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Rainfall Runoff Modelling Using Fuzzy Rule Based Approach

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ABSTRACT: The problem of transformation of rainfall into runoff has been a subject of scientific investigations throughout the evolution of the subject of hydrology. A number of investigators have tried to relate runoff with the different characteristics which affect it. Various researchers have attempted to address this modelling issue either using knowledge based models or data-driven models. However, simulating the real-world relationships using these Rainfall-Runoff models is not a simple task since the various hydrological processes that involve the transformation of rainfall into discharge are complex and variable. In recent years, data-driven soft computing techniques e.g. artificial neural network and fuzzy logic have gained significant attention in hydrological modelling. In the present paper fuzzy rule based approach is chosen for developing a rainfall-runoff model for Manot sub-basin of Narmada River system. Further, the model performance has been examined using global model performances indices. The results of the study indicate that the choice of the model input structure certainly has an impact on the model prediction accuracy. The fuzzy model has improved with the increase in the number of input combinations up to a certain extent. The study presents an efficient methodology developed for rainfall runoff modeling over the medium size catchment with limited data.

INTRODUCTION

The rainfall-runoff process is highly nonlinear, time-varying, spatially distributed, and not easily described by simple models. Therefore, the problem of transformation of rainfall into runoff has been a subject of scientific investigations throughout the evolution of the subject of hydrology. Hydrologists are mainly concerned with evaluation of catchment response for planning, development and operation of various water resources schemes. A number of investigators have tried to relate runoff with the different characteristics which affect it (Dooge, 1959; Rodriguez-Iturbe and Valdes, 1979; Stedinger and Taylor, 1982; Chow *et al*., 1988; Van der Tak and Bras, 1990; Bevan *et al*., 1995; Muzik, 1996a, b; Bevan, 2000; Rajurkar *et al*., 2004). Various attempts have been made to address this modelling issue either using knowledge based models or data-driven models. A knowledge based model aims to reproduce the system and its behaviour in a physically realistic manner and are generally called physically-based model. The physically based models generally use a mathematical framework based on mass, momentum and energy conservation equations in a spatially distributed model domain, and parameter values that are directly related to catchment characteristics. For the purpose of rainfall-runoff process simulation, conceptual and physical based models are widely used. However, simulating the real-world relationships using these rainfall-runoff models is not a simple task since the various hydrological processes that involve the transformation of rainfall into runoff are complex and variable. Many of the conceptual models widely used in rainfall-runoff modeling are lumped one and the factors in generating runoff are not represented clearly by these models. The time required to construct these models is enormous and thus an alternative modelling technique is sought when detailed modelling is not required in cases such as streamflow forecasting. The linear regression or linear time series models such as ARMA (Auto Regressive Moving Average) have been developed to

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handle such situations because they are relatively easy to implement. However, such models do not attempt to represent the non-linear dynamics inherent in the hydrologic processes, and may not always perform well. Recent developments and use of fuzzy logic in hydrological modeling indicates that more applications and research is needed to support the utility of fuzzy technique in the area of daily rainfall-runoff modelling and to help in establishing their full practical use in dealing the real world problems. This study presents the development of intelligence models based on fuzzy logic for prediction of runoff. The fuzzy relations between the input and output variables were inferred from the measured data and they are laid out in the form of IF-THEN statements. The performance of the developed models is evaluated using various model performance indices.

FUZZY LOGIC—AN OVERVIEW

Zadeh (1965) showed that fuzzy logic unlike classical logic can realize values between zero and one and thus he transformed the crisp set into a continuous set. A sharp set is a sub set of a fuzzy set where the membership function can take only the values 0 and 1 (Babuška, 1998). Fuzzy logic is a superset of classical logic with the introduction of "degree of membership". The membership μ is a value between 0 and 1. Fuzzy logic was originally meant to be a technique for modeling the human thinking and reasoning, which is done by rules expressed as: IF (antecedent) THEN (consequent). The premise (IF part) of each rule describes a certain input data situation. The inference system evaluates all premises and calculates a truth value for each rule out of the membership values of the fuzzy sets contained in the premise. The consequent (THEN part) of all rules are calculated where the truth value of the premise is greater than zero. The results of each consequent are then used to compute the overall result, weighted by the truth-value of the rule. In rule based fuzzy systems, the relationships between variables are represented by means of fuzzy if-then rules e.g. "*If* antecedent proposition *then* consequent proposition". On the basis of the structure of the consequent proposition, the fuzzy models are classified into three groups: (i) Linguistic (Mamdani type) fuzzy model (Zadeh, 1973; Mamdani, 1977) (ii) Fuzzy relational model (Pedrycz, 1984; Yi and Chung, 1993) (iii) Takagi-Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). In modeling using a fuzzy inference system, interpretation of fuzzy rules is one of the important tasks. Three principal ways to obtain the fuzzy rules are (i) experts knowledge (ii) data driven approach and (iii) combination of (i) and (ii). In situations when the human experts are not available or may not provide sufficient number of rules, data driven approach for rule extraction is very useful. A data driven *TS* fuzzy model (Takagi and Sugeno, 1985), where the consequents are (crisp) functions of the input variables, is relatively easy to identify and their structure can be readily analyzed (Lohani *et al.*, 2005a).

FUZZY INFERENCE SYSTEM

The process of formulating the mapping from a given input to an output using fuzzy logic is called fuzzy inference. The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership function to rules, rules to a set of output characteristics, output characteristics to output membership functions, and the output membership function to a single-valued output or a decision associated with the output (Jang *et al.*, 2002). A fuzzy rule-based model suitable for the approximation of many systems and functions is the *Takagi–Sugeno* (TS) fuzzy model (Takagi and Sugeno, 1985). The TS fuzzy model result in smooth transition between linear sub-models (Figure 1), which are responsible for separate sub-space of states. A TS fuzzy model is defined as,

$$Ri\text{: IF } x \text{ is } A_i \text{ THEN } y_i = a_i^T x + b_i,\ i = 1, 2, \ldots, M \quad \ldots (1)$$

Where $x \in \Re^n$ is the antecedent and $y_i \in \Re$ is the consequent of the i^{th} rule. In the consequent, a_i is the parameter vector and b_i is the scalar offset. The number of rules is denoted by M and A_i is the (multivariate) antecedent fuzzy set of the i^{th} rule defined by the membership function,

$$\mu_i(x): \Re^n \rightarrow [0, 1] \quad \ldots (2)$$

The fuzzy antecedent in the *TS* fuzzy model is normally defined as an and-conjunction by means of the product operator,

$$\mu_i(x) = \prod_{j=1}^{p} \mu_{ij}(x_j) \quad \ldots (3)$$

where x_j is the j^{th} input variable in the p dimensional input data space, and μ_{ij} the membership degree of x_j to the fuzzy set describing the j^{th} premise part of the i^{th} rule. $\mu_i(x)$ is the overall truth value of the i^{th} rule.

For the input x the total output y of the TS model is computed by aggregating the individual rules contributions,

$$y = \sum_{i=1}^{M} u_i(x).y_i \qquad \ldots (4)$$

where u_i is the normalized degree of fulfillment of the antecedent clause of rule R_i,

$$u_i(x) = \frac{\mu_i(x)}{\sum_{i=1}^{M} \mu_i(x)} \qquad \ldots (5)$$

The y_i's are called consequent functions of the M rules and are defined by,

$$y_i = w_{i0} + w_{i1}x_1 + w_{i2}x_2 + \ldots + w_{ip}x_p \qquad \ldots (6)$$

where w_{ij} are the linear weights for the i^{th} rule consequent function.

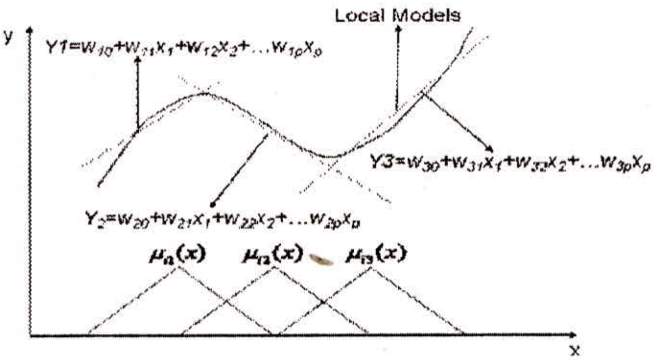

Fig. 1: Piece-wise linear approximation on non-linear function by TS fuzzy model

Identification of rules by manual inspection has its limitations. The data driven approach based on subtractive clustering has shown promising results in various hydrological modeling applications (Lohani 2005a, 2005b). The purpose of subtractive clustering is to identify natural grouping of the data from a large data set and finally to produce a concise representation of a systems behavior. The subtractive clustering approach is used in the present study to determine the number of rules and antecedent membership functions by considering each cluster center (d_i) as a fuzzy rule. Further, each data point of a set of N data points $\{x_1, \ldots, x_N\}$ in a p-dimensional space is considered as the candidate for cluster centers. Density measure at data point x_i is computed from the normalized and scaled data points on the basis of its location with respect to other data points and expressed as,

$$d_i = \sum_{j=1}^{N} \exp\left(-\left(\frac{2}{r_a}\right)^2 . \left\| x_i - x_j \right\|^2 \right) \qquad \ldots (7)$$

where r_a is a positive constant called cluster radius.

The data point (x_1^*) with highest density measure (d_1^*) is considered as first cluster center. After excluding the influence of the first cluster center, the density measure of all other data points is recalculated as,

$$d_i = d_i - d_1^* . \mu(x_i^*) \qquad \ldots (8)$$

$$\mu(x_i^*) = \exp\left(-\frac{\left\| x_i - x_i^* \right\|^2}{(r_b/2)^2} \right) \qquad \ldots (9)$$

where r_b ($r_b > r_a \geq 0$) is a positive constant that results in a measurable reduction in potential of neighborhood data points and thus avoids closely spaced cluster centers.

After revising the density measure for each data point, the data point with the highest remaining density measure is selected and set as the next cluster center. The process is repeated until a sufficient number of clusters are generated and finally the process is stopped considering the criterion suggested by Chiu (1994). The identified cluster centers (d_i^*, $i = 1, k$) are used as the centers of the fuzzy rules' premise of input data vector x. Finally, the degree to which rule i is fulfilled is defined by Gaussian membership function,

$$\mu_{ij}(x_i) = \exp\left(\frac{(x_i - x_i^*)^2}{(r_a/2)^2} \right) \qquad \ldots (10)$$

STUDY AREA AND DATA USED

The Narmada River emanates at Amarkantak in the Shahdol district of Madhya Pradesh in Central India at an elevation of 1057 m.s.l. The river travels a distance of 1312 km before it falls into Gulf of Cambay in the Arabian Sea near Bharuch in Gujarat. The Narmada basin extends over an area of 98,796 sq. km and lies between longitudes 72° 32′ E to 81° 45′ E and latitudes 21° 20′ N to 23° 45′ N. In the present study the upper Narmada basin upto Manot G&D site has been selected for rainfall-runoff modelling (Figure 2). Validated and processed data of Narmada catchment up to Manot gauging site covering an area of 4300 sq. km. have been selected for rainfall-runoff modeling. Daily rainfall at Narayanganj, Bichhia, Baihar, Palhera, Manot, Gondia and Nimpur stations and daily discharge at Manot gauging site have been considered. The available data were divided into two sets, one for calibration and other for validation. The daily rainfall and discharge data from June to September (monsoon period) of the years 1993 and 1996 were used for calibration of the fuzzy model because these four years

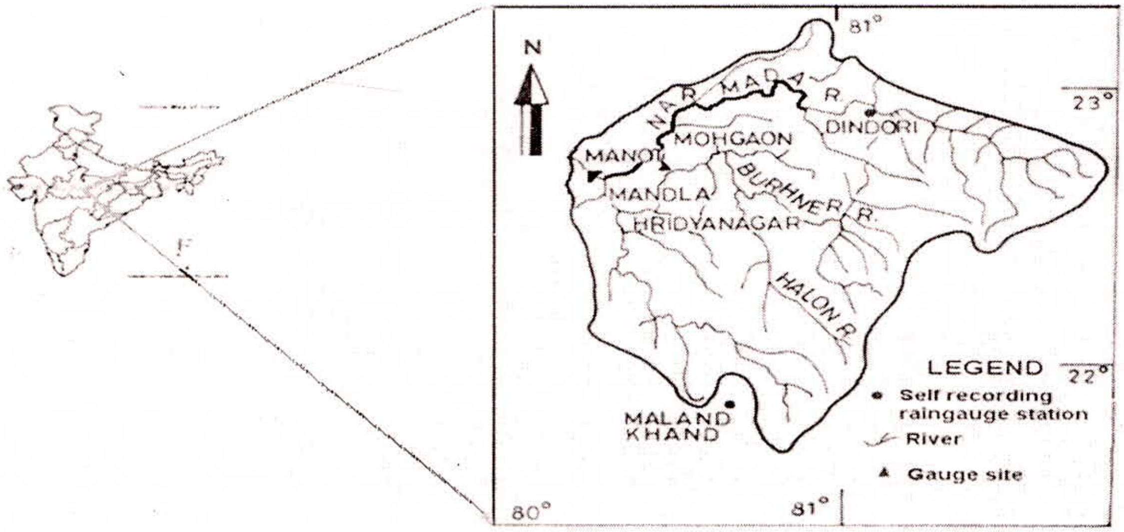

Fig. 2: Index map of the study area

of data represent the extreme values of rainfall and discharge. The data of year 1997 and 1998 were used for the validation of the model.

FUZZY MODEL DEVELOPMENT AND RESULTS

Selection of the input and output variables is the first step in development of a fuzzy rule based rainfall-runoff model. The range of model input values, which are judged necessary for the description of the situation, can be portioned into fuzzy sets (Hellendoorn and Driankov, 1997). In case of a rainfall-runoff model with minimum available data, the output variable describes the runoff that is to be predicted and possible input variables are measured rainfall and runoff data. The following four combinations of input data vectors have been considered:

1. Only rainfall as input,

M1 $Q_t = f(P_t, P_{t-1}, P_{t-2}, P_{t-3})$

2. Rainfall and Runoff as input,

M2 $Q_t = f(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1})$

M3 $Q_t = f(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1}, Q_{t-2})$

M4 $Q_t = f(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1}, Q_{t-2}, Q_{t-3})$

where Q_t and P_t are the runoff and precipitation at time t respectively.

The evaluation of a set of fuzzy rules (or rule base) in a fuzzy rule based model for the determination of the runoff value is an important task. The basis of fuzzy logic is to consider hydrologic variables in a linguistically uncertain manner, in the form of sub-groups, each of which is labeled with successive fuzzy word attachments such as "low", "medium", "high" etc. In this way, the variable is considered not as a global and numerical quantity but in partial groups which provided better room for the justification of sub-relationship between two or more variables on the basis of fuzzy words (Sen and Altunkayank, 2003). Since rainfall-runoff relationship in general, has a direct proportionality feature, it is possible to write the following rule base. for the description of Takagi-Sugeno fuzzy rainfall-runoff model.

1. Only rainfall as input,

M1 Rule $R_i : IF(P_t, P_{t-1}, P_{t-2}, P_{t-3})$ is C_i then

$$Q_t = a_{1i}P_t + a_{2i}P_{t-1} + a_{3i}P_{t-2} + a_{4i}P_{t-3} + c_i \quad \ldots (11)$$

2. Rainfall and Runoff as input,

M2 Rule $R_i : IF(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1})$ is C_i then

$$Q_t = a_{1i}P_t + a_{2i}P_{t-1} + a_{3i}P_{t-2} + a_{4i}P_{t-3} + a_{5i}Q_{t-1} + c_i \quad \ldots (12)$$

M3 Rule

$R_i : IF(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1}, Q_{t-2})$ is C_i then

$$Q_t = a_{1i}P_t + a_{2i}P_{t-1} + a_{3i}P_{t-2} + a_{4i}P_{t-3} + a_{5i}Q_{t-1} + a_{6i}Q_{t-2} + c_i \quad \ldots (13)$$

M4 Rule

$R_i : IF(P_t, P_{t-1}, P_{t-2}, P_{t-3}, Q_{t-1}, Q_{t-2}, Q_{t-3})$ is C_i then

$$Q_t = a_{1i}P_t + a_{2i}P_{t-1} + a_{3i}P_{t-2} + a_{4i}P_{t-3} + a_{5i}Q_{t-1} + a_{6i}Q_{t-2} + a_{7i}Q_{t-3} + c_i \quad \ldots (14)$$

where a_{ji} and c_i are the parameters of the consequent part of rule R_i.

Using the linear consequent part of the fuzzy rainfall-runoff model, subtractive clustering based identification method has been applied. The model performance is examined by means of Nash and Sutcliffe (NS) model efficiency (Nash and Sutcliffe, 1970) and Root Mean Square Error (RMSE) criteria. In order to find the optimal model, the parameters of the subtractive clustering algorithm were finalized after a number of trial runs. In the trials, the parameters of subtractive clustering were varied from 0.5 to 2 for quash factor and 0.1 to 1 for the cluster radius (r_a), accept ratio and reject ratio with steps of 0.01. The cluster centers and thus the Gaussian membership function identified for each case were used to compute consequent parameters through a linear least square method and finally a TS fuzzy model was developed. The developed model gives crisp output value for a given input data. Fuzzy model developed form the actual data sets have different rules ranging from 4 to 7. Performance indices such as Root Mean Square Error (RMSE) between the computed and observed runoff, correlation coefficient and NS efficiency were used to finalize the optimal parameter combination of the model. The effect of error in peak and low observations are taken care by the criteria viz. correlation coefficient and NS efficiency. The error in time to peak is another criterion which is not considered here as it is normally considered in storm studies. In rainfall-runoff modeling, accurate estimation of total volume is an important aspect. Therefore, another criterion known as volumetric error (Kachroo and Natale, 1992) has been considered in this study to hydrologically evaluate the performance of the models under consideration. The volumetric error (V_{er}) is expressed as,

$$V_{er} = \frac{\sum_{i=1}^{n}(Q_{ci} - Q_{oi})}{\sum_{i=1}^{n} Q_{oi}} \times 100 \qquad \text{... (15)}$$

where Q_{ci}, Q_{oi} and n are computed runoff, observed runoff and number of data sets.

Fuzzy rule based models developed using model structures presented through Equation 11 to Equation 14 were compared using various statistical model performance indices e.g. RMSE, coefficient of correlation, NS efficiency and volumetric error. Table 1 presents these performance indices of all the four model structure defined as M1 to M4. Model results (Table 1) show that the models M2, M3 and M4 performs better than M1 model both during calibration (M1: 0.672, 0.584, 237.2) and validation (M1: 0.683, 0.577, 214.1). Model M2 to M4 were developed for different combinations of precipitation and runoff and they show coefficient of correlation in the range of 0.823 to 0.842, NS efficiency in the range of 0.663 to 0.675, RMSE in the range of 108.7 to 109.4 and volumetric error in the range of –9.66 to 10.3 during validation. It is observed that the inclusion of previous day runoff in the model input shows a significant improvement in the model performance both during calibration and validation.

As more and more information (input) is added to the model, generally the coefficient of correlation improves and RMSE is reduced (Lohani *et al.*, 2006, 2007a). This may be due to auto correlation and cross correlation structure in the input data vector. It is observed that increase in inputs beyond a certain limit in the model cause reduction in model performance. This is due to reduction in auto correlation and cross correlation in the input data vector. From Table 1, it is apparent that model M3, which consists of one current rainfall, three antecedent precipitation values and two antecedent runoff values as input, showed the highest coefficient of correlation (0.842), minimum RMSE (108.7) and maximum model efficiency (0.675), and it is selected as the best fit model for daily rainfall-runoff relationship for Manot gauging sites. Further, it is observed that model M1 and M2 over estimates the runoff and Model M3 and M4 underestimate the runoff both during calibration and validation period. Model M2 overestimate the runoff by 11.7% during calibration and by 10.3% during validation while, model M3 underestimate runoff volume by 11% during calibration and by 9.66% during validation. Therefore, if the purpose of rainfall-runoff modeling is the computation of total runoff from rainfall, the model performance indices volumetric error plays an important role. The estimations of daily runoff for the validation period are compared with the observed runoff values in the form of hydrographs for all the four models through Figures 3 to 6.

Table 1: Statistical Performances Indices—Fuzzy Models

Model	*Calibration*				*Validation*			
	Coefficient of Correlation	*NS*	*RMSE*	V_{er}	*Coefficient of Correlation*	*NS*	*RMSE*	V_{er}
M1	0.672	0.584	237.2	23.6	0.683	0.577	214.1	21.2
M2	0.870	0.634	170.1	11.7	0.823	0.663	109.2	10.3
M3	0.878	0.647	168.7	–11.0	0.842	0.675	108.7	–9.66
M4	0.871	0.642	169.3	–11.6	0.838	0.672	109.4	–10.2

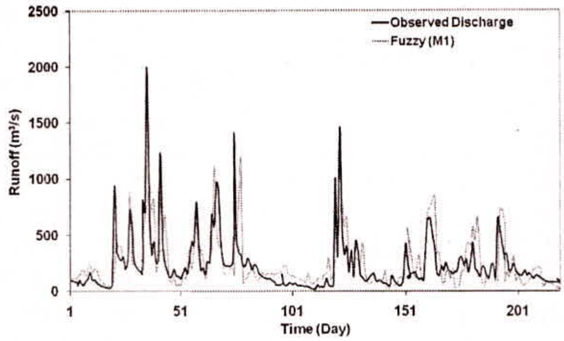

Fig. 3: Time series of Observed Runoff and Model Predicted Runoff—Fuzzy Model (M1)

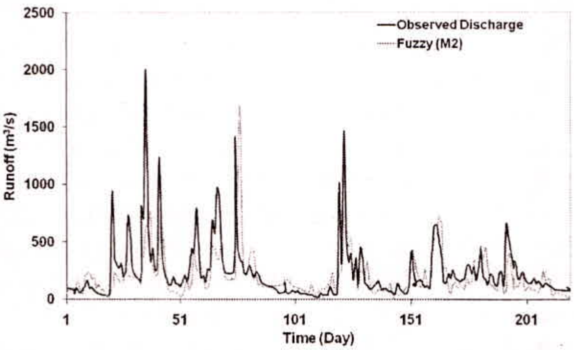

Fig. 4: Time series of Observed Runoff and Model Predicted Runoff—Fuzzy Model (M2)

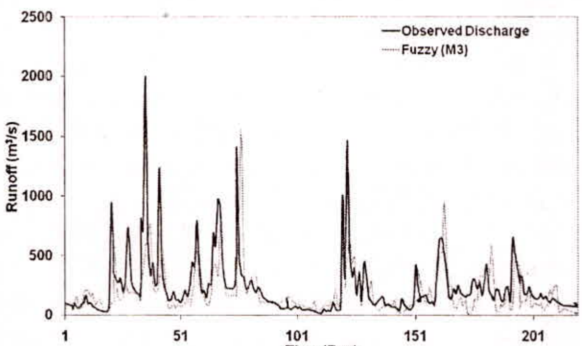

Fig. 5: Time series of Observed Runoff and Model Predicted Runoff—Fuzzy Model (M3)

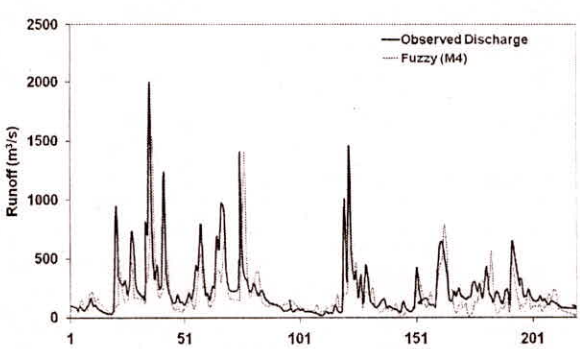

Fig. 6: Time series of Observed Runoff and Model Predicted Runoff—Fuzzy Model (M4)

CONCLUSIONS

In this study fuzzy rule based technique has been used to develop models for the prediction of runoff using rainfall-runoff models for Narmada catchment upto Manot gauging site. Potential of fuzzy rule based technique for modeling of rainfall-runoff process is investigated by selecting different combinations of input vectors and comparing the results with observed runoff. The daily rainfall and runoff data of the monsoon season (Mid June to September) from 1993 to 1998 were considered for the development (calibration and validation) of models. Rainfall-runoff models developed using four major input vectors to identify a fuzzy model for the Manot gauging site. The study suggests a suitable model structure for the study area and concludes that the fuzzy rule based approach is a useful soft computing technique for developing a daily rainfall-runoff model.

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