TN-13

REGIONAL AQUIFER SIMULATION

SATISH CHANDRA

DIRECTOR

STUDY GROUP

P V SEETHAPATHI

NATIONAL INSTITUTE OF HYDROLOGY
JAL VIGYAN BHAVAN
ROORKEE-247667(UP)INDIA

1984-85

CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Over the past two decades ground water has become topic of discussion and investigation and as a result the knowledge and understanding of ground water conditions have radically improved. The ground water in a basin is in a state of continuous movement. Its volume decreases by processes like discharge into natural streams or springs or evaporation or abstraction from wells, etc., causing the water table to go down. At the same time, its volume increases by recharge from rainfall or from surface water bodies causing the water table to rise. When considered over a long period of time, the water table will be nearly stationary indicating a state of hydrological equilibrium, the average recharge equalling the average discharge from the system. For a short duration observations, seasonal fluctuations around the average water table conditions may be noticed and at times these fluctuations may be significant and important.

The flow of groundwater is governed by certain laws. Solutions for the groundwater flow problems are obtained by solving the differential equation that can describe hydrological relationship within an aquifer. However, it is pre-requisite to have knowledge about geometry of the study area, hydraulic characteristics, initial and boundary conditions. It is attempted in this report to identify these various parameters that are required to be specified.

Mathematical models of ground water systems are primarily intended as means of investigating and predicting flow processes within an aquifer. A review on the models which have been developed to solve the groundwater problems is presented. Data which are obtained from various sources form the basis for building a model. Other information

required for a model, boundary conditions, the recharge abstraction components, the historical data are discussed. Methods for estimating the aquifer parameters using inverse methods are indicated. Some of the commonly used solution techniques are also given. The formulation of a mathematical model and using a specific numerical technique for solving such a mathematical model is also presented. A specific case study using one of the finite difference schemes, conducted for one of the doabs in India is also described.

1.0 INTRODUCTION

Groundwater comprises of 98% of the total available fresh water resources in the entire world. The phenomenal rise in the demands for water for different purposes in the recent past, has made it imperative to exploit the ground water resources to the fullest extent possible. Though, it is necessary to exploit the groundwater for sustaining the present growth of civilisation and industrialisation, it is also imperative that utmost economy must be exercised while using these resources. The formulation of a project for the optimal management of this resource demands a knowledge of the relationship between the pumpages to the drawdowns in the aquifers in spatial and temporal coordinates. Such relationships are available from the several analytical solutions, the notable among those being by Theis and Hantush. However, these solutions have been derived under highly idealised conditions and at point locations in space. Thus, there exists a need to refine the analytical solution by incorporating actual field conditions to the extent possible.

During the past two decades, it has been fully realised that quantitative assessment of the water resources is needed before using them. Computation for the available resources has brought about an awareness about its management. Before the ground water resources can be managed, they must be quantitatively assessed. Proper planning and management of this important resource requires the testing of all possible segments and apprising of the relative merits of various alternatives. It is concerned with the sustaining yields of aquifers, interrelation between well and well fields, the interrelationship between surface and sub-surface water and the quality of water. Thus, the need of the day is proper and realistic assessment of the water resources especially the groundwater resources.

The hydro-geologic properties, dimension of aquifers and existing aquitards, and the resourcesfulness of aquifers are of utmost importance in relating cause and effect. This relationship cannot be established until proper hydro-geologic maps are available. A water pumping stratum is called an aquifer that may be either confined, semi confined or unconfined. A confined aquifer is a permeable bed confined in between two impermeable layers. Semi-confined aquifer also termed as leaky aquifer is the one which is compounded above and/or below by a semi-pervious layer having low permeability. This semi-pervious layer is generally known as aquitard. An unconfined aquifer is permeable bed which is exposed to the atmospheric pressure on the top and may be bounded by an impervious layer at the bottom.

There have been a series of developments between 1856 and until now which lead to the groundwater resource evaluation as a quantitative science. Darcy(1856) did experimental works on the flow of water in sand and developed a formula known as Darcy's law which serves as basis for numerous quantitative methods in the field of groundwater flow. In 1935 Theis, first of all analysed and solved the case of unsteady flow in respect of flow towards wells in confined aquifers. The hydraulics of flow, that has been developed until now, could lead to the estimation of the aquifer properties. However, the properties determined by such methods, like pump tests, give point values for the aquifer properties and do not reflect the behaviour of the aquifer system as a whole.

It is commonly known that the groundwater in a basin is not at rest. Rather it is in a state of continuous movement. Its volume increases by the increase in percolation of surface water, causing the water table to rise. At the same time its volume decreases by the natural phenomena, like evaporation, evapotranspairation or by means of the man-made causes, like extraction through wells, thus causing the water table to fall. When considered for a long period of time, the recharge and the discharge components may counter-balance each other keeping the water table by and large stationary. If man interferes in the hydraulics of the natural phenomenon, he may create undersirable side-effects. For example, if the abstraction from well is more than the recharge from rainfall and/or from surface water, the discharge component causes the water table to gradually decrease. On the other hand, if the abstraction from the ground water is reduced and in turn surface water irrigation is created abundantly, the recharge components would increase compared to the discharge components and cause the water table to rise. This may create the problem of water logging.

It can thus be seen that in spite of the greater understanding available in the literature over the groundwater movement, the greater details concerning estimation of the water table rise or fall in relation to the recharge and abstraction components over a basis are yet to be studied and analysed. Unfortunately, there are many groundwater flow problems for which the analaytical solutions are difficult to obtain, since these problems are complex in nature and possess non-linear features. These features, such as, variations in an aquifer's hydrogeologic conductivity, boundary conditions that change with time, can not be considered by analytical means and hence, can not determine the long-term time depending effects. Though, certain times the analytical solutions are applied to such problems by oversimplifying the complex hydro-geologic situations, the solutions thus obtained are untrue. Thus, it is obvious that such results will be inaccurate or at times may be even totally erroneous.

Owing to the difficulties of obtaining analytical solutions to complex groundwater flow problems, there has been a need to find alternate procedures which may lead to meaningful solutions. With the advent of fast computers, such techniques exist now. They are in the the form of numerical modelling which can be used for understanding and simulating the aquifer behaviour under different stress conditions. Though, the technique of solving groundwater flow problems by numerical methods is not new, it has gained momentum over the past two decades.

Subsequent sections deal with the formulation of governing differential equation for transient groundwater movement and some of the various types of models that are developed, giving emphasis to numerical models. The data requirement, for conducting a model study is discussed. Estimation of aquifer parameters from the field data and from parameter estimation methods is also discussed briefly. The commonly adopted methods for solving the system of equations derived from the use of digital models in subsurface hydrology are presented in the section dealing with solution techniques. Finally, ground water model studies that were conducted at the institute are also presented.

2.0 REVIEW

2.1 The Natural System Behaviour

Although groundwater is traditionally defined as that body of water that exists in the saturated zone below the ground surface, its phase of the hydrologic cycle does not operate in isolation of all other components. The occurrence and movement of groundwater should be considered both on regional scale and local scale. When it is viewed on regional scale, the interest is not concentrated at any one point of the aquifer but overall water balance of a large aquifer with known physical boundaries and is aimed at for large scale planning of the groundwater resource. In the latter case region of interest is limited, say, in the vicinity of a well of well field. The groundwater moves from the place of recharge to the place of discharge. Under the natural conditions and over a long period of time, the groundwater system is in a state of dynamic equilibrium condition wherein recharge is equalling the discharge keeping water table levels stationary. However, with the man made interferences, the ground water levels are no more in a state of equilibrium. At places where there is an abundant irrigation and less of pumping ,water tables are on rising trend tending the land to become waterlogged. In regions where there is excessive pumping, the reverse is the trend causing alarming problems like salt water intrusion near the sea coasts, land subsidence etc.

Recharge to the aquifer may result basically from the infiltration of rainfall and by seepage from streams or other surface water bodies. At times, recharge to an aquifer may take place through the lateral or vertical movement of another groundwater body. The natural mode of discharge of an aquifer is to rivers, springs and lakes, and evaporation to atmosphere. Besides these,with the increase in demand for the additional supplies of fresh water for sustaining the developmental needs of the society, heavy pumpages are made from the groundwater system. Depending upon the location of the well, it may draw off water from the storage or intercept some natural discharge.

The movement of water in the soil is governed by two forces, namely, the driving force mainly due to hydraulic gradient and the opposing force due to friction between moving water and soil particles. While determination of the former is rather easy, it is difficult to determine the latter forcé, since the soil matrix is generally heterogeneous and anisotropic in nature, making its exact structure impossible to define. Model complexity is usually proportional to the extent to which actual conditions are taken into consideration. In planning for most uses of groundwater, not only the amount of water available but also its quality is of great importance. A ground water model, therefore, should after processing certain given information, be able to give to the planner the quantity and quality of water that is available at the required point in space and time. However, the quantity and quality are not always modelled concurrently in most of the present day models. The majority of models are deterministic in nature and only recently have some statistical stochastic models appeared in the literature.

2.2 Groundwater Flow

The Darcy law states that the velocity of groundwater flow is proportional to the hydraulic gradient, subjected to the condition that the flow is laminar. Skichter(1899) showed the validity of Darcy law to flow in any direction. He later applied the continuity equation with the assumption that no external stresses are acting and derived an equation describing the movement of ground water under steady state conditions. The equation resembled the famous Laplace equation which facilitated the solution of many steady state groundwater problems on the direct analogy with the corresponding heatflow problems. The unsteady state problems(transient state problems) in any easstic antosian aquifer was first analysed by Theis(1935) when he derived an equation giving the relationship between the lowering of head and discharge of a well using groundwater storage. The concepts of coefficient of storage and transmissivity were introduced by him(1938). Later in the three dimensional unsteady state problems were analysed by Jacob(1950), De Weist (1966), Cooper (1966) which lead to the governing differential equation in cartesian coordinate system for groundwater flow, as·

$$\frac{\partial^2 h}{\partial x^2} \qquad \frac{\partial^2 h}{\partial y^2} \qquad \frac{\partial^2 h}{\partial z^2} \qquad \frac{S_s}{K} \qquad \frac{\partial h}{\partial t} \quad \ldots(1)$$

where S_s = $\rho\, g(\alpha + \theta\beta$) and is termed as Specific Storage, L^{-1}

K = hydraulic conductivity, LT^{-1}

h = head above datum, L

α = compressibility of the aquifer medium, LT^2M^{-1}

β = compressibility of water, LT^2M^{-1}

θ = porosity of the medium(dimensionless)

x,y,z = Space coordinates, L

g = acceleration due to gravity, LT^{-2}

ρ = Sp.wt. of water, ML^{-3}

Multiplying S_s and K by saturated thickness of aquifer,b the equation (1) and using the Laplacian operator, it can be rewritten as

$$\nabla^2 h = \frac{S}{T} \frac{\partial h}{\partial t} \qquad \ldots(3)$$

where S = storage coefficient (dimensionless)

T = Transmissivity, $L^2 T^{-1}$

S,T are defined as the aquifer parameters. Though, equation(3) is derived for flow through confined aquifers of homogeneous and isotropic nature, the same can be used for flow through unconfined aquifers also, provided that the vertical component of flow is negligible and that the saturated thickness of the aquifer is large enough compared to the drawdown(Davis and Dewiest,1966).

In the case of unconfined flow, the storage coefficient, S is to be replaced by the specific yield term, S_y. Considering that there is an external stress on the aquifer system, the equation(3) for the unconfined flow through non-homogeneous and anistropic medium would be

$$\frac{\partial}{\partial x}\left(T_{xx}\cdot\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\cdot\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(T_{zz}\frac{\partial h}{\partial z}\right) = S_y\cdot\frac{\partial h}{\partial t} + \qquad \ldots.(4)$$

Where Q is the external stress on the system.

2.3 Analytical Solutions

Complex boundary conditions, non-homogeneous and anisotropic nature of the system and wide variations in space and time of the external recharge/discharge (Q) term(as is the case of a real life system) make the analytical solution of the equation 4 practically impossible. Even for the cases of steady state flow, analytical solution is possible only if the geometry of the boundaries and conditions imposed along these boundaries are simple. In general, these restrictions are not compatible with the majority of ground water problems that are encountered in real life situations and as such analytical solutions rather become impracticable. However, there are number of analytical solutions available in the literature for many different and defined boundary conditions which are of great help in understanding the behaviour, though, they can not be used for the management of large basins.

2.4 Simulation Studies by Models

A model is termed as a system which can by and large duplicate the response of a groundwater reservoir. The state of art on modelling techniques for the evaluation of groundwater potential was presented by Prickett(1975). He also gave exhaustive references of almost all the widely used groundwater models. Various aspects of groundwater studies like, groundwater resource evaluation, groundwater quality, groundwater development and its management was published by Journal of Groundwater (1963 to 1976). An overview of the literature reveals that the models can be grouped under the following heads:

a) Physical models

b) Analytical models

c) Analogue models

d) Digital models

2.4.1 Physical models

These models are the first ones used in the field of subsurface flow. The prototype aquifer system is scaled down and physically represented. The boundaries and aother features are also truly represented. Usually these models are prepared in circular containers and the sides of containers are made of transparent material. Generally sand, crushed stone, graded river sand or glass spheres are used as porous media. The container is packed with either of the abovesaid materials under water to achieve a homogeneous and isotropic medium. Todd(1959), Lahr(1963),Prickett(1975) give a detailed account of these models.

Though the prototype aquifer system is physically represented by these models, there are many limitations due to the following:

a) Scale reduction generally poses a problem

b) Capillary action, entrapped air and organic growth create trouble.

c) Overall modelling of large and complex basins is difficult.

d) Changes for different aspects of study cause inconvenience.

e) Good laboratory facilities are needed.

2.4.2 Analytical models

It is possible to evaluate the response of wells with analytical methods by devising approximate methods of analysis based on idealised aquifer situations. The case histories where the analytical models are used suggest that the predicted behaviour of the aquifer system may not deviate substantially from the observed one, provided that the idealisation does not drastically alter the physical system. However, the analytical models can not describe in detail hydrogeologic systems having complex geometry and great variations in the hydraulic properties. In such cases, it is necessary to go in for other types of models.

The well field problems are solved by image well theory, method of superposition and the appropriate theoretical ground water equation. Prickett(1975) gives a summary of some selected applications of analytical models. However, with the present day development in the field of numerical modelling, the analytical modelling for solving the problems on a regional basis have become outdated.

2.4.3 Analogue models

Analogue models can be either viscous flow models or electrical analogue models. In the former case, the analogy between flow of groundwater and that of viscous flow between two closely placed parallel plates is used. These parallel plates can either be horizontal or vertical. The comprehensive articles describing these models are presented by Santing(1958) and Collins et.al(1972). Generally viscous flow models can simulate two dimensional flow problems. Cecer and Omay(1973) indicated the use of single plate viscous flow model to simulate three dimensional flow problems as well.

Electrical analogue models can be divided into two categories: a) continuous models b) discrete models. While in the continuous models, an electric conducting medium, like a resistance paper is used, in the latter case an assemblage of discrete electrical components are used. These are also termed as network models. They consist of regular arrays of resistors and capacitors. In the resistor-capacitor network, resistors are approximately considered as inversely proportional to the hydraulic conductivity of the aquifer, and the capacitors store electrostatic energy in a manner analogous to the storage of water within an aquifer. The electrical network resembles closely to an aquifer and can be considered as scaled down version of it. The network model is connected to an exitation-response apparatus consisting of power supply, wave form generator, pulse generator and oscilloscope. This apparatus forces electrical energy in the proper time phase into the model and measures energy levels within the energy-dissipative resistor-capacity network. Oscilloscope traces are analogous to time drawdown graphs that would result after a stop function type change in discharge or head. A catalogue of time voltage graphs provide data for construction of a series of water-level-change maps.

These models can deal with problems of much greater complexity than is practicable with analytical models. The accuracy, flexibility and speed give added advantage to analyse rapidly almost any conveivable aquifer system. Transient and complex conditions can easily be handled. The recent publications on the usage of these models are by Meyer etal Jong enson(1975)Walton(1970) gives details of these models and also mentions certain case studies. A digital computer working in conjunction with network model is used for instantaneous solution of groundwater flow problems. In spite of the fact that these models can truly represent hydrogeologic nature of an aquifer system, they became outdated with the advent of fast digital computers. The summary on the analogue models with their advantages and disadvantages is given by Prickett(1975).

2.4.4 Digital models

The digital models gained momentum for solving groundwater flow problems since early 1970s. These models offer several advantages over the other types of models, because of which they gained popularity. The convenience with which they can be modified for solving varied problems and the accuracy of the solutions are some of the advantages. There are several numerical techniques available in literature, (Remson et.al 1971 ; Pinder and Gray, 1977; Brebbia,1978), the commonest being finite difference and finite element methods. The models that are widely adopted for solving the groundwater problems, at present times, use either of these techniques. (Bredhoeft and Pinder,1970 ; Pinder and Frind,1972 ; Trescott 1973 and 1975). Wide variety of aquifer conditions were considered in formulating the digital models like two and three dimensional flow in non-homogeneous and anisotropic media under confined and unconfined conditions. Besides the above, for other problems, like involving evapotranspiration(Prickett and Linquist,1971) coupled saturated-unsaturated flow (Freeze,1971), combined watershed-aquifer-stream system(Groon et al.,1973), mass transport(Kanikov and Bredehoeft,1978), induced infiltration from rivers and channels(Chaturvedi and Srivastava 1979), coupled flow and heat-transport(Mercer and Pinder,1975;Voss, 1984) and combined evapotranspiration, stream-aquifer interaction(Mc Donald,1984),these techniques are used .

a) Finite difference method

The governing differential equation is replaced by difference equation, in the finite difference scheme. The region is discretized into rectangular grids or polygons with each grid/polygon centering about a node with the basic assumption that over the area represented by a grid, the recharge, abstraction, hydrogeologic properties remain unchanged and are represented at its nodal point. This assumption facilitates the discretization of aquifer parameters and other variables which are really continuous. Thus, the governing differential equation involving the head values at the nodal points is solved by applying suitable boundary conditions and solving the set of equations. There are several efficient methods in the literature for solving the system of simultaneous equations(Todd,1962; Ralston & Wilf,1965; Smith,1965; Richtmeyer and Morton,1967; Remson, et.al.,1971). Of those Alternating Direct Implicit Scheme, Strongly Implicit procedure, and slice Successive Over relaxation scheme have been widely used. The Gauss elimination scheme, though rather simple, requires larger computer memory(Eshett and Logenbough,1965). Gauss-Seidel iteration technique is also used widely. Generally, a relaxation factor is used for faster convergence. Trescott and Larson(1977) compared the relative efficiencies amongst the fair popularly used methods and indicated that strongly Implicit procedure is best for solving linear simultaneous equations with adequate rate of convergence. Others who worked in this area include Ruston(1974) Stone(1968), Fiering(1964).

b) Finite element method

The application of finite element method to subsurface problems followed distinctly two paths, a) Galerkin approach b) variational approach. The Galerkin finite element approaoch is generally used in petroleum industry, the first of its kind being introduced by Price et.al(1968). Although, the objective was to overcome problems generally associated with finite different solutions to the convection-dominated transport equation, it has greater potential appeal. Subsequently, this method was applied to a two phase flow water flooding problems by Douglas et al(1969). McMichael and Thomas (1973) extended this concept to two dimensional two phase flow. Because of their computational inefficiency, this approach is non- competitive with finite difference algorithms, though solutions are highly accurate.

In groundwater hydrology, finite element theory has been employed using triangular and isoparametric elements. Mostly, first degree Lagrange polynomials as basis functions are used in ground water simulations. The classic papers by Javandel and Witherspoon(1968) and Zienkiewiez et.al(1966) appear to be the first two publications describing the use of triangular finite element theory for flow through porous media. Shortly thereafter Neuman and Wither spoon(1970,71) and Taylor and Brown(1967) demonstrated the application of this methodology to the analysis of freesurface Darcy flow. Seethapathi(1979), Huyakorn(1973) and Volker extended to deal with non Darcy flow. Finite element method is particularly suited to free surface flow problems becuase of the simplicity of deforming the mesh and updating the element matrix coefficients to accommodate the changing geometry of the solution domain and changing element properties due to nonlinear material behaviour. This is more evident in the case of land subsidence problems. The unsaturated flow problem that exhibits a dynamic air phase does not appear to have been solved by finite element approach.

The finite element approach consists essentially of the following steps:

a) Discretization of the region and defining the nodal points and elements.

b) Defining the flow matrix and flow load vector corresponding to a single element derived on the basis of appropriate choice of element type and basis function.

c) Assembly of element matrices and load vectors to obtain global matrix.

d) Solution of the system of equations for nodal values.

Pricket(1975) presented selected application of finite element method in groundwater upto 1973. Prasad (1981) presented a list of later works. In spite of the fact that finite element approaoch is more powerful than finite difference methods, the application models for real life situations still lacks momentum.

c) The boundary element method

The boundary element approach is a relative newcomer. One of the early papers in this area was written by Jaswon and Ponter(1963) who applied this method to solid mechanics problem. In groundwater hydrology, this method was introduced by Liggett(1977) for solving the problems of free surface flow. Though this method is advantageous for solving elliptic equations, it loses much of its appeal for parabolic problems or problems involving variable coefficients. Unlike in the case of finite difference, finite element methods , the matrix generated in the boundary element method is full, because of which it loses the computational efficiency. Brebbia and Walker(1980) presents a comprehensive discussion about this method.

d) The collocation method

Although collocation method has been used since the 1930s, for the solution of engineering problems(Finlyson,1972), it is only during the last decade that the technique has been employed over subspaces in a manner analogus to finite element method. This method has been applied to groundwater flow problems by Pinder et.al(1978). The detailed theory is given by Frind and Pinder(1979). Later, it was extended to solve transport equation by Celia and Pinder(1980). The relative merits of the collocation method as compared to other methods for solving subsurface flow problems are yet to be established. It appears that real life simulation problems are yet to be attempted by using either Boundary element method or collocation method.

3.0 PROBLEM DEFINITION

Whenever a groundwater model study is to be conducted for any basin, it is highly essential to know the purpose for which the study is to be conducted. The scope of the study will define the problem formulation and the extent to which the solution is to be accurate. As it can be seen from the review, there are number of models which have been developed and used for real life situations. However, each model has its own limitations and advantages. A particular model can be less accurate but it may be simple in its formulation and faster in convergence. In order to choose a model, it is necessary to understand the limitations, the assumptions made in its formulation. Also, it is essential to know the requirements of data for the use of a specific model. Most of the times, sufficient data with regard to the hydrologic characteristics like aquifer parameters and topographical characteristics like boundary conditions to be incorporated, may not be available. Even if such data are available, at times, the data may not be accurate enough. Thus, the analysis of data plays an important part. Sometimes, the data may be available at few discrete points only which may have to be regionalised. Rational procedures for such analysis and refinement of data have to be adopted. These situations make the ground water modeller to have an indepth knowledge of the available techniques that are available in literature with respect to the types of models, the methods for the estimation of the values for the aquifer parameters and the various techniques of solution procedures.

To provide the necessary background for simulating an aquifer on a regional scale, the various methods available in literature for parameter estimation and the different criterion functions and their implications are discussed.

The data requirement, in general for a model development is also indicated. The different solution techniques for solving the system of equations and their capabilities are discussed. In the end, a case study conducted by the Institute for an alluvial basin is presented to demonstrate the process which has to be followed for simulating an aquifer system on a regional basis.

4.0 DATA REQUIREMENT AND COLLECTION

Data collection forms the initial phase of any model study. All geological and hydrological data are required to be collected before initiating a groundwater problem. This will include information on surface and subsurface geology, water tables, precipitation, evapotranspiration, stream flows, land use, vegetative cover on the surface, extraction from wells, aquifer boundaries, irrigation, aquifer characteristics,etc. In case a little or no data are available, the first effort should be directed towards field work for collection of data. This is very much essential to develop a conceptual model on a rational basis with its various inflow and outflow components. A conceptual model is based on a number of assumptions that must be verified in a later phase of the study. Developing and testing a model requires a set of quantitative hydrogeological data which can be broadly grouped into three classifications, namely:

1. Physical characteristics
2. The exitations on the system,
3. Other relevant information.

The different requirement of data under these classifications are given in table 1.

It is customary to present the results of hydrogeological investigations in the form of maps, geological sections and tables.

4.1 Physical Characteristics

4.1.1 Topographical map

An accurate topographical map of the groundwater basin to be

Table 1 - Data requirements for groundwater model study

Physical Characteristics	Exitation on the System	Other information
a) Topographical map showing surface water bodies,other features.	a) Water table map	a) Legal and administrative rules
b) Hydrogeological map showing areal extent of boundaries, boundary conditions and types of aquifers.	b) Type and extent of recharge and discharge areas	b) Environmental factors.
c) Lithological variations within aquifer.	c) Rates of recharge and discharge	c) Planned changes in water and land use
d) Aquifer parameters and their distribution	d) Ground Water balance	d) Economic information of water supply.

modelled is a pre-requisite. The scope of the study and size of the basin determines the scale of the topographical map. A scale of 1: 50,000 to 1:75,000 will suffice generally. This map should indicate all surface water bodies, streams, big lakes, and other water transportation systems. It should also indicate the ground level contours with a contour interval of 5 to 10 m.

An inventory of dug wells, deep tube wels and types of pumping structures in the area should be made. Two observation wells which are used to define the watertable elevations should also be identified on the map. Proper numbering of these wells for identification should be made. The datum level, the ground level after proper levelling for each of the observation wells should be marked on the map. The measured water levels should be converted into water levels above/below a particular datum level, say, mean level by conducting a systematic levelling.

4.1.2 Hydrogeological maps

Intensive geological and geomorphological studies of the ground water basin will be required to delineate its geomorphological features or land forms and to evaluate the manner and degree in which they contribute to the basin's hydrology. The type of the material forming the aquifer system and confining material, location and nature of the aquifer's impermeable base, the hydraulic characteristics of the aquifer and location of any structures affecting groundwater movement are of special importance.

Groundwater basins, which are usually defined as hydrological units containing one large aquifer or several connected and interrelated aquifers, may be classified on the basis of their main depositional environment. For a proper understanding of the basin's hydrology, it is necessary to recognize and delineate the morphological features, which are grouped into topographical high lands and topographical low lands. The high lands are generally the recharge areas, characterized by a downward flow of water and the low lands are usually the discharge areas,a characterized by an upward flow of water.

The geological history of the basin must be known as the resulting geological structure controls the occurrence and movement of groundwater. The number and type of water-bearing formations, their depth, interconnections, hydraulic properties and out crop patterns are all the results of the basin's geological history. The study of the subsurface geology is required to find out the type of materials that make up the groundwater basin, their depositional environment and age, and their structural deformation, if any. The geological information has to be related to the occurrence and movement of the groundwater by translating it in terms of water bearing formations, confining layers, leaky aquifers. Also, isopach maps of the aquifers are to be prepared which will indicate the thickness of the aquifers. From logs of the wells, the configuration and elevation of impermeable base of the aquifer can be determined.

The condition at the boundaries of the aquifer must be properly defined. Different types of boundaries exist, which may or may not be a function of time. They are: a) zero flow (noflow) boundaries, b) head-controlled boundaries, c) flow controlled boundaries.

The model of the basin requires that external noflow boundaries be delineated and indicated on a map. It also requires that the configuration and elevation of the impermeable base, which is an internal no flow boundary, be determined. Groundwater divides, which acts as no flow boundaries must be indicated. In mathematical terms, the condition at a no flow boundary is $\partial h / \partial n = 0$, where h is the ground water potential and n is the direction normal to the boundary.

A head controlled boundary is a boundary with a known potential or hydraulic head, which may or may not be a function of time. Examples of this type of boundary are: large water bodies like lakes, occeans whose water levels may not drastically change with the events, water courses and irrigation canals with fixed known water levels. Mathematically, a head controlled boundary that changes with time is expressed as $h= f(x,y,t)$ if it varies with time or $h=f(x,y)$ if it is independent of time. Similar to the no-flow boundaries, head controlled boundaries can be differentiated as internal boundaries or as external boundaries.

A flow controlled boundary is a boundary through which a certain volume of groundwater enters/leaves the aquifer under study per unit of time from adjacent basin, whose hydraulic head and/or transmissivity are not known. Flow controlled boundaries are simulated by setting the hydraulic conductivity at the boundary equal to zero, and entering the underflow into the model as a recharge/discharge term. Mathematically, the flow is represented, for a steady state, by the normal gradient dh/dn, taking a specified value for it.

While modelling, it is preferable to delineate the basin for study at the no-flow boundaries or head controlled boundaries. If it is not feasible to do so, it is desirable to choose an arbitrary boundary and estimate the flow across it by knowing the heads on either side of the boundary.

4.1.3 Lithological variations in an aquifer

No aquifer is lithologically uniform over the entire basin.

Both lateral and vertical variations do occur. Since the grain size of the aquifer material has bearing on the hydraulic conductivity and porosity, and thus on the flow and storage of groundwater, the preparation of sand percentage map forms an important part of modelling.

4.1.4 Aquifer parameters

The magnitude and distribution of the aquifer characteristics must be specified. These characteristics depend on the type of the aquifer. They may be:

a) hydraulic conductivity(for all types of aquifers),

b) storage coefficient(for confined and leaky aquifers),

c) specific yield(for unconfined aquifers)

d) hydraulic conductivity for leaky aquifer, if exists.

There are various field, laboratory and numerical methods to determine or estimate these parameters.

i) Estimation of hydraulic conductivity

Aquifer in-situ tests are the most reliable methods of determination of these parameters,though they are costly. In view of this, only few tests can be conducted in regional aquifer studies. The data, thus collected may not be adequate to draw the hydraulic conductivity maps or storage coefficient distribution maps. Supplementary data have to be collected by conducting tests, like, well tests, slug tests and point tests. A well test consists of pumping an existing small diameter well at a constant rate and measuring the drawdown in the well. When the steady flow conditions are obtained, the transmissivity of the aquifer can be determined using the following modified Theis equation, as:

$$KD = 1.22Q/s_w \qquad ...(5a)$$

where,

K is the hydraulic conductivity in m/day

D is the thickness of the aquifer in m

s_w is the drawdown in the well in m

Q is the constant well discharge in m^3/day

This test can be used either for confined or unconfined aquifers. However, if it is applied for unconfined aquifers, the drawdown is suitably corrected as $s'_w = s_w - (s^2_w/2D)$ where s'_w is the corrected drawdown and D is the saturated thickness of the unconfined aquifer. The values of transmissivity thus obtained may not be accurate, especially when well construction information is not available or when the well screen is partly clogged.

A slug test consists of abruptly removing a certain volume of water from a well and measuring the rate of rise of the water level in the well. Sufficient drawdown is to be created for this test to be effective. Bouwer(1978) gave the formulae for estimating the transmissivity and specific yield for partially and fully penetrating wells in an unconfined aquifer. Using these formulae an estimate can be made for the aquifer parameters.

A point test is a permeability test made while drilling an exploratory bore hole. This is also termed as a packer test. When the hole has reached a certain depth, a small screen whose length equals its diameter is lowered into the hole. After the casing has been pulled up over a certain distance and a packer is placed to close the annual space, the water level in the pipe is lowered by a compressor. When the water level stabilizes, the pressure is released and the rate of rise of water level is measured. These are plotted against corresponding time on a semi-log paper. The points fall on a straight line, the slope of which can be determined. Using the following formula, the hydraulic conductivity of the aquifer material at the depth of the screen can be determined:

$$K = 0.575 \frac{r_c^2}{r_s} \cdot \frac{1}{\Delta t} \qquad \ldots(5b)$$

where, r_c = radius of the casing pipe in m

r_s = radius of the screening pipe in m.

However, when applying this formula , the resistance of the screen, the well storage are to be properly accounted.

The hydraulic conductivity can also be estimated using the grain size distribution. Aquifer material do not generally consist of uniform particular of one single diameter but of particles of different sizes to be grouped in fractions, each with certain limits of particle size based on which classification of sands is made. It was found that the hydraulic conductivity is inversely proporotional to the square of the specific surface of the aquifer material.

Other methods like flow net method can also be used in estimating the hydraulic conductivity. In regional aquifer simulation, it is more important to know the order of magnitude of hydraulic conductivity at as many places as possible in the study area rather than very accurate values at few places. Hence, estimation of hydraulic conductivity even by less accurate methods will be of great importance for model studies, which will provide quantitative data. Thus the data obtained by any of the methods should be used to compile a transmissivity map from which hydraulic conductivity map can be prepared. For this purpose, isopach map can be used.

ii) Estimation of specific yield

It is a dimension-less parameter. It characterizes the storage capacity of unconfined aquifers. It can be determined from the data of an aquifer test. But these tests may have to last for days together for the value of specific yield to be reliable. This can also be measured by other techniques, like by determining in the laboratory the difference between volumetric water content at saturation and the water content when most of it has drained from the pores (water content at field capacity). The other method consists of measuring the drop in the water table and the amount of water drained from the field. Then the specific yield is determined by dividing the quantity of drained water per unit area by the drop in water table. This method is often used in experimental fields. However, these methods are time consuming and costly because of which they can be performed at few selected locations only. While conducting groundwater model studies, where the distribution of specific yield is of more interest, it may not be possible to have so much data by these methods only for obtaining fair and realistic distribution of this parameter over the entire basin. So generally using the grain size distribution, the estimates will be made. The table 2 gives the orders of magnitude and ranges for specific yield of different materials.

iii) Estimation of storage coefficient

The aquifer test data has to be for the determination of the storage coefficient for case of confined and leaky aquifers. Van der Gun(1979) has presented an empirical expression for finding out the order of magnitude of the storage coefficient knowing the depths upper and lower surface of the aquifer below ground surface. The information obtained from the well logs, thus, can be used in case the wells are fully penetrating.

In spite of the best efforts, the distribution maps of the aquifer

Table 2 - Orders of magnitude and ranges for specific yield of different materials(after Morris and Johnson 1967)

Type of material	Specific yield(percent)	
	Range	Mean
Coarse gravel	13-25	21
Medium gravel	17-44	24
Fine gravel	13-40	28
Coarse sand	18-43	30
Medium sand	16-46	32
Fine sand	1-46	33
Silt	1-39	20
Clay	1-18	6
Loess	14-22	18
Eolian sand	32-47	38
Tuff	2-47	21
Sandstone(fine)	2-40	21
Sandstone(medium)	12-41	27
Siltstone	1-33	12

parameters can not be completed due to the complexity of the aquifer systems. It is, therefore, necessary to adjust these values at the time of model calibration which is discussed in the subsequent chapter.

4.2 Exitation on the System

4.2.1 Water table map/Piezometric map

The exitations caused due to the infiltration,stream-bed percolation, evapotranspiration, pumping from wells etc. create hydrological stress on the aquifer system and reflects in the form of a change in the configuration and fluctuations in the water-table. Thus,preparation of water table contour map is essential which requires realistic water table data. Appropriate network of observation wells/piezometers are needed depending upon the size and nature of the basin to study the magnitude and distribution of the hydraulic head. Proper selection of sites for observation wells have to be made depending upon the variation in the aquifer thickness, lateral and vertical variations in the lithology. The data thus obtained is used for the preparation of water table map/piezometric contour map. Many times other maps like depth to water table maps, water table change maps, head difference maps are also prepared to give sufficient insight into the hydrogeological conditions of the basin and its ground water regime. Depending upon the time discretization adopted in the model study, these maps are prepared accordingly, i.e.if the time unit is a month, the maps are to be prepared on monthly basis.

4.2.2 Type and extent of recharge and discharge areas

Recharge areas are areas where the aquifer gains water through infiltration,stream-bed percolation, surface runoff from adjacent hilly terrain, percolation from irrigated areas, seepage from canal conveyance systems etc. On the similar lines, there will be discharge areas forom where the aquifer losses water through springs, evaporation and pumpage etc. A study of topographical maps and aerial photographs, in addition to field surveys will be able to identify the type and extent of recharge and discharge areas.

4.2.3 Rate of recharge and discharge

Several methods exist for the determination of the recharge and discharge components. After proper identification of the factors that contribute to the recharge and discharge, they are to be quantitatively assessed.

a) recharge components

The main source of recharge to the aquifer is the recharge from precipitation. It is, thus, necessary to have a detailed knowledge of the amount of precipitation and its distribution spatially and temporally. The rate of recharge can then be assessed either by using the water table fluctuation method or by the empirical formulae which have been derived earlier for the region under study. Similarly, the recharge from the irrigation canals, deep percolation from irrigated fields, stream bed percolation have to be properly assessed. Various methods, like lysimeter studies, insertion of tensiometers, isotope studies can be used for this purpose.

The sources of discharge are springs, outflow towards rivers/ streams, evapotranspiration and pumpages. Springs are the most common form of groundwater discharge. Their occurrence is governed by local hydrogeological conditions. When the water table is cut by the ground level or due to faults, springs are generally originated. The outflow through springs can be assessed using runoff hydrographs.

The evapotranspiration is a combined effect of evaporation from soil and transpiration by natural vegetation and cultivated crops. Assistance of agronomists may be necessary in realistic assessment of this component. The basic factors which govern this component are: climate, soils, soil water availability, crops, their intensity and pattern, environment and exposure, methods of irrigation etc. The data with regard to land use, irrigation practices, crop survey, climatological information are, thus, needed for proper estimation of evapotranspiration.

Due to the fast growing needs of the society, the groundwater pumpage is on an increasing trend. The assessment of this component, thus,requires a realistic approach. The data pertinent to the total number of wells, different types of structures and rate of pumping from each of these structures at different times forms an important source of information. Inquiries about the time of operation of wells can be made and the rate can be assessed. Alternatively, data on fuel or electricity consumption can be used to assess the rate of pumpage.

4.3 Groundwater balance

The groundwater basin must be in dynamic hydraulic balance during a time period of any duration. This condition can be written as:

$$R + E + I + O + \frac{\Delta S}{\Delta T} = 0 \qquad \ldots(6)$$

where, R = total recharge

E= total extraction

I = lateral inflows at the boundaries

O = lateral outflows at the boundaries

Δ S = change in storage over a time period ΔT

When hydrogeological investigations have been completed and the inflow and outflow components of the aquifer have been quantified, an overall groundwater balances of the basin must be assessed, which will serve as a verification of the results of the model study. Also, the water balance technique can be used as a valuable tool for quantifying certain components of the equation 6 which are otherwise difficult to determine. It is very much essential to note that while using this approach, the quantification of the other components should be precise, otherwise, the results may be misleading.

The data requirement for Water Balance Study is summarised below:

1. Precipitation: Daily rainfall data at raingauge stations lying within and around the study area.
2. Stage and discharge of all rivers flowing within the basin and rivers forming external boundaries at various control points.
3. Monthly discharge in the main branch canals at the off take points.
4. Monthly discharge at various sections of the main branch canals.
5. Cross section and longitudinal sections of the canals and its distributaries, depth of water in the canal.
6. The dates or days on which water is supplied to the canals and the running hours.
7. Ground Water Table Data: Monthly water table data observed at the observation wells available in the study area and corresponding ground elevations.
8. Water table hydrographs near raingauge stations, if available.
9. Ground Water Withdrawal Data: Number and capacity of Ground

Water draft structures areawise: Sample survey of drafts and running hours.

10. Test pumping data to evaluate specific yield and transmissivity.
11. Infiltrometer test data at few places within the study area.
12. Seepage from canal estimated either by inflow-outflow method or ponding method.
13. Grain size distribution of the soil Sample obtained from canal bed.
14. Land Use Data- forests, orchards and tall vegetations, urban area, waterlogged area, cultivated area, canal irrigated area, well irrigated area, and unirrigated area.
15. Existing Cropping Pattern.
16. Daily pan evaporation values.
17. Daily minimum and maximum temperature, minimum and maximum rela-humidity, average wind speed,sunshine hours.
18. Well log data.
19. Irrigation practices over the area.

5.0 INVERSE PROBLEMS

The problem of calibration is identified by various names, such as system identification, inverse problem etc. all of which is effect refer to the process of making a chosen model operational. For example, if equation 4 is considered to represent the flow of water through an aquifer, assignment of initial and boundary conditions and values to S and T is required before the model is put to operation.

Rewriting equation 4 as

$$\frac{\partial h}{\partial t} \nabla \left(\frac{T}{S}\right) h(X,t) + Q = 0, \quad X \in \Omega \times \Gamma \quad \text{and } t > 0 \qquad \ldots(7)$$

where

$$\Delta = \sum_{1}^{n} \frac{T}{S} \frac{\partial^2}{\partial X_i} (.)$$

Ω = metric space

Γ = boundary of a metric space, Ω

If an initial condition

$$h(X,O) = \xi (X), \quad x \in \Omega \qquad \ldots(8)$$

and a boundary condition

$$h(X,t) = \eta(X,t), \; x \in \Gamma, \; t > 0 \qquad \ldots(9)$$

are also given, the solution of equation 7 through 9 with suitable hypothesis to ξ and η uniquely determines $h(X,t)$, $x \in \Omega$, $t > 0$. This determination of the dependent variable is called direct problem.

The inverse problem arises when $h(X,t)$ has been actually observed for some $X_i \in \Omega$ at $t_i \in (O,T)$ and it is required to solve equation 7 for either of the following:

a) Parameters, T and S or T/S

b) Initial conditions, ξ

c) Boundary conditions, η

d) The inputs, Q

e) A combination of the above in the event decomposition is not feasible.

In the inverse problem, the dependent variable of the direct problem becomes the independent variable and thus, the problem is to be solved in reverse. Like in the case of direct problem, inverse problems may be either continuous or discrete type. If in equation 7, h is completely known as a continuous function X and t, then it is a continuous inverse problem. On the other hand, if h is known only at few discrete points in the basin,(as is the real life situation where h is measured at discrete points) then it is discrete inverse problem. In a real life situation, the problem should always be properly posed, yet, in the usual mathematical formulation the inverse problem is inherently ill posed. This is mainly because of insufficient and inaccurate data as well as because of the fact that different combinations of the variables(T,S, ξ, η and Q) may produce the same result, thus giving rise to non-uniqueness. Because the data are inexact and solution is ordinarily non-unique, the history of the system becomes utmost important for an accurate solution.

5.1 Parameter Estimation

It is observed from literature that out of the five types of problems which can be visualized, only parameter estimation(problem cited at 'a' above) has received greater attention for investigation.

Generally, the following steps are used to solve an inverse problem.

a) Assume certain values for the unknown parameters.

b) Solve the direct problem with this assumed value.

c) Compare the results obtained in step(2) with the actual observations.

d) If the two do not correspond within a certain limit, with the help of a suitable algorithm modify initially assumed values of the parameters.

e) Repeat steps(2) to (4) until satisfactory value of criterion function, expressing the difference between the observed and the computed values, is obtained.

The methods for solving the inverse problem differ with the different algorithms chosen for step 'd' and with the different criterion function adopted at step'e'.

5.2 Criterion Function

Different criteria can be used for optimality i.e.if λ is a true parameter vector and $\bar{\lambda}$ the estimated one, then

a) The minimization of some function of (λ $\bar{\lambda}$). As λ is unknown, the minimization is possible for the expected value of this difference if sufficient prior knowledge is available.

b) The minimization of some functions or functional of $(h-\bar{h})$ where h is the measured head and $\bar{h}$ is the computed head. This error may be used because $(h-\bar{h})$ is measurable.

c) The minimization of some functional containing the measurable process outputs and the estimates of the state vector and the parameter vector.

Often the second criterion is used because the correspondence of input-output relations are considered as important than parameter correspondence while trying to minimize a functional related to the difference between observed and calculated response. It is also required that the resulting parameter distributions to be rather smooth and uniform.

In order to demonstrate, the following example is considered. Let us say, the aquifer is discretized into rectangular grids as shown in Fig.1a and consider a cell with its balance components(Fig.1b). The balance equation for the cell, then would be

$$T_{i-1/2,j} \cdot \Delta y_j \frac{h^{k+1/2}_{i-1,j} - h^{k+1/2}_{ij}}{} + T_{i+1/2\ j} \cdot$$

$$\Delta y_j \frac{h^{k+1/2}_{i+1.j} - h^{k+1/2}_{ij}}{x_{i+1/2}} + T_{ij-1/2}\, \Delta x_i \frac{h^{k+1/2}_{i.j-1} - h^{k+1/2}_{ij}}{y_{j-1/2}}$$

$$+ T_{i,j+1/2} \cdot \Delta x_i \frac{h^{k+1/2}_{ij+1} - h^{k+1/2}_{ij}}{y_{i+1/2}} + R^{k+1/2}_{ij}$$

$$= S_{ij} \frac{h^{k+1}_{ij} - h^{k}_{ij}}{(\Delta t)^{k+1/2}} \quad \ldots(10)$$

Where

T is the transmissivity

h is the head

R is the net recharge/discharge

S is the storavity

i is the row

j is the column

Δx is discretization in x direction(grid length in x direction)

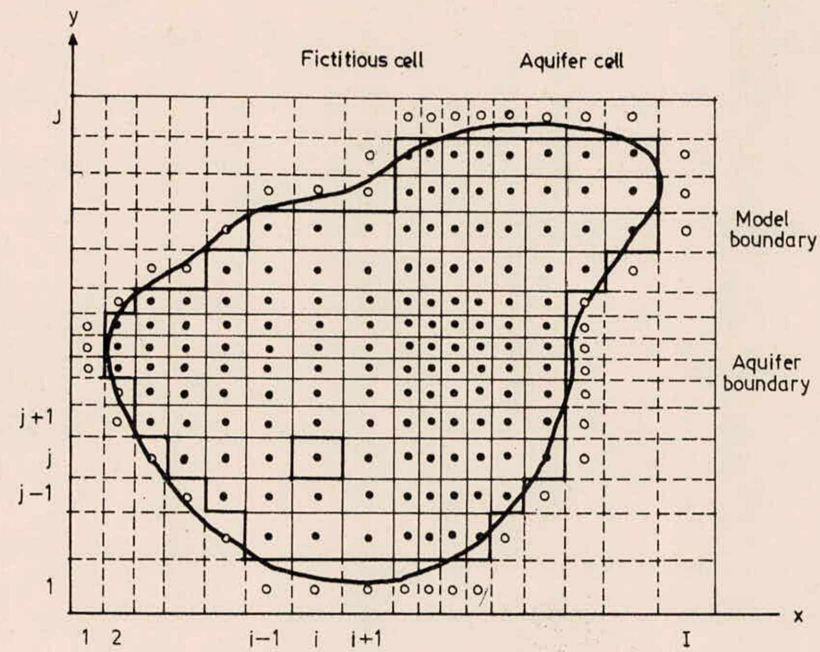
Δy discretization in y direction(grid length in y direction).

k indicates the time step.

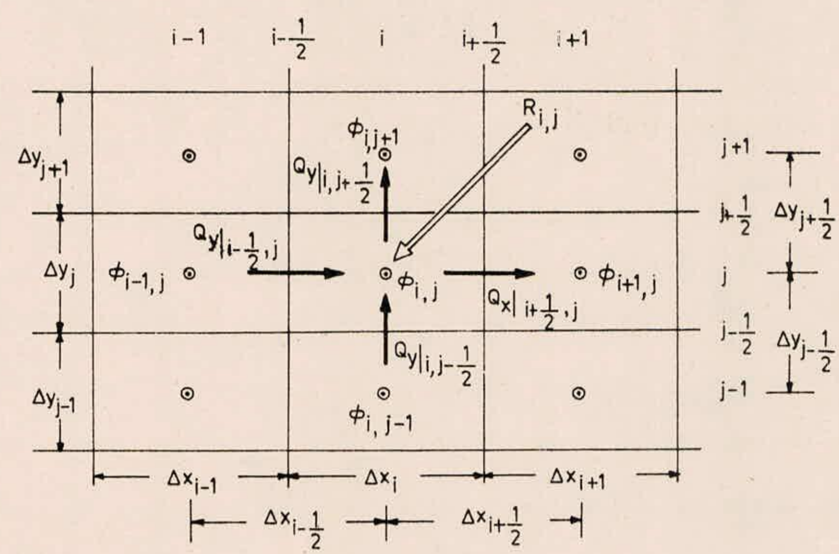
and the transmissivities(T) at the cell boundaries are given by

$$T_{i+1/2j} = (T_{ij} + T_{i+1j})/2 \quad \ldots(11)$$

and gradients of head(h) at the mid point of the time interval are



a. Layout of cells



b. Water balance components for i, j

FIGURE 1 - A MULTIPLE CELL AQUIFER MODEL

given by

$$(\Delta t)^{k+1/2} = t^{k+1} - t^{k} \ , \ t^{k} = \Sigma_{k} (\Delta t)^{k} , h_{ij}^{k+1/2} = (h_{ij}^{k} + h_{ij}^{k+1})/2 \qquad \ldots(12)$$

While the values of T_{ij}, S_{ij}, R_{ij} are known, the value of h_{i}^{k+1} can be determined provided h_{ij}^{k} is known, which is the case in forecasting problem. While in the inverse problem, the values T_{ij} and S_{ij} for each cell have to be determined by writing the equation 10 for all the cells and for every time interval during the calibration period and while the following are specified; shape of model, its division into cells, boundary conditions in terms of heads at the boundary nodal points, values of heads in all cells at the begining and end of each time interval and the net recharge for all the cells and for all the time intervals. Hence, the problem becomes one of obtaining best set of parameters, i.e.,the set which will satisfy the equation most closely. In other words, an optimal solution for T and S is to be obtained for the given set of equations.

Rewriting equation 10 in a form which will emphasize that T and S are unknown to be identified;

$$\sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} \cdot S_{ij} = r_{ij}^{k+1/2} \quad \text{for all } i,j.k. \qquad \ldots(13)$$

where

$$\sum_{m=1}^{n} g_m T_m = a_{ij}^{k+1/2} T_{ij-1} + b_{ij}^{k+1/2} T_{i-1j} + c_{ij}^{k+1/2} T_{ij}$$

$$+ d_{ij}^{k+1/2} . T_{i+1j} + e_{ij}^{k+1/2} T_{ij+1} \qquad \ldots(14)$$

and

$$a_{ij}^{k+1/2} = \frac{-\Delta h_{ij-1/2}^{k+1/2}}{2\Delta y_i \cdot \Delta y_{i-1/2}} , \quad b_{ij}^{k+1/2} = - \frac{\Delta h_{i-1/2j}^{k+1/2}}{2\Delta x_i \cdot \Delta x_{i-1/2}}$$

$$c_{ij}^{k+1/2} = \frac{1}{2\Delta x_i} \left[\frac{\Delta h_{i+1/2\ j}^{k+1/2}}{\Delta x_{i+1/2}} - \frac{\Delta h_{i-1/2j}^{k+1/2}}{\Delta x_{i-1/2}} \right]$$

$$+ \frac{1}{2\Delta y_j} \left[\frac{\Delta h_{ij+1/2}^{k+1/2}}{\Delta y_{j+1/2}} - \frac{\Delta h_{ij-1/2}^{k+1/2}}{\Delta y_{j-1/2}} \right]$$

$$d_{ij}^{k+1/2} = \frac{1}{2\Delta x_i} \left[\frac{\Delta h_{i+1/2.j}^{k+1/2}}{\Delta x_{i+1/2}} \right]$$

$$e_{ij}^{k+1/2} = \frac{1}{2\Delta y_j} \left[\Delta h_{ij+1/2}^{k+1/2} / \Delta y_{j+1/2} \right]$$

$$f_{ij}^{k+1/2} = - \frac{h_{ij}^{k+1} - h_{ij}^{k}}{\Delta t^{k+1/2}}$$

$$r_{ij}^{k+1/2} = \frac{r_{ij}^{k+1/2}}{\Delta x_i \Delta y_i}$$

and n is the total number of cells.

Even when the solution is obtained and if it is substituted back into the equations, only a part of the equations(represented by 13) will hold good. For the remaining part, there will be some deviation between the left hand and right hand terms. There are five different possible criteria for approaching an equality between the two sides of the equations.

5.2.1 Maximum absolute deviation

If X is the maximum absolute deviation, then all other deviations are less than X, i.e.,

$$\sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} - r_{ij}^{k+1/2} \leq X \text{ for all } i,j,k \qquad \ldots(15)$$

Using this criterion, the problem may be restated as, determine T_{ij}, S_{ij} and X from

$$\min \quad F = X \qquad \ldots(16)$$

subject to the constraints

$$\left.\begin{array}{ll} \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} \cdot S_{ij} - X \leqslant r_{ij}^{k+1/2} & \text{for all } i,j,k \\ \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} + X \geqslant r_{ij}^{k+1/2} & \text{for all } i,j,k \end{array}\right\} \qquad \ldots.(17)$$

and $$T_{ij}, S_{ij}, X \geqslant 0 \quad \text{for all } i,j \qquad \ldots(18)$$

This is a typical statement of linear programming.

5.2.2 The sum of the absolute values of the maximum deviations in all time intervals. If $Y^{k+1/2}$ is the maximum absolute deviation at any time interval, the criterion defines the problem as

$$\text{Min } F = \sum_{k=0}^{N-1} Y^{k+1/2} \qquad \ldots(19)$$

where N is the number of equations corresponding to each time interval. Subject to the constraints

$$\left.\begin{array}{ll} \sum_{m=1}^{n} g_m T_m + f_{ij} \quad S_{ij} - Y^{k+1/2} \leqslant r_{ij}^{k+1/2} & \text{for all } i,j,k \\ \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} + Y^{k+1/2} \geqslant r_{ij}^{k+1/2} & \text{,all } i,j,k \end{array}\right\} \qquad \ldots(20)$$

and $T_{ij}, S_{ij}, Y^{k+1/2} \geqslant 0$ for all i,j,k $\qquad \ldots(21)$

5.2.3 The sum of the absolute values of the maximum deviations of all cells. If Z_{ij} is the maximum of the absolute values of the deviations occuring in the N equations of a cell, then the criterion defines the problem as

$$\text{Min} \quad F = \sum_i \sum_j Z_{ij} \qquad \ldots(22)$$

subject to the constraints

$$\left.\begin{array}{ll} \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} - Z_{ij} \leqslant r_{ij}^{k+1/2} & \text{for all } i,j,k \\ \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} s_{ij} + Z_{ij} \geqslant r_{ij}^{k+1/2} & \text{for all } i,j,\ k \end{array}\right\} \ldots(23)$$

and $$T_{ij},\ S_{ij},\ Z_{ij} \geqslant 0 \quad \text{for all } i,j. \qquad \ldots(24)$$

5.2.4 The sum of absolute **values** of all deviations. If U_{ij} is designated as the absolute value of the deviations for each equation (for each i,j,k), then the criterian defines the problem as

$$\text{Min} \quad F = \sum_k \sum_j \sum_i U_{ij}^{k+1/2} \qquad \ldots(25)$$

subject to the constraints

$$\left.\begin{array}{ll} \sum_{m=1}^{n} g_m T_m + F_{ij}^{k+1/2} S_{ij} - U_{ij}^{k+1/2} \leqslant r_{ij}^{k+1/2} & \text{for all } i,j,k \\ \sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} + U_{ij}^{k+1/2} \geqslant r_{ij}^{k+1/2} & \text{for all } i,\ ,k \end{array}\right\} \ldots(26)$$

and $$T_{ij},\ S_{ij},\ U_{ij}^{1+1/2} \geqslant 0 \quad \text{for } i,j,k \qquad \ldots(27)$$

However, it can be seen here that the number of unknowns to be solved one less than the number of equations and hence there will be redundancy which will only burden the solution. To avoid this the deviation in each equation is expressed as a difference between two positive variables say V and W, as

$$U_{ij}^{k+1/2} = V_{ij}^{k+1/2} - W_{ij}^{k+1/2} \qquad \ldots(28)$$

thus, the minimization would be

$$\text{Min } F = \sum_k \sum_j \sum_i (V_{ij}^{k+1/2} + W_{ij}^{k+1/2}) \qquad \ldots(29)$$

$$\sum_{m=1}^{n} g_m T_m + f_{ij}^{k+1/2} S_{ij} - V_{ij}^{k+1/2} + W_{ij}^{k+1/2} = r_{ij}^{k+1/2}$$

for all i,j,k ...(30)

and $T_{ij}, S_{ij}, V_{ij}^{k+1/2}, W_{ij}^{k+1/2} \geqslant 0$ for all i,j,k ...(31)

The basic advantage of this modified form would be the reduction in constraints to half which increases the computational efficiency.

5.2.5 The sum of squares of the deviations. A quadratic objective function may be defined by taking the sum of the squares of all the deviations between the two sides of the equations for all cells and at all time intervals.

The problem is reduced to,

$$\text{Min } F = \sum_k \sum_j \sum_i (g_m T_m + f_{ij}^{k+1/2} S_{ij} - r_{ij}^{k+1/2})^2 \qquad \ldots(32)$$

subject to the constraint

$$T_{ij}, S_{ij} \geqslant 0 \qquad \ldots(33)$$

This is a typical quadratic programming problem, with a quadratic objective function with linear constraints.

The important aspect in this methodology is that it is required to solve the equation repeatedly which causes a number of difficulties, such as, complicated cases may be difficult to solve, initial and boundary conditions and inputs will have to be exactly known.

6.0 SOLUTION TECHNIQUES

Once the set of equations is formulated, the next stage would be to solve them for the unknowns. There are numerous methods available in literature. Considering the methodology adopted to solve these equations, broadly the procedures can be divided into two categories, as direct methods and indirect (iterative) methods. Direct methods yield an exact solution in a finite number of operations if they are no roundoff errors. Iterative methods, on the other hand, begin with an approximate solution and obtain an improved solution with each step of iteration, but require infinite number of steps to obtain an exact solution in the absence of roundoff error. The accuracy of the solution, thus, depends on the number of iterations performed. At times, direct and indirect methods can be used in combination. The following subsections deal with some of the well known direct and iterative methods.

6.1 Direct methods

6.1.1 Methods based on triangularisation

Procedures based on triangularisation as well as those based on diagonalisation are probably the most commonly used, since, they are easy to program and comparatively yield accurate results. The theory behind these procedures is that if A is nonsingular coefficient matrix, then it can be expressed as:

$$A = LDU$$

where L is lower triangular
L is lower triangular
D is diagonal

U is upper triangular

The only real difference in the various methods lies in the way D is partioned.

i.e.

$$\begin{bmatrix} l_1, & 0 & 0 & 0 & 0 & \ldots 0 \\ l_2, & l_{22} & 0 & 0 & 0 & \ldots 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ l_n & l_{n2} & l_{n3} & l_{n4} & \ldots & l_{nn} \end{bmatrix} \begin{bmatrix} u_1, & u_{12}, & U_{13} \ldots & U_{1n} \\ 0 & u_{22} & U_{23} \ldots & U_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & U_{nn} \end{bmatrix}$$

$$\begin{bmatrix} a_{11}, & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{bmatrix}$$

In the above partioning it is not specific whether D belongs to L,U or both. If the equations are solved, they result for l_{ij}, $i > j$ and U_{ij}, $i < j$ and a_{ij} known.

$$\text{i.e.} \quad \sum_{k=1}^{\min(i,j)} l_{ik}\, u_{kj} = a_{ij} \qquad \ldots(36)$$

The main error arising in the solution by trinagularization is that resulting from the decomposition of A to LDU, rather than from the solution of the two triangular systems obtained from such decomposition.

a) Gaussian Elimination

If the system of equations are represented by the matrix relation as AX=B, in this method. Elementary row operations are performed on A and on B to annihiliate successive elements of A in such way to reduce

A to upper form U. The product of the matrices effecting the elementary row operations on A is a lower triangular matrix L with unit diagnomal. Then LA = U or $A=L^{-1}U$. Since L^{-1} is again lower triangular, A has been expressed in LDU form with D included in U.

For solving AX = B,

The process is as follows. At the kth stage, a pivotal element $a_{ij}^{k-1} \neq 0$ of $A^{(k-1)}$ is chosen.

Then the ith and kth rows and the jth and kth columns of $A^{(k-1)}$ are interchanged so that a_{ij}^{k-1} becomes $a_{kk}^{(k)}$. The ith and kth rows of $B^{(k-1)}$ are also interchanged. The pivot $a_{kk}^{(k)}$ is then used to obtain zeros in all positions in its column below the diagonal.

Assuming at stage K = 0 represents the original A and B, the desired triangular form is obtained at the conclusion of stage k=n-1. The upper triangular system is solved by back substitution which gives the desired result.

b) Crouts method

The method depends explicitly on the triangular resolution A=LU. It is termed as compact scheme since the elements in the final triangular form are obtained by accumulation, dispensing with the computation and recording of intermediate a_{ij} coefficients and reducing roundoff errors. Setting A=LU, the equations for the elements of L and U are

$$\sum_{k=1}^{\min(i,j)} l_{ik} U_{kj} = a_{ij} \qquad \ldots(37)$$

Letting u_{kk} as 1 for k=1 to n, the following equations are obtained.

$$u_{kk} = 1$$

$$l_{ik} = a_{ik} - \sum_{m=1}^{k-1} l_{im} U_{mk} \quad \text{for } i = k \text{ to } n$$

$$u_{ij} = \frac{1}{l_{kk}} \left(a_{kj} - \sum_{m=1}^{k-1} l_{km} u_{mj}\right) \quad \text{for } j = k+1 \text{ to } n$$

$$l_{ik} = 0 \text{ for } i < k, \quad u_{kj} = 0 \text{ for } j < k$$

As the elements of l_{ik} and u_{kj} are computed $AX = B$ is solved by writing LUX=B, which is then equivalent to solving the triangular systems

$$L\xi = B \text{ for } \xi \text{ and } UX = \xi \text{ for } X, \text{ thus}$$

$$\xi_{ij} = \frac{1}{l_{ii}} \left(b_{ij} - \sum_{k=1}^{i-1} l_{ik} \cdot \xi_{kj}\right) \qquad \ldots(38)$$

for j=1 to r and i=1 to n

$$x_{ij} = \left(\xi_{ij} - \sum_{k=i+1}^{n} u_{ik} x_{kj}\right) \qquad \ldots(39)$$

for j=1 to r and i=n to 1

c) Symmetric Cholesky method

If A is a symmetric matrix such that $A^T = A$, then A can be written as $A=LL^T$ or $A=U^TU$ provided A is non singular matrix, and where L^T is transpose of L

i.e. from $\sum_{k=1}^{\min(i,j)} l_{ik} u_{ij} = a_{ij}$ ....it can be written as

$$\sum_{k=1}^{\min(i,j)} u_{ki} \cdot u_{kj} = a_{ij} \qquad \ldots(40)$$

The elements of U are given by

$$u_{ii} = \sqrt{a_{11}}$$

$$u_{ij} = a_{ij}/u_{11} \text{ for } j = 2 \text{ to } n$$

$$u_{ij} = (a_{ii} - \sum_{k=1}^{i-1} u^2_{ki})^{1/2} \quad \text{for } i=2 \text{ to } n$$

$$l_{ij} = \frac{1}{u_{ii}} (a_{ij} - \sum_{k=1}^{i-1} u_{ki} u_{kj}) \quad \text{for } j= i+1 \text{ to } n \qquad \ldots(41)$$

and i=2 to n

Then $AX = U^T UX=B$

and $X= (U^{-1})(U^{-1})^T B$ where U^{-1} can easily be found.

6.1.2 Gauses-Jordan method

Any matrix A is equivalent to a diagonal matrix and this diagonal matrix is obtained from A by a finite number of operations on the flows and/or columns of A. This method is similar to Gaussian elimination, but in this case, the elements above the diagonal are also eliminated as well so that back substitution is no more necessary.

The result of elimination is

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - (\frac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}} \cdot a_{kj}^{(k-1)}$$

for i=1 to k-1 and k+1 to n

and j=k to n

$$b_{ij}^{(k)} = b_{ij}^{(k-1} - (\frac{b_{ik}^{(k-1)}}{b_{kk}^{(k-1)}} \; b_{kj}^{(k-1)} \qquad \ldots(42)$$

for i=1 to k-1 and k+1 to n

j=1 to r

$$a_{ij}^{(k)} = a_{ij}^{(k-1)}$$

for i = 1 to n and j = 1 to k-1

If stage K=0 represents the original status of A and B, the desired diagonal form is obtained at the conclusion of stage k=n-1. Then the solutions of Ax=B are

$$x_{ij} = \frac{b_{ij}^{(n-1)}}{a_{ii}^{(n-1)}} \qquad \ldots(43)$$

Some of the other direct methods include the procedures based on othogonalization, block decomposition like bordering method etc. which are not generally used in solving the problems relating to subsurface flow.

6.2 Iterative Methods

In general iterative methods are preferred for solving a single large sparse system of linear equations for which convergence is known to be rapid. Iterative methods are less useful for finding inverses or for solving systems with many right-hand sides. Iterative procedures begin with an approximate solution to a linear system and obtain and improved solution with each step of the process. An iterative process would require an infinite number of steps to obtain an exact solution; the accuracy of the solution obtained depends upon the number of iterations performed(and/or the convergence rate), the condition of the matrix as well as its size, the accuracy of the arithmetic performed by the computer and the particular algorithm used.

It is purported that iterative methods use the matrix in its original form for each iteration, hence tend to be self-correcting and to minimize roundoff error. Iterative methods do have the advantage of preserving the zero elements of the matrix.

Iterative methods are particularly efficient for solving systems of linear equations arising from finite-difference approximations for elliptic partial differential equations. The associated matrices in such cases are characterized by a great many zero elements and by the fact that the nonzero elements occur in some systematic pattern. In addition, such matrices are usually symmetric positive definite and irreducible. For very large order matrices, direct methods wwould be too laborious, require too much storage, and give limited accuracies. Iterative methods, on the other hand are ideally suited to such matrices, particularly when the pattern of nonzeros is a cyclic one needing little storage even for large order matrices.

The total amount of work involved in using an iterative method depends upon the convergence rate and the desired accuracy. Usually an iteration is used only in problems for which convergence is known to be rapid. Slow or irregular convergence would be a considerable drawback. The rate of convergence for an illconditioned matrix will be poor.

The storage required for an iterative method will vary with the matrix used. Some of the iteration techniques are discussed herein briefly.

6.2.1 Explicit iteration methods

These are also termed as single step iteration or point iteration. In this, each $x_i^{k+1)}$ of the new iteration vector $x^{(k+1)}$ can be determined by itself without the necessity of determining a group of other components $x_j^{(k+1)}$ simultaneously.

a) Jaccbi method

It can be expressed as

$$x^{(k+1)} = x^{(k)} + C^{(k)} r^{(k)} \qquad \ldots(44)$$

With $C^{(k)} = C$ = diagonal, where it is assumed that $a_{ii} \neq 0$ for any i=1 to n. The method consists of solving ith equation of the system Ax=b for $x_i^{(k+1)}$ using the values of $x_j^{(k)}$, $i \neq j$ for the remaining variables, that is

$$X_i^{(k+1)} = \frac{1}{-a_{11}} \left(b_i - \sum_{\substack{j=1 \\ j \neq 1 \\ j = i}}^{n} a_{ij} \; x_j^{(k)} \right)$$

$$\text{for} \quad i = 1 \text{ to } n \qquad \ldots(45)$$

The Jacobi iteration method converges for every $x^{(o)}$ (for any assumed initial value of x) and for any order of the equation to the solution Ax=b.

b) Gauss-Siedel iteration by successive displacement

This scheme may be written as

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \; b_i - \sum_{j=i+1}^{n} , x_j^{(k)} - \sum_{j=1}^{k-1} a_{ij} \; x_j^{(k+1)} \qquad \ldots(46)$$

This method assumes values for $x_2^{(o)}$ .... $x_n^{(o)}$ and computes $x_1^{(1)}$ from the first equation, uses $x_1^{(o)}, x_3^{(o)}$ $x_n^{(o)}$ in the second equation to compute $x_2^{(1)}$ etc., by cycling through the set of n equations. IN general, $x_i^{(k+1)}$ is computed from the ith equation using $x_{i-1}^{(k+1)}$, $x_{i+1}^{(k)}$ ... $x_n^{(k)}$. It is not necessary that the equations are to be used in the order 1,2,....n. However, the order must remain fixed in order for the iteration matrix to remain constant. The equations must be ordered so that $a_{ii} = 0$ for any i. This method converges for every $x^{(o)}$, and for any order of the equations, to the solution Ax= b. The limitations of the method is that the convergence is linear(though twice that of Jacobi scheme).

(c) Successive overrelaxation(SOR) method

In the case of Gauss siedel scheme the relaxation parameter

(w) is considered as unity, while in the case of SOR method the relaxation factor is more than unity. This method was developed by Young and Frankel(1950) and has proved particularly useful in solving system of linear equations arising from difference equations for solution of elliptic partial differential equation. The successive over relaxation for solving Ax=b can be written as

$$x_i^{(K+1)} = - \left(\frac{w}{a_{ii}} \right) \left(\sum_{j=1}^{i-1} a_{ij}\, x_i^{k+1} - \sum_{j=i+1}^{n} a_{ij} . x_j^{(k)} - b_i \right) - (w-1)\, x_i^{(k)} \qquad \ldots (47)$$

for i = 1 to n

6.2.2 Implicit Iterative Methods

In block itermative schemes several unknowns are connected together in the iteration formula in such a way that a linear subsystem must be solved before any one of them can be determined. The equations are divided into groups and the subsystem of equations belonging to a given group is solved for the corresponding unknowns using approximate values for the other unknowns.

For example, suppose the unknowns are divided into N groups so that $x_1 \ldots\ldots x_m$, belong to group X_1, $x_{m_i+1} \ldots, x_{m_2}$ belong to group x_2; etc. In general, $x_{m_{k-1}+1} \ldots, x_{m_k}$ belong to group x_k. The matrix A is similarly divided into blocks A_{ii}, where the submatrix A_{ij} has m_i rows and $(m_j - m_{j-1})$ columns, and b vector is divided into N groups $\beta_1 \ldots\ldots \beta_M$. Then the system A_x = b can be written

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_N \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_N \end{bmatrix}$$

Naturally the blocks are chosen so that solving each subsystem is as simple as possible.

The block-iterative methods considered are the alternating-direction-implicit schemes, which are particularly effective when dealing with linear systems arising from elliptic partial differential equations.

a) Peaceman-Rackford alternate direction implicit method

Let $A= A_1 + A_2 + D$, where D is a non negative diagonal matrix, A_1 and A_2 are symmetrix, positive definite, have positive diagonal elements but non positive off-diagonal elements. Then the equation Ax=b is equivalent to each of the equations

$$(A_1 + D + E_1) = b (A_2 - E_1)x \qquad ...(48)$$

$$(A_2 + D + E_2)x = b- (A_1 - E_2)x \qquad ...(49)$$

provided the matrices $(A_1 + D + E_1)$ and $(A_2 + D + E_2)$ are nonsingular. The two equations 48 and 49 provide the basis for the AD1 methods in which an intermediate vector $x^{(k+1/2)}$ is calculated from $x^{(k)}$ and in turn used to compute $x^{(k+1)}$. In Peaceman-Rachford method

The iteration formulae are

$$x^{(k+1/2)} = [A_1 + D + {}_kI]^{-1}[b-(A_2-w_kI)x^{(k)}] \qquad ...(50)$$

$$x^{(k+1)} = [A_2 + D + {}_kI]^{-1}[b-(A_1-w'_kI)x^{(k+1/2)}] \qquad ...(51)$$

Each equation can be solved, provided the matrices which have to be inverted are similar to positive definite, well conditioned triangular matrices. The parameters w_k and w'_k are chosen so as to accelerate convergence. The parameters can also be $w_k = w'_k = w$ in which case the process become stationary.

b) Strongly implicit scheme

When a complex problem is attempted, the advantage of the ADI method is considerably reduced. In such cases where the flow occurs in heterogeneous or anisotropic media, the strongly implicit method offers a faster convergence. It is also termed as 'an approximate factorization technique'. It can be described heuristically in the following manner. If solution is sought to a system of equations represented by Ax=b and when it is difficult to solve, the coefficient matrix is substituted by a modified matrix, A+ λ , in such a manner that A + λ is in some sense close to A. Then the modified matrix should be an acceptable basis for the iterative technique.

Thus the set of equations which require solutions is

$$[A +\lambda] x = b + [\lambda] x \qquad ...(52)$$

In order to maintain the original equation [λ]x is added on the right hand side as well. As it is stated [A + λ] can be factorized in lower and upper diagonal matrices without affecting the sparseness of the original matrix. The solution can easily be obtained for the equation(52) provided the value of [λ]x is known. Since, the value of x is unknown and is to be determined an approximate value of x is used. This would give rise to the iterative equation

$$[A +\lambda] x^{(k)} = b + [\lambda) x^{(k-1)} \qquad ...(53)$$

where, k indicates the index of iteration.

At the first iteration i.e., when k=1, $x^{(k+1)}$ are the assumed initial values from which $x^{(1)}$ are computed using equation 53. For subsequent iterations the previous iteration solution values can be used and then, the values of x can be refined until satisfactory solution is arrived. Thus, the solution of original equation is reduced in finding the matrix which when added to the coefficient matrix A, can easily be decomposed in lower and upper diagonal matrices with the sparseness being maintained. A suitable acceleration parameter, the value of which can range between 0 to 1 can be used for faster convergence.

7.0 CASE STUDY

The model studies of Upper Ganga Canal Irrigation system in Western Uttar Pradesh, India involves developing, implementing and validating a computer model for the study area. This model was then used for studying and inferring suitable strategies for the development of surface water system and in conjunctive utilisation of surface water and ground water for irrigation purposes.

The study is quite complex which was to deal with an area of 24,000 sq.km with a considerable variation of rainfall(1000 mm in the northern part and 500 mm in southern part of the study area), canal supplies, ground water extraction and cropping pattern. Further complications are introduced due to interaction of ground water system with major and minor rivers and canals surrounding the study area and flowing through it.

7.1 Aquifer System

The aquifers in the study area are part of Indo-Gangetic alluvium consisting mainly of sand and silt, interspersed with clay and extending to depths of 1 to 2 km. There are nearly 2500 public tubewells with depths of extraction ranging between 35 m and 105 m(termed as deep wells) and nearly 30,000 private wells whose depth range is between 5 m and 35 m(known as shallow wells). Using the well log data of these deep wells, the aquifer parameters viz.permeability and specific yield have been estimated. The sand percentages at each of the extraction centres have been used to draw maps, which indicated that over the shallow depth(0m - 35 m) the distribution of percentage of sand in the aquifer varied from 70% in the east of the study area to 40% in the west whereas at the deeper depths (35 m to 104.5m), this variation is 70% in the north to 40% in the south.

7.2 Ground Water System

The spring levels from the shallow wells have indicated that over most of the study area, the depth to water table is of the order of 1 m to 5 m with a small patches of deep water tables. The seasonal (monsoon and non-monsoon) fluctuation of these levels is found to be of the order of 1m to 2m whereas the annual change, over a period of 10 years, is noticed to be very marginal, indicating that the ground water system is in a state of dynamic equilibrium.

The piezometric data obtained from the deep tubewells, when compared with those of water table indicated that there is a difference in pressure and the vertical flow component can not be ignored while modelling.

7.3 Surface Water(Canal)System

The study area is well covered with a close net work of a perennial canal system having a main canal with a design discharge of 10500 cusecs and with three branch canals with design capacities of 1380 cusecs, 1950 cusecs and 1200 cusecs. The main and branch canals are being located along the ridge lines and hence through most of their lengths they are either in cutting or partial cutting. The observations of spring levels just near these canals indicated that there is a continuity between the water level in the canal and the water table level at the adjoining places.

These main/branch canals supply water to the fields on a roster basis through distributories/minors and through the field channels. These small canals which have design capacities of the order of 100 to 600 cusecs run intermittently.

7.4 Land Use

The study area is having varying irrigation intensity of about 80% to 30% during monsoon season and 60% to 20% during non-monsoon season. The crops that are generally grown during monsoon season include paddy, maize etc., and during non-monsoon season, the crops are wheat, barseem,pulses, potatoes etc. Besides these crops, sugarcane(a ten month crop) is grown in abundance over the northern portion of the study area with the crop percentage of about 40.

7.5 **Modelling** the Study Area

The following decisions were taken from the analysis of the available data.

a) The aquifer should be treated as two layer system; the top layer extending from 0 m to 34.5 m and the 2nd layer from 34.5 to 104.5 m and separated by an aquitard whose thickness varies depending upon the sand percentage. The permeability of the aquifer is 38 m/day with a specific capacity of 12% at the zone of fluctuation of water table. The ratio of permeabilities between the aquifer and aquitard is to be estimated by the model, through simulation of average steady state conditions.

b) The ground water system is in a state of dynamic equilibrium and hence the calibration of the model may be done on a mean year condition.

c) The major rivers which envelope 80% of area are considered as head fixed boundaries for both layers. The other part of the external boundary consists of major canals which are treated as 'head fixed' for upper layer and 'gradient' for the lower layer. The internal main/branch canals and medium sized rivers are treated as 'head fixed' for both layers.

d) Tyson Weber approach of the finite difference method is adopted for simulating the ground water system. Accordingly, the study area is divided into 351 polygons with an average polygonal area of about 70 sq.km, keeping in view the internal boundaries.

e) The recharge/abstraction components have been identified and they are transferred to these polygonal areas.

7.6 Recharge-Abstraction Components

The various recharge and abstraction components in the study area have been identified. They are:

a) recharge components

(i) recharge from rainfall during monsoon period only.

(ii) recharge from seepage losses from the canal conveyance system.

(iii) recharge from seepage losses from ground water conveyance system.

(iv) recharge from irrigated areas.

(b) abstraction components

(i) extraction from private and public tube wells.

(ii) consumptive utilisation of scattered trees.

(iii) direct evaporation from the shallow river bed areas.

(iv) consumptive use by crops and vegetation from shallow water table areas.

Each of the above components have been estimated using the available data.

7.7 Model Calibration

For calibration of the ground water model the following criteria have been adopted:

(a) Matching of mean year piezometry for both the layers and for both the seasons.

b) Matching the change in storage with observed values.

c) Comparing the baseflows in rivers and the main and branch canal seepages with the corresponding estimated values.

To solve the transient problems, the time discretization has been done using the finite difference expression. In the present case, the model is simulated by two season approach. The entire model programme is structured so that unnecessary calculations and repetitions are avoided wherever it is possible, and to achieve this it was divided into three modules viz.,

a) Programme dealing with the geometry of the area, the polygonal net work and the aquifer parameters.

b) Programme dealing with the recharge and discharge components from various sources excluding main and branch canal seepage and river baseflows.

c) Programme dealing with the solution of differential equation. Gauss-Siedel iteration technique is used for solving the equations.

Suitable strategy has been adopted for the calibration of the model using the criteria mentioned earlier.

After the calibration of the model with the mean year data(all the data pertaining to piezometric levels, crops, recharge components and extraction components reduced to corresponding mean values), the model is validated using the historical data from 1975 to 1979 in a continuous sequence.

Using this validated model, future forecast runs for arriving at developmental strategies like lining of various portions of the canals, increase in the shallow/deep pumping and introduction of new canal system in the present existing system have been studied.

8.0 CONCLUSIONS

Proper management of groundwater resources requires a realistic quantification of these resources, which in turn can be made only through proper understanding the behaviour of the aquifer system. Regional aquifer simulation is the only way by means of which the aquifer behaviour can be predicted under different stress condition. However, the simulation of a complex aquifer system poses varied problems. Huge amount of data are to be collected and inferences have to be drawn using these data. The different types of data, the procedures to be adopted for such collection have been indicated. Depending on the site conditions and scope of the study either of the methods or combination of methods can be used for the collection of relevant data.. The various approaches for the modelling are indicated. The parameter estimation using inverse problem techniques was also discussed. The commonly used solution techniques for solving the resulting algebraic equations are mentioned. A case study conducted by the Institute is presented to indicate the philosophy of the application of the simulation technique to a real life situation. Though, the process/steps to be followed for simulating a basin, it is to be realised that modelling is the art and science of applying various investigatory methods, checking their results against one another, and representing the complexity of nature in a simplified form that is amenable to mathematical treatment.

REFERENCES

1. Brebbia,C.A.,(1978),"The Boundary Element Method for Engineers", Pentech Press,London.

2. Brebbia,C.A., and S.Walker(1980),"Boundary Element Techniques in Engineering", Butterworths, London.

3. Bredehoeft,J.D. and G.F.Pinder(1970),"Digital Analysis of Areal Flow in Multiaquifer Groundwater System:a Quasi Three Dimensional Approach", Water Resour.Res.,Vol.6,No.3.

4. Cecen,K and E.Omay(1973),"Three Dimensional Viscous Flow Analogy", Proc.XV Congress of I.A.H.R.Vol.3,Istanbul,Turkey.

5. Celia,M.A., and G.F.Pinder,(1980),"Alternating direction collection solution to the transport equation", In Proceedings of the 3rd International Conference on Finite Elements in Water Resources, S.Y.Wang,C.V.Alsonso,C.A.Brebbia,W.G.Gray,and G.F. Pinder,eds., pp.3.36-3.48,University of Mississippi.

6. Chaturvedi,M.C. and V.K.Srivastava(1979),"Induced Groundwater Recharge in the Ganges Basin", Water Resources Research,Vol.15, No.5.

7. Collins,M.A.,L.W.Galhar and J.L.Wilson III(1972),"Hele Shaw Model of Long Island Aquifer System", Proc.A.S.C.E.,Journal of the Hydraulic Division,Vol.98,HY9.

8. Cooper,H.H.,Jr(1966),"The Equation of Groundwater Flow in Fixed and Deforming Coordinates", J.Geophys.Res.71(20):4785-4790.

9. Darcy,H.(1856),"Les Fontaines Publiques de La Ville de Dijon", V.Dalmont,Paris.

10. Davis,S.N. and R.J.M.De Weist(1966),"Hydrogeology", John Wiley and Sons,New York.

11. De Weist,R.J.M.(1966),"On the Storage Co-efficient and the Equations of Groundwater Flow,J.Geophys.Res.,71:1117-1122.

12. Douglas,J.,Jr.,Dupont,T., and Rachford,H.H.Jr.(1969). The application of variational methods to water flooding problems,J.Can. Pet.Technol.8,79-85.

13. Eshett,A., and R.A.Logenbough(1965),"Mathematical Model for Transient Flow in Porous Media", Progress Report,Civil Engineering Section,Colorado State University, Fort Collins,Colorado.

14. Fiering,M.B.(1964),"A Digital Computer Solution for Well Field Draw Down;Bull.Intern.Assoc.Sci.Hydrol.,9(4).

15. Finlayson,B.A.(1972),"The Method of Weighted Residuals and Variational Principles", Academic Press, New York.

16. Freeze,R.A.(1971),"Three Dimensional, Transient, Saturated-Unsaturated Flow in a Groundwater Basin", Water Resour.Res.,Vol.7, No.2.

17. Frind,E.O., and Pinder,G.F.(1979),"A Collection finite element method for potential problems in irregular domain, Int.J.Num. Methods Eng.14,681-701.

18. Green,D.W.,E.C.Pogge,R.M.Knapp,B.Schorn and C.Stanford(1973),"The Development and Field Testing of Basin Hydrology Simulator", The University of Kansas Centre for Research,Inc.,Lawrence,Kansas,U.S.A.

19. Huyakorn,P.S.(1973),"Finite element solution of two-regime flow toward wells", Water Research Lab.,Report No.137, University of New South Wales,Australia.

20. Huyakorn,P.S. and G.F.Pinder(1983),"Computational methods in subsurface flow,"Academic Press, New York.

21. Jacob,C.E.(1950),"Chapter 5 In: Engineering Hydraulics(Ed),H. Rouse,John Wiley and Sons,New York,P.321-386.

22. Jaswon,M.A., and Ponter,A.R.(1963),"An integral equation solution of the torsion problem," Proc.R.Soc.London,Ser.A.273,237-246.

23. Javandel,I., and Witherspoon,P.A.(1968),"Application of the finite element method to transient flow in porous media,Soc.Pet. Eng.J.8,241-252.

24. Jorgenson,D.G.(1975),"Analog Model Studies of Groundwater Hydrology in the Houston District,Texas,Texas Water Development Board, Report 190.

25. Konikov,L.F. and J.D.Bredehoeft(1978),"Computer Model of Two Dimensional Solute Transport and Dispersion in Groundwater", Chapter C 2,Book 7,U.S.Geol.Surv.Reston,Virgina.

26. Lahr,J.A.(1963),"Ground Water Flow Models Simulating Sub-surface conditions", Journal of Geological Education,Vol.11.

27. Liggett,J.A.(1977),"Location of sub surface in porous media,ASCE J,Hydraul.Div.,103,(HY4),353-365.

28. Mc Donald,M.G. and A.W.Harbaugh(1984)," A Modular 3-dimensional Finite Difference Ground Water Flow Model USGS open file report 83-875.

29. McMichael,C.L., and Thomas,G.W.(1973),"Reservoir simulation by Galerkin's method,Soc.Pet.Eng.J.13(3),125-138.

30. Mercer,J.W. and G.F.Pinder(1975),"A Finite Element Model of Two Dimensional Single Phase Heat Transport in a Porous Medium", U.S.Geol.Surv.Open File, Report 75-574.

31. Meyer,W.,J.P.Reussow, and D.C.Gillies(1975),"Availability of Groundwater in Marion Country, Indiana, U.S.G.S.Open File Report 75-312.

32. Neuman,S.P., and Witherspoon,P.A.(1970),"Finite element method of analyzing steady seepage with a free surface, Water Resour. Res.6,889-897.

33. Neuman,S.P., and Witherspoon,P.A.(1971),"Analysis of nonsteady flow with a free surface using the finite element method, Water Resour.Res.7,611-623.

34. Pinder,G.F. and E.O.Frind(1972),"Application of Galerkin's Procedure to Aquifer Analysis" Water Resources Research 8(1):108-120.

35. Pinder,G.F., and W.G.Gray(1977),"Finite Element Simulation in Surface and Subsurface Hydrology", Academic Press, New York.

36. Pinder,G.F.,E.O.Frind and M.A.Celia(1978),"Groundwater flow simulation using collection finite elements", In Proceedings of the 2nd International Conference on Finite Elements in Water Resources", C.A.Brebbia,W.G.Gray and G.F.Pinder,eds.,1.171-1.185, Pentech Press,Plymouth,England.

37. Prasad,R.K.(1981),"Finite element simulation of groundwater system,"Ph.D.thesis submitted to Indian Institute of Technology, Delhi, India.

38. Price,H.S.,R.S.Varga and J.E.Warren(1968),"Numerical Methods of Higher Order Accuracy for Diffusion-Convection Equations", Soc.Petrol.Eng.J.8(3):293-303.

39. Prickatt,T.A. and C.G.Lonnquist(1971),"Selected Digital Computer Techniques for Groundwater Resource Evaluation",Bull.No.55,Illinois State Surv.Urbana,U.S.A.

40. Prickett,T.A.(1975),"Modelling Techniques for Groundwater Evaluation" in Advances in Hydroscience, Vol.10,edited by V.T.Chow,-Academic Press.

41. Remson,U.,G.M.Harnberger and F.J.Molz(1971),"Numerical Methods in Subsurface Hydrology", Wiley and Sons,Inc.,New York.

42. Richtmeyer,R.D. and K.W.Morton(1967),"Difference Methods for Initial Problems", Inter Science, New York.

43. Rushton,K.R.(1974),"Critical Analysis of the Alternative Direction Implicit Method of Aquifer Analysis", Journal of Hydrology, No.21.

44. Santing,G.(1958),"A Horizontal Scale Model Based on the Viscous Flow Analogy for Studying Groundwater Flow in an Aquifer Having Storage", I.A.S.H.Publication, 43.

45. Seethapathi,P.V.(1979),"Finite element solutions of Darcy and Forchheimeur flows around well with storage",Ph.D.thesis submitted to Indian Institute of Technology, Kharagpur, India.

46. Slichtler,C.S.(1899),"Theoretical Investigation of the Motion of Ground Water", U.S.Geological Survey, 19th Ann.Rept.Pt.2:259-389.

47. Smith,G.D.(1965),"Numerical Solution of Partial Differential Equations", Oxford University Press, New York.

48. Stone,H.L.(1968),"Iterative Solution of Implicit Approximations of Multi-Dimensional Partial Differential Equations", SIAM-Jour. Numr.Analysis,Vol.5.

49. Taylor,R.L., and C.B.Brown(1967),"Darcy flow solutions with a free surface," Proc.Am.Soc.Civ.Eng.93(HY2),25-33.

50. Theis,C.V.(1935),"The Relation between Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage", Trans.Amer.Geophys.Union 11:519-524.

51. Theis,C.V.(1938),"The Significance and Nature of the Cone of Depression in Ground Water Bodies", Econ.Geol,33:889-920.

52. Todd,D.K.(1959),"Ground Water Hydrology", John Wiley and Sons, New York.

53. Todd,J.(ed),(1962),"Survey of Numerical Analysis", McGraw Hill Book Company, New York.

54. Trescott,P.C.(1973),"Iterative Digital Model for Aquifer Evaluation", U.S.Geol.Survey Open File Report.

55. Trescott,P.C.(1975),"Documentation of Finite Difference Model for Simulation of Three Dimensional Ground Water Flow", U.S.Geol. Survey Open File Report 75-438.

56. Trescott,P.C. and S.P.Larson(1977),"Comparison of Iterative Medthods of Solving Two Dimensional Groundwater Equation", Water Resour.Res.Vol.13,No.1.

57. Volker,R.E.(1969),"Nonlinear flow through porous media by finite elements,ASCE J.Hydraul.Div.95 (HY6),2093-2114.

58. Voss,C.I.(1984),"Saturated Unsaturated Transport-A finite element simulation model for saturated-unsaturated,fluid-density-dependent ground-water flow with energy transport of chemically-reactive single-species solute transport,U.S.Geological Survey Water Resources Investigation Report No.84-4369.

59. Walton,W.C.(1970),"Ground Water Resource Evaluation",McGraw Hill Book Company.

60. Zienkiewicz,O.C.,P.Meyer, and Y.K.Cheung (1966),"Solution of anisotropic seepage problems by finite elements, Proc.Am.Soc.Civ. Eng.92(EM1),111-120.