

STATISTICAL MODELLING OF ANNUAL MAXIMUM PEAK FLOODS

1.0 INTRODUCTION

In many problems in hydrology, the data consists of measurements on a single random variable; hence we must deal with univariate analysis and estimation. The objective of univariate analysis is to analyse measurements on the random variable, which is called sample information, and identify the statistical population from which we can reasonably expect the sample measurements to have come. After the underlying population has been identified, one can make probabilistic statements about the future occurrences of the random variable, this represents univariate estimation. It is important to remember that univariate estimation is based on the assumed population and not the sample, the sample is used only to identify the population.

Hydrologic processes such as rainfall, snowfall, floods, droughts etc. are usually investigated by analysing their records of observations. Many characteristics of these processes may not represent definite relationship. For example, if you plot instantaneous peak discharges from each year for a river, a rather erratic graph is obtained. The variation of peak discharge from one year to another can not be explained by fitting a definite relationship, which we call as deterministic relationship. For the purpose of hydrologic analysis, the annual peak discharge is then considered to be a random variable. Methods of probability and statistics are employed for analysis of random variables. In this lecture, some elementary concepts of probability and statistics are presented, which are used for frequency analysis in hydrology.

Frequency analysis is performed to determine the frequency of the likely occurrence of hydrologic events. This information is needed in the solution of a variety of water resources problems. Some pertinent examples include, design of reservoirs, flood ways, bridges, culverts, highways, levees, urban drainage systems, air field drainage, irrigation system, stream-control works, water supply systems, and hydro-electric power plants; zoning of flood plain land for industrial, residential and recreational use; economic evaluation of flood protection projects; setting of flood insurance premium; drought-mitigation problems etc. Although, the frequency analysis of virtually every component of the hydrologic is required, our emphasis here will be on flood frequency analysis only.

Estimation of flood magnitudes and their frequencies for planning and design of water resources projects have been engaging attention of the engineers the world over since time immemorial. Estimation of magnitudes of likely occurrence of floods is of a great importance for solution of a variety of water resources problems such as design of various hydraulic structures, urban drainage systems, flood plain zoning and economic evaluation of flood protection works etc.

Objectives

After going through this lecture participants will be able to:

- ☛ define some of the important terms related to statistical and probabilistic methods in hydrology.
- ☛ compute the statistics representing the measure of location, measure of dispersion and measure of symmetry
- ☛ compute the standard errors of some of the important sample statistics such as mean, standard deviation and coefficient of skewness
- ☛ fit some of the theoretical frequency distributions such as Normal, Log normal, Extreme value Type-I, Pearson Type-III and Log Pearson type-III distribution to the site data
- ☛ estimate the parameters of some of the distributions particularly by the method of moments

- ☛ compute standard errors as well as confidence limits over the flood estimates
- ☛ identify the robust frequency distribution based on some of the goodness of fit tests.

The following assumptions are implicit in frequency analysis in order to have meaningful estimates from flood frequency analysis:

- i. The data to be analysed describe random events.
- ii. It is homogeneous.
- iii. The population parameters can be estimated from the sample data.
- iv. It is of good quality.

If the data available for analysis, do not satisfy any of the above listed assumptions then, much reliability can not be attached to the estimates.

For flood frequency analysis, either annual flood series or partial duration series may be used. The requirements with regard to data are that,

- i. it should be relevant,
- ii. it should be adequate and,
- iii. it should be accurate.

In general, an array of annual peak flood series may be considered as a sample of random and independent events. The non-randomness of the peak series will, however, increases the degree of uncertainty in the derived frequency relationship. Various tests are available to check the randomness of the peak flow data. The annual maximum flood series can generally be regarded as consisting of random events as the mean interval of each observed flood peak is 1 year. However, in the case of data used for partial duration series analysis, the independence among the data is doubtful. The peaks are selected in such a way that they constitute a random sample.

The term relevant means that data must deal with problem. For example, if the problem is of duration of flooding then data series should represent the duration of flows in excess of some critical value. If the problem is of interior drainage of an area then data series must consist of the volume of water above a particular threshold.

The term adequate primarily refers to length of data. The length of data primarily depends upon variability of data and hence there is no guideline for the length of data to be used for frequency analysis.

The term accurate refers primarily to the homogeneity of data and accuracy of the discharge figures. The data used for analysis should not have any effect of man made changes. Changes in the stage discharge relationship may render stage records non-homogeneous and unsuitable for frequency analysis. It is therefore preferable to work with discharges and if stage frequencies are required then most recent rating curve is used. Watershed history and flood records should be carefully examined to ensure that no major watershed changes have occurred during the period of record. Only records, which represent relatively constant watershed conditions should be used for frequency analysis.

2.0 FUNDAMENTAL TERMS

In this section of the lecture some of the important statistical terms are defined:

(a) Population

A population is a collection of persons or objects, e.g.(i) the pupils in a school, the workers in a factory, the people in a country, (ii) motor cars produced in a factory. Each unit of the population has many different possible attributes associated with it. These attributes might be (i) height, volume or weight which are measurable on a scale, or (ii) colour, condition which may not be numerically measurable

(b) Sample data

Sample data are available data from the observation of an event.

(c) Random events

Events whose occurrence is not influenced by the occurrence of the same event earlier.

(d) Probability density function

Probability density function (P.D.F.) is the probability of occurrence of an event.

(e) Cumulative density function

Cumulative density function (C.D.F.) is the probability of occurrence of all the events that are equal to or less than an event.

(f) Probability paper

A probability paper is a special graph paper on which the ordinate usually represents the magnitude of the variate and the abscissa represents the probability P, or the return period T. The ordinate and abscissa scales are so designed that the distribution plots more nearly a straight line permitting better definition of the upper and lower parts of the frequency curve. The probability paper is used to linearize the distribution so that data to be fitted appear close to the straight line. For example, the extreme value and the log normal probability papers are used for linearization of the extreme value and log normal distribution.

(g) Plotting position

Determining the probability to assign a data point is commonly referred to as determining its plotting position.

(h) Return Period

Return period (T) or recurrence interval is the time that elapses on an average between two events that equal or exceed a particular level. For example, T year flood will be equaled or exceeded on an average once in T years.

(i) Probablity of Exceedence

If a coin is tossed once a year. On the average, its head will appear once in 2 years, or probability of exceedance(P) is $P = 1/2 = 0.5$. The reciprocal of probablity of occurrence is termed as return period, T .

$$P = 1/T$$

Thus a storm that has been exceeded on the average once in 10 years has a probability of exceedance in any one year $P = 1/10$ or 0.1.

(j) Probablity of Non-Exceedence

The probablity that the storm will not occur is termed as the probablity of non-exceedance (P').

$$P' = 1 - P = 1 - 1/T$$

(k) Peak Annual Discharge

The peak annual discharge is defined as the maximum instantaneous volumetric rate of discharge during a year.

(l) Annual Flood Series

The annual flood series is the sequence of the peak annual discharges for each year of the record.

(m) Partial Duration Flood Series

The partial series consists of all recorded floods above a particular threshold regardless of the number of such floods occurring each year.

(n) Design Flood

Design flood is the maximum flood which any structure can safely pass. It is the adopted flood to control the design of a structure.

3.0 SAMPLE STATISTICS

In any analysis of statistical data in general and of hydrolytic data in particular, certain calculations are usually made in order to determine some of the basic properties inherent in the data. For instance, the sample mean and variance are two statistics defining the most important characteristics of a given set of statistical data. In general sample statistics provide the basic information about the variability of a given data set. The most useful sample statistics measure the following characteristics.

- (i) the central tendency or value around which all other values are clustered.
- (ii) the spread of the sample values around mean
- (iii) the asymmetry or skewness of the frequency distribution and
- (iv) the flatness of the frequency distribution.

These statistical properties are determined by sample statistics as described below:

3.1 Measure of Central Tendency or Measures of Location

In statistics various measures of location are described. One of the important measure of central tendency is mean.

(i) Mean

The sample mean is the measure of central tendency of a given data set. If $X_1, X_2, X_3, \dots, X_n$ represent a sequence of observations, the mean of the sequence is given by:

$$\bar{X} = \frac{\sum_{i=1}^N X_i}{N} \quad (1)$$

3.2 Measure of Dispersion or Variation

(i) Standard deviation

The unbiased estimate of population standard deviation (S) from the sample is given as:

$$S = [1/(N-1) \sum_{i=1}^N (x_i - \bar{x})^2]^{1/2} \quad (2)$$

(ii) Variance

Variance represents dispersion about the mean. Mathematically for sample it is expressed as:

$$Variance = S^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N} \quad (3)$$

(iii) The coefficient of variation

The coefficient of variation C is a dimensionless dispersion parameter and is equal to the ratio of the standard deviation and the mean

$$C_v = S/\bar{X} \quad (4)$$

This coefficient is extensively used in hydrology particularly as a regionalisation parameter.

The range and mean deviation have the same units (dimension) as the original data. The variance has the square of the units of the original data and hence can not be directly compared with the data. Therefore, the standard deviation is used because its dimensions are that of the data.

In many samples of hydrological data, especially in flood hydrology the largest value is very much larger than the second largest. Therefore the range R might not be a good indicator of the scatter inherent in the data as a whole.

The mean deviation is a good measure of spread but can not be handled easily in mathematical statistics because of the absolute value sign while the same applies to the interquartile range. The variance is more easily handled mathematically and holds a prominent place. The interquartile range is easy to evaluate but is very difficult from a mathematical point of view and hence is not much used even though it is quite good at describing spread.

3.3 Measures of Symmetry

If the data are exactly symmetrically displaced about the mean than the measure of symmetry should be zero. If the data to the right of the mean (larger) are more spread out from the mean than those on the left then, by convention, the asymmetry is positive and vice versa for negative asymmetry.

(i) Skewness Coefficient

The skewness coefficient or coefficient of skewness represents a non-dimensional measure the asymmetry of the frequency distribution of the data. An unbiased estimate of the coefficient is given by:

$$C_s = \frac{N \sum_{i=1}^N (x_i - \bar{x})^3}{(N-1)(N-2)S^3} \quad (5)$$

The skewness coefficient has an important meaning since it gives indication of the symmetry of the distribution of the data. Symmetrical frequency distributions have very small or negligible sample skewness coefficient C_s , while asymmetrical frequency distributions have either positive or negative coefficients. Often a small value of C_s , indicate that the frequency distribution of the sample may be approximated by the normal distribution function since $C_s = 0$ for this function.

Note that because of the third Central Moment has dimension equal to the cube of the data, it is not of direct use. It also depends on the units of the original data. The co-efficient of skewness does not have this disadvantage and is therefore preferred. The interquartile measure of symmetry (I_{as}) is not dimensionless.

3.4 Measures of Peakedness or Flatness

The Kurtosis coefficient measures the peakedness or the flatness of the frequency distribution near its center. An unbiased estimate of this coefficient is given by:

$$C_k = \frac{N^2 \sum_{i=1}^N (x_i - \bar{x})^4}{(N-1)(N-2)(N-3)S^4} \quad (6)$$

A related coefficient called the excess coefficient denoted by E is defined by:

$$E = C_k - 3 \quad (7)$$

Positive values of E indicate that a frequency distribution is more peaked around its center than the normal distribution. Frequency distribution is known as LEPTOKURTIC the negative values of E indicate that a given frequency distribution is more flat around its center than the normal. Frequency distribution is known as PLATYKURTIC.

Normal distribution is said to be MESOKURTIC. Both kurtosis and excess coefficient are seldom used in statistical hydrology.

4.0 PROBABILITY DISTRIBUTION

A distribution is an attribute of a statistical population. If each element of a population has a value of X then the distribution describes the constitution of the population as seen through its X values. It tells whether they are in general very large or very small , that is their location on the axis . It tells whether they are are bunched together or spread out and whether they are symmetrically disposed on the X axis or not. These three things are described by the mean, standard deviation and skewness.

Distribution also telis the relative frequency or proportion of various X values in the population in the same way that a histogram gives that information about a sample. These relative frequencies are also probabilities and hence the distribution tells us the probability, $\Pr(X < x)$, that the X value on an element drawn randomly from the population would be less than a particular value x. Knowing $\Pr(X < x)$ for all x values , the laws of probability may then be used to deduce the probability of any proposition about the behaviour of a random sample of X values drawn from the population.

When the population is sufficiently large the histogram of its X values can be made with very small class intervals and the histogram can be replaced by a smooth curve , the area enclosed by any two vertical ordinates being the relative frequency or probability of x values between those ordinates.

Because of this probability interpretation , a relative frequency distribution is also called a probability distribution and the curve describing it is called a probability density function (p.d.f) whose cumulative function is called the distribution function (d.f.).

In flood frequency analysis, the sample data is used to fit probability distribution which in turn is used to extrapolate from recorded events to design events either graphically or analytically by estimating the parameters of the distribution. A large number of peak flow distributions are available in literature.

A large number of frequency distributions are available in literature. Some of the commonly used probability distributions in the area of hydrology and water resources are: normal, log normal (two parameters, and three-parameters), Extreme value type-I (Gumbel or EV1), Pearson type-III , Log Pearson type-III ,General extreme Value , Gamma and Exponential distributions. The probability density functions (P.D.F.), Cumulative density functions (C.D.F.) and other properties of these "Theoretical Frequency Distributions" are given in Appendix-I. Here the normal, log normal (two parameters), Extreme value type-I (Gumbel or EV1), Pearson type-III and Log Pearson type-III distributions have been discussed. The probability density functions (P.D.F.), Cumulative density functions (C.D.F.) and other properties of these distributions are given below:

4.1 Normal Distribution

The normal distribution is one of the most important distribution in statistical hydrology. This is a bell shaped symmetrical distribution having coefficient of skewness equal to zero. The normal distribution enjoys unique position in the field of statistics due to central limit theorem. This theorem states that under certain very-broad conditions, the distribution of sum of random variables tends to a normal distribution irrespective of the distribution of random variables, as the number of terms in the sum increases. The Probability Density Function (PDF) and Cumulative Density Function (CDF) of the distribution are given in Appendix-I.

4.2 Log Normal Distribution

The causative factors for many hydrologic variables act multiplicatively rather than additively and so the logarithms of these variables which are the product of these causative factors follow the normal distribution.

If $Y = \log_e(X)$ follows normal distribution, the X is said to follow log normal distribution. If the variable X has a lower bound X_o , different from zero and the variable $Y = \log_e(X-X_o)$ follows normal distribution then X is log normally distributed with three parameters. The PDF and CDF of the distribution are given in Appendix.

4.3 Pearson type-III Distribution (PT3)

Pearson type III distribution is a three parameter distribution. This is also known as Gamma distribution with three parameters. The PDF and CDF of the distribution are given in Appendix-I.

4.4 Exponential Distribution

Exponential distribution is a special case of Pearson type III distribution when shape parameter $\gamma = 1$. The PDF and CDF are given in appendix-I.

4.5 Gumbel Extreme Value (Type-I) Distribution (EV1)

One of the most commonly used distributions in flood frequency analysis is the double exponential distribution (known as Gumbel distribution or extreme value type I or Gumbel EV1 distribution). The PDF and CDF of the distribution are given in Appendix-I.

4.6 Log Pearson Type-III Distribution (LP3)

If $Y = \log_e(X)$ follows Pearson type III distribution then X is said to follow log Pearson type III distribution. In 1967, the U.S. water Resources Council recommended that the log Pearson type III distribution should be adopted as the standard flood frequency distribution by all U.S. federal government agencies. The PDF and CDF of the distribution are given in Appendix-I.

4.7 Extreme Value Distribution

Just as there is a family of Pearson type III distributions, each member being characterised by a value of γ , there is also a family of EV distributions, each member of which is characterised by the value of a parameter denoted by k . The family can be divided into three classes, corresponding to different ranges of k values. The three classes are referred as Fisher Tippett type 1, type 2 and type 3. They

are also known as EV-1, EV-2 and EV-3 distributions. In practice, k value lie in the range -0.6 to + 0.6. For EV-1 distribution k is zero and coefficient of skewness is equal to 1.139. For EV-2 distribution, the values of k is -ve and skewness is greater than 1.139. For EV3 distribution the value of k is +ve and coefficient of skewness is less than 1.139. EV-1 and EV-2 distributions are known as Gumbel and Frechet distributions respectively.

5.0 METHODS OF PARAMETER ESTIMATION

There are four well known parameter estimation techniques viz:

- (i) Graphical
- (ii) Least squares
- (iii) Method of moments , and
- (iv) Method of maximum likelihood.

5.1 Graphical Method

In graphical method of parameter estimation, the variate under consideration is regarded as a function of a reduced variate of a known distribution. The steps involved in the graphical method are as follows:

- (i) Arrange the variates of annual maximum flood series in ascending order and assign different ranks to the individual variates.
- (ii) Assign the plotting positions to each of the variates. The plotting position formula may be used depending upon the type of distribution being fitted. The recommended plotting position formulae for normal, log normal, Gumbel EV-I, Pearson type-III and Log Pearson type-III distributions are given in table. 1

Table 1: Unbiased Plotting Position Formulae

Distributions(s)	Recommended plotting position formula	Form of the plotting position formulae
Normal and Log Normal	Blom	$(m-3/8)$ $F(X \leq x) = \frac{(m-3/8)}{(N+1/4)}$
Gumbel EV-I	Gringorton	$(m-0.44)$ $F(X \leq x) = \frac{(m-0.44)}{(N+0.12)}$
Pearson type III and Log Pearson type III	Cunnane	$(m-0.40)$ $F(X \leq x) = \frac{(m-0.40)}{(N+0.20)}$

where, m indicates the rank number, m=1 for the smallest observation and m=N for the largest observation when flood series is arranged in ascending order.

(iii) Estimate the reduced variates for the selected Distribution corresponding to different plotting positions, which represent the probability of non-exceedence.

The reduced variates for normal and log normal distributions are computed with the help of table 2.

For Gumbel (EV1) Distribution, the reduced variates are computed using eq. (15) of Appendix-I, where $F(z)$ represents probability of non-exceedence which have been computed using the suitable plotting position formula, and z is the corresponding reduced variates.

In case of Pearson type-III and Log Pearson type-III which are three parameter distributions different sets of reduced variates are obtained for different coefficients of skewness.

At this stage it is essential to introduce the concept of frequency factor as it has been found easier to develop probability paper based on the frequency factor, K for Pearson type-III and Log Pearson type-III distributions on comparison to that developed based on the reduced variate.

Frequency Factor

A general frequency equation generally applicable for different distributions was proposed by Chow in the following form:

$$X = m + Ks \quad (8)$$

where,

X = the magnitude of the different variates of the peak flood series, or the magnitude of the flood at required return period T .

K = the frequency factor corresponding to X . For Pearson Type-III and Log Pearson Type-III distribution, the values of frequency factors are given in appendix-II at different probability of exceedence levels corresponding to the various coefficient of skewness values.

m and s = mean and standard deviation of the population which would be replaced by the sample statistics.

(iv) Plot the sample data as a series of discrete points on an ordinary graph paper with the ordinate being the variate and the abscissa being the reduced variate or frequency factor. Such plot can be prepared for different distribution.

(v) Draw a best fit line through the plotted points. The slope and intercept of the line provide the estimates for the parameters of the respective distributions. This straight line can be projected to arrive at the flood magnitudes of desired return periods.

In graphical estimation procedure the line is subjectively placed and could vary with analyst. This subjectivity is regarded as a major drawback by hydrologists.

The following example provides the procedure for estimating the reduced variate corresponding to a given probability level for Normal, EV1 and Pearson Type- III distributions:

Example 1:

- (i) Find out the value of the reduced variates for Normal and EVI distributions corresponding to probability of non-exceedence of 0.30 from Appendix-II.
- (ii) Find out the frequency factor K for Pearson Type-III distribution corresponding to non-exceedence probability of 0.30 and coefficient of skewness of 1.3 from Appendix-III.

Solution:

(i) For Normal distribution use table given as appendix-II to estimate the reduced variate corresponding to probability of non-exceedence equal to 0.30. In this table, areas under the normal curve are given. Since the probability of non-exceedence, $F(z)$ is 0.3 which is less than 0.5, the value of Z will be a negative quantity.

Find out a value of Z' which has non-exceedance probability = 0.30.

$$Z' = -0.5244.$$

The reduced variate corresponding to $F(Z)=0.3$ can be obtained for EVI distribution using eq.(15) given in Appendix-I.

$$0.30 = e^{-e^{-z}}$$

$$Z = -\ln(-\ln(0.30)) = -0.186$$

(ii) Table given as Appendix-III may be used to estimate the frequency Factor, K for Pearson Type III distribution.

$$\text{Probability of exceedence} = 1 - 0.30 = 0.70$$

$$\text{and coefficient of skewness} = 1.3$$

$$\text{Frequency Factor, } K = -0.634. \text{ (From the table of appendix-III)}$$

5.2 Least Squares Method

In the method of least squares for the parameter estimation the steps from (i) to (ii) of the Graphical method may be repeated. In this technique a simple linear regression equation is fitted between the variate under consideration and the corresponding reduced variate or frequency factor, rather than drawing a best fit line subjectively on a simple graph paper.

This method has not been accepted as a standard method in practice as it involves the use of plotting position formula to determine the reduced variate or the frequency factor and due to the assumption that the error variance remain same for all observations. The defect due to the former assumption could be eliminated by using the appropriate plotting positions formula given in table 1. However, the later assumption makes the method more defective as the higher events recorded have more error variance than the recorded lower events. All these assumptions affect the correct estimation of the slope and intercept of the line, which represent the parameters of the distribution.

5.3 Method of Moments

The method of moments makes use of the fact that if all the moments of a distribution are known then everything about the distribution is known. For all the distributions in common usage, four moments

or fewer are sufficient to specify all the moments. For instance, two moments, the first together with any moment of even order are sufficient to specify all the moments of the Normal Distribution and therefore the entire distribution. Similarly, in the Gumbel EV Type-I Distribution, the first two moments are sufficient to specify all the moments and hence the distribution. For Pearson Type III Distribution three moments, always taken as the first three required to specify all the moments. In these cases the number of moments needed to specify all the moments and hence the distribution equals the number of parameters.

The method of moments estimation is dependent on the assumption that the distribution of variate values in the sample is representative of the population distribution. Therefore, a representation of the former provides an estimate of the later. Given that the form of the distribution is known or assumed, the distribution which the sample follows is specified by its first two or three moments calculated from the data.

Normal Distribution

The parameters of the normal distribution, μ and σ , which describe the characteristics of the given set, are computed as:

$$\mu \equiv X = \frac{1}{N} \sum_{i=1}^N X_i \quad (9)$$

$$\sigma \equiv s = \frac{\sqrt{\sum(X_i - \bar{X})^2}}{(N-1)} \quad (10)$$

After computing the parameters, μ and σ , the T-year flood estimates can be obtained using the Normal Distribution. The steps are:

- (i) Compute the \bar{X} and s from the sample data using eq. (1) and (2). \bar{X} and s provide estimates for the parameter, μ and σ .
- (ii) Compute the probability of non-exceedence using the relation:

$$F(X < x) = 1 - 1/T \quad (11)$$

- (iii) Compute the Normal Reduced Variate Z_T from the table given as Appendix-II corresponding to the probability of non-exceedence computed at step (ii).

- (iv) Estimate the flood for T-year recurrence interval using the following equation:

$$X_T = \mu + \sigma Z_T \quad (12)$$

Log Normal Distribution (two parameters)

In log normal two parameter distribution the variates are transformed to the log domain by taking log of each variate, and then the mean and standard deviation of the transformed series are computed as given by eq. (1) and (2). The flood at required recurrence interval can be computed in the following steps after fitting the log normal distribution with the sample data.

- (i) Transform the original series of peak flood data into log domain by taking log of each variate to the base e.
- (ii) Compute the mean, \bar{Y} and standard deviation, S_Y , from the log transformed series using eq.(1) and (5) respectively. \bar{Y} and S_Y provide estimates for the parameters, μ_Y and σ_Y , respectively.
- (iii) Compute the probability of non-exceedance for the given recurrence interval (T).
- (iv) Compute the normal reduced variate, Z_T from the table given as Appendix-II which corresponds to the probability of non-exceedance computed at step (iii).
- (v) Estimate the flood for T-year recurrence interval in log domain using the following equation:

$$Y_T = \mu_Y + \sigma_Y Z_T \quad (13)$$

- (iv) Transform the estimated T-year flood in original domain by computing its exponent i.e.

$$X_T = e^{Y_T} \quad (14)$$

Gumbel's Extreme value type-I Distribution

u and α are the two parameters of EV1 distribution. The relationship between the parameters and the statistical moments of the data are given by

$$\mu = u + 0.5772 \alpha \quad (15)$$

$$\alpha = \alpha \pi \sqrt{6} \quad (16)$$

Solving equations (15) and (16) for u and α , we get

$$\alpha = 0.7797 \sigma \quad (17)$$

$$u = \mu - 0.45 \sigma \quad (18)$$

The population statistics, μ and σ would be replaced by the sample statistics, \bar{X} and S computed as given by eq. (1) and (2) respectively. The step by step procedure for computing the T-year flood using EV1 distribution is given below:

- (i) Compute mean, \bar{X} and standard deviation, S , from the sample using eq. (1) and (2) respectively. \bar{X} and S provide estimates for μ and σ respectively.
- (ii) Compute the parameter u and α using eq. (17) and (18).
- (iii) Compute the probability of non-exceedence $F(Z)$ corresponding to T-year recurrence interval
- (iv) Compute the EV1-reduced variate, Z_T , using the relationship;

$$Z_T = -\ln(-\ln F(Z)) \quad (19)$$

- (v) Estimate T-year recurrence interval flood using the EV1 distribution as follows:

$$X_T = u + \alpha Z_T \quad (20)$$

Pearson Type -III Distribution (PT-III)

PT-III distribution is a three parameter distribution. Three moments are, therefore, needed for computing the parameters. Mean, standard deviation and skewness computed from the sample data describe the measures for first three moments of the sample data.

The following steps are usually involved in computing the T-year flood using the Pearson Type-III Distribution.

- (i) Compute the mean, \bar{X} and standard deviation S, using eq (1) and (2) respectively. Compute the coefficient of skewness g from the sample using the following equation:

$$g = \frac{N}{(N-1)(N-2)} \sum_{i=1}^N \frac{(X_i - \bar{X})^3}{S} \quad (21)$$

- (ii) Compute the probability of exceedence for the given recurrence interval, T, which equals to $1/T$.
- (iii) Estimate the frequency factor, K_T from the table given as Appendix-III which corresponds to the computed coefficient of skewness g, and the probability of exceedence.
- (iv) Estimate T-year flood using the equation;

$$X_T = \bar{X} + S K_T \quad (22)$$

Log Pearson Type-III Distribution (LP-III)

While fitting the log Pearson Type-III Distribution, each variate of the annual maximum peak flood series are transformed into log domain. The T-year flood is estimated in log domain after fitting the PT3 distribution to the log transformed series. Subsequently, the T-year flood in real space can be estimated taking the antilog of the T-year flood estimated in log domain. The step by step procedure for computing T-year flood using LP3 distribution is given below:

- (i) Transform the annual maximum peak flood series into the log domain by taking log of each variate to the base e.
- (ii) Compute the mean, \bar{Y} standard deviation, S and coefficient of skewness , g using eq.(1), (2) and (5) respectively.
- (iii) Compute the probability of exceedence corresponding to the given recurrence interval, T which equals to $1/T$.

- (iv) Estimate the frequency factor K_T from table given as Appendix-III which corresponds to the computed skewness, g_Y and the probability of exceedence.
- (v) Estimate the T-year flood in log domain using the following equation.

$$Y_T = \bar{Y} + S_Y K_T \quad (23)$$

- (vi) Estimate T-year flood in real domain as follows:

$$X_T = e^{Y_T} \quad (24)$$

Example 2:

The mean, standard deviation and coefficient of skewness of original and log transformed annual maximum peak flood series of a typical gauging site are given below:

	Original Series	Log transformed series
Mean(m^3/s)	506.843	6.157
Standard dev. (m^3/s)	211.087	0.372
Coefficient of skewness	1.564	0.556

Estimate 1000-years floods assuming that the peak discharge data follow: (i) Log Normal Distribution, (ii) Log Pearson Type-III, (iii) Pearson Type-III and (iv) Gumbel EV1 Distribution.

Solution:

- (i) Flood estimate Assuming Log Normal Distribution

$$\text{Since } \mu_Y = \bar{Y} = 6.157$$

$$\sigma_Y = S_Y = 0.372 \text{ and,}$$

$$\text{Probability of non-exceedence, } F(Z) = 1 - 1/1000 = 0.999$$

$$\therefore Z_T = 3.10 \text{ for } T = 1000 \text{ years (using Table given in Appendix-II)}$$

$$\therefore Y_T = \mu_Y + \sigma_Y Z_T$$

$$Y_{1000} = 6.157 + 0.372 \times 3.10 \\ = 7.3102$$

$$\therefore X_{1000} = e^{Y_{1000}} = e^{7.3102} = 1495 \text{ m}^3/\text{s}$$

- (ii) Flood Estimate Assuming Log Pearson Type -III Distribution

$$\text{Since } \mu_Y = \bar{Y} = 6.157$$

$$\sigma_Y = S_Y = 0.372 \text{ and,}$$

$$\gamma_Y = g_Y = 1.564$$

Probability of exceedence $1-F(Z) = 1/1000 = 0.001$

Frequency Factor, $K_T = 3.8919712$ (using table given in appendix-III)

$$\therefore Y_T = \mu_Y + \sigma_Y K_T$$

$$Y_{1000} = 6.157 + 0.372 \times 3.8919712 = 7.6048$$

$$X_{1000} = e^{7.6048} = 2007.8 \text{ m}^3/\text{s}$$

(iii) Flood estimate assuming Pearson type-III Distribution

$$\text{Since } \mu = \bar{X} = 506.843$$

$$\sigma = S = 211.087 \text{ and,}$$

$$g = g_X = 1.564$$

Probability of exceedence, $1-F(Z) = 1/1000 = 0.001$

Frequency factor $K_T = 5.3214276$ (using table given in Appendix-III)

$$\begin{aligned} \therefore X_T &= \mu + \sigma K_T \\ &= 506.843 + 211.08 \times 5.3214276 \\ &= 1630 \text{ m}^3/\text{s} \end{aligned}$$

(iii) Flood Estimate Assuming Gumbel EV1 Distribution

$$\text{Since } \mu = \bar{X} = 506.843$$

$$\sigma = S = 211.087 \text{ and,}$$

$$\alpha = 0.7797 \times \sigma = 0.7797 \times 211.087 = 164.585$$

$$u = \mu - 0.45 \sigma$$

$$\begin{aligned} u &= 506.843 - 0.45 \times 211.087 \\ &= 411.854 \end{aligned}$$

$$Z_T = -\ln(-\ln(F(Z)))$$

$$F(Z) = 1 - 1/1000$$

$$Z_{1000} = -\ln(-\ln(0.999)) = 6.907255$$

$$X_{1000} = u + \alpha Z_{1000}$$

$$\begin{aligned} \therefore X_{1000} &= 411.854 + 164.585 \times 6.907255 \\ &= 1548.68 \text{ m}^3/\text{s} \end{aligned}$$

6.0 TEST GOODNESS OF FIT

The validity of a probability distribution function proposed to fit the frequency distribution of a given sample may be tested graphically or by analytical methods. Graphical approaches are usually based on comparing visually the probability density function with the corresponding empirical density function of the sample under consideration. In other words model CDF is compared with empirical CDF. Often these CDF graphs are made on specifically designed paper such that the model CDF plots

as a straight line. An example of this is the Gumbel paper. If empirical CDF plots as a straight line on the Gumbel paper it is an indication that the Gumbel distribution may be valid model for the data at hand. Often graphical approaches for judging how good a model is, are quite subjective.

A number of analytical tests have been proposed for testing the goodness of fit of proposed models. One of these tests namely D-index method is presented here under.

D-Index Method

The D-index for the comparison of the fit of various distributions in upper tail is given as:

$$D\text{-index} = \frac{\sum_{i=1}^n \text{Abs}(x_i - \hat{x})^2}{\bar{x}} \quad (25)$$

where, x_i and \hat{x} are the i th highest observed and computed values for the distribution. The distribution giving the least D-index is considered to be the best fit distribution.

7.0 REMARKS

The purpose of the frequency analysis is to estimate the design flood for desired recurrence interval assuming the sample data follow a theoretical frequency distribution. It is assumed that the sample data is a true representative of the population. It is generally seen that minimum 30 to 40 years of records are needed in order to carry out flood frequency analysis to the site data for estimating the floods in extrapolation range, somewhat, within the desired accuracy. In case the length of records are too short, it represents inadequate data situation and at site flood frequency analysis fails to provide the reliable and consistent flood estimates. The regional flood frequency curves together with at site mean is generally able to provide more reliable and consistent estimates of floods under the inadequate data situation. For ungauged catchment, the regional flood frequency analysis approach is the only way to estimate the flood, for desired recurrence interval for which a regional relationship between mean annual peak flood and catchment characteristics is developed along with the regional frequency curves.

Most of the goodness of fit tests used for judging the best fit distribution represent the descriptive ability of the theoretical distributions. Generally the frequency distributions show larger deviations in extrapolation range. Thus a distribution which may pass the goodness of fit criteria not necessarily be able to estimate the floods for higher recurrence intervals to the desired accuracy. In order to judge the performance of the theoretical distributions in predicting the higher recurrence interval floods the predictive ability tests must be taken up using the Monte Carlo experiments with the data, generated by the selected distributions based on the descriptive ability criteria. Such generation studies provide a better opportunity for understanding the characteristics of the theoretical distributions. Those who are interested further in the area of flood frequency analysis may refer the relevant literatures appeared in national as well as international journals dealing with hydrology and water resources.

FURTHER READINGS

1. Chow, V.T., 1964. *Hand book of Applied Hydrology*. Mc-Graw Hill New York.
2. Haan, C.T., 1977. *Statistical Methods in Hydrology*. The Iowa State University Press, U.S.A.
3. Kite, G.W., 1977. *Frequency and Risk Analysis in Hydrology*. Water Resources Publications, Colorado.
4. Linseley, R.K., Kohler, H.A. and Paulhus, J.L.H., 1975. *Hydrology for Engineers*. Mc-Graw Hill, International Book Company.
5. McGuess, R.H. and Snyder, W.M., 1985. *Hydrologic Modelling, Statistical Methods and Applications*. Prentice-Hall, Englewood Cliffs, New Jersey.
6. Yevjevich, V., 1972. *Probability and Statistics Hydrology*. Water Resources Publications, Fort Collins, Colorado.

APPENDIX-I

Normal Distribution:

$$P.D.F. : f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \quad \dots (1)$$

$$C.D.F. : F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \quad \dots (2)$$

Parameters: μ = location parameter
 σ = scale parameter

$$\text{Reduced Variate: } Z = \frac{(x-\mu)}{\sigma} \quad \dots (3)$$

$$P.D.F.: f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \dots (4)$$

$$C.D.F.: F(z) = - \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

$$f(x) = \frac{1}{\sigma_y\sqrt{2\pi x}} \exp\left[-\frac{1}{2}\left(\frac{\log_e x - \mu}{\sigma_y}\right)^2\right] \quad \dots (5)$$

Mean of the reduced variate: $\bar{z} = 0$

Standard deviation of reduced variate $\sigma_z = 1$

Coefficient of skewness of the reduced variates = $g_z = 0$

Log Normal Distribution (Two Parameters):

$$P.D.F.: f(x) = \frac{1}{\sigma_y\sqrt{2\pi x}} \exp\left[-\frac{1}{2}\left(\frac{\log_e x - \mu_y^2}{\sigma_y}\right)^2\right] \quad \dots (6)$$

$$C.D.F.: F(x) = \frac{1}{\sigma_y\sqrt{2\pi}} \int_{x_0}^x \exp\left[-\frac{1}{2}\left(\frac{\log_e x - \mu_y^2}{\sigma_y}\right)^2\right] dx \quad \dots (7)$$

where,

$y = \log_e x$

μ = Mean of Y series

σ_y = Standard deviation of Y-series

Parameters: u_y = location parameter
 σ_y = scale parameter

$$\text{Reduced variate } Z = \frac{\log e^x - \sigma_y}{\sigma_y} \quad \dots(8)$$

$$\text{P.D.F.: } f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^{2/2}} \quad \dots(9)$$

$$\text{C.D.F.: } F(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-z^{2/2}} dz \quad \dots(10)$$

Mean of the reduced variate: $\bar{z} = 0$

Standard deviation of reduced variate: $\sigma_z = 1$

Coefficient of skewness of the reduced variates = $g_z = 0$

Gumbel Extreme Value (Type-I) Distribution(EV1):

$$\text{P. D. F. } f(x) = \frac{1}{\alpha} \exp \left[-\left(\frac{x-u}{\alpha} \right) - \exp \left\{ -\left(\frac{x-u}{\alpha} \right) \right\} \right] \quad \dots(11)$$

$$\text{C.D.F.: } F(x) = e^{-e^{-\left(\frac{x-u}{\alpha} \right)}} \quad \dots(12)$$

$$\text{Reduced Variate : } z = \frac{x-u}{\alpha} \quad \dots(13)$$

$$\text{P.D.F.: } f(z) = \exp(-z - \exp(-z)) \quad \dots(14)$$

$$\text{C.D.F. : } F(z) = e^{-e^{-z}} \quad \dots(15)$$

Mean reduced variate $Z = 0.5772$

Standard deviation of reduced variate $\alpha_z = \frac{\pi}{\sqrt{6}} = 1.2825$

Coefficient of skewness of reduced variate $g_z = 1.14$

Pearson type-III Distribution (PT3):

$$\text{P.D.F. : } f(x) = \frac{(x-x_o)^{\gamma-1} e^{-(x-x_o)/\beta}}{\beta^\gamma \sqrt{\gamma}} \quad \dots(16)$$

$$C.D.F. F(x) = \int_{x_o}^x \frac{(x-x_o)^{\gamma-1} e^{-(x-x_o)/\beta}}{\beta^\gamma \sqrt{\gamma}} dx \quad \dots \dots (17)$$

Parameters:
 x_o = Location parameter
 β = scale parameter
 γ = shape parameter

$$\text{Reduced variate } Z = \frac{x-x_o}{\beta} \quad \dots \dots (18)$$

$$\text{P.D.F.: } f(z) = \frac{1}{|\beta| \sqrt{\gamma}} (z)^{\gamma-1} e^{-z} \quad \dots \dots (19)$$

$$\text{C.D.F.: } F(z) = \int_0^z \frac{1}{|\beta| \sqrt{\gamma}} (z)^{\gamma-1} e^{-z} dz \quad \dots \dots (20)$$

Mean of the reduced variate: $\bar{z} = \gamma$

St. Deviation of the reduced variate: $\sigma_z = \sqrt{\gamma}$

Coefficient of skewness of the reduced variate: $g_z = 2/\sqrt{\gamma}$

Log Pearson Type-III Distribution (LP3):

$$P.D.F.: f(x) = \frac{(\log_e x - y_o)^{\gamma-1} e^{-(\log_e x - y_o)/\beta}}{|\beta| \sqrt{\gamma} x} \quad \dots \dots (21)$$

$$\text{C.D.F.: } F(x) = \int_{y_o}^x f(x) dx \quad \dots \dots (22)$$

Parameters:
 y_o = Location parameter
 β = scale parameter
 γ = shape parameter

$$\text{Reduced variate: } Z = \frac{\log_e x - y_o}{\beta} \quad \dots \dots (23)$$

$$\text{P.D.F.: } f(z) = \frac{1}{|\beta| \gamma} (z)^{\gamma-1} e^{-z} \quad \dots \dots (24)$$

$$\text{C.D.F.: } F(z) = \int_0^z f(z) dz \quad \dots \dots (25)$$

Mean of reduced variate = $\bar{z} = \gamma$

Standard dev. of reduced variate: $\sigma_z = \sqrt{\gamma}$

Coefficient of skewness of reduced variate: $g_z = 2/\sqrt{\gamma}$

General Extreme Value Distribution:

$$P.D.F.: f(x) = \frac{1}{\alpha} [1 - K(\frac{x-u}{\alpha})]^{(1/K)-1} e^{-[1-K(\frac{x-u}{\alpha})]^{1/K}} \quad \dots \dots (26)$$

$$C.D.F.: F(x) = \text{Exp} [-(1 - K(\frac{x-u}{\alpha}))^{1/K}] \quad \dots \dots (27)$$

Parameters:
 u = Location parameter
 α = Scale parameter
 K = Shape Parameter

If $K = 0$ it leads to EV-I distribution,
 $K < 0$ it leads to EV-II distribution
 $K > 0$ it leads to EV-III distribution

$$\text{GEV reduced variate : } w = \frac{x-u}{\alpha} \quad \dots \dots (28)$$

Here $w = (l_e - kz)/K$
 $Z = \text{EV-I reduced variate}$

Gamma Distribution:

It is a special case of PT3 distribution.

$$P.D.F.: f(x) = \frac{(x)^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \sqrt{\gamma}} \quad \dots \dots \dots \dots (29)$$

$$C.D.F.: F(x) = \int_0^x \frac{(x)^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \sqrt{\gamma}} dx \quad \dots \dots (30)$$

Parameters: β = Scale Parameter
 γ = Shape parameter

$$\text{Gamma Reduced Variate } z = \frac{x}{\beta} \quad \dots \dots (31)$$

$$P.D.F.: f(z) = \frac{1}{|\beta| \sqrt{\gamma}} (z)^{\gamma-1} e^{-z} \quad \dots \dots (32)$$

$$C.D.F.: F(z) = \int_0^z \frac{1}{|\beta| \sqrt{\gamma}} (z)^{\gamma-1} e^{-z} dz \quad \dots \dots (33)$$

Mean of the reduced variate: $\bar{z} = \gamma$
St. Deviation of the reduced variate: $\sigma_z = \sqrt{\gamma}$
Coefficient of skewness of the reduced variate: $g_z = 2/\sqrt{\gamma}$

Exponential Distribution:

$$\text{P.D.F.: } f(x) = \frac{1}{\beta} e^{-\frac{(x-x_o)}{\beta}} \quad \dots(34)$$

$$\text{C.D.F.: } F(x) = 1 - e^{-\frac{(x-x_o)}{\beta}} \quad \dots(35)$$

Parameters: x_o = Location Parameter

β = Scale Parameter

$$\text{Reduced Variate : } z = \frac{x-x_o}{\beta} \quad \dots(36)$$

$$\text{P.D.F.: } f(z) = e^{-z} \quad \dots(37)$$

$$\text{C.D.F.: } F(Z) = 1 - e^{-z} \quad \dots(38)$$

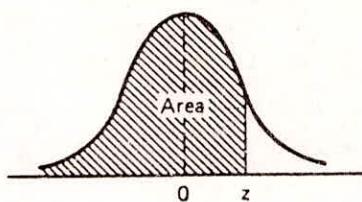
Mean of the reduced variate: $\bar{z} = 1$

St. Deviation of the reduced variate $\sigma_z = 1$

Coefficient of skewness of the reduced variate : $g_z = 2$

TABLE Normal Distribution Probabilities

APPENDIX X - II



<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0214	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0334	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0416	.0409	.0401	.0393	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0745	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2559	.2534	.2504	.2476
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3226	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3850
-.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.0	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	.5395
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6066	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6626	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8236	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9543
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9646	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9785	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9866	.9867	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9942	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9966	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9986	.9986	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9994	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9997	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

APPENDIX-II*

TABLES OF K-VALUES FOR PEARSON TYPE-III DISTRIBUTION

P	G = 0.0	G = 0.1	G = 0.2	G = 0.3	G = 0.4	G = 0.5	G = 0.6
0.9999	-3.71902	-3.50703	-3.29921	-3.09631	-2.89907	-2.70836	-2.52507
0.9995	-3.29053	-3.12767	-2.96698	-2.80889	-2.65390	-2.50257	-2.35549
0.9990	-3.09023	-2.94834	-2.80786	-2.66915	-2.53261	-2.39867	-2.26780
0.9980	-2.87916	-2.75706	-2.63672	-2.51741	-2.39942	-2.28311	-2.16884
0.9950	-2.57583	-2.48187	-2.38795	-2.29423	-2.20092	-2.10825	-2.01644
0.9900	-2.32635	-2.25258	-2.17840	-2.10394	-2.02933	-1.95472	-1.88029
0.9800	-2.05375	-1.9973	-1.94499	-1.88959	-1.83361	-1.77716	-1.72033
0.9750	-1.95996	-1.91219	-1.86360	-1.81427	-1.76427	-1.71366	-1.66253
0.9600	-1.75069	-1.71580	-1.67999	-1.64329	-1.60574	-1.56740	-1.52830
0.9500	-1.64485	-1.61594	-1.58607	-1.55527	-1.52357	-1.49101	-1.45762
0.9000	-1.28155	-1.27037	-1.25824	-1.24516	-1.23114	-1.21618	-1.20028
0.8000	-0.84162	-0.84611	-0.84986	-0.85285	-0.85508	-0.85653	-0.85718
0.7000	-0.52440	-0.53624	-0.54757	-0.55839	-0.56867	-0.57840	-0.58757
0.6000	-0.25335	-0.26882	-0.28403	-0.29897	-0.31362	-0.32796	-0.34198
0.5704	-0.17733	-0.19339	-0.20925	-0.22492	-0.24037	-0.25558	-0.27047
0.5000	0.0	-0.01662	-0.03325	-0.04993	-0.06651	-0.08302	-0.09945
0.4296	0.17733	0.16111	0.14472	0.12820	0.11154	0.09478	0.07791
0.4000	0.25335	0.23763	0.22168	0.20552	0.18916	0.17261	0.15589
0.3000	0.52440	0.51207	0.49927	0.48600	0.47228	0.45812	0.44352
0.2000	0.84162	0.83639	0.83044	0.82377	0.81638	0.80829	0.79950
0.1000	1.28155	1.29178	1.30105	1.30936	1.31671	1.32309	1.32850
0.0500	1.64485	1.67279	1.69971	1.72562	1.75048	1.77428	1.79701
0.0400	1.75069	1.78462	1.81756	1.84949	1.88039	1.91022	1.93896
0.0250	1.95996	2.00688	2.05290	2.09795	2.14202	2.18505	2.22702
0.0200	2.05375	2.10697	2.15935	2.21081	2.26133	2.31084	2.35931
0.0100	2.32635	2.39961	2.47226	2.54421	2.61539	2.68572	2.75514
0.0050	2.57583	2.66965	2.76321	2.85636	2.94900	3.04102	3.13232
0.0020	2.87815	2.99978	3.12169	3.24371	3.36566	3.48737	3.60872
0.0010	3.09023	3.23322	3.37703	3.52139	3.66608	3.81090	3.95567
0.0005	3.29053	3.45513	3.62113	3.78820	3.95605	4.12443	4.29311
0.0001	3.71902	3.93453	4.15301	4.37394	4.59687	4.82141	5.04718

P — Probability of exceedance

G — Coefficient of skewness

ρ	$G = 0.7$	$G = 0.8$	$G = 0.9$	$G = 1.0$	$G = 1.1$	$G = 1.2$	$G = 1.3$
0.9999	-2.35015	-2.14444	-2.02891	-1.88410	-1.75053	-1.62838	-1.51752
0.9995	-2.21328	-2.07661	-1.94611	-1.82241	-1.70603	-1.59738	-1.49673
0.9990	-2.14053	-2.01739	-1.89894	-1.78572	-1.67825	-1.57695	-1.48216
0.9980	-2.05701	-1.94806	-1.84244	-1.74062	-1.64305	-1.55016	-1.46232
0.9950	-1.92580	-1.83660	-1.74919	-1.66390	-1.58110	-1.50114	-1.42439
0.9900	-1.80621	-1.73271	-1.66001	-1.58838	-1.51808	-1.44942	-1.38267
0.9800	-1.66125	-1.60604	-1.54886	-1.49188	-1.43529	-1.37929	-1.32412
0.9750	-1.61099	-1.55914	-1.50712	-1.45507	-1.40314	-1.35153	-1.30042
0.9600	-1.48852	-1.44813	-1.40720	-1.36584	-1.32414	-1.28225	-1.24028
0.9500	-1.42345	-1.38855	-1.35299	-1.31684	-1.28019	-1.24313	-1.20578
0.9000	-1.18347	-1.16574	-1.14712	-1.12762	-1.10726	-1.08608	-1.06413
0.8000	-0.85703	-0.85607	-0.85426	-0.85164	-0.84809	-0.84369	-0.83841
0.7000	-0.59615	-0.60412	-0.61146	-0.61815	-0.62415	-0.62944	-0.63400
0.6000	-0.35565	-0.36889	-0.39186	-0.39434	-0.40638	-0.41794	-0.42899
0.5704	-0.28516	-0.29961	-0.31368	-0.32740	-0.34075	-0.35370	-0.36620
0.5000	-0.11579	-0.13199	-0.14807	-0.16397	-0.17968	-0.19517	-0.21040
0.4296	0.06097	0.04397	0.02693	0.00987	-0.00719	-0.02421	-0.04116
0.4000	0.13901	0.12199	0.10485	0.08763	0.07032	0.05297	0.03560
0.3000	0.42951	0.41309	0.39729	0.38111	0.36458	0.34772	0.33054
0.2000	0.79002	0.77986	0.76902	0.75752	0.74537	0.73257	0.71915
0.1000	1.33294	1.33640	1.33899	1.34039	1.34092	1.34047	1.33904
0.0500	1.81864	1.83916	1.85856	1.87683	1.89395	1.90992	1.92472
0.0400	1.96660	1.94311	2.01849	2.04269	2.06573	2.08758	2.10823
0.0250	2.26790	2.30764	2.34623	2.38354	2.41984	2.45482	2.48855
0.0200	2.40670	2.45298	2.49811	2.54206	2.58480	2.62631	2.66657
0.0100	2.82359	2.89101	2.95735	3.02256	3.08650	3.14944	3.21103
0.0050	3.22281	3.31243	3.40109	3.48874	3.57530	3.66073	3.74497
0.0020	3.72957	3.84981	3.96932	4.08902	4.20582	4.32263	4.43839
0.0010	4.10022	4.24339	4.38807	4.53112	4.67344	4.81492	4.95549
0.0005	4.46189	4.63057	4.79899	4.96701	5.13449	5.30130	5.46735
0.0001	5.27389	5.50124	5.72899	5.95691	6.18480	6.41249	6.63980

P	G = 1.4	G = 1.5	G = 1.6	G = 1.7	G = 1.8	G = 1.9	G = 2.0
0.9999	-1.41753	-1.32774	-1.24728	-1.17520	-1.11054	-1.05239	-0.99990
0.9995	-1.40413	-1.31944	-1.24235	-1.17240	-1.10901	-1.05159	-0.99950
0.9990	-1.39408	-1.31275	-1.23805	-1.16974	-1.10743	-1.05068	-0.99900
0.9980	-1.37981	-1.30279	-1.23132	-1.16534	-1.10465	-1.04898	-0.99800
0.9950	-1.35114	-1.28167	-1.21618	-1.15477	-1.09749	-1.0427	-0.99499
0.9900	-1.31915	-1.25611	-1.19680	-1.14042	-1.08711	-1.03695	-0.98995
0.9800	-1.26999	-1.21716	-1.16584	-1.11628	-1.06864	-1.02311	-0.97980
0.9750	-1.25004	-1.20059	-1.15229	-1.10537	-1.06001	-1.01640	-0.97468
0.9600	-1.19842	-1.15682	-1.11566	-1.07513	-1.03543	-0.99672	-0.95916
0.9500	-1.16827	-1.13075	-1.09338	-1.05631	-1.01973	-0.98381	-0.94971
0.9000	-1.04144	-1.01810	-0.99418	-0.96977	-0.94496	-0.91988	-0.89464
0.8000	-0.83223	-0.82516	-0.81720	-0.80837	-0.79868	-0.78816	-0.77686
0.7000	-0.63779	-0.64080	-0.64300	-0.64436	-0.64488	-0.64453	-0.64333
0.6000	-0.43949	-0.44942	-0.45873	-0.46739	-0.47538	-0.48265	-0.48917
0.5704	-0.37824	-0.38977	-0.40075	-0.41116	-0.42095	-0.43008	-0.43854
0.5000	-0.22535	-0.23996	-0.25422	-0.26808	-0.28150	-0.29443	-0.30685
0.4296	-0.05803	-0.07476	-0.09132	-0.10769	-0.12381	-0.13964	-0.15116
0.4000	0.01824	0.0092	-0.01631	-0.03344	-0.05040	-0.06718	-0.08371
0.3000	0.31307	0.29535	0.27740	0.25925	0.24094	0.22250	0.20397
0.2000	0.70512	0.69050	0.67532	0.65959	0.64335	0.62662	0.60944
0.1000	1.33665	1.33330	1.32900	1.32376	1.31760	1.31054	1.30259
0.0500	1.93836	1.95083	1.96213	1.97227	1.98124	1.98906	1.99573
0.0400	2.12768	2.14591	2.16243	2.17873	2.19332	2.20670	2.21888
0.0250	2.52102	2.55222	2.58214	2.61076	2.63810	2.66413	2.68888
0.0200	2.70556	2.74325	2.77964	2.81472	2.84848	2.88091	2.91202
0.0100	3.27134	3.33035	3.39804	3.44438	3.49935	3.55295	3.60517
0.0050	3.82798	3.90973	3.99016	4.06926	4.14700	4.22336	4.29832
0.0020	4.55304	4.66651	4.77875	4.88971	4.99937	5.10768	5.21461
0.0010	5.09505	5.23353	5.37087	5.50701	5.64190	5.77549	5.90776
0.0005	5.63252	5.79673	5.95990	6.12196	6.28285	6.44251	6.60090
0.0001	6.86661	7.09277	7.31816	7.54272	7.76632	7.98888	8.21034

ρ	$G = 2.1$	$G = 2.2$	$G = 2.3$	$G = 2.4$	$G = 2.5$	$G = 2.6$	$G = 2.7$
0.9999	-0.95234	-0.90908	-0.86956	-0.83333	-0.80000	-0.76923	-0.74074
0.9995	-0.95215	-0.90899	-0.86952	-0.83331	-0.79999	-0.76923	-0.74074
0.9990	-0.95188	-0.90885	-0.86945	-0.83324	-0.79998	-0.76922	-0.74074
0.9980	-0.95131	-0.90854	-0.86929	-0.83320	-0.79994	-0.76920	-0.74073
0.9950	-0.94945	-0.90742	-0.86863	-0.83283	-0.79973	-0.76909	-0.74067
0.9900	-0.94607	-0.90521	-0.86723	-0.83196	-0.79921	-0.76878	-0.74049
0.9800	-0.93878	-0.90009	-0.86371	-0.82959	-0.79765	-0.76779	-0.73987
0.9750	-0.93495	-0.89728	-0.86169	-0.82817	-0.79667	-0.76712	-0.73943
0.9600	-0.92295	-0.88814	-0.85486	-0.82315	-0.79306	-0.76456	-0.73765
0.9500	-0.91458	-0.88156	-0.84976	-0.81927	-0.79015	-0.76242	-0.73610
0.9000	-0.86938	-0.84422	-0.81929	-0.79472	-0.77062	-0.74709	-0.72422
0.8000	-0.76482	-0.75211	-0.73880	-0.72495	-0.71067	-0.69602	-0.68111
0.7000	-0.64129	-0.63833	-0.63456	-0.62999	-0.62463	-0.61854	-0.61176
0.6000	-0.49494	-0.49991	-0.50409	-0.50744	-0.50999	-0.51171	-0.51263
0.5704	-0.44628	-0.5329	-0.45953	-0.46499	-0.46966	-0.47353	-0.47660
0.5000	-0.31872	-0.32999	-0.34063	-0.35062	-0.35992	-0.36852	-0.37640
0.4296	-0.17030	-0.18504	-0.19933	-0.21313	-0.22642	-0.23915	-0.25129
0.4000	-0.09997	-0.11590	-0.13148	-0.14565	-0.16138	-0.17564	-0.18939
0.3000	0.18540	0.16682	0.14827	0.12979	0.11143	0.09323	0.07523
0.2000	0.59183	0.57383	0.55549	0.53683	0.51789	0.49872	0.47934
0.1000	1.29377	1.28412	1.27365	1.26240	1.25039	1.23766	1.22422
0.0500	2.00128	2.00570	2.00903	2.01128	2.01247	2.01263	2.01177
0.0400	2.22986	2.23967	2.24831	2.25581	2.26217	2.26743	2.27160
0.0250	2.71234	2.73451	2.75541	2.77506	2.79345	2.81062	2.82658
0.0200	2.94181	2.97028	2.99744	3.02330	3.04787	3.07116	3.09320
0.0100	3.65600	3.70543	3.75347	3.80013	3.84540	3.88930	3.93183
0.0050	4.37186	4.44398	4.51467	4.58393	4.65176	4.71815	4.78313
0.0020	5.32014	5.42426	5.52694	5.62818	5.72796	5.82629	5.92316
0.0010	6.03865	6.16016	6.29626	6.42292	6.54814	6.67191	6.79421
0.0005	6.75798	6.91370	7.06804	7.22098	7.37250	7.52258	7.67121
0.0001	8.43064	8.64971	8.86753	9.08403	9.29920	9.51301	9.72543

P	6 = 2.8	6 = 2.9	6 = 3.0	6 = 3.1	6 = 3.2	6 = 3.3	6 = 3.4
0.9999	-0.71429	-0.69966	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9995	-0.71429	-0.68966	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9990	-0.71429	-0.68965	-0.66667	-0.64516	-0.62500	-0.60605	-0.58824
0.9980	-0.71428	-0.68955	-0.66667	-0.64516	-0.62500	-0.60606	-0.58824
0.9950	-0.71425	-0.68964	-0.66666	-0.64516	-0.62500	-0.60606	-0.58824
0.9900	-0.71415	-0.68959	-0.66663	-0.64514	-0.62499	-0.60606	-0.58823
0.9800	-0.71377	-0.68435	-0.66649	-0.64507	-0.62495	-0.60603	-0.58822
0.9750	-0.71348	-0.68417	-0.66638	-0.64500	-0.62491	-0.60601	-0.58821
0.9600	-0.71227	-0.68836	-0.66585	-0.64465	-0.62469	-0.60587	-0.58812
0.9500	-0.71116	-0.68759	-0.66532	-0.64429	-0.62445	-0.60572	-0.58802
0.9000	-0.70209	-0.68075	-0.66023	-0.64056	-0.62175	-0.60379	-0.58666
0.8000	-0.66603	-0.65086	-0.63569	-0.62060	-0.60567	-0.59096	-0.57652
0.7000	-0.60434	-0.59634	-0.58783	-0.57887	-0.56953	-0.55989	-0.55000
0.6000	-0.51276	-0.51212	-0.51073	-0.50863	-0.5085	-0.50244	-0.49844
0.5704	-0.47888	-0.48037	-0.48109	-0.48107	-0.48033	-0.47890	-0.47682
0.5000	-0.38353	-0.38991	-0.39554	-0.40041	-0.40454	-0.40792	-0.41058
0.4296	-0.26282	-0.27372	-0.28395	-0.29351	-0.30238	-0.31055	-0.31802
0.4000	-0.20259	-0.21523	-0.22726	-0.23868	-0.24946	-0.25958	-0.26904
0.3000	0.05746	0.03997	0.02279	0.00596	-0.01050	-0.02654	-0.04215
0.2000	0.45980	0.44015	0.42040	0.40061	0.38081	0.36104	0.34133
0.1000	1.21013	1.14539	1.18006	1.16416	1.14772	1.13078	1.11337
0.0500	2.00992	2.00710	2.00335	1.99869	1.99314	1.98674	1.97951
0.0400	2.27470	2.27676	2.27780	2.27785	2.27693	2.27506	2.27229
0.0250	2.84134	2.85492	2.86735	2.87865	2.88884	2.89795	2.90599
0.0200	3.11399	3.13356	3.15143	3.16911	3.18512	3.20000	3.21375
0.0100	3.97301	4.01286	4.05138	4.08859	4.12452	4.15917	4.19257
0.0050	4.84669	4.90884	4.96959	5.02897	5.08697	5.14362	5.19892
0.0020	6.01858	6.11254	6.20506	6.29613	6.38578	6.47401	6.56084
0.0010	6.91505	7.03443	7.15235	7.26881	7.38382	7.49739	7.60953
0.0005	7.81839	7.96411	8.10836	8.25115	8.39248	8.53236	8.67079
0.0001	9.93643	10.14602	10.35418	10.56090	10.76618	10.97001	11.17239

P	G = 3.5	G = 3.6	G = 3.7	G = 3.8	G = 3.9	G = 4.0	G = 4.1
0.9999	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9995	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9990	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9980	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9950	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9900	-0.57143	-0.555556	-0.54054	-0.52632	-0.51282	-0.50000	-0.48780
0.9800	-0.57142	-0.555555	-0.54054	-0.52631	-0.51282	-0.50000	-0.48780
0.9750	-0.57141	-0.555555	-0.54054	-0.52631	-0.51282	-0.50000	-0.48780
0.9600	-0.57136	-0.555552	-0.54052	-0.52630	-0.51281	-0.50000	-0.48780
0.9500	-0.57130	-0.555548	-0.54050	-0.52629	-0.51281	-0.49999	-0.48780
0.9000	-0.57035	-0.55483	-0.54006	-0.52600	-0.51261	-0.49986	-0.48772
0.8000	-0.56242	-0.54867	-0.53533	-0.52240	-0.50990	-0.49784	-0.48622
0.7000	-0.53993	-0.52975	-0.51952	-0.50929	-0.49911	-0.48902	-0.47906
0.6000	-0.49391	-0.48888	-0.48342	-0.47758	-0.47141	-0.46496	-0.45828
0.5704	-0.47413	-0.47088	-0.46711	-0.46286	-0.45619	-0.45314	-0.44777
0.5000	-0.41253	-0.41381	-0.41442	-0.41441	-0.41381	-0.41265	-0.41097
0.4296	-0.32479	-0.33085	-0.33623	-0.34092	-0.34494	-0.34831	-0.35105
0.4000	-0.27782	-0.28592	-0.29335	-0.30010	-0.30617	-0.31159	-0.31635
0.3000	-0.05730	-0.07195	-0.08610	-0.09972	-0.11279	-0.12530	-0.13725
0.2000	0.32171	0.30223	0.29290	0.26376	0.24484	0.22617	0.20777
0.1000	1.09552	1.07726	1.05863	1.03965	1.02036	1.00079	0.98096
0.0500	1.97147	1.95266	1.95311	1.94283	1.93186	1.92023	1.90796
0.0400	2.26862	2.26409	2.25872	2.25254	2.24558	2.23786	2.22940
0.0250	2.91299	2.91898	2.92397	2.92799	2.93107	2.93324	2.93450
0.0200	3.22641	3.23800	3.24853	3.25803	3.26653	3.27404	3.28060
0.0100	4.22473	4.25569	4.28545	4.31403	4.34147	4.36777	4.39296
0.0050	5.25291	5.30559	5.35698	5.40711	5.45598	5.50362	5.55005
0.0020	6.64627	6.73032	6.81301	6.89435	6.97435	7.05304	7.13043
0.0010	7.72024	7.82954	7.93744	8.04395	8.14910	8.25289	8.35534
0.0005	8.80779	8.94335	9.07750	9.21023	9.34158	9.47154	9.60013
0.0001	11.37334	11.57284	11.77092	11.96757	12.16280	12.35663	12.54906

P	G = 4.2	G = 4.3	G = 4.4	G = 4.5	G = 4.6	G = 4.7	G = 4.8
0.9999	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9995	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9990	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9980	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9950	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9900	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9800	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9750	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9600	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9500	-0.47619	-0.46512	-0.45455	-0.44444	-0.43478	-0.42553	-0.41667
0.9000	-0.47614	-0.46508	-0.45452	-0.44443	-0.43477	-0.42553	-0.41667
0.8000	-0.47504	-0.46428	-0.45395	-0.44402	-0.43448	-0.42532	-0.41652
0.7000	-0.46927	-0.45967	-0.45029	-0.44114	-0.43223	-0.42357	-0.41517
0.6000	-0.45142	-0.44442	-0.43734	-0.43020	-0.42304	-0.41590	-0.40880
0.5704	-0.44212	-0.43623	-0.43016	-0.42394	-0.41761	-0.41121	-0.40477
0.5000	-0.40881	-0.40621	-0.40321	-0.39985	-0.39617	-0.39221	-0.38800
0.4296	-0.35318	-0.35473	-0.35572	-0.35619	-0.35616	-0.35567	-0.35475
0.4000	-0.32049	-0.32400	-0.32693	-0.32928	-0.33108	-0.33236	-0.33315
0.3000	-0.14861	-0.15939	-0.16958	-0.17918	-0.18819	-0.19661	-0.20446
0.2000	0.18967	0.17189	0.15445	0.13737	0.12067	0.10436	0.08847
0.1000	0.96090	0.94064	0.92022	0.89964	0.87895	0.85817	0.83731
0.0500	1.89508	1.98160	1.86757	1.85300	1.83792	1.82234	1.80631
0.0400	2.22024	2.21039	2.19988	2.18874	2.17699	2.16465	2.15174
0.0250	2.93489	2.93443	2.93314	2.93105	2.92818	2.92455	2.92017
0.0200	3.28622	3.29092	3.29473	3.29767	3.29976	3.30103	3.30149
0.0100	4.41706	4.44009	4.46207	4.48303	4.50297	4.52192	4.53990
0.0050	5.59529	5.63934	5.68224	5.72400	5.76464	5.80418	5.84265
0.0020	7.20654	7.28138	7.35497	7.42733	7.49847	7.56842	7.63718
0.0010	8.45646	8.55627	8.65479	8.75202	8.84800	8.94273	9.03623
0.0005	9.72737	9.85326	9.97784	10.10110	10.22307	10.34375	10.46318
0.0001	12.74010	12.92977	13.11808	13.30504	13.49066	13.67495	13.85794

P	$G = 4.9$	$G = 5.0$	$G = 5.1$	$G = 5.2$	$G = 5.3$	$G = 5.4$	$G = 5.5$
0.9999	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9995	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9990	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9980	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9950	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9900	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9800	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9750	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9600	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9500	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.9000	-0.40816	-0.40000	-0.39216	-0.38462	-0.37736	-0.37037	-0.36364
0.8000	-0.40806	-0.39993	-0.39211	-0.38458	-0.37734	-0.37036	-0.36363
0.7000	-0.40703	+0.39914	-0.39152	-0.38414	-0.37701	-0.37011	-0.36345
0.6000	-0.40177	-0.39482	-0.38799	-0.38127	-0.37469	-0.36825	-0.36196
0.5704	-0.39833	-0.39190	-0.38552	-0.37919	-0.37295	-0.36680	-0.36076
0.5000	-0.38359	-0.37901	-0.37428	-0.36945	-0.36453	-0.35956	-0.35456
0.4296	-0.35343	-0.35174	-0.34972	-0.34740	-0.34481	-0.34198	-0.33895
0.4000	-0.33347	-0.33336	-0.33284	-0.33194	-0.33070	-0.32914	-0.32729
0.3000	-0.21172	-0.21843	-0.22458	-0.23019	-0.23527	-0.23984	-0.24391
0.2000	0.07300	0.05798	0.04340	0.02927	0.01561	0.00243	-0.01028
0.1000	0.81641	0.79548	0.77455	0.75364	0.73277	0.71195	0.69122
0.0500	1.78982	1.77292	1.75563	1.73795	1.71992	1.70155	1.68287
0.0400	2.13829	2.12432	2.10985	2.09490	2.07950	2.06365	2.04739
0.0250	2.91508	2.90930	2.90283	2.89572	2.88796	2.87459	2.87062
0.0200	3.30116	3.30007	3.29823	3.29567	3.29240	3.28844	3.28381
0.0100	4.55694	4.57304	4.58823	4.60252	4.61594	4.62850	4.64022
0.0050	5.88004	5.91639	5.95171	5.98602	6.01934	6.05169	6.08307
0.0020	7.70479	7.77124	7.83657	7.90078	7.96390	8.02594	8.08691
0.0010	9.12852	9.21961	9.30952	9.39827	9.48586	9.57232	9.65766
0.0005	10.58135	10.69829	10.81401	10.92853	11.04186	11.15402	11.26502
0.0001	14.03963	14.22004	14.39918	14.57706	14.75370	14.92912	15.10332

P	G = 5.6	G = 5.7	G = 5.8	G = 5.9	G = 6.0	G = 6.1	G = 6.2
0.9999	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9995	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9990	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9980	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9950	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9900	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9800	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9750	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9600	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9500	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.9000	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.8000	-0.35714	-0.35088	-0.34483	-0.33898	-0.33333	-0.32787	-0.32258
0.7000	-0.35700	-0.35078	-0.34476	-0.33893	-0.33330	-0.32787	-0.32258
0.6000	-0.35583	-0.34985	-0.34402	-0.33836	-0.33285	-0.32750	-0.32230
0.5704	-0.35484	-0.34903	-0.34336	-0.33782	-0.33242	-0.32215	-0.32202
0.5000	-0.34955	-0.34455	-0.33957	-0.33463	-0.32974	-0.32492	-0.32016
0.4296	-0.33573	-0.33236	-0.32886	-0.32525	-0.32155	-0.31780	-0.31399
0.4000	-0.32514	-0.32285	-0.32031	-0.31759	-0.31472	-0.31171	-0.30859
0.3000	-0.24751	-0.25064	-0.25334	-0.25562	-0.25750	-0.25901	-0.26017
0.2000	-0.02252	-0.03427	-0.04553	-0.05632	-0.06662	-0.07645	-0.08580
0.1000	0.67058	0.65006	0.62966	0.60941	0.58933	0.56942	0.54970
0.0500	1.66390	1.64464	1.62513	1.60538	1.58541	1.56524	1.54487
0.0400	2.03073	2.01369	1.99629	1.97455	1.96048	1.94210	1.92343
0.0250	2.86107	2.85096	2.84030	2.82912	2.81743	2.80525	2.79259
0.0200	3.27854	3.27263	3.26610	3.25848	3.25128	3.24301	3.23419
0.0100	4.65111	4.66120	4.67050	4.67903	4.68680	4.69382	4.70013
0.0050	6.11351	6.14302	6.17162	6.19333	6.22616	6.25212	6.27723
0.0020	8.14683	8.20572	8.26359	8.32046	8.37634	8.43125	8.48519
0.0010	9.74190	9.82505	9.90713	9.98815	10.06812	10.14706	10.22499
0.0005	11.37487	11.48360	11.59122	11.69773	11.80316	11.90752	12.01082
0.0001	15.27632	15.44813	15.61878	15.78826	15.95660	16.12380	16.26959

<i>P</i>	<i>G</i> = 6.3	<i>G</i> = 6.4	<i>G</i> = 6.5	<i>G</i> = 6.6	<i>G</i> = 6.7	<i>G</i> = 6.8	<i>G</i> = 6.9
0.9999	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9995	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9990	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9980	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9950	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9900	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9800	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9750	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9600	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9500	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.9000	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.8000	-0.31746	-0.31250	-0.30769	-0.30303	-0.29851	-0.29412	-0.28986
0.7000	-0.31745	-0.31249	-0.30769	-0.30303	-0.29850	-0.29412	-0.28985
0.6000	-0.31724	-0.31234	-0.30757	-0.30294	-0.29844	-0.29407	-0.28982
0.5704	-0.31702	-0.31216	-0.30743	-0.30283	-0.29835	-0.29400	-0.28977
0.5000	-0.31549	-0.31090	-0.30639	-0.30198	-0.29766	-0.29344	-0.28931
0.4296	-0.31016	-0.30631	-0.30246	-0.29862	-0.29480	-0.29101	-0.28726
0.4000	-0.30538	-0.30209	-0.29875	-0.29537	-0.29196	-0.28854	-0.28511
0.3000	-0.26097	-0.26146	-0.26167	-0.26160	-0.26128	-0.26072	-0.25995
0.2000	-0.09469	-0.10311	-0.1107	-0.11859	-0.12566	-0.13231	-0.13953
0.1000	0.53019	0.51089	0.49182	0.47299	0.45440	0.43608	0.41003
0.0500	1.52434	1.50365	1.48281	1.46186	1.44079	1.41963	1.39839
0.0400	1.90449	1.88528	1.86584	1.84616	1.82627	1.80618	1.78591
0.0250	2.77947	2.76591	2.75191	2.73751	2.72270	2.70751	2.69195
0.0200	3.22484	3.21497	3.20460	3.19374	3.18241	3.17062	3.15838
0.0100	4.70571	4.71061	4.71482	4.71836	4.72125	4.72350	4.72512
0.0050	6.30151	6.32497	6.34762	6.36948	6.39055	6.41086	6.43042
0.0020	8.53820	8.59027	8.64142	8.69167	8.74102	8.78950	8.83711
0.0010	10.30192	10.37785	10.45281	10.52681	10.59986	10.67197	10.74316
0.0005	12.11307	12.21429	12.31450	12.41370	12.51190	12.60913	12.70539
0.0001	16.45487	16.61875	16.78156	16.94329	17.10397	17.26361	17.42221

P	G = 7.0	G = 7.1	G = 7.2	G = 7.3	G = 7.4	G = 7.5	G = 7.6
0.9999	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26657	-0.26316
0.9995	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9990	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9980	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9950	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9900	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9800	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9750	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9600	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9500	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.9000	-0.28571	-0.28169	-0.27778	-0.27397	-0.27027	-0.26667	-0.26316
0.5704	-0.28565	-0.28164	-0.27774	-0.27394	-0.27025	-0.26665	-0.26315
0.5000	-0.28528	-0.28135	-0.27751	-0.27376	-0.27010	-0.26654	-0.26306
0.4296	-0.28355	-0.27990	-0.27629	-0.27274	-0.26926	-0.26584	-0.26248
0.4000	-0.28169	-0.27829	-0.27491	-0.27156	-0.26825	-0.26497	-0.26175
0.3000	-0.25899	-0.25785	-0.25654	-0.25510	-0.25352	-0.25183	-0.25005
0.2000	-0.14434	-0.14975	-0.15478	-0.15942	-0.16371	-0.16764	-0.17123
0.1000	0.40026	0.38277	0.36557	0.34868	0.33209	0.31582	0.29986
0.0500	1.37708	1.35571	1.33430	1.31297	1.29141	1.26995	1.24850
0.0400	1.76547	1.74487	1.72412	1.70325	1.68225	1.66115	1.63995
0.0250	2.67603	2.65977	2.64317	2.62626	2.60905	2.59154	2.57375
0.0200	3.14572	3.13263	3.11914	3.10525	3.09099	3.07636	3.06137
0.0100	4.72613	4.72653	4.72635	4.72559	4.72427	4.72240	4.71998
0.0050	6.44924	6.46733	6.48470	6.50137	6.51735	6.53264	6.54727
0.0020	8.88387	8.92979	8.97488	9.01915	9.06261	9.10526	9.14717
0.0010	10.81343	10.88281	10.95129	11.01890	11.08565	11.15154	11.21658
0.0005	12.80069	12.89505	12.98848	13.08095	13.17258	13.26328	13.35209
0.0001	17.57979	17.73636	18.04652	18.20013	18.35278	18.50447	18.50447

P	G = 7.7	G = 7.8	G = 7.9	G = 8.0	G = 8.1	G = 8.2	G = 8.3
0.9999	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9995	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9990	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9980	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9950	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9900	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9800	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9750	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9600	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9500	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.9000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.8000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.7000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.6000	-0.25974	-0.25641	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.5704	-0.25973	-0.25640	-0.25316	-0.25000	-0.24691	-0.24390	-0.24096
0.5000	-0.25966	-0.25635	-0.25312	-0.24996	-0.24689	-0.24388	-0.24095
0.4296	-0.25919	-0.25596	-0.25280	-0.24970	-0.24667	-0.24371	-0.24081
0.4000	-0.25857	-0.25544	-0.25236	-0.24933	-0.24637	-0.24345	-0.24060
0.3000	-0.24817	-0.24622	-0.24421	-0.24214	-0.24003	-0.23571	
0.2000	-0.17450	-0.17746	-0.18012	-0.18249	-0.18459	-0.18803	
0.1000	0.28422	0.26892	0.25394	0.23929	0.22494	0.21101	0.19737
0.0500	1.22706	1.20565	1.18427	1.16295	1.14168	1.12048	1.09936
0.0400	1.61867	1.59732	1.57591	1.55444	1.53294	1.51141	1.48985
0.0250	2.55569	2.53737	2.51881	2.50001	2.48099	2.46175	2.44231
0.0200	3.04604	3.03038	3.01439	2.99810	2.98150	2.96462	2.94746
0.0100	4.71704	4.71358	4.70961	4.70514	4.70019	4.69476	4.68867
0.0050	6.56124	6.57456	6.58725	6.59931	6.61075	6.62159	6.63193
0.0020	9.18828	9.22863	9.26823	9.30709	9.34521	9.38262	9.41931
0.0010	11.28080	11.34419	11.40677	11.46855	11.52953	11.58974	11.64917
0.0005	13.44202	13.53009	13.61730	13.70366	13.78919	13.87389	13.95778
0.0001	18.65522	19.80504	18.95393	19.10191	19.24898	19.39517	19.54046

P	G = 0.4	G = 0.5	G = 0.6	G = 0.7	G = 0.8	G = 0.9	G = 0.90
0.9999	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9995	-0.23610	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9990	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9980	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9950	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9900	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9800	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9750	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9600	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9500	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.9000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.8000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.7000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.6000	-0.23810	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.5704	-0.23809	-0.23529	-0.23256	-0.22989	-0.22717	-0.22472	-0.22222
0.5000	-0.23808	-0.23528	-0.23255	-0.22988	-0.22717	-0.22472	-0.22222
0.4296	-0.23797	-0.23520	-0.23248	-0.22982	-0.22717	-0.22472	-0.22222
0.4000	-0.23779	-0.23505	-0.23236	-0.22972	-0.22717	-0.22472	-0.22222
0.3000	-0.23352	-0.23132	-0.22911	-0.22690	-0.22717	-0.22472	-0.22222
0.2000	-0.18939	-0.19054	-0.19147	-0.19221	-0.19277	-0.19316	-0.19338
0.1000	0.18408	0.17113	0.15851	0.14624	0.13431	0.12272	0.11446
0.0500	1.07832	1.05738	1.03654	1.01581	0.99519	0.97471	0.95435
0.0400	1.46829	1.44673	1.42518	1.40364	1.38213	1.36065	1.33922
0.0250	2.4226A	2.40287	2.38248	2.36273	2.34242	2.32197	2.30138
0.0200	2.93002	2.91234	2.89440	2.87622	2.85782	2.83919	2.82035
0.0100	4.68252	4.67573	4.66850	4.66085	4.65277	4.64429	4.63541
0.0050	6.64148	6.65056	6.65907	6.66703	6.67443	6.68130	6.69763
0.0020	9.45530	9.49060	9.52521	9.55915	9.59243	9.62504	9.65701
0.0010	11.70785	11.76576	11.82294	11.87938	11.93509	11.99009	12.04437
0.0005	14.04086	14.12314	14.20463	14.28534	14.36528	14.44446	14.52284
0.0001	19.68489	19.82845	19.97115	20.11300	20.25402	20.34420	20.53356

P	G = -0.0	G = -0.1	G = -0.2	G = -0.3	G = -0.4	G = -0.5	G = -0.6
0.9999	-3.71902	-3.93453	-4.15301	-4.37394	-4.59687	-4.82141	-5.04718
0.9995	-3.29053	-3.45513	-3.62113	-3.78820	-3.95605	-4.12443	-4.29311
0.9990	-3.09023	-3.23322	-3.37703	-3.52139	-3.66608	-3.81090	-3.95567
0.9980	-2.87316	-2.99978	-3.12169	-3.24371	-3.36566	-3.48737	-3.60872
0.9950	-2.57583	-2.66965	-2.76321	-2.85636	-2.94900	-3.04102	-3.13232
0.9900	-2.32636	-2.39961	-2.47226	-2.54421	-2.61539	-2.68572	-2.75514
0.9800	-2.05375	-2.10697	-2.15935	-2.21081	-2.26133	-2.31084	-2.35931
0.9750	-1.95996	-2.00688	-2.05290	-2.09795	-2.14202	-2.18505	-2.22702
0.9600	-1.75069	-1.78462	-1.81756	-1.84949	-1.88039	-1.91022	-1.93896
0.9500	-1.64485	-1.67279	-1.69971	-1.72562	-1.75048	-1.77428	-1.79701
0.9000	-1.28155	-1.29178	-1.30105	-1.30936	-1.31671	-1.32309	-1.32850
0.8000	-0.84162	-0.83639	-0.83044	-0.82377	-0.81638	-0.80829	-0.79950
0.7000	-0.52440	-0.51207	-0.49927	-0.48600	-0.47228	-0.45812	-0.44352
0.6000	-0.25335	-0.23763	-0.22168	-0.20552	-0.18916	-0.17261	-0.15589
0.5704	-0.17733	-0.16111	-0.14472	-0.12820	-0.11154	-0.09478	-0.07791
0.5000	0.0	0.01662	0.03325	0.04993	0.06651	0.08302	0.09945
0.4296	0.17733	0.19339	0.20925	0.22492	0.24037	0.25558	0.27047
0.4000	0.25335	0.26882	0.28403	0.29897	0.31362	0.32796	0.34198
0.3000	0.52440	0.53624	0.54757	0.55839	0.56867	0.57840	0.58757
0.2000	0.84162	0.84611	0.84986	0.85285	0.85508	0.85653	0.85718
0.1000	1.28155	1.27037	1.25824	1.24516	1.23114	1.21618	1.20028
0.0500	1.64485	1.61594	1.58607	1.55527	1.52357	1.49101	1.45762
0.0400	1.75069	1.71580	1.67999	1.64329	1.60574	1.56740	1.52830
0.0250	1.95996	1.91219	1.86360	1.81427	1.76427	1.71366	1.66253
0.0200	2.05375	1.99973	1.94499	1.88959	1.83361	1.77716	1.72033
0.0100	2.32635	2.2525A	2.17840	2.10394	2.02933	1.95472	1.88029
0.0050	2.57583	2.48187	2.38795	2.29423	2.20092	2.10825	2.01644
0.0020	2.87816	2.7570B	2.63672	2.51741	2.39942	2.28311	2.16884
0.0010	3.09023	2.94834	2.80786	2.66915	2.53261	2.39867	2.26780
0.0005	3.29053	3.12767	2.96698	2.80889	2.65390	2.50257	2.35549
0.0001	3.71072	3.50703	3.29921	3.09631	2.89907	2.70836	2.52507

P	G1=-0.7	G1=-0.8	G1=-0.9	G1=-1.0	G1=-1.1	G1=-1.2	G1=-1.3
0.9999	-5.27389	-5.50124	-5.72899	-5.95691	-6.18480	-6.41249	-6.63980
0.9995	-4.46189	-4.63057	-4.79899	-4.96701	-5.13449	-5.30130	-5.46735
0.9990	-4.10022	-4.24439	-4.38807	-4.53112	-4.67344	-4.81492	-4.95549
0.9980	-3.72957	-3.84981	-3.96932	-4.0802	-4.20582	-4.32263	-4.43839
0.9950	-3.22281	-3.31243	-3.40109	-3.4874	-3.57530	-3.66073	-3.7497
0.9900	-2.82359	-2.89101	-2.95735	-3.02256	-3.08660	-3.14944	-3.21103
0.9800	-2.40670	-2.45298	-2.49811	-2.54205	-2.58480	-2.62631	-2.66657
0.9750	-2.26790	-2.30764	-2.34623	-2.38364	-2.41984	-2.45482	-2.48855
0.9600	-1.96660	-1.99311	-2.01848	-2.04269	-2.06573	-2.08758	-2.10823
0.9500	-1.81864	-1.83916	-1.85856	-1.87683	-1.89395	-1.90992	-1.92472
0.9000	-1.33294	-1.33640	-1.33889	-1.34039	-1.34092	-1.34047	-1.33904
0.8000	-0.79002	-0.77986	-0.76902	-0.75752	-0.74537	-0.73257	-0.71915
0.7000	-0.42851	-0.41309	-0.39729	-0.38111	-0.36458	-0.34772	-0.33054
0.6000	-0.13901	-0.12199	-0.10486	-0.08763	-0.07032	-0.05297	-0.03560
0.5704	-0.06097	-0.04397	-0.02693	-0.00987	0.00719	0.02421	0.04116
0.5000	0.11578	0.13199	0.14807	0.16397	0.17968	0.19517	0.21040
0.4296	0.28516	0.29961	0.31368	0.32740	0.34075	0.35370	0.36620
0.4000	0.35565	0.35889	0.38186	0.39434	0.40638	0.41794	0.42899
0.3000	0.59615	0.60412	0.61146	0.61815	0.62415	0.62944	0.63400
0.2000	0.85703	0.85607	0.85426	0.85161	0.84809	0.84369	0.83441
0.1000	1.18347	1.16574	1.14712	1.12762	1.10726	1.08608	1.05413
0.0500	1.42345	1.38655	1.35299	1.31684	1.28019	1.24313	1.20578
0.0400	1.48852	1.44813	1.40720	1.36584	1.32414	1.28225	1.24024
0.0250	1.61099	1.55914	1.50712	1.45507	1.40314	1.35153	1.30042
0.0200	1.66325	1.60604	1.54886	1.49188	1.43529	1.37929	1.32412
0.0100	1.80621	1.73271	1.66001	1.58838	1.51808	1.444942	1.38267
0.0050	1.92580	1.83660	1.74519	1.66390	1.58110	1.50114	1.42439
0.0020	2.05701	1.94806	1.84244	1.74062	1.64305	1.55016	1.46232
0.0010	2.14053	2.01739	1.99894	1.78572	1.67825	1.57695	1.48216
0.0005	2.21328	2.07661	1.94611	1.82241	1.70603	1.59738	1.49673
0.0001	2.35015	2.18448	2.02891	1.988410	1.75053	1.622838	1.51752

	$\mu = -1.4$	$\mu = -1.5$	$\mu = -1.6$	$\mu = -1.7$	$\mu = -1.8$	$\mu = -1.9$	$\mu = -2.0$
0.9999	-6.86661	-7.09277	-7.31918	-7.54272	-7.76632	-7.98898	-8.21034
0.9995	-5.63252	-5.79673	-5.95940	-6.12196	-6.28285	-6.44251	-6.60090
0.9990	-5.09505	-5.23353	-5.37097	-5.50701	-5.64190	-5.77549	-5.90776
0.9980	-4.55304	-4.66651	-4.78775	-4.88971	-4.99937	-5.0768	-5.21461
0.9950	-3.82798	-3.90973	-3.9016	-4.06926	-4.14700	-4.22336	-4.29832
0.9900	-3.27134	-3.33035	-3.38804	-3.44438	-3.49935	-3.55295	-3.60517
0.9800	-2.70556	-2.74325	-2.7964	-2.81472	-2.84848	-2.88091	-2.91202
0.9750	-2.52102	-2.55222	-2.58214	-2.61076	-2.63810	-2.66413	-2.68888
0.9600	-2.12768	-2.14591	-2.16293	-2.17873	-2.19332	-2.20670	-2.21888
0.9500	-1.95083	-1.95083	-1.96213	-1.97227	-1.98124	-1.98906	-1.99573
0.9000	-1.33665	-1.33330	-1.32900	-1.32376	-1.31760	-1.31054	-1.30259
0.8000	-0.70512	-0.69050	-0.67532	-0.65959	-0.64335	-0.62662	-0.60944
0.7040	-0.31307	-0.29535	-0.27740	-0.25925	-0.24094	-0.22250	-0.20397
0.6000	-0.01824	-0.00992	0.01631	0.03344	0.05040	0.06718	0.08371
0.5704	0.05803	0.07476	0.09132	0.10769	0.12381	0.13964	0.15516
0.5000	0.22535	0.23996	0.25422	0.26808	0.28150	0.29443	0.30685
0.4296	0.37824	0.38977	0.40075	0.41116	0.42095	0.43008	0.43854
0.4000	0.43949	0.44942	0.45873	0.46739	0.47538	0.48265	0.48917
0.3000	0.63779	0.64080	0.64300	0.64436	0.64488	0.64453	0.64333
0.2000	0.83223	0.82516	0.81720	0.80837	0.79868	0.78816	0.77686
0.1000	1.04144	1.01810	0.99418	0.96977	0.94496	0.91988	0.89464
0.0500	1.16827	1.13075	1.09388	1.05631	1.01973	0.98381	0.94871
0.0400	1.19842	1.15682	1.1566	1.07513	1.03543	0.9672	0.95914
0.0250	1.25004	1.20059	1.15229	1.10537	1.06001	1.01640	0.97468
0.0200	1.26999	1.21716	1.16584	1.11628	1.06864	1.02311	0.97980
0.0100	1.31815	1.25611	1.19680	1.14042	1.08711	1.03695	0.98995
0.0050	1.35114	1.28167	1.21618	1.15477	1.09749	1.04427	0.99499
0.0020	1.37981	1.30279	1.23132	1.16534	1.10465	1.04898	0.99800
0.0010	1.39408	1.31275	1.23805	1.16974	1.10743	1.05068	0.99900
0.0005	1.40413	1.31944	1.24235	1.17240	1.10901	1.05159	0.99950
0.0001	1.41753	1.32774	1.24728	1.17520	1.11054	1.05239	0.99990

P	6 = -2.1	6 = -2.2	6 = -2.3	6 = -2.4	6 = -2.5	6 = -2.6	6 = -2.7
0.9999	-8.43064	-8.64971	-8.80753	-9.08403	-9.29920	-9.51301	-9.72543
0.9995	-6.75798	-6.91370	-7.06804	-7.22098	-7.37250	-7.52258	-7.67121
0.9990	-6.03865	-6.16816	-6.29626	-6.42292	-6.54814	-6.67191	-6.79421
0.9980	-5.32014	-5.42426	-5.52694	-5.62818	-5.72796	-5.82629	-5.92316
0.9950	-4.37186	-4.44398	-4.51407	-4.58393	-4.65176	-4.71815	-4.78313
0.9900	-3.65600	-3.70543	-3.75347	-3.80013	-3.84540	-3.88930	-3.93183
0.9800	-2.94181	-2.97028	-2.99744	-3.02330	-3.04787	-3.07116	-3.09320
0.9750	-2.71234	-2.73451	-2.75541	-2.77506	-2.79345	-2.81062	-2.82658
0.9600	-2.22986	-2.23967	-2.24831	-2.25581	-2.26217	-2.26743	-2.27160
0.9500	-2.00128	-2.00570	-2.00903	-2.01128	-2.01247	-2.01263	-2.01177
0.9000	-1.29377	-1.28412	-1.27305	-1.26240	-1.25039	-1.23766	-1.22422
0.8000	-0.59183	-0.57383	-0.55549	-0.53683	-0.51789	-0.49872	-0.47934
0.7000	-0.18540	-0.16682	-0.14827	-0.12974	-0.11143	-0.09323	-0.07523
0.6000	0.09997	0.11590	0.13148	0.14665	0.16138	0.17564	0.18939
0.5704	0.17030	0.18504	0.19933	0.21313	0.22642	0.23915	0.25129
0.5000	0.31872	0.32999	0.34063	0.35062	0.35992	0.36852	0.37640
0.4296	0.44628	0.45329	0.45953	0.46494	0.46966	0.47353	0.47660
0.4000	0.49494	0.49991	0.50409	0.50744	0.50999	0.51171	0.51263
0.3000	0.64125	0.63833	0.63456	0.62999	0.62463	0.61854	0.61176
0.2000	0.76482	0.75211	0.73830	0.72495	0.71067	0.69602	0.68111
0.1000	0.86938	0.94422	0.94429	0.9472	0.77062	0.74709	0.72422
0.0500	0.91458	0.88156	0.84976	0.81927	0.79015	0.76242	0.73610
0.0400	0.92295	0.88814	0.85486	0.82315	0.79306	0.76456	0.73765
0.0250	0.93495	0.89728	0.86169	0.82817	0.79667	0.76712	0.73943
0.0200	0.93878	0.90009	0.86371	0.82959	0.79765	0.76779	0.73987
0.0100	0.94607	0.90521	0.86723	0.83196	0.79921	0.76878	0.74049
0.0050	0.94945	0.90742	0.86863	0.83283	0.79973	0.76909	0.74067
0.0020	0.95131	0.90854	0.86929	0.83320	0.79994	0.76920	0.74073
0.0010	0.95188	0.90885	0.86945	0.83328	0.79998	0.76922	0.74074
0.0005	0.95215	0.90899	0.86952	0.83331	0.79999	0.76923	0.74074
0.0001	0.95234	0.90908	0.86956	0.83333	0.80000	0.76923	0.74074

P	$G = -2.8$	$G = -2.9$	$G = -3.0$	$G = -3.1$	$G = -3.2$	$G = -3.3$	$G = -3.4$
0.9999	-9.93643	-10.14602	-10.35418	-10.56096	-10.76618	-10.97001	-11.17235
0.9995	-7.81839	-7.96411	-8.10936	-8.25115	-8.39248	-8.53236	-8.67087
0.9990	-6.91505	-7.03443	-7.15235	-7.26881	-7.38382	-7.49739	-7.60853
0.9980	-6.01858	-6.11254	-6.20506	-6.29613	-6.38578	-6.47401	-6.56884
0.9950	-4.84669	-4.90884	-4.96959	-5.02897	-5.08697	-5.14362	-5.19892
0.9900	-3.97301	-4.01286	-4.05138	-4.08859	-4.12452	-4.15917	-4.19257
0.9800	-3.11399	-3.13356	-3.15143	-3.16911	-3.18512	-3.20000	-3.21375
0.9750	-2.94134	-2.85492	-2.86735	-2.87865	-2.88884	-2.89795	-2.90569
0.9600	-2.27470	-2.27676	-2.27740	-2.27785	-2.27693	-2.27506	-2.27229
0.9500	-2.00902	-2.00710	-2.00335	-2.00335	-2.00335	-2.00335	-2.00335
0.9000	-1.21013	-1.19539	-1.19006	-1.16416	-1.14772	-1.13078	-1.11337
0.8000	-0.45980	-0.44015	-0.42040	-0.40061	-0.38081	-0.36104	-0.34133
0.7000	-0.05746	-0.03997	-0.02794	-0.00596	0.01050	0.02654	0.04215
0.6000	0.20259	0.21523	0.22726	0.23868	0.24946	0.25958	0.26904
0.5704	0.26282	0.27372	0.28395	0.29351	0.30238	0.31055	0.31802
0.5000	0.38353	0.38991	0.39554	0.40041	0.40454	0.40792	0.41058
0.4296	0.47988	0.48037	0.48109	0.48107	0.48033	0.47890	0.47682
0.4000	0.51276	0.51212	0.51073	0.50863	0.50585	0.50244	0.49844
0.3000	0.60434	0.59634	0.58783	0.57887	0.56953	0.55989	0.55000
0.2000	0.66603	0.65086	0.63569	0.62060	0.60567	0.59096	0.57652
0.1000	0.70209	0.68075	0.66023	0.64056	0.62175	0.60379	0.58666
0.0500	0.71116	0.68759	0.66532	0.64429	0.62445	0.60572	0.58802
0.0400	0.71227	0.68836	0.66585	0.64465	0.62469	0.60587	0.58812
0.0250	0.71348	0.68917	0.66538	0.64500	0.62491	0.60601	0.58821
0.0200	0.71377	0.68935	0.66649	0.64507	0.62495	0.60603	0.58822
0.0100	0.71415	0.68959	0.66663	0.64514	0.62499	0.60606	0.58823
0.0050	0.71425	0.68964	0.66666	0.64516	0.62500	0.60606	0.58824
0.0020	0.71428	0.68965	0.66667	0.64516	0.62500	0.60606	0.58824
0.0010	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824
0.0005	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824
0.0001	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824

P	$G = -3.5$	$G = -3.6$	$G = -3.7$	$G = -3.8$	$G = -3.9$	$G = -4.0$	$G = -4.1$
0.9999	-11.37334	-11.57284	-11.77092	-11.96757	-12.16280	-12.35663	-12.54906
0.9995	-8.80779	-8.94335	-9.07750	-9.21023	-9.34158	-9.47154	-9.60013
0.9990	-7.72024	-7.82994	-7.93744	-8.04395	-8.14910	-8.25289	-8.35534
0.9980	-6.64627	-6.73032	-6.81301	-6.89435	-6.97435	-7.05304	-7.13043
0.9950	-5.25291	-5.30559	-5.35648	-5.40711	-5.45598	-5.50362	-5.55005
0.9900	-4.22473	-4.25569	-4.28545	-4.31403	-4.34147	-4.36777	-4.39296
0.9800	-3.22641	-3.23800	-3.24853	-3.25803	-3.26653	-3.27404	-3.28060
0.9750	-2.91299	-2.91848	-2.92397	-2.92799	-2.93107	-2.93324	-2.93450
0.9600	-2.26862	-2.26409	-2.25812	-2.25254	-2.24558	-2.23786	-2.22940
0.9500	-1.97147	-1.96266	-1.95311	-1.94283	-1.93186	-1.92023	-1.90796
0.9000	-1.04552	-1.07726	-1.05863	-1.03965	-1.02036	-1.00079	-0.98096
0.8000	-0.32171	-0.30223	-0.28290	-0.26376	-0.24484	-0.22617	-0.20777
0.7000	0.05730	0.07195	0.04610	0.0972	0.11279	0.12530	0.13725
0.6000	0.27782	0.28592	0.29335	0.30010	0.30617	0.31159	0.31635
0.5704	0.32479	0.33085	0.33623	0.34092	0.34494	0.34831	0.35105
0.5000	0.41253	0.41381	0.41442	0.41441	0.41381	0.41265	0.41097
0.4296	0.47413	0.47098	0.46711	0.46286	0.45819	0.45314	0.44777
0.4000	0.49391	0.48888	0.474742	0.47758	0.47141	0.46496	0.45828
0.3000	0.53993	0.52975	0.51452	0.50924	0.49911	0.48902	0.47906
0.2000	0.56242	0.54867	0.53533	0.52240	0.50990	0.49784	0.48922
0.1000	0.57035	0.55483	0.54076	0.52601	0.51261	0.49498	0.48772
0.0500	0.57130	0.55548	0.54050	0.52629	0.51281	0.49999	0.48780
0.0400	0.57136	0.55552	0.54052	0.52630	0.51281	0.50000	0.48780
0.0250	0.57141	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780
0.0200	0.57142	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780
0.0100	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0050	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0005	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780
0.0001	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780

	$G = -4.2$	$G = -4.3$	$G = -4.4$	$G = -4.5$	$G = -4.6$	$G = -4.7$	$G = -4.8$
0.9999	-12.74010	-12.92977	-13.11808	-13.30504	-13.49066	-13.67495	-13.85794
1.9995	-9.72737	-9.85326	-9.97784	-10.10110	-10.22307	-10.34375	-10.46318
0.9990	-8.45646	-8.55627	-8.65479	-8.75202	-8.84800	-8.94273	-9.03623
0.9980	-7.20654	-7.28138	-7.35497	-7.42733	-7.49847	-7.56842	-7.63718
0.9950	-5.59528	-5.63934	-5.68224	-5.72400	-5.76464	-5.80418	-5.84265
0.9900	-4.41706	-4.44009	-4.46207	-4.48303	-4.50297	-4.52192	-4.53990
0.9800	-3.28622	-3.29092	-3.29473	-3.29767	-3.29976	-3.30103	-3.30149
0.9750	-2.93489	-2.93443	-2.93314	-2.93105	-2.92818	-2.92455	-2.92017
0.9600	-2.22024	-2.21039	-2.19988	-2.18874	-2.17699	-2.16465	-2.15174
0.9500	-1.89508	-1.88160	-1.86757	-1.85300	-1.83792	-1.82234	-1.80631
0.9000	-0.96090	-0.94064	-0.92022	-0.89964	-0.87895	-0.85817	-0.83731
0.8000	-0.18967	-0.17189	-0.15445	-0.13737	-0.12067	-0.10436	-0.08847
0.7000	0.14861	0.15939	0.16958	0.17918	0.18819	0.19661	0.20446
0.6000	0.32049	0.32400	0.32693	0.32928	0.33108	0.33236	0.33315
0.5704	0.35318	0.35473	0.35572	0.35619	0.35616	0.35567	0.35475
0.5000	0.40881	0.40621	0.40321	0.39985	0.39617	0.39221	0.38800
0.4296	0.44212	0.43623	0.43016	0.42394	0.41761	0.41121	0.40477
0.4000	0.45142	0.44442	0.43734	0.43020	0.42304	0.41590	0.40880
0.3000	0.46927	0.45967	0.45029	0.44114	0.43223	0.42357	0.41517
0.2000	0.47504	0.46428	0.45395	0.44402	0.43448	0.42532	0.41652
0.1000	0.47614	0.46508	0.45452	0.44443	0.43477	0.42553	0.41666
0.0500	0.47619	0.46511	0.45454	0.44444	0.43478	0.42553	0.41667
0.0400	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0250	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0200	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0100	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0050	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0010	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667
0.0001	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667

P	G = -4.9	G = -5.0	G = -5.1	G = -5.2	G = -5.3	G = -5.4	G = -5.5
0.9999	-14.03963	-14.22064	-14.39918	-14.57706	-14.75370	-14.92912	-15.10332
0.9995	-10.59135	-10.69829	-10.81401	-10.92853	-11.04186	-11.15402	-11.26502
0.9990	-9.12852	-9.21961	-9.30952	-9.39827	-9.48536	-9.57232	-9.65766
0.9980	-7.70479	-7.77126	-7.83657	-7.90078	-7.96390	-8.02594	-8.08691
0.9950	-5.88004	-5.91639	-5.95171	-5.98602	-6.01934	-6.05169	-6.08307
0.9900	-4.55694	-4.57304	-4.58823	-4.60252	-4.61594	-4.62850	-4.64022
0.9800	-3.30116	-3.30007	-3.29823	-3.29567	-3.29240	-3.28844	-3.28381
0.9750	-2.91508	-2.90930	-2.90283	-2.89572	-2.88796	-2.87062	-2.87062
0.9600	-2.13829	-2.12432	-2.10985	-2.09490	-2.07950	-2.06365	-2.04739
0.9500	-1.78982	-1.77292	-1.75563	-1.73795	-1.71992	-1.70155	-1.68287
0.9000	-0.81641	-0.79548	-0.77455	-0.75364	-0.73277	-0.71195	-0.69122
0.8000	-0.07300	-0.05798	-0.04340	-0.02927	-0.01561	-0.00243	0.01028
0.7000	0.21172	0.21843	0.22458	0.23019	0.23527	0.23984	0.24391
0.6000	0.333347	0.333336	0.33284	0.33194	0.33070	0.32914	0.32729
0.5704	0.35343	0.35174	0.34972	0.34740	0.34481	0.34198	0.33895
0.5000	0.38359	0.37901	0.37428	0.36945	0.36453	0.35956	0.35456
0.4296	0.39833	0.39190	0.38552	0.37919	0.37295	0.36680	0.36076
0.4000	0.40177	0.39482	0.38799	0.38127	0.37469	0.36825	0.36196
0.3000	0.40703	0.39914	0.39152	0.38414	0.37701	0.37011	0.36345
0.2000	0.40806	0.39994	0.39211	0.38454	0.37734	0.37036	0.36363
0.1000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0500	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0400	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0250	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0200	0.40416	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0100	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0050	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0020	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0010	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0005	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364
0.0001	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364

P	G = -5.6	G = -5.7	G = -5.8	G = -5.9	G = -6.0	G = -6.1	G = -6.2
0.9999	-15.27632	-15.44813	-15.61878	-15.78826	-15.95660	-16.12380	-16.28969
0.9995	-11.37487	-11.48360	-11.59122	-11.69773	-11.80316	-11.90752	-12.01082
0.9990	-9.74190	-9.82505	-9.90713	-9.98815	-10.06812	-10.14706	-10.22499
0.9980	-8.14693	-8.20572	-8.26359	-8.32046	-8.37634	-8.43125	-8.48519
0.9950	-6.11351	-6.14302	-6.17162	-6.19933	-6.22616	-6.25212	-6.27723
0.9900	-4.65111	-4.66120	-4.67050	-4.67903	-4.68680	-4.69382	-4.70133
0.9800	-3.27854	-3.27203	-3.26610	-3.25398	-3.25128	-3.24301	-3.23419
0.9750	-2.86107	-2.85096	-2.84030	-2.82912	-2.81743	-2.80525	-2.79239
0.9600	-2.03073	-2.01369	-1.99629	-1.97855	-1.96048	-1.94210	-1.92343
0.9500	-1.66390	-1.64464	-1.62513	-1.60538	-1.58541	-1.56524	-1.54487
0.9000	-0.67058	-0.65006	-0.62966	-0.60941	-0.58933	-0.56942	-0.54970
0.8000	0.02252	0.03427	0.04553	0.05632	0.06662	0.07645	0.08580
0.7000	0.24751	0.25064	0.25334	0.25562	0.25750	0.25901	0.26015
0.6000	0.32519	0.32285	0.32031	0.31759	0.31472	0.31171	0.30859
0.5704	0.33573	0.33236	0.32886	0.32525	0.32155	0.31780	0.31399
0.5000	0.34955	0.34455	0.33957	0.33463	0.32974	0.32492	0.32016
0.4296	0.35484	0.34903	0.34336	0.33782	0.33242	0.32715	0.32202
0.4000	0.35583	0.34985	0.34402	0.33836	0.33285	0.32750	0.32230
0.3000	0.35700	0.35078	0.34476	0.33893	0.33330	0.32784	0.32256
0.2000	0.35714	0.35087	0.34483	0.33898	0.33333	0.32787	0.32259
0.1000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32259
0.0500	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0400	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0050	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0020	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0010	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0005	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258
0.0001	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258

P	Q = -6.4	Q = -6.3	Q = -6.2	Q = -6.1	Q = -6.0	Q = -5.9	Q = -5.8	Q = -5.7	Q = -5.6	Q = -5.5	Q = -5.4	Q = -5.3	Q = -5.2
0.9999	-16.45487	-16.61875	-16.78156	-16.94329	-17.10397	-17.26361	-17.42221						
0.9995	-12.11307	-12.21429	-12.31450	-12.41370	-12.51190	-12.60913	-12.70539						
0.9990	-10.30192	-10.37785	-10.45281	-10.52681	-10.59986	-10.67197	-10.74316						
0.9980	-8.533920	-8.59027	-8.64142	-8.69167	-8.74102	-8.78950	-8.83711						
0.9950	-6.30151	-6.32497	-6.34762	-6.36948	-6.39055	-6.41086	-6.43042						
0.9900	-4.70571	-4.71061	-4.71482	-4.71836	-4.72125	-4.72350	-4.72512						
0.9800	-3.22484	-3.21497	-3.20460	-3.19374	-3.18241	-3.17062	-3.15939						
0.9750	-2.777947	-2.76591	-2.75191	-2.73751	-2.72270	-2.70751	-2.69195						
0.9600	-1.90449	-1.88528	-1.86584	-1.84616	-1.82627	-1.80618	-1.78591						
0.9500	-1.52434	-1.50365	-1.48281	-1.46186	-1.44079	-1.41963	-1.39839						
0.9000	-0.53019	-0.51089	-0.49182	-0.47299	-0.45440	-0.43608	-0.41803						
0.8000	0.09469	0.10341	0.11107	0.11859	0.12566	0.13231	0.13853						
0.7000	0.26097	0.26146	0.26167	0.26160	0.26128	0.26072	0.25995						
0.6000	0.30538	0.30209	0.29875	0.29537	0.29196	0.28854	0.28511						
0.5704	0.31016	0.30631	0.30246	0.29862	0.29480	0.29101	0.28726						
0.5000	0.31549	0.31090	0.30639	0.30198	0.29766	0.29344	0.28931						
0.4296	0.31702	0.31216	0.30743	0.30283	0.29835	0.29400	0.28977						
0.4000	0.31724	0.31234	0.30757	0.30294	0.29844	0.29407	0.28982						
0.3000	0.31745	0.31249	0.30769	0.30303	0.29850	0.29412	0.28985						
0.2000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.1000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0500	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0400	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0250	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0200	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0100	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0050	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0020	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0010	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0005	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						
0.0001	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986						

P	G = -7.0	G = -7.1	G = -7.2	G = -7.3	G = -7.4	G = -7.5	G = -7.6
0.9999	-17.57979	-17.73636	-17.89193	-18.04652	-18.20013	-18.35278	-18.50447
0.9995	-12.80069	-12.89505	-12.98848	-13.08098	-13.17258	-13.26328	-13.35309
0.9990	-10.81343	-10.88281	-10.95129	-11.01890	-11.08565	-11.15154	-11.21658
0.9980	-8.88387	-8.92979	-8.97488	-9.01915	-9.06261	-9.10528	-9.14717
0.9950	-6.44924	-6.46733	-6.48470	-6.50137	-6.51735	-6.53264	-6.54727
0.9900	-4.72613	-4.72653	-4.72635	-4.72559	-4.72427	-4.72240	-4.71998
0.9800	-3.14572	-3.13263	-3.11914	-3.10525	-3.09099	-3.07636	-3.06137
0.9750	-2.67603	-2.65977	-2.64317	-2.62626	-2.60905	-2.59154	-2.57375
0.9600	-1.76547	-1.74487	-1.72412	-1.70325	-1.68225	-1.66115	-1.63995
0.9500	-1.37708	-1.35571	-1.33430	-1.31287	-1.29141	-1.26995	-1.24850
0.9000	-0.40026	-0.38277	-0.36557	-0.34868	-0.33209	-0.31582	-0.29986
0.8000	0.14434	0.14975	0.15478	0.15942	0.16371	0.16764	0.17123
0.7000	0.25899	0.25785	0.25654	0.25510	0.25352	0.25183	0.25005
0.6000	0.28169	0.27829	0.27491	0.27156	0.26825	0.26497	0.26175
0.5704	0.28355	0.27990	0.27629	0.27274	0.26926	0.26584	0.26248
0.5000	0.28528	0.28135	0.27751	0.27376	0.27010	0.26654	0.26306
0.4296	0.28565	0.28164	0.27774	0.27394	0.27025	0.26665	0.26315
0.4000	0.28569	0.28167	0.27776	0.27396	0.27026	0.26666	0.26315
0.3000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.2000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.1000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0500	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0400	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0250	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0020	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0010	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0005	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316
0.0001	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316

P	$G = -7.7$	$G = -7.8$	$G = -7.9$	$G = -8.0$	$G = -8.1$	$G = -8.2$	$G = -8.3$
0.9999	-18.65522	-18.80504	-18.95393	-19.10191	-19.24898	-19.39517	-19.54046
0.9995	-13.44202	-13.53009	-13.61730	-13.70366	-13.78919	-13.87389	-13.95778
0.9990	-11.34419	-11.40677	-11.46855	-11.52953	-11.58974	-11.64917	-11.69193
0.9980	-9.18828	-9.22863	-9.26823	-9.30704	-9.34521	-9.38262	-9.41931
0.9950	-6.56124	-6.57456	-6.58725	-6.59931	-6.61075	-6.62159	-6.63183
0.9900	-4.71704	-4.71358	-4.70901	-4.70514	-4.70019	-4.69476	-4.68887
0.9800	-3.04604	-3.03038	-3.01439	-2.99810	-2.98150	-2.96462	-2.94746
0.9750	-2.55569	-2.53737	-2.51881	-2.50004	-2.48099	-2.46175	-2.44231
0.9600	-1.61867	-1.59732	-1.57591	-1.55444	-1.53294	-1.51141	-1.48985
0.9500	-1.22706	-1.20565	-1.18427	-1.16295	-1.14168	-1.12048	-1.09936
0.9000	-0.28422	-0.26892	-0.25394	-0.23929	-0.22498	-0.21101	-0.19737
0.8000	0.17450	0.17746	0.18012	0.18249	0.18459	0.18643	0.18803
0.7000	0.24817	0.24622	0.24421	0.24214	0.24003	0.23788	0.23571
0.6000	0.25857	0.25544	0.25236	0.24933	0.24637	0.24345	0.24060
0.5704	0.25919	0.25596	0.25280	0.24970	0.24667	0.24371	0.24081
0.5000	0.25966	0.25635	0.25312	0.24996	0.24689	0.24388	0.24095
0.4296	0.25973	0.25640	0.25316	0.25000	0.24691	0.24390	0.24096
0.4000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.3000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.2000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.1000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0500	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0400	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0250	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0200	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0100	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0050	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0020	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0010	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0005	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096
0.0001	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096

P	$G = -8.4$	$G = -8.5$	$G = -8.6$	$G = -8.7$	$G = -8.8$	$G = -8.9$	$G = -9.0$
0.9999	-19.68489	-19.82845	-19.97115	-20.11300	-20.25402	-20.39420	-20.53356
0.9995	-14.04096	-14.12314	-14.20463	-14.28534	-14.36528	-14.44446	-14.52288
0.9990	-11.70785	-11.76576	-11.82294	-11.87938	-11.93509	-11.99009	-12.04437
0.9980	-9.45530	-9.49060	-9.52521	-9.55915	-9.59243	-9.62504	-9.65701
0.9950	-6.64148	-6.65056	-6.65907	-6.66703	-6.67443	-6.68130	-6.68763
0.9900	-4.68252	-4.67573	-4.66850	-4.66085	-4.65277	-4.64429	-4.63541
0.9800	-2.93062	-2.91234	-2.89440	-2.87622	-2.85782	-2.83919	-2.82035
0.9750	-2.42264	-2.40287	-2.38288	-2.36273	-2.34242	-2.32197	-2.30138
0.9600	-1.46429	-1.44673	-1.42518	-1.40364	-1.38213	-1.36065	-1.33922
0.9500	-1.07432	-1.05738	-1.03654	-1.01581	-0.99519	-0.97471	-0.95435
0.9000	-0.18408	-0.17113	-0.15851	-0.14624	-0.13431	-0.12272	-0.11146
0.8000	0.19939	0.19054	0.19147	0.19221	0.19277	0.19316	0.19338
0.7000	0.23352	0.23132	0.22911	0.22690	0.22469	0.22249	0.22030
0.6000	0.2779	0.25505	0.23236	0.22972	0.22714	0.22461	0.22214
0.5704	0.23747	0.23520	0.23248	0.22982	0.22722	0.22468	0.22219
0.5000	0.23904	0.23526	0.23255	0.22986	0.22727	0.22472	0.22222
0.4296	0.23409	0.23529	0.23256	0.22984	0.22727	0.22472	0.22222
0.4000	0.23810	0.23529	0.23256	0.22984	0.22727	0.22472	0.22222
0.3000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.2000	0.23811	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.1000	0.23910	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0500	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0400	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0250	0.23410	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0200	0.23410	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0100	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0050	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0020	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0010	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0005	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222
0.0001	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222