TWO-DIMENSIONAL MODELLING OF OVERLAND FLOW FOR URBAN SUBCATCHMENT

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ABSTRACT

The world growth of urbanisation over the years is very high. This rapid process of urbanisation has tremendous effect on hydrology of the region. With increase in impervious area due to urbanisation, there has been increase in runoff peak, runoff volume and decrease in time to peak. Design of drainage facilities, which do not account for this increased runoff volume, are inadequate and therefore, it is important to develop an overland flow model for urban catchment in order to estimate the exact runoff and time to peak. In this study, a two-dimensional overland flow model has been developed considering random roughness coefficients and irregular boundaries for the analysis of surface flow component. One-dimensional subsurface flow equation has been used for calculating infiltration from pervious area. The surface and subsurface flow components of the model are linked explicitly at the ground surface through the processes of infiltration. The present model is validated separately for surface flow and subsurface flow using earlier results. It is used to simulate a hypothetical urban subcatchment.

1.0 INTRODUCTION

Urbanisation is defined as the concentration of people in urban settlements and the process of change in land use occupancy resulting from the conversion of rural lands into urban, suburban and industrial communities (Davis, 1965). Urbanisation includes the transformation of a rural set-up to an urban set-up, development of a sub-urban area to an urban area and rural-urban migration. The rate of urban growth is especially high in developing countries like India. In India, urban population increased from approximately 20 % to about 25 % during 1981 to 1991. During this period, urban population has gone up from 160 million in 1981 to 218 million in 1991. By 2001, the urban population is estimated to go up to 330 million and by 2011, it may be about 405 million. The higher demand of water results in higher urban discharge, because of increase in residential and commercial facilities such as building, pavements and parking lots, the built up or impervious areas in the urban area increases. There is an increase in runoff peak and runoff volume, decrease in time to peak, decrease in infiltration and reduction of recharge to ground water. Design of drainage facilities, which do not account for this increased runoff volume, are inadequate and may result in heavy damage and loss of property. Therefore, there is a need for correct estimation of storm runoff and design of suitable drainage system. Thus, it is important to estimate the exact runoff, time to peak and to check the adequacy of the existing drainage facilities.

An extensive literature review on modelling of urban hydrological processes has been carried out for different developed and developing countries by Shukla and Soni (1993). The study shows that in almost all the countries authentic long-term data on discharge through urban drains are not available. Storm Water Management Model (SWMM), developed by the US Environmental Protection Agency, is a package of models linked together and divided into a number of blocks. It is a comprehensive model covering both quantity and quality aspects. However, it is unwieldy to apply in Indian hydrological conditions. Design of urban drainage system in India is based on rational formula, because of lack of adequate continuous records of precipitation and stream flow (UNESCO, 1978). Ramaseshan (1983) has also reported that the urban hydrologic problems of India differ from those of developed countries in several important points such as lateral rather than vertical development, limited amounts of paved area, initial interaction between urban drainage and flood control, preference for open drains over closed ones, limited availability of continuous records of precipitation, stream flow and water quality, limited number of sewer connections and hence shifting of combined sewer, high cost of construction and modification and limited capacity of financial investment. The urban drainage index adopted in urban storm water runoff modelling of Rohini, Delhi was 3.5 cumec/sq. km with 35 mm/hr rainfall intensity with once in two years recurrence interval. Since the rate of urbanisation in Rohini is expected to increase from 55 % to 84 %, the urban drainage index needs to be modified to 5.5 cumec/sq km (Chakraborti, 1989). Yen (1987) has presented the evolution of urban storm drainage technology by a schematic diagram.

Traditionally, overland flow modelling of a catchment is performed using hydrological concepts. However, in this study, an attempt has been made to model the urban catchment using hydraulic approach. The main objective of this study is to develop a numerical model for an urban subcatchment, considering two-dimensional surface flow and one-dimensional subsurface flow for an irregular boundary and variable roughness. Generally, an urban catchment is divided into subcatchments. Runoff from each subcatchment is collected from branch drains to the main drain. The flow in the main drain is routed until the catchment outlet is reached. In this study, runoff from a subcatchment is only considered. This model will form a component of the complete urban catchment model, which is under progress.

2.0 GOVERNING EQUATIONS

2.1 Surface Flow Equations

The two-dimensional depth averaged shallow water flow equations are (Chaudhry 1993):

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \qquad (1)$$

in which, **U**, **E**, **G** and **S** are vectors and are defined as;

$$\mathbf{U}=\begin{Bmatrix} h \\ uh \\ vh \end{Bmatrix},\ \mathbf{E}=\begin{Bmatrix} uh \\ u^2h+\frac{1}{2}gh^2 \\ u\,vh \end{Bmatrix},\ \mathbf{G}=\begin{Bmatrix} vh \\ uvh \\ v^2h+\frac{1}{2}gh^2 \end{Bmatrix},\ \mathbf{S}=\begin{Bmatrix} R-I \\ gh(S_{ox}-S_{fx}) \\ gh(S_{oy}-S_{fy}) \end{Bmatrix} \qquad (2)$$

where, h = flow depth; u, v = depth averaged velocity components along x- and y-directions; R = volumetric rate of rainfall per unit area; I = volumetric rate of infiltration per unit area; g = acceleration due to gravity; S_{ox} and S_{oy} = catchment slopes in x- and y-directions respectively; S_{fx} and S_{fy} = friction slopes in x- and y-directions, respectively.

2.2 Friction Slope

The friction slope is computed using the Darcy-Weisbach equation as given below.

$$S_{fx}=\frac{f_d u\sqrt{u^2+v^2}}{8gh}, \qquad S_{fy}=\frac{f_d v\sqrt{u^2+v^2}}{8gh} \qquad (3)$$

where, f_d = frictional resistance coefficient and is computed using,

$$f_d=\frac{\kappa\nu}{h\sqrt{\imath^2+v^2}} \qquad (4)$$

In Eq. 4, κ = surface roughness, and ν = kinematic viscosity of liquid.

2.3 Subsurface flow equations

One-dimensional, single phase, incompressible, and transient unsaturated flow equation in an isotropic porous medium is (Freeze and Cherry 1979):

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial Z}\left[K(\psi)\left(\frac{\partial\psi}{\partial Z}-1\right)\right] \qquad (5)$$

where, θ = volumetric moisture content; $K(\psi)$ = unsaturated hydraulic conductivity, ψ = suction head. The above parameters ($K(\psi)$ and θ) are estimated by

$$K(\psi)=K_s\frac{A}{A+|\psi|^m}, \qquad \theta=\theta_r+\frac{B(\theta_s-\theta_r)}{B+|\psi|^n} \qquad (6)$$

where, K_S = saturated hydraulic conductivity, θ_S = saturated moisture content, θ_r = residual soil moisture. A, B, m and n are constants of the characteristic curve.

The parameters for the soil moisture characteristic should be specified a-priori for field application of the subsurface flow equation. These values may be obtained from field observations using parameter estimation models. In this study, soil moisture characteristics derived from experimental and field data are employed.

3.0 NUMERICAL METHOD

The governing equations are a set of non-linear partial differential equations for which generalised analytical solutions are not available. Therefore, they are solved using numerical schemes. In the present study, a finite-volume method is used to solve the surface flow equations and an implicit finite-difference method for the subsurface flow equation. The numerical solution procedure is explained through the following sequential modules.

INPUT : Co-ordinates of the study area, no. of grids in x-, y- and z-directions, impervious cells in the x-y plane, Courant number, time of computation, initial surface flow depth and discharge, Bed slopes in x- and y-directions, rainfall intensity, duration of rainfall, kinematic viscosity, initial moisture content for subsurface, and physical parameters of soil are used as input parameters.

INITIAL: The input values, for computational grid, are used to compute individual cell-sizes and the normal unit vectors along all sides. Initial values ($t = 0$) of flow depth and discharge for surface flow, and moisture content for subsurface flow, are prescribed at all the grid points.

STABILITY: Courant condition for two-dimensional case is used to compute the time step (Δt) for numerical stability (Singh, 1998). Time level is updated using this value of time step (i.e. $t = t + \Delta t$).

SURFACE: Governing equations for surface flow (Eq. 1) are solved using a predictor-corrector, explicit and second order finite volume method based on the Essentially Non-Oscillating (ENO) approach (Singh and Bhallamudi, 1997). Suitable boundary conditions (line inflow, corner inflow and no flow) are also implemented.

SUBSURFACE: In order to determine the infiltration rate, I, in the continuity equation for the surface flow, the subsurface flow equation has to be solved along with an appropriate boundary condition at the ground surface. In the present study, a recently developed strongly implicit finite-difference method (Hong et al. 1994, Singh and Bhallamudi 1998) is used to solve the subsurface flow equation. The subsurface flow is assumed to occur only in the vertical direction. In this study, the infiltration rate at any distance x is determined using the one-dimensional subsurface flow equation with the surface flow depth at that point as the top boundary condition. Two types of boundary conditions (Flux type and Pressure-head type) are implemented. Second-order forward finite-difference analog is used to determine the infiltration rate.

INTERACTION: Surface and subsurface flow components are interrelated by a common pressure head and the infiltration at the ground surface. The top boundary condition for the subsurface flow is determined by the surface flow depth. In turn, the infiltration term in the surface flow equation is controlled by the subsurface flow conditions (Singh 1998).

RESET: Computed values of all the variables for the unknown time level are reset for the known time level.

The above computation procedure is repeated till the time of computation is attained and then results are obtained as outputs.

4.0 RESULTS AND DISCUSSION

4.1 Validation of surface flow component

The two-dimensional surface flow model was compared with the results of Zhang and Cundy (1989) for overland flow on a rectangular domain with spatial variation of surface roughness. Zhang and Cundy (1989) conducted the simulation with a rainfall of 15 cm/hr and infiltration rate of 5 cm/hr. Infiltration was continued after the end of rainfall at 3.5 minutes. The rectangular domain was 12 m long and 12 m wide with a 5 % gradient tilting in the x-direction.

Zhang and Cundy (1989) generated the κ values randomly from a log-normal function with a mean of 512 and standard deviation of 154. The values of κ assigned to each of the finite-volume grids on the plane are given in Table-1. The above test case was simulated using the present model with a grid size of $\Delta x = \Delta y = 1.0$ m. Initial flow depth on the plane, h_{ini} is 1.0 E^{-4} m and Courant number, C_n is 0.8.

Table 1: Spatial distribution of surface roughness

j/i	1	2	3	4	5	6	7	8	9	10	11
1	295	288	312	650	698	287	486	710	460	744	538
2	472	591	603	450	276	529	585	405	418	543	332
3	631	404	728	494	616	510	648	752	412	744	264
4	521	385	458	426	599	345	677	775	262	556	647
5	557	723	372	690	634	593	271	466	396	286	358
6	701	256	692	761	458	373	248	531	468	361	498
7	710	272	721	402	688	549	561	502	759	570	755
8	533	313	678	275	457	388	521	284	264	826	497
9	451	414	691	612	732	498	663	298	526	570	366
10	491	653	627	584	727	481	266	748	699	440	298
11	405	639	650	578	357	450	340	663	447	252	545

Figure 1 shows the comparison of numerical results for the outflow hydrograph, obtained using the variable roughness surface. Although the hydrograph peaks simulated using the present model and by Zhang and Cundy (1989) match satisfactorily, the present model does not show any oscillations.

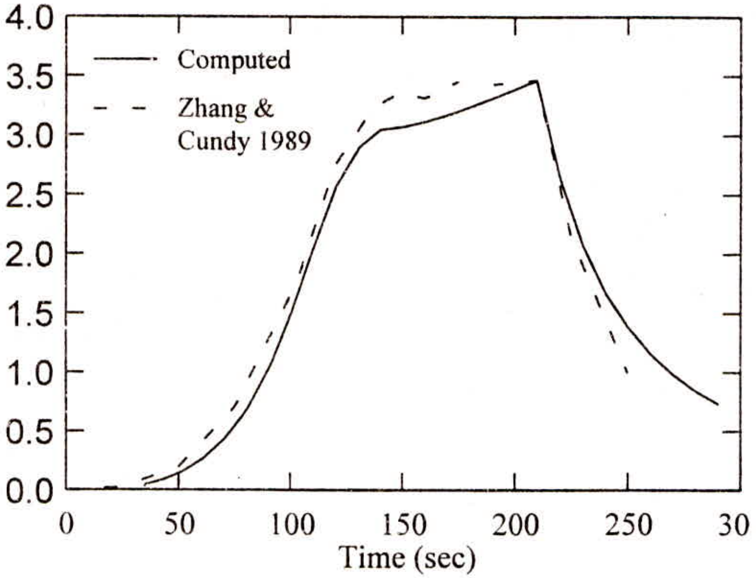

Fig. 1 : Comparison of outflow hydrograph from surface flow model

4.2 Validation of subsurface flow component

In order to validate the subsurface flow component of the present model, a case of one-dimensional infiltration into a uniform sand column, for which an analytical solution by Philips (Hong et al. 1994) is available, has been considered. The input parameters used in the above study are: $K_s = 34$ cm/hr; $A = 1.175\ E^6$; $m = 4.74$; $\theta_s = 0.287$; $\theta_r = 0.075$, $B = 1.611\ E^6$, $n = 3.96$, $\theta\ (z, 0) = 0.1$, $\psi\ (0, t) = -20.73$ cm, $\Delta Z = 1$ cm and $w = 1$.

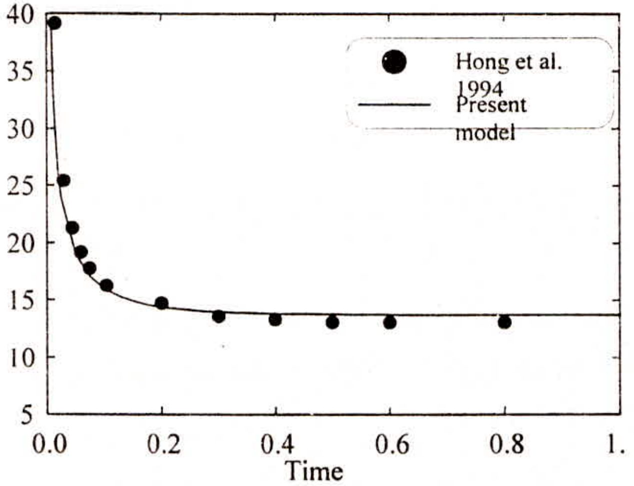

Fig. 2 : Comparison of Infiltration rate with earlier results

The lower boundary condition has been considered as free drainage and upper boundary condition has been considered as the specified pressure head boundary. The infiltration rate obtained by present model and Hong et al. 1994 are presented in Fig. 2. It shows good agreement between the two results.

4.3 Simulation of hypothetical urban subcatchment

Present model is used to demonstrate the applicability of the model for the urban catchment. For this purpose, a hypothetical urban subcatchment has been considered having length = width = 16 m., and, longitudinal slope of 0.05 in x-direction only. The viscosity = $1.94E^{-4}$ m^2/s and the rainfall intensity is considered to be 250 mm/hr over a duration of 10 minutes. The model has been run for duration of 12 minutes. The physical properties, (K_s, θ_s, θ_r, K - ψ and θ - ψ relationship etc.), of soil for hypothetical urban catchment are taken from Singh (1998). The values are; K_s = 566.9 cm/day, θ_s=0.46, θ_r=0.02, constants =2.03 and 8.09. The finite-volume grid spacing in the x-,y- and z-directions are 1.0 m, 1.0 m, and 2.74 cm respectively. Courant number, C_n = 0.65. A very small value of initial flow depth of 0.2 mm is taken to overcome the singularity problem. The numerical parameters for subsurface flow computations are: w = 1.0, and $\varepsilon = 10^{-3}$.

Figure 3 shows the runoff hydrograph at the downstream end for three different cases of imperviousness. It can be seen from Fig. 3 that as the imperviousness increases the runoff increases. The runoff hydrograph for 50 % imperviousness does not show that much increment as the 100 % impervious curve shows. Because, the impervious area has been considered in the centre of the catchment and there is a pervious area after the impervious area, so some water infiltrates into the ground. At the end of the rainfall, there is an increase in runoff hydrograph in the 100 % impervious case, this may be due to the impact of the rainfall on the friction factor. Though not presented, the model can be used for different orientations of the impervious area. In addition, irregular boundary problems can also be simulated using the present model.

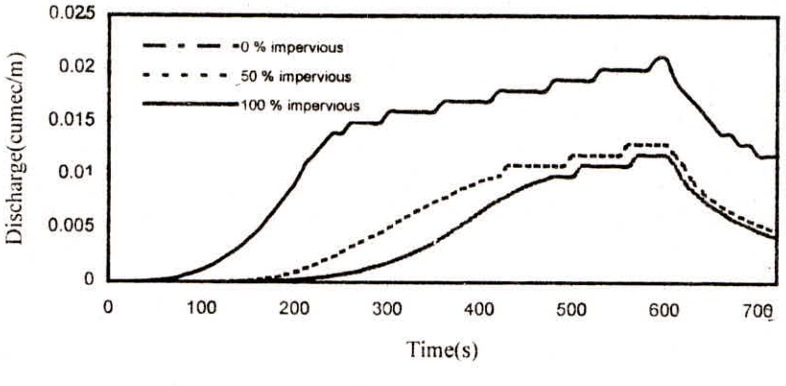

Fig. 3 : Effect of urbanisation on runoff hydrograph

Figure 4 shows the infiltration rate from the pervious surface of the catchment. The computed infiltration rate is based on the physical parameters. It may be noted that infiltration rate curves for 0 % and 50 % impervious cases will be same and there will be no infiltration in case of 100 % impervious case.

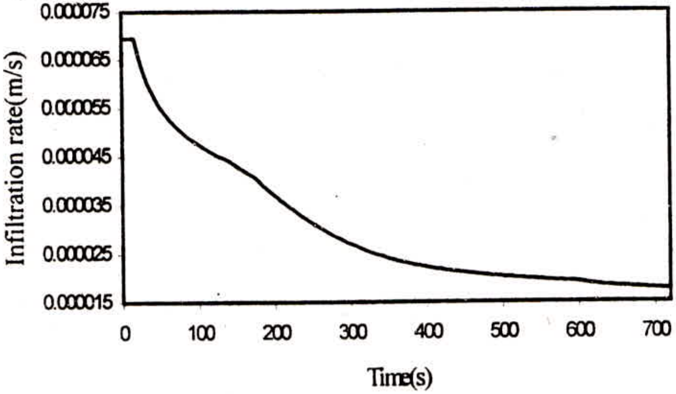

Fig. 4 : Infiltration rate for pervious surface

5.0 CONCLUSION

In this study, a two-dimensional surface flow model was developed along with the one-dimensional subsurface flow model for infiltration, to simulate the overland flow in urban subcatchment. In this model, two-dimensional surface flow equations were solved using a simple, explicit finite-volume scheme. The subsurface flow component in the present model was represented by one-dimensional subsurface equation, which was solved by a recently developed strongly implicit finite-difference scheme. Present model was validated separately for surface and subsurface flow components. It was also used to simulate a hypothetical urban subcatchment to show its applicability. The effect of imperviousness i.e. extent of urbanisation, on the runoff hydrograph was presented. In this model, the uniform infiltration rate was assumed all over the catchment. This model can be used for the exact computation of the surface runoff from the urban catchment considering the soil moisture characteristic of that catchment.

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