi_0 \quad z=0, \quad t \ge 0 \end{aligned} \quad \ldots (2)$$

In Eqn. (2), L is the length of the soil, ψ_0 is the initial pressure head and ψ_t is the pressure head applied at the top.

SOIL HYDRAULIC RELATIONS

Eqn. (1) needs constitutive K–ψ and θ–ψ relations for the solution. The relationships proposed by Van Genucheten's (1980) is used in the present studies which are as follows,

θ-ψ Relationship

$$\begin{aligned} S_e &= \left(\frac{1}{1+\alpha|\psi|^n}\right)^m \quad \psi<0 \\ &=1 \quad \psi \ge 0 \end{aligned} \quad \ldots (3)$$

Where, α and n are unsaturated soil parameters with $m = 1-(1/n)$ and S_e is the effective saturation defined as,

$$S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r} \quad \ldots (4)$$

Where, θ_s is the saturated moisture content and θ_r, is the residual moisture content of the soil respectively.

K - θ Relationship

$$\begin{aligned} K &= K_s S_e^{1/2}\left\{1-\left(1-S_e^{1/m}\right)^m\right\}^2 \quad S_e \le 1 \\ &= K_s \quad S_e = 1 \end{aligned} \quad \ldots (5)$$

Where, K_s is the saturated hydraulic conductivity.

FORMULATION OF THE INVERSE PROBLEM

The inverse problem involves the estimation of unsaturated soil parameters K_s, α, n, θ_s, θ_r from the infiltration experiments. The infiltration experiments are chosen since they are simple and easy to conduct in the field (Russo *et al.*, 1991). The inverse problem is formulated as a nonlinear optimization problem, i.e., the parameters are estimated by minimizing a suitable objective function which expresses the discrepancy between observed and predicted infiltration rates as,

$$\min_b \Phi = \left(q^* - q(b)\right)^T W \left(q^* - q(b)\right) \quad \ldots (6)$$

Where, Φ is the objective function of the model parameters, $q^* = \{q_1^*, q_2^*, \ldots\}^T$ represents the observation vector of measured infiltration rates, $q(b) = \{q_1, \ldots q_n(b_n)\}^T$ represents the model predicted infiltration rates for a given parameter vector b and b is the parameter vector, $b = \{b_1, b_2, \ldots, b_m\}^T$. In the present case, $b = \{K_s, \alpha, n, \theta_s, \theta_r\}^T$. It is to be noted here that $q(b)$ is obtained by solving eqn. (1), subjected to initial and boundary condition(2). Among the five parameters involved in the parameter estimation procedure, θ_s is usually taken as the porosity of the soil and θ_r is a fitting parameter to the θ–ψ curve at very low moisture contents and as such are not estimated in the present study. Following Kool and Parker (1988), a parameter optimization code is developed to estimate the unsaturated soil parameters by coupling an optimization module with an implicit finite difference model. Levenberg-Marquardt algorithm is used in the optimization. A compute code is developed in FORTRAN 77 for this purpose.

TESTING OF COMPUTER CODE

To test the correctness of the optimization code, hypothetical infiltration rate data are generated with the following set of parameters; K_s = 5 cm/hr, α = 0.02 cm^{-1}, n = 2.3, θs = 0.38 and θ_r = 0.03. The length L of the soil profile is considered as 1m. The initial pressure head in the soil profile is taken as –10 m. A pressure head of 0m is applied at the top and the pressure head at the bottom of the soil is held at –10 m. Synthetic infiltration data are generated by solving

Eqn. (1), with appropriate boundary conditions (2) using an implicit finite difference method and computing the infiltration at the soil surface. These infiltration data are used as observed infiltration rates in the optimization. The parameters K_s, α and n are estimated by starting from an arbitrary initial values. Table 1 shows the true parameters and the corresponding estimates by the inverse procedure. It is clear from Table 1 that the inverse procedure estimates the parameters correctly, thus validating the correctness of the optimization procedure.

Table 1: Parameter Estimates for the Hypothetical Infiltration Data

Parameter	*True Value of Parameter*	*Estimated Parameter*
α (cm^{-1})	0.02	0.019999
n	2.3	2.30001
K_s (cm/hr)	5.0	5.000001

EXPERIMENTAL STUDY

Having checked the optimization code for its correctness, it is used to estimate the unsaturated parameters from a field infiltration experiment. The infiltration experiment was conducted at the field experimental site at Civil Engineering Department of Civil Engineering of Indian Institute of Technology, Roorkee. The experimental set up consists of Double Ring infiltrometer as shown in Figure 1, having 60 cm outer ring diameter, 44 cm inner ring diameter. The height of the ring being 30 cm. in order to insert the ring in to the ground, a driving plate and a heavy driving hammer had been used. The level of the cylinder had been checked frequently to keep it oriented properly. A point gauge had been used for measuring the water surface elevation in the cylinder. The experiment was started with the soil being at a very dry condition before the experiment. A small positive head is applied at the soil surface in the infiltrometer and the infiltration rate is measured at different times. Table 2 provides the textural properties of the soil at the field site. Table 3 provides the time versus infiltration data of the experiments. This infiltration data are used in the inverse procedure to estimate the unsaturated soil parameters. To check the accuracy of the parameter estimates, the parameters are also obtained from the textural data using the relationships provided by Rawls *et al.* (1993). Table 4 shows the comparison of the parameters estimated by the inverse procedure and those estimated using Rawls *et al's* equations. It is seen from Table 4, that the parameters estimated by the inverse procedure are in close agreement with those obtained by Rawls *et al's* equations. Figure 2 shows the observed infiltration rate curve.

Table 2: Textural Properties of the Soil at the Field Site

Textural Property	*Value*
Percentage of sand	90
Percentage of clay	5.3
Porosity	0.3268
Specific gravity	2.3
Bulk density (gm/cc)	1.7
Dry density of soil (gm/cc)	1.5483

Table 3: Time Versus Infiltration Data

Time (mts)	*Infiltration (cm/hr)*
1	76.2
2	50.1
3	42.3
4	36.72
5	34.2
6	33.4
7	32.0
8	29.65
9	28.35
10	26.66
15	23.4
20	22.0
25	20.76
30	19.5
35	18.3
40	17.6
45	16.7
50	16.23
55	15.85
60	15.0
65	14.4
75	13.0
90	12.0
110	11.4
130	10.8
150	10.2
180	9.9
210	9.7
240	9.7
270	9.7
300	9.7

Table 4: Comparison of the Parameters Estimated by the Inverse Procedure

Parameters	*Experimental Value*	*Estimated Values*
Van Genucheten's parameter, α (cm^{-1})	0.006446	0.006116
Van Genucheten's parameter (n)	1.7674	1.6733
Saturated hydraulic conductivity (cm/hr)	9.6555	9.2101

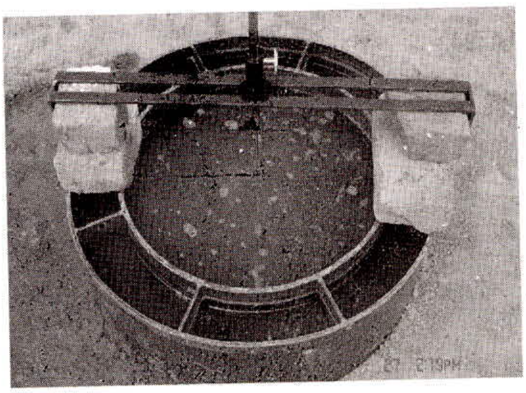

Fig. 1: Double ring infiltrometer

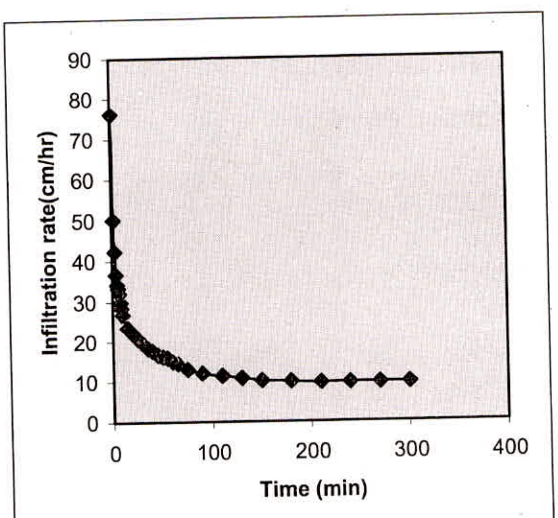

Fig. 2: Field observed infiltration rate curve

CONCLUSIONS

The present study discusses the efficacy of the inverse procedures in estimating the unsaturated soil parameters from the infiltration experiments. An optimization model is developed using Levenberg-Marquardt algorithm for this purpose. The correctness of the optimization model is tested with a hypothetically generated infiltration data. The model is used to estimate the unsaturated soil parameters from a field infiltration data. The comparisons of the parameter estimates with those obtained by the other methods are found to be in good agreement. It is concluded that the inverse procedures can be effectively used for the estimation of unsaturated soil parameters which saves time and effort in conducting laboratory experiments.

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