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# Estimation of Partially Penetrated Aquifer Parameters from Pumping Test Data by Genetic Algorithm Optimization Technique

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**ABSTRACT:** Adequate and reliable estimates of aquifer parameters are of utmost importance for proper management of vital groundwater resources. The pumping (aquifer) test is the standard technique for estimating various hydraulic properties of aquifer systems, viz., transmissivity (T), hydraulic conductivity (K), storage coefficient (S), and thickness of the aquifer (H), for which the graphical method is widely used. In the present study, the efficacy of the Genetic Algorithm (GA) optimization technique is assessed in estimating aquifer parameters from the time-drawdown pumping test data. Computer codes were developed to optimize various aquifer parameters of partially penetrated confined aquifer by using the GA technique. Applicability, adequacy, and robustness of the developed codes were tested using two sets field test data. The aquifer parameters were also estimated by the graphical method, and were compared with those obtained by the GA technique. The GA technique yielded significantly low values of the Sum of Square Errors (SSE) for both the datasets under study. The results revealed that the GA technique is an efficient and reliable method for estimating various aquifer parameters, especially in the situation when the graphical matching is poor. Also, it was found that because of its inherent characteristics, GA avoids the subjectivity, long computation time and ill-posedness often associated with conventional optimization techniques.

## INTRODUCTION

Groundwater is a treasured earth's resource and it has several inherent advantages over the surface water resource. In view of the galloping population and unabated pollution worldwide, there is an urgent need to manage the vital but shrinking groundwater resource efficiently to ensure its sustainable utilization. Groundwater processes being insidious and complex in nature, modeling plays a pivotal role in the design and management of groundwater systems. A prior knowledge of aquifer parameters is indispensable for successful and reliable modeling results, and thereby ensuring proper management of vital groundwater resources. For the estimation of hydraulic parameters of aquifers, viz., transmissivity (T), storage coefficient (S), hydraulic conductivity (K), and thickness of the aquifer (H), pumping tests (or aquifer tests) are the most widely used technique. Time-drawdown pumping test data are analyzed by using the nonlinear analytical models such as Theis model, Theis-Hantush model, Jacob-Hantush model, Hantush model, Neuman model, and so on depending on the type of aquifer and the penetration of the wells in which the test is conducted. The analysis is performed by the graphical method, which is deemed as a standard method for estimating aquifer parameters. However, the curve matching is often approximate and a perfect match is not expected because of the large difference between the assumptions made in the theories and the actual field condition. Furthermore, the graphical method is very cumbersome, time-consuming and subjective despite the availability of the computer software Aquifer Test developed by the Waterloo Hydrogeologic, Inc., Canada.

## MODELS FOR ANALYZING PUMPING TEST DATA

According to Hantush (1961a) the drawdown in a piezometer due to pumping in a confined aquifer can be described by the following equation,

$$s = \frac{Q}{4\pi KH}\left[W(u) + F_s\right] \qquad \text{... (1)}$$

where,

$$W(u) = E_1(u) = \int_u^{\infty} \frac{1}{y}\exp(-y)\,dy \qquad \text{... (2)}$$

$$u = \frac{r^2 S}{4KHt} \qquad \text{... (3)}$$

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$$f_s = \frac{2H}{\pi(b-d)} \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) W\left(u, \frac{n\pi r}{H}\right) \left[\sin\left(\frac{n\pi b}{H}\right) - \sin\left(\frac{n\pi d}{H}\right)\right] \cos\left(\frac{n\pi a}{H}\right) \quad \ldots (4)$$

$$W\left(u, \frac{n\pi r}{H}\right) = \int_u^{\infty} \frac{1}{y} \exp\left(-y - n^2\pi^2 r^2 / 4H^2 y\right) dy \quad \ldots (5)$$

Note that the angles are expressed in radians. (For an explanation of these symbols, see Figure 1.)

Values of the functions $W(u)$ and $W(u, n\pi r/H)$ can usually be found in handbooks on well-flow analysis, or can be obtained numerically. The above solutions are exact, but rather difficult to handle. Approximations are possible for both short and long pumping times.

## THEIS-HANTUSH METHOD

For a relatively short period of pumping, $t < [(2H - b - a)^2(S_s)]/20K$, the general equation can be simplified (Hantush, 1961a, b). The drawdown in a piezometer at distance $r$ from a partially penetrating well is approximately,

$$s = \frac{Q}{8\pi K(b-d)} E\left(u, \frac{b}{r}, \frac{d}{r}, \frac{a}{r}\right) \quad \ldots (6)$$

where

$$E\left(u, \frac{b}{r}, \frac{d}{r}, \frac{a}{r}\right) = M(u, B_1) - M(u, B_2) + M(u, B_3) - M(u, B_4) \quad \ldots (7)$$

$$u = \frac{r^2 S_s}{4Kt} \quad \ldots (8)$$

$S_s = \frac{S}{H} =$ Specific storage of the aquifer

$B_1 = (b + a)/r$

$B_2 = (d + a)/r$

$B_3 = (b - a)/r$

$B_4 = (d - a)/r$

$$M(u, B) = \int_u^{\infty} \frac{e^{-y}}{y} erf\left(B\sqrt{y}\right) dy$$

Because erf $(-x) = -$erf $(x)$, it follows that $M(u,-B) = -M(u, B)$. Numerical values of the function $M(u, B)$ can be found in most handbooks.

## JACOB-HANTUSH METHOD

According to Hantush (1961 a), Eqn. (4) reduces for relatively long pumping times ($t > H^2S/2KH$) to,

$$f_s = \frac{4H}{\pi(b-d)} \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) K_O\left(\frac{n\pi r}{H}\right) \left[\sin\left(\frac{n\pi b}{H}\right) - \sin\left(\frac{n\pi d}{H}\right)\right] \cos\left(\frac{n\pi a}{H}\right) \quad \ldots (9)$$

where $K_o(x)$ is the modified Bessel function of the second kind and zero order,

$$K_O\left(\frac{n\pi r}{H}\right) = \int_u^{\infty} \cos\left(\frac{n\pi r}{H} \sinh y\right) dy \quad \ldots (10)$$

Numerical values of the function $K_o(n\pi r/H)$ can also be found in most handbooks.

It can be seen that, for a given set of parameters, Eqn. (9) yields a constant value. The Hantush modification of the Jacob method was based on this phenomenon. Hantush (1961a) showed that for this case, Eqn. (1) shows a straight-line segment that can be approximated by,

$$s = \frac{2.3Q}{4\pi KH} \left(\log \frac{2.25KHt}{r^2 S} + f_s\right) \quad \ldots (11)$$

If we use drawdown observations at a particular distance $r$ from the pumped well, a plot of drawdown $s$ *versus* the logarithm of time $t$ will show a straight line, provided that the time of pumping $t$ was long enough. If the slope of this straight-line segment is expressed as the drawdown difference ($\Delta s = s_1 - s_2$) per $^{10}$log cycle of time ($\log t_2/t_1 = 1$), rearranging Eqn. (11) gives,

$$KH = \frac{2.3Q}{4\pi \Delta s} \quad \ldots (12)$$

If the straight line is extended until it intercepts the time axis where $s = 0$, the interception point has the

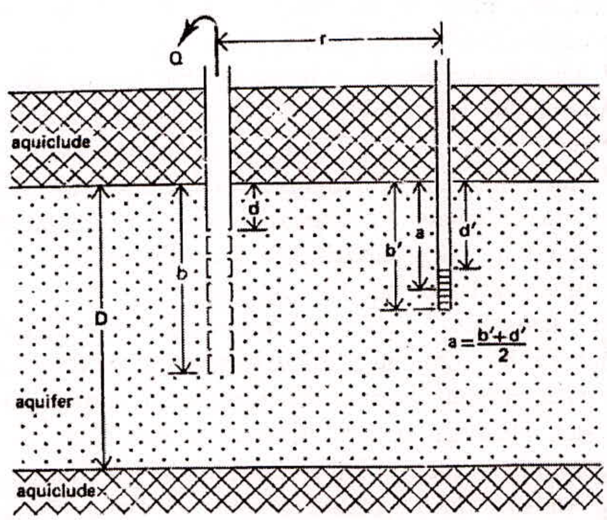


**Fig. 1:** Nonleaky artesian aquifer with partially penetrating wells

coordinates $s = 0$ and $t$ = to. Substituting these values into Eqn. (11) gives $\log\left(2.25KHt_o e^{f_s}/r^2S\right)=0$ or $\left(2.25KHt_o e^{f_s}/r^2S\right)=1$. The storativity can thus be found from,

$$S=\frac{2.25KHt_o}{r^2}e^{f_s} \qquad \ldots (13)$$

## APPLICATION TO FIELD DATA PROBLEMS

Genetic algorithm optimization technique is applied to analyze a set of reported data of partially penetrated confined aquifer invoking the well hydraulics model described earlier. The data set refer to a pumping test conducted in partially penetrated confined aquifer in June 1976 at "Janpur" site located in Indus basin (Lahore, Pakistan) by courtesy of Water and Power Development Authority (WAPDA). At Janpur confined aquifer of large thickness is partially penetrated by a pumping well and two observation wells for conducting the pumping test. The pumping test was conducted for a period of 6000 minutes during which 38 time-draw-down observations were recorded at each observation wells at a constant discharge of 6350.4 $m^3$/day. The summarized time-drawdown data of both the observation wells I and II located at a distance of 30.5 meters and 91.5 meters are shown in Table 1 and 2 respectively.

**Table 1:** Aquifer Test Data 'Janpur', Indus Plain, Pakistan ($r$ = 30.5)

| *t (min)* | *s (m)* | *t (min)* | *s (m)* | *t (min)* | *s (m)* |
|---|---|---|---|---|---|
| 0 | 0 | 30 | 0.518 | 500 | 0.613 |
| 1 | 0.177 | 40 | 0.533 | 600 | 0.619 |
| 2 | 0.25 | 50 | 0.543 | 750 | 0.634 |
| 3 | 0.32 | 60 | 0.549 | 1000 | 0.64 |
| 4 | 0.344 | 75 | 0.555 | 1250 | 0.643 |
| 6 | 0.372 | 100 | 0.555 | 1500 | 0.649 |
| 8 | 0.427 | 125 | 0.57 | 1750 | 0.658 |
| 10 | 0.445 | 150 | 0.576 | 2000 | 0.674 |
| 12 | 0.457 | 175 | 0.578 | 2500 | 0.68 |
| 15 | 0.472 | 200 | 0.579 | 3000 | 0.695 |
| 18 | 0.488 | 250 | 0.582 | 4000 | 0.716 |
| 21 | 0.497 | 300 | 0.588 | 5000 | 0.722 |
| 25 | 0.509 | 400 | 0.61 | 6000 | 0.728 |

**Table 2:** Aquifer Test Data 'Janpur', Indus Plain, Pakistan ($r$ = 91.5)

| *t ( min)* | *s (m)* | *t ( min)* | *s (m)* | *t ( min)* | *s (m)* |
|---|---|---|---|---|---|
| 0 | 0 | 30 | 0.518 | 500 | 0.613 |
| 1 | 0.177 | 40 | 0.533 | 600 | 0.619 |
| 2 | 0.25 | 50 | 0.543 | 750 | 0.634 |
| 3 | 0.32 | 60 | 0.549 | 1000 | 0.64 |
| 4 | 0.344 | 75 | 0.555 | 1250 | 0.643 |
| 6 | 0.372 | 100 | 0.555 | 1500 | 0.649 |
| 8 | 0.427 | 125 | 0.57 | 1750 | 0.658 |
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| 15 | 0.472 | 200 | 0.579 | 3000 | 0.695 |
| 18 | 0.488 | 250 | 0.582 | 4000 | 0.716 |
| 21 | 0.497 | 300 | 0.588 | 5000 | 0.722 |
| 25 | 0.509 | 400 | 0.61 | 6000 | 0.728 |

## ESTIMATING PARTIALLY PENETRATED CONFINED AQUIFER PARAMETERS BY GRAPHICAL METHOD

The two sets of the time-drawdown data, each for observation well I and II of partially penetrated confined aquifer were also analyzed graphically. The cumbersome matching of type curve with the field-data curve and the match point selection is made according to the procedures described in J. Boonstra (1991) and summarized in Tables 3 and 4.

**Table 3:** Results of Graphical Analysis According to Theis-Hantush Model

| *r (m)* | *K (m/day)* | *S* | *H (m)* | *SSE* |
|---|---|---|---|---|
| 30.5 | 34.2 | 0.041 | 1144 | 1.7750 |
| 91.5 | 34.7 | 0.047 | 1178 | 0.4951 |

**Table 4:** Results of Graphical Analysis According to Jacob-Hantush Model

| *r (m)* | *K (m/day)* | *S* | *H (m)* | *SSE* |
|---|---|---|---|---|
| 30.5 | 34.4 | 0.013 | 410 | 0.0811 |
| 91.5 | 35.9 | 0.018 | 470 | 0.0647 |

## EFFICACY OF GENETIC ALGORITHM TECHNIQUE IN ESTIMATING PARTIALLY PENETRATED CONFINED AQUIFER PARAMETERS

### Genetic Algorithm Optimization Technique versus Graphical Method

The results of the time-drawdown pumping test data analysis by the conventional graphical method and the

GA optimization technique are compared based on the SSE as shown in Table 5. The optimum values of hydraulic conductivity, storage coefficient and thickness of the aquifer for both the datasets of partially penetrated confined aquifer are determined by using the developed GA based computer program as shown in Table 5. It should be noted that the values of SSE are considerably less in the case of GA technique for both the datasets (Table 5) as compared to the graphical method.

**Table 5:** Comparison of Partially Penetrated Confined Aquifer Parameters Obtained by Graphical Method and Genetic Algorithm Optimization Technique

| *r (m)* | *Method* | *Hydraulic Conductivity (m/day)* | *Storage Coefficient* | *Thickness of Aquifer (m)* | *SSE ($m^2$)* |
|---|---|---|---|---|---|
| 30.5 | Graphical method | 34.4 | 0.013 | 410 | 0.0811 |
| | GA technique | 27.8 | 0.032 | 461 | 0.0466 |
| 91.5 | Graphical method | 35.9 | 0.018 | 470 | 0.0647 |
| | GA technique | 27.1 | 0.029 | 458 | 0.0083 |

## Visual Inspection of Computed Time-Drawdown Curves

The time–drawdown curve based on the aquifer parameters yielded by GA technique is reasonably close to the observed time-drawdown curve Figures 2, 3. Thus, the visual inspection of computed time–drawdown curves further confirms the findings described in preceding sections. That is, the GA optimization technique is an efficient and reliable method for estimating the hydraulic parameters of partially penetrated confined aquifer, especially in the situation when the graphical matching is poor.

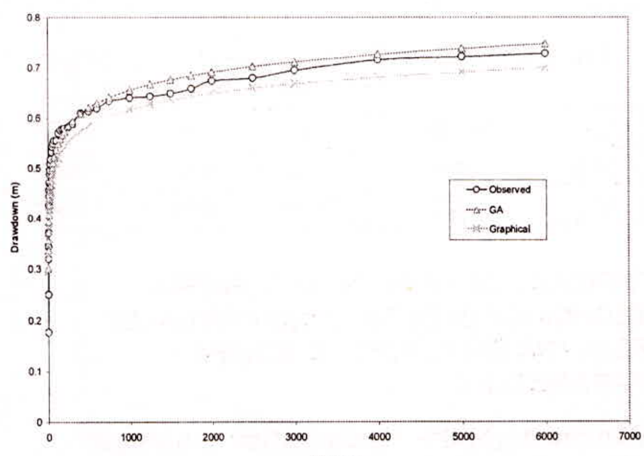


**Fig. 2:** Comparison of computed time-drawdown curves with observed ones for datasets of partially penetrated confined aquifer ($r$ = 30.5 m)

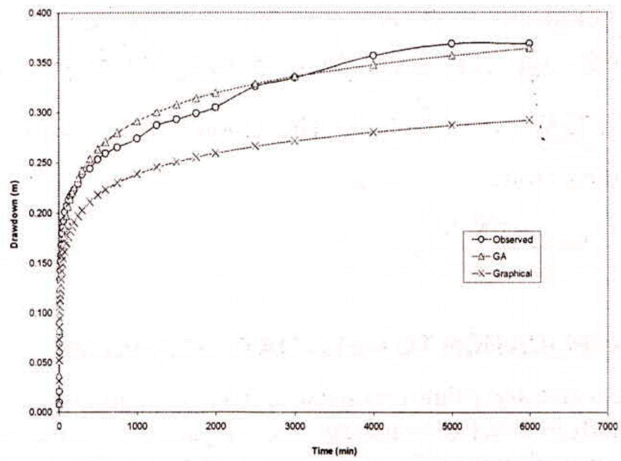


**Fig. 3:** Comparison of computed time-drawdown curves with observed ones for datasets of partially penetrated confined aquifer ($r$ = 91.5 m)

## CONCLUSIONS

1. The GA technique proved to be an efficient and reliable method for estimating the hydraulic parameters of partially penetrated confined aquifer, particularly in the situation when the graphical matching is poor.
2. The GA optimization technique yielded consistent parameter values for the aquifer tested from two different datasets indicating that it's more reliable.
3. Overall, the GA optimization technique is easy, less time consuming, reliable, robust and efficient in estimating various aquifer parameters from the time-drawdown data. With the availability of high speed and large memory PCs these days, the use of GA technique is recommended for determining aquifer parameters from the pumping test data instead of the traditional, cumbersome and subjective graphical method.

## NOTATION

The following symbols are used in this paper:

$r_w$ = well radius
$Q$ = constant rate of pumping ($L^3 T^{-1}$)
$r$ = distance of observation well from pumping well (L)
$s$ = drawdown at observation well (L)
$S$ = storage coefficient of aquifer (dimensionless)
$T$ = transmissivity of aquifer ($L^2 T^{-2}$)
$t$ = time measured since start of pumping (T)
$u$ = $r^2s/4Tt$ = argument of well function (dimensionless)
$W(u)$ = well function (dimensionless)
$s_c[\,]$ = calculated drawdown in observation well
$s_0$ = observed drawdown in observation well

$K_0(x)$ = modified Bessel function of second kind and of zero order

$a$ = penetration depth of piezometer (L)

$b$ = penetration depth of pumped borehole (L)

$d$ = penetration depth of non-screened part of pumped borehole (L)

$f_s$ = correction factor for partial penetration

$H$ = aquifer thickness (L)

K = hydraulic conductivity of the aquifer ($LT^{-1}$)

erf $(x)$ = error function.

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