DETERMINATION OF HYDRAULIC CONDUCTIVITY USING GEOPHYSICAL PARAMETERS IN A DITCH DRAINAGE SYSTEM

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ABSTRACT

A steady state subsurface drainage problem between two parallel ditches / streams reaching up to the impermeable layer in a two layered horizontally heterogeneous aquifer, receiving constant rainfall/replenishment, was formulated. The analytical solution given by Shiv Kumar and Chauhan (1999) for phreatic surface in a two layered horizontally heterogeneous aquifer receiving constant replenishment, based on the steady state form of Boussinesq's equation, has been utilized to determine the hydraulic conductivity. The experiments were conducted on the Hele-Shaw model which was simulated for three sets of horizontal heterogeneity, three rates of replenishment and two ditch water levels. Dupit's relationship has also been used to develop the flow relationship for the two flow regions and determining the hydraulic conductivity. The theoretically predicted hydraulic conductivities for both the regions were compared with the model hydraulic conductivity. The two procedures of determining hydraulic conductivity from the drain flow data based on the proposed approaches were found to give reasonable results.

1.0 INTRODUCTION

Hydraulic conductivity is the most important aquifer property for predicting water table profile. One of the methods could be, taking core samples and determining hydraulic conductivity in the laboratory by conventional methods. The other procedure could be determining hydraulic conductivity by in-situ method. Both these methods are useful but involve less soil volume in the flow process. A method could be possible in which the flow parameters like water table heights, aquifer geometry and discharge are also involved and the flow process involves larger soil volume.

Hele-Shaw model technique has been used in the simulation of ground water studies by a number of researchers in the past. Dachler (1936) used the Hele-Shaw model for the first time for ground water investigation. Parr (1937) described the Hele-Shaw model similar to fluid flow analyzer to show laminar flow around objects. Gunther (1940) reported on the extensive investigation of model analogy as applied to flow in porous media with different boundary conditions. Santing (1951) discussed its adoptability to study the sea water intrusion problems in coastal aquifers. Todd (1954) used the Hele-Shaw model for the study of unsteady flow in porous media. De Wiest (1962) used the vertical Hele-Shaw model for conducting the studies on free surface flow in homogeneous porous medium. Yen (1972) developed the vertical Hele-Shaw model to study the flow in porous media with a free surface. Mishra (1984) used the Hele-Shaw model for the two layered phreatic aquifer by keeping uniform spacing of 2 mm for upper layer and 1 mm for lower layer. Sewa Ram and Chauhan (1987) conducted experiments on Hele-Shaw model for subsurface drainage of phreatic aquifer receiving time variable replenishment resting over a sloping impervious barrier. Khan *et al.* (1989) used the Hele-Shaw model with non-uniform spacing between the parallel plates. Shiv Kumar and Chauhan (1999) used the Hele-Shaw model to represent a two layered vertically stratified soil by keeping a perticular spacing between the parallel plates in part length, and higher spacing in the remaining length of the model.

Keeping earlier studies in view, the present study was undertaken to evolve a method for determining the hydraulic conductivity which involves the geophysical parameters like water table heights, aquifer geometry and discharge of drains.

2.0 PROBLEM FORMULATION

In the solutions of the problems of subsurface drainage or seepage, the soils are generally assumed to be isotropic. In nature however, soils with heterogeneity, vertical or horizontal, do occur. In the present study, the problem of subsurface drainage / seepage between two parallel ditches or two water bodies reaching up to the impermeable layer in a two layered horizontally heterogeneous aquifer, receiving a constant recharge, was simulated on a vertical Hele-Shaw model. Analytical solutions for steady state water table profiles using Boussinesq equation obtained by Shiv Kumar and Chauhan (1999) under above conditions were utilized to determine the hydraulic conductivity.

Boussinesq's differential equation has been assumed to characterise the flow in the present drainage problem. The flow system is given in Fig. 1 and described as below:

1. Two parallel ditches are spaced at length L reaching up to the impermeable layer.

2. The soil between the two drains are vertically stratified in two layers of length L_1 and $L - L_1$ having hydraulic coductvity K_1 and K_2, respectively, ($K_1 < K_2$), which are homogeneous and isotropic in themselves.

3. Boussinesq equation is valid to describe the phreatic surface in the unconfined aquifer.

4. Rainfall or irrigation is applied at a constant rate P.

5. A constant rainfall replenishment creats a water divide in the flow region located at a distance d from the left ditch. The seepage process is divided in two main flow regions consisting of two different hydraulic coductivities K_1 and K_2.

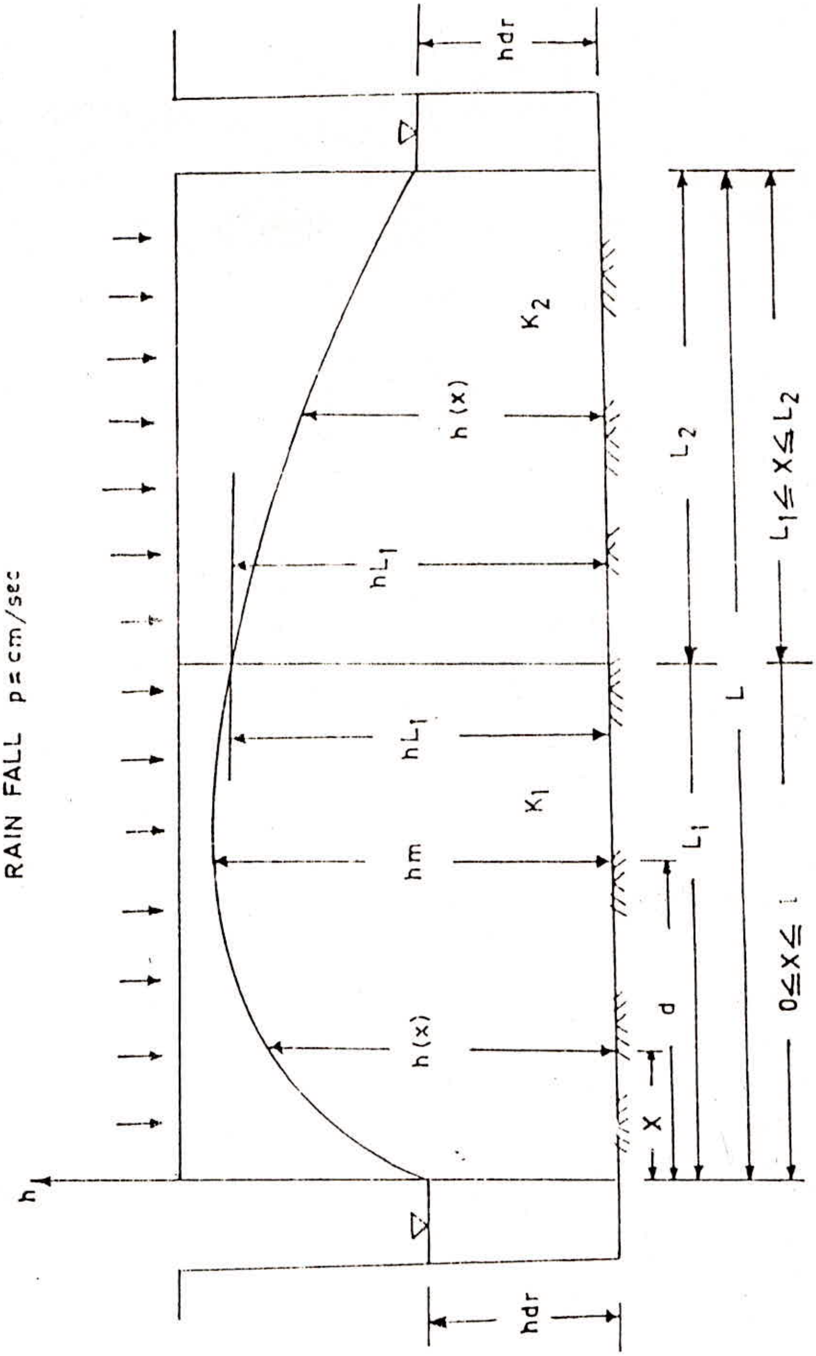

Fig. 1 : Steady State Water Table with Constant Recharge between Two Parallel Drainage Ditches with Vertically Stratified Soil

The first flow region lying to the left of the interface in the layer with hydraulic coductivity K_1 draining to the left ditch and the second lying away from the interface to the right ditch with hydraulic conductivity K_2 which drains to the right ditch. The Boussinesq's equation including the replenishment term P may be written as:

$$\frac{\partial}{\partial X}\left\{Kh\frac{\partial h}{\partial X}\right\} - \alpha\left\{\frac{\partial h}{\partial X}\right\} + P = f\frac{\partial h}{\partial t} \quad \ldots(1)$$

For a steady state flow when drains are placed on the horizontal impermeable layer, equation (1) may be written as

$$\frac{\partial}{\partial X}(Kh\frac{\partial h}{\partial X}) + P = 0 \quad \ldots(2)$$

The phreatic surface using differential equation (2) as obtained by Shiv Kumar and Chauhan (1999) for region I may be reproduced as below :

$$h^2 = \frac{P(d^2 - X^2)}{K_1} + \frac{(h_{L1}{}^2 - h_m{}^2)}{K_1}(X - d) + h_m{}^2 \quad \ldots(3)$$

Equation (3) with boundary coditions $h(0) = h_{dr1}$ and $h(X) = h_1$ may be written as

$$h^2 = -\frac{PX^2}{K_1} + \left[\frac{h_1{}^2 - h_{dr1}{}^2}{X_1} + \frac{PX_1}{K_1}\right]X + h_{dr1}{}^2 \quad \ldots(4)$$

Applying boundary condition $h(X_1) = h_1$ and $h(X_2) = h_2$ in equation (4), the expression for h_1^2 and h_2^2 can be obtained, substracting one from another and simplifying,the expression for hydraulic conductivity for region I may be written as

$$K_1 = \frac{P(X_2 - X_1)(X_1X_2)}{X_1(h_1{}^2 - h_2{}^2) + (X_2 - X_1)(h_1{}^2 - h_{dr1}{}^2)} \quad \ldots(5)$$

The phreatic surface using differential equation (2) as obtained by Shiv Kumar and Chauhan (1999) for region II may be reproduced as below:

$$h^2 = -\frac{P(X - L_1)^2}{K_2} + \left[\frac{h_{dr2}{}^2 - h_{L1}{}^2}{L - L_1} + \frac{P(L - L_1)}{K_2}\right](X - L_1) + h_{L1}{}^2 \quad \ldots(6)$$

Applying boundary conditions $h(X_1) = h_1$ and $h(X_2) = h_2$ in equation (6), the expression for h_1^2 and h_2^2 can be obtained, substracting one from another and simplifying, the expression for hydraulic conductivity for region II may be written as :

$$K_2 = \frac{[(X_2 - L_1)^2 - (X_1 - L_1)^2 + (L - L_1)\ (X_1 - X_2)]}{(h_1^2 - h_2^2) - (h_{dr2}^2 - h_{L1}^2 / L - L_1)(X_1 - X_2)} \qquad ...(7)$$

Determination of hydraulic conductvity by Dupit's formula

$$q = Kh\frac{dh}{dx} \qquad ...(8)$$

Integrating equation (8) and applying boundary conditions $h\ (X_1) = h_1$, $h\ (X_2) = h_2$, the expression for K may be obtained as:

$$K = \frac{2q(X_2 - X_1)}{h_2^2 - h_1^2} \qquad ...(9)$$

We know q = P.L, where $L = X_2 - X_1$, therefore, equation (9) may be written as:

$$K = \frac{2P(X_2 - X_1^2)^2}{h_2^2 - h_1^2} \qquad ...(10)$$

3.0 EXPERIMENTAL INVESTIGATION

The experiments were conducted on a closely spaced parallel plate viscous flow model commonly known as the vertical Hele-Shaw model. The model representing vertically stratified soil was designed and fabricated in the workshop of the College of Technology, G.B. Pant University of Agri. and Tech., Pantnagar, U.P. and assembled in " Flow Through Porous Media " laboratory of the Department of Irrigation and Drainage Engineering. The hydraulic conductvity of the model is given by the equation as :

$$K_m = \frac{b^2 g}{12v}$$

To represent a two layered vertically stratified soil, a particular spacing between the parallel plates in part length and larger spacing in the remaining length of the model was maintained. The spacing was varied by affixing one thin plexi glass sheet with one of the parallel plates on the portion of the length where spacing was to be reduced and then assembling the model with hydraulic conductvity K_1 in some length and K_2 ($K_1 < K_2$) in the remaining length. The following three cases of vertical stratification between the drains were considered.

Case I	$K_1 < K_2$,	$3L_1$	=	L_2	=	3L/4,	Heterogeneity A
Case II	$K_1 < K_2$,	L_1	=	L_2	=	L/2	Heterogeneity B
Case III	$K_1 < K_2$,	L_1	=	$3L_2$	=	3L/4	Heterogeneity C

The model was first set up for case I, $K_1 < K_2$, $3L_1 = L_2 = 3L/4$ and the reservoirs having drains were fixed at both the ends. The oil supply from the over head tank was first regulated to the recharge manifold and then to main body through drain cock nozzles. The Gear oil HP 140 of Indian Oil Corporation was used as the fluid. The experiments were conducted at an average temperature of 29.5^{o}C and for three different replenishment rates for each level of water in the drains. The heights of the drains were kept zero and h/3 for each case. The zero level of the drains corresponded to 2.5 cm from the bottom of the model and the h/3 level of drain corresponded to 18.5 cm from the bottom of the model. For the purpose of observation, the heights of the steady state phreatic surface of the fluid at different points between drains were measured at 2 cm, L/8, 2L/8, 3L/8, 4L/8, 5L/8, 6L/8, 7L/8 and L, respectively, along the length of the model.

4.0 RESULTS AND DISCUSSION

The theoretically predicted hydraulic conductivities, K_1 for region - I and K_2 for region - II, using equations (5) and (7) are given in Table 1. It may be observed that computed values of hydraulic conductivity K_1 in region - I was found to have values in the range of 0.07432 to 0.08160 cm/s which may be considered reasonably close to the model hydraulic conductivity in the region-I, K_{ml} = 0.08166 cm/s.

The theoretically computed hydraulic conductivity from equation (7) in region - II was found to be in the range of 0.30019 to 0.36087 cm/s. The predicted values of hydraulic conductivity for heterogeneity A to C were found to be fairly close to the model hydraulic conductivity K_{m2} = 0.32664 cm/s in region - II. For heterogeneity B, particularly for equal finite drain water level, the predicted hydraulic conductivity was found to be higher than the model hydraulic conductivity K_{m2} = 0.32664 cm/s.

4.1 Dupit's Approach

The predicted hydraulic conductivity K_1 and K_2 for region- I and II, respectively, using equation (10) are given in Table 2. It may be observed that the hydraulic conductivity predicted from equation (10) for region - I is found to have values in the range of 0.07864 to 0.11829 cm/s. The values predicted from equation (10) for heterogeneity B and C were found to be in the range of 0.07864 to 0.08591 cm/s, which may be considered to be reasonably close to the model hydraulic conductivity K_{ml} = 0.08166 cm/s in region - I.

Table 1 : Computed hydraulic Conductivity K_1 for region I, using equation (5) and K_2 for region II using equation (7). Model K_1 = 0.08166, K_2 = 0.32664.

Heterogeneity	Replenishment (cm/s)	Hydrraulic conductivity for region-I (cm/s)	Hydraulic conductivity for region- II (cm/s)
Heterogeneity A			
Equal finite drain water level	0.0118206	0.07998	0.31129
	0.0271700	0.07876	0.32548
	0.0425000	0.07909	0.30019
Equal zero drain water level	0.0118206	0.08065	0.29979
	0.0271700	0.07569	0.32464
	0.0425000	0.07623	0.31579
Heterogeneity B			
Equal finite drain water level	0.0118206	0.07678	0.35564
	0.0271700	0.07769	0.35679
	0.0425000	0.08160	0.36087
Equal zero drain water level	0.0118206	0.08023	0.33786
	0.0271700	0.07965	0.33809
	0.0425000	0.08032	0.32917
Heterogeneity C			
Equal finite drain water level	0.0118206	0.07989	0.31995
	0.0271700	0.08116	0.32659
	0.0425000	0.07899	0.31918
Equal zero drain water level	0.0118206	0.07916	0.32079
	0.0271700	0.07998	0.31658
	0.0425000	0.07932	0.30928

The predicted hydraulic conductivity for region- II was found to be in the range of 0.28567 to 0.36215 cm/s. The values of predicted hydraulic conductivity for heterogeneity B is found in the range of 0.31987 to 0.33048 cm/s, which may be considered close to the hydraulic conductivity of the model K_{m2} = 0.32664 in the region-II. Some of the factors that may be the cause of variation between the predicted and the model hydraulic conductivity for different heterogeneities for region I and II in both the approaches may be the following:

1. It is practically difficult to mechanically maintain uniform gap between two parallel plates by means of washers of equal thickness at different grids.

2. There is capillary rise because of viscous liquid flowing through the plateswith small gap and also a minor temperature variation.

Table 2 : Computed hydraulic Conductivity K_1 for region I and K_2 for region II using equation (10). Model K_1 = 0.08166, K_2 = 0.32664.

Heterogeneity	Replenishment (cm/s)	Hydrraulic conductivity for region-I (cm/s)	Hydraulic conductivity for region- II (cm/s)
Heterogeneity A			
Equal finite drain water level	0.0118206	0.09187	0.30074
	0.0271700	0.11829	0.29594
	0.0425000	0.09849	0.30167
Equal zero drain water level	0.0118206	0.09969	0.28567
	0.0271700	0.10674	0.31026
	0.0425000	0.10928	0.29842
Heterogeneity B			
Equal finite drain water level	0.0118206	0.08061	0.32654
	0.0271700	0.08591	0.31979
	0.0425000	0.07927	0.32089
Equal zero drain water level	0.0118206	0.07896	0.31987
	0.0271700	0.07967	0.33048
	0.0425000	0.08112	0.32764
Heterogeneity C			
Equal finite drain water level	0.0118206	0.07864	0.34567
	0.0271700	0.08156	0.35187
	0.0425000	0.08039	0.34012
Equal zero drain water level	0.0118206	0.08149	0.36215
	0.0271700	0.08210	0.35918
	0.0425000	0.07992	0.34179

5.0 CONCLUSION

The two procedures of determining hydraulic conductivity from the drain flow data based on the solutions given by Shiv Kumar and Chauhan (1999) and the Dupuit's formula were found to give reasonable results using experimental observations. Therefore, a method for finding out hydraulic conductivity could be possible in which geophysical parameters like water table heights, aquifer geometry and discharge are also involved and the flow process involves larger soil volume.

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