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Application of Shallow Water Equations Models for Water Hazards Problems

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ABSTRACT: The Shallow Water Equations (SWEs) are applicable to vast range of practical problems where water flows dominantly in the horizontal directions. The finite volume method is effective for numerically solving the SWEs in complex topography. A numerical model using a Total Variation Diminishing (TVD) finite volume scheme is developed for simulating two-dimensional transcritical surface flows. The model is applied to the valley bottoms of a West African inland valley. The valley bottoms are seasonal wetlands where rain-fed rice is cultivated, and there are several earthen dams across the valley bottoms to harvest surface water in rainy seasons and to recharge unconfined aquifers, where dug wells are installed to provide water for domestic purposes. Thus, simulation analysis of surface water flows in the valley bottoms during rainstorms is important to predict flooding of the rice fields and consequences of possible breaching of the earthen dams. A simple hydraulic groundwater flow model is proposed to explain hydrological processes in inland valleys. To implement computations, geographical, infrastructural, and rural environmental data are collected from sheet maps, satellite images, and field surveys. Simulation results suggest suitable rehabilitation works to be done on the earthen dams to reduce risk of water hazards. It is concluded that numerical analysis using reliable numerical models with appropriate data acquisition procedures is the unique prediction tool for water hazards that could happen in extreme conditions.

INTRODUCTION

Water, which is a vital substance for life and environment, sometimes induces hazards to human beings. Water resources development is to reduce occurrence of water hazards such as floods and droughts, but it also causes environmental degradation that may be regarded as extra water hazards. Today, integrated application of mathematical and numerical models is considered very effective in comprehensive understanding of complex hydraulic phenomena involving both beneficial and disastrous aspects of water.

The Shallow Water Equations (SWEs) are partial differential equations to model hydrodynamics of surface water flowing dominantly in the horizontal directions, and they are applicable to vast range of practical problems where transcritical flows and abrupt changes in water depths may occur. Several computational procedures have been proposed for numerically solving the SWEs, and the finite volume method, which conserves mass and momentum, has turned out to be competent for solving complex practical problems (Mohamadian *et al.*, 2004; Namin *et al.*, 2004; Tseng, 2004). Here, a numerical model using the finite volume method for the Two-Dimensional (2D) SWEs is developed to simulate hydraulic phenomena including water hazards problems such as flooding and dam breaching. The computational scheme is equipped with Total Variation Diminishing (TVD) properties (Delis and Skeels, 1998).

The numerical model is applied to practical problems in a West African inland valley. According to the Inland Valley Consortium (IVC), inland valleys in general are defined as the upper reaches of river systems, in which river alluvial sedimentation processes are completely or almost absent (IVC, 2007). They cover about 8% of the land area in Sub-Saharan Africa, being strategic in terms of food security and poverty alleviation. However, hydrological feature of the inland valley to be studied here is slightly different from what the IVC describes; the land is so flat that hydromorphic fringes are rather marginal. As a result, the valley bottoms are almost contiguous to the upland slopes and become seasonal wetlands where rain-fed rice is cultivated. There are several earthen dams across the valley bottoms to harvest surface water in rainy seasons and to recharge unconfined aquifers. Water from dug wells is used for domestic purposes. Thus, simulation analysis of surface water flows in the valley bottoms in rainstorms is important to predict the

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processes of rainwater harvesting in the dams, flooding of the rice fields, and possible breaching of the dams.

TWO-DIMENSIONAL SHALLOW WATER EQUATIONS

Governing Equations System

The 2D SWEs represent conservation laws of mass and horizontal momentum. Taking x and y as the horizontal coordinates, the conservation laws are written in the vector form as,

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=\mathbf{S} \quad \ldots (1)$$

where t is the time, for the state vector,

$$\mathbf{U}=\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}=\begin{pmatrix} h \\ p \\ q \end{pmatrix} \quad \ldots (2)$$

where h is the water depth, p and q are the unit width discharges in x and y directions, respectively. The flux vectors are,

$$\mathbf{F}=\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}=\begin{pmatrix} p \\ \dfrac{p^2}{h}+\dfrac{gh^2}{2} \\ \dfrac{pq}{h} \end{pmatrix},$$

$$\mathbf{G}=\begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix}=\begin{pmatrix} q \\ \dfrac{pq}{h} \\ \dfrac{q^2}{h}+\dfrac{gh^2}{2} \end{pmatrix} \quad \ldots (3)$$

where g is the gravitational acceleration. The source term is,

$$\mathbf{S}=\begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}=\begin{pmatrix} \varphi \\ -gh\dfrac{\partial z_b}{\partial x}-gh\dfrac{n^2 p\sqrt{p^2+q^2}}{h^{10/3}} \\ -gh\dfrac{\partial z_b}{\partial y}-gh\dfrac{n^2 q\sqrt{p^2+q^2}}{h^{10/3}} \end{pmatrix} \quad \ldots (4)$$

where ϕ is the vertical inflow rate, z_b the ground surface elevation, and n is the Manning's roughness coefficient.

Finite Volume Model

Computational procedures are constructed so as to be carried out over unstructured meshes consisting of triangular cells (Unami *et al.*, 2006). Discretization of Eqn. (1) in the spatial domain is based on the finite volume formulation,

$$\frac{d}{dt}\int_{\Omega_i}\mathbf{U}d\Omega_i+\int_{\Gamma_i}\left(\mathbf{F}n_x+\mathbf{G}n_y\right)d\Gamma_i=\int_{\Omega_i}\mathbf{S}d\Omega_i \quad \ldots (5)$$

where Ω_i is the domain of the ith cell, Γ_i the boundary of Ω_i, and $\left(n_x,n_y\right)^T$ is the outward unit normal vector on Γ_i. Eqn. (5) is further arranged as the ordinary differential equation,

$$\frac{d\mathbf{U}_i}{dt}=-\frac{1}{A_i}\int_{\Gamma_i}\left(\mathbf{F}n_x+\mathbf{G}n_y\right)d\Gamma_i+\mathbf{S}_i \quad \ldots (6)$$

where $\mathbf{U}_i$ is the state vector $\mathbf{U}$ attributed to the i^{th} cell, A_i the area of the i^{th} cell, and $\mathbf{S}_i$ is the source term $\mathbf{S}$ evaluated in the i^{th} cell. The boundary integral $\int_{\Gamma_i}\left(\mathbf{F}n_x+\mathbf{G}n_y\right)d\Gamma_i$ is the sum of the integrals along the three edges of the i^{th} cell. Therefore, developing a numerical model focuses on evaluation of the flux $\mathbf{F}n_x$ + $\mathbf{G}n_y$ along the boundary of each cell and on treatment of the source term $\mathbf{S}$ within each cell. Firstly, a special technique is employed for reconstructing water surface elevations to deal with a wide range of water depths. The uniform depth is used as a criterion for interpolating the water depth h within each cell. Then, a flux splitting scheme similar to the Liou-Steffen splitting scheme (Liou and Steffen, 1993) is applied for evaluating the flux on the edge that two cells share to fulfill the TVD properties. The TVD scheme, which does not necessitate the characteristic decompositions (Yu and Liu, 2001), refers to the state vectors in the adjacent two cells only, so that the flux values in those cells bound those evaluated on the edge. Therefore, the finite volume formulation Eqn. (5) becomes consistent with the original 2D SWEs Eqn. (1) as the meshes are refined. An upwind scheme operated on the reconstructed water surface levels determines the values of $\mathbf{S}_i$. The temporal integration is performed using the 4th order Runge-Kutta method with a specified time-step Δt. However, when the water depth in a cell falls below ε = 0.001 m, it is replaced by ε in every calculation of the right hand side of Eqn. (6). Furthermore, if the revised water depth after the temporal integration of Δt is still less than ε, then the unit width discharges are reset as 0 while the water level is left less than ε.

SITE DESCRIPTION

The Guinea Savanna Zone and Inland Valleys

The Guinea savanna agro-ecological zone, where the length of the growing period is 165–270 days, constitutes a strip between the equatorial forest zone and the Sudan savanna zone in West Africa. Water scarcity due to global desertification and climate change is adversely affecting the region, whose largest sector of economy is subsistence agriculture. In Ghana, the Guinea savanna agro-ecological zone covers most part of the northern three Regions: Upper East, the Upper West, and Northern. These regions are the poorest parts of the nation. However, in contrast with two Upper Regions, the flat and low-lying land of Northern Region is also vulnerable to floods and water-borne diseases. Figure 1 shows monthly distribution of rainfall in Tamale, the capital city of Northern Region (BBC Weather, 2007). The annual rainfall pattern is monomodal with the single rainy season from May to September. The mean annual rainfall is about 1,050 mm.

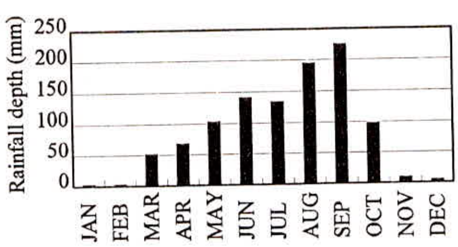

Fig. 1: Distribution of average monthly rainfall in Tamale

One of the characteristics of land use in Northern Region is intensive agricultural production in inland valleys. Inland valleys in Northern Region comprise upland slopes of high unsaturated hydraulic conductivity and valley bottoms appearing as wetlands during rainy seasons. Traditionally, slash-and-burn farming system is established on the upland slopes while the valley bottoms are cultivated for rice production. There is a considerable pool of genetic diversity in rice, enabling robust production against climatic variability, pests, and diseases (Tanzubil *et al.*, 2004). Cattle, goats, and free-range guinea fowl are commonly seen.

Hydrological processes in inland valleys are not well established in literatures. Here, a simple hydraulic groundwater flow model is proposed, assuming that the rain vertically infiltrating into the upland slopes recharges the unconfined aquifer and then flows horizontally until seeping up to the ground surface of the valley bottoms. When the surface soil of the valley bottoms is saturated, the seepage flow rate ψ per unit area is given by,

$$\psi = k_H \nabla \eta \cdot \nabla z_b \qquad \ldots (7)$$

where k_H is the hydraulic conductivity in horizontal directions, and η is the surface water level. If this hydrological process is steady in an inland valley, then the mass balance of water is expressed as,

$$\int_{\Omega_{US}} r_{in}\, dxdy = \int_{\Omega_{VB}} k_H \nabla \eta \cdot \nabla z_b\, dxdy \qquad \ldots (8)$$

where r_{in} is the intensity of rain infiltrating into the soil, and Ω_{US} and Ω_{VB} are the projection of the upland slopes and the valley bottoms onto the *x*-*y* plane, respectively.

Study Area

Bontanga River is one of the tributaries of White Volta and flows through Tolon/Kumbungu District of Northern Region. An inland valley, which is a sub-basin of the catchment basin of Bontanga River, is chosen as the study area. It ranges from 09 26 08 N to 09 29 29 N in latitude, from 001 00 41 W to 001 03 45 in longitude, and from EL129.3 m to EL182.9 m in altitude. Bontanga River has no perennial flow in the part forming the downstream boundary of the study area. The Bontanga Irrigation Project expanding along the Bontanga River is a gravity-fed irrigation project with a dam of about 20 million m^3 storage capacity. However, the project is not for the benefit of the inland valley. Therefore, in the study area rain-fed agriculture is dominant, and the valley bottoms are used for rice cultivation in the rainy season. The soil of the upland slopes is classified as Dystric Plinthosols while that of the valley bottoms is Dystric Planosols (FAO, 1998). When $\eta = z_b$ in Ω_{VB}, the ratio of k_H to r_{in} is identified as 1.320×10^5 to 1 from the topographical data. Soil moisture data are automatically collected in two sites near the study area to suggest that r_{in} is about 5 mm/hour at maximum. Infiltration tests in situ also prove high unsaturated hydraulic conductivity in the upland slopes.

Geographical, infrastructural, and rural environmental data are collected from a sheet map (Survey of Ghana, 1967), QuickBird satellite images with 0.6 m resolution, and field surveys conducted several times during 2005 through 2007. The valley bottoms can be identified from the satellite images as many threshing fields are found in them. Plane view of the inland valley with ground surface elevation of the valley bottoms is shown in Figure 2. The total area is about 6.44 km^2. There are 6 earthen dams (Dam 0 through 5) constructed in 1990s, and 4 of them (Dam 2 through 5) are equipped with concrete spillways. Dam 0 and Dam 1 are rather simple dug-outs. Key parameters of the dams such as Dead-Water Level (DWL), width *B* of concrete spillway, Full Water Level (FWL), and freeboard (FB), are summarized in Table 1. Rural communities, whose names are mentioned in Figure 2, lay over upland slopes, and most of the residents are subsistence farmers. The water stored in the dams is primarily used for domestic purposes in the communities. However, drinking water directly from the dams is not recommended since pollution due to the animals is serious and possibility of being infected with water-borne diseases such as dracunculiasis (guinea worm) and schistosomiasis (bilharzia) is still high. Fetching water from the dug wells that are scattered in the valley bottoms is more hygienic, but they often dry up in dry seasons. The dams are also expected to raise ground water levels so as to retard depletion of water in the dug wells. On the other hand, construction of the dams results in creation of backwater areas sacrificing the rice fields in the valley bottoms. Water of the dams is not used for irrigation purposes with a few exception of vegetable gardening. Minor fishery targeting *Oreochromis niloticus* (nile tilapia) and *Clarias anguillaris* (mudfish) is also practiced in the dams. Feeder roads are embanked across the valley bottoms, and culverts of circular or rectangular cross sections are installed beneath the roads to avoid inundation. Dimensions of the culverts are shown in Tables 2 and 3.

Table 1: Key Parameters of Earthen Dams

No.	*Benefiting Communities*	*DWL (m)*	*B (m)*	*FWL (m)*	*FB (m)*
0.	Gbullung	EL134.20	No concrete spillway		
1.	Tonjing, Gbullung	EL135.20	No concrete spillway		
2.	Kunkulun, Jakpahi-Nayili	EL142.70	24.5	EL144.85	0.65
3.	Jakpahi-Kukuo, Jakpahi-Bagong, Gizaa	EL156.30	15.0	EL157.60	0.90
4.	Gizaa-Gundaa, Gizaa	EL160.60	20.2	EL163.15	0.55
5.	Cheshegu, Gizaa-Gundaa	EL166.40	14.7	EL168.00	0.50

Table 2: Locations and Dimensions of Culverts with Circular Cross-sections

Latitude	*Longitude*	*Number of Ducts*	*Diameter (m)*
09 28 29.5 N	001 00 26.0 W	2	0.90
09 28 48.6 N	001 00 36.8 W	1	0.90
09 28 49.1 N	001 00 27.0 W	3	0.90
09 28 11.2 N	001 02 11.6 W	1	0.90
09 27 42.1 N	001 02 52.9 W	1	0.90

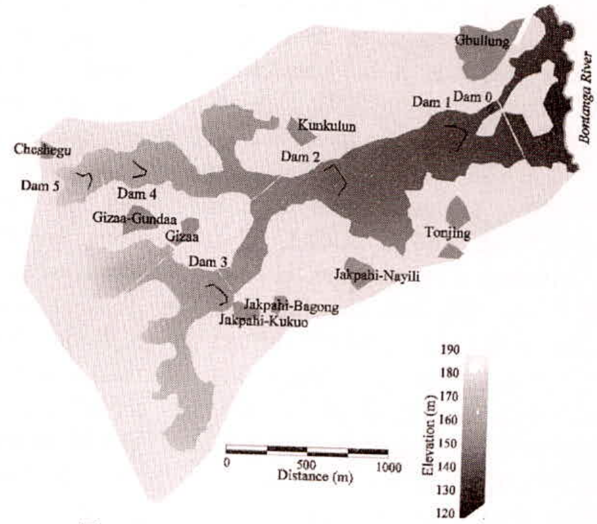

Fig. 2: Plane view of the inland valley

Table 3: Locations and Dimensions of Culverts with Rectangular Cross-sections

Latitude	*Longitude*	*Width (m)*	*Height (m)*
09 28 35.4 N	001 00 30.0 W	1.00	0.55
09 28 49.6 N	001 00 37.4 W	2.00	2.00
09 28 50.0 N	001 00 37.6 W	1.95	1.30
09 28 50.8 N	001 00 37.9 W	1.00	0.55
09 28 15.9 N	001 02 06.4 W	1.30	0.60
09 27 36.0 N	001 03 00.1 W	3.90	1.00
09 27 36.8 N	001 02 26.7 W	1.90	1.20
09 27 34.9 N	001 02 25.1 W	0.95	1.20
09 27 33.9 N	001 02 24.4 W	2.00	1.25

SIMULATION OF SURFACE FLOWS IN RAINSTORMS

The finite volume model is applied for simulating surface flows during rainstorm events in the study area. The valley bottoms are assumed to be saturated, and Ω_{VB} is taken as the computational domain to be divided into unstructured triangular meshes with 7,094 cells and 4,187 nodes. Critical flow boundary conditions are imposed on the edges along the bank of Bontanga River, since the river bed is eroded almost vertically. Other boundary edges are regarded as wall boundaries. The hydraulic structures and the roads are treated as internal boundary conditions. For the culverts, the Manning's formula is employed to estimate the cross-sectional average velocity V as,

$$V = \frac{1}{n_c}\sqrt{\|\nabla \eta\|}R^{2/3} \qquad \dots (9)$$

where n_c is the Manning's roughness coefficient for concrete, R the hydraulic radius, and $\nabla\eta$ is evaluated from η of the adjacent two cells. The spillways of the dams and the roads when water overflows them are hydraulically regarded as broad-crested weirs. The unit width discharge q_w flowing over a broad-crested weir is calculated as,

$$q_w = 0.326\sqrt{2gH_d} \quad \dots (10)$$

where H_d is the overflow depth (Hager and Schwalt, 1992). Once the water level in the reservoir of a dam exceeds the top of the dam embankment, the FWL, which is the spillway crest level, is instantaneously reduced to the ground surface level to represent breaching of the dam in computations.

A raingauge with 0.2 mm tipping-bucket connected to a pulse logger is installed at latitude 09 29 55.9 N, longitude 000 59 18.0 W, and time series data of rainfall intensity r actually observed on May 15th, 2006, are used as the input to the numerical model. The initial time is 1:28 a.m. Simulation is firstly conducted for 6 hours to examine possibility of harvesting rainwater in the dams within a single normal rainstorm event. The total rainfall depth is 62.0 mm, of which 47.8 mm concentrates in the first 1 hour. The Manning's roughness coefficient n is taken as 0.045 $m^{-1/3}s$ in the whole valley bottoms, and n_c is assumed to be 0.015 $m^{-1/3}s$ in the culverts. The seepage flow from ground to surface is assumed to be zero so that $\varphi = r$. Simulated water depths of the dams are delineated in Figure 3, together with the hyetograph. The rainwater flowing over ground surface is successfully harvested in the dams. Excess water overflows from the spillway of Dam 3. Few parts of the roads are inundated because capacity of the culverts is not enough. Oscillations in the water depths are due to the finite volume meshing that cannot capture exact positions of flow discontinuities.

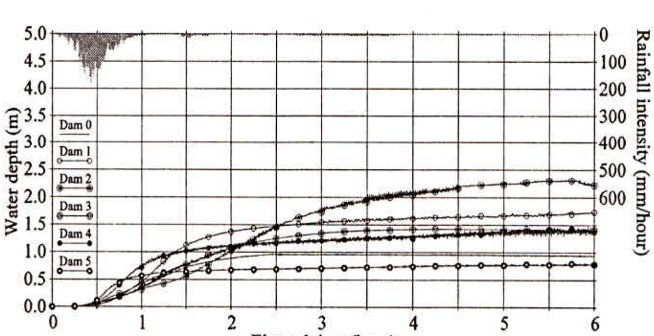

Fig. 3: Hyetograph and simulated water depths of dams

Next, another case where the rainfall intensity is 2.5r in the temporal domain scaled up by $\sqrt{2.5}$ is considered. The maximum hourly rainfall depth is 110.5 mm. Note that the maximum hourly rainfall depth of 100 years return period is 100.6 mm in Tamale and is 115.1 mm in Yendi, situated about 100 km east of Tamale (Dankwa, 1974). The seepage flow rate ψ per unit area is set in accordance with Eqn. (7) where the hydraulic conductivity k_H is equal to 3.142×10^{-3} m/s in horizontal directions. This value of the hydraulic conductivity corresponds to r_{in} = 5 mm/hour. Thus, the source term of the continuity equation is $\varphi = r + \psi$. The magnified hyetograph and simulated water depths of the dams are shown in Figure 4. Breaching of Dam 3 occurs at t = 5,294 s, resulting in a surge to bring about cascading collapse of Dam 2 at t = 6,255 s. Figure 5 delineates the

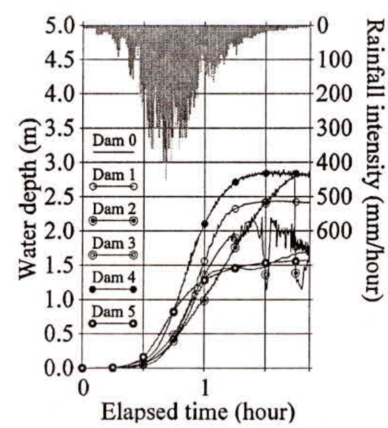

Fig. 4: Magnified hyetograph and simulated water depths of dams

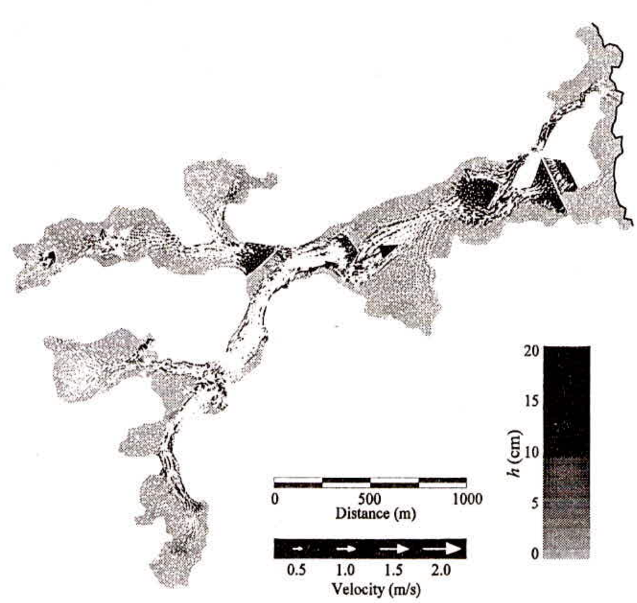

Fig. 5: Computed flow fields at t = 6,720 s

computed flow fields at t = 6,720 s, which is the terminal time of the computation. In the Figure, black arrows and white arrows for velocities represent supercritical and subcritical flows, respectively, but the arrow for velocity is omitted when the flow is supercritical and is deeper than 70 cm in the cell. The surge would devastate the rice fields in the downstream half of the study area. It is easy to imagine these losses of the dams and the farm lands turning out catastrophic shortage of food and water in the communities. Enhancement of the concrete spillways of the dams, especially for Dam 3, is strongly recommended.

SUMMARY

Applicability of the 2D SWEs to modelling surface water flows is demonstrated. The numerical model is capable of computational reproduction of hydraulic phenomena in water hazards. The study site is very strategic in terms of food security and poverty alleviation. The computational results suggest suitable rehabilitation works to be done on the dams. Numerical analysis using reliable numerical models with appropriate data acquisition procedures is the unique prediction tool for water hazards that could happen in extreme conditions.

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