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RECHARGE FROM LARGE DEPRESSION STORAGE

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ABSTRACT

The transient analysis of the groundwater flow around a lake of square cross section and having uniform depth has been analysed for a hypothetical setting of the boundaries (rivers on the lateral sides of the lake and no flow boundary at the other two sides; both the boundaries being at the same distance from the lake) using a three dimensional finite difference model. Similarity approach has been employed to develop a type curve, which enables the assessment of the rate of recharge from the lake to groundwater reservoir, with the help of observed head change in an observation well situated anywhere within the influence area of the lake and vise-versa. Ideal location for an observation well has also been suggested.

1.0 INTRODUCTION

1.1 General

Lakes are a highly visible features on earth surface in many parts of the world, because they are pleasing aesthetically and important economically for water supply etc., and are foci for urban development. The other benefits that might occur from a lakes/reservoirs are flood cushion, pisiculture, etc. Hence owing to its importance in various hydrologic and economic fields, it is necessary to maintain their quality and existance in its pure form. Understanding of the water balance the lake management. Hydrologic studies on lakes include lake manipulation, general hydrologic description, lake level fluctuation analysis and effect of floods and evaporation on lakes and the interaction with the aquifer system underneath it.

Inspite of early recognition of lake management studies, very few attempts have made to the understanding of the interaction of lake and ground-water reservoir. Even with the well established science of limnology, groundwater system around lake have not received the attention it deserves. Since the groundwater is often a very significant component of water budget of lake, the quantification of ground water flow system around a lake is generally necessary for the accurate preparation of lakes hydrologic, chemical or energy budget In most of the studies of lake water balance, groundwater is often calculated as residual of the water balance equation, which lead to inaccurate water balance due to the reasons that overland runoff and non channelised surface flow in the lake, is often lumped into the residual with groundwater and that the errors in the measurements of different components are not considered.

The process of estimating lake seepage rate, based on the observed flow system, makes use of the simple application of Darcy's law in one or two dimensions, thus, over simplyfying most real lakes system. Spacial variability of ground water flow is a function of nearly boundary, variable lake depth thickness of lake sediment, hydraulic conductivity, aquifer homogenity and isotropy and hence, it is obvious that manual flow net calculations based on Darcy's law will be combersom and inaccurate. Therefore, proper theoretical and field studies of lake and ground-water interaction are needed, former will define the principles underlying this interaction and later will help in varifying the theoretical models and can be used to develop practical field measurement techniques.

With the advent of very high speed computers, various numerical techniques have been evolved for solving non linear problems with complex boundary conditions. The three dimensional analysis of complex flow is now possible with the help of mathematical modelling. Thus, the mathematical modelling has been proved to be the only tool for assessing the groundwater flow pattern near the lake, as they are useful for ident-ifying most important parameters affecting lake-seepage rates, thereby lending direction to field studies of ground water system around the lake.

1.2 Scope of the Present Study

The present study was aimed at understanding of the behaviour of the ground water system in conjunction with lake (as a recharge source) on one side and a river (as a discharging area) on the other side.

The cross sectional view of the problem of lake-aquifer-river interaction is shown in Fig. 1. At one side of the lake constant head

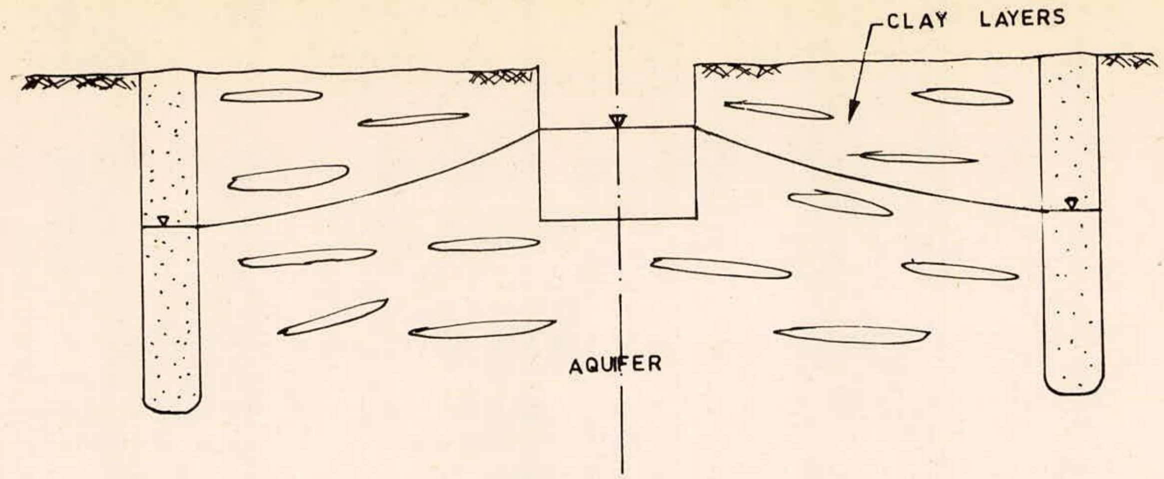

FIG. 1 – CROSS SECTIONAL VIEW OF THE PROBLEM OF LAKE RIVER-AQUIFER INTERACTION.

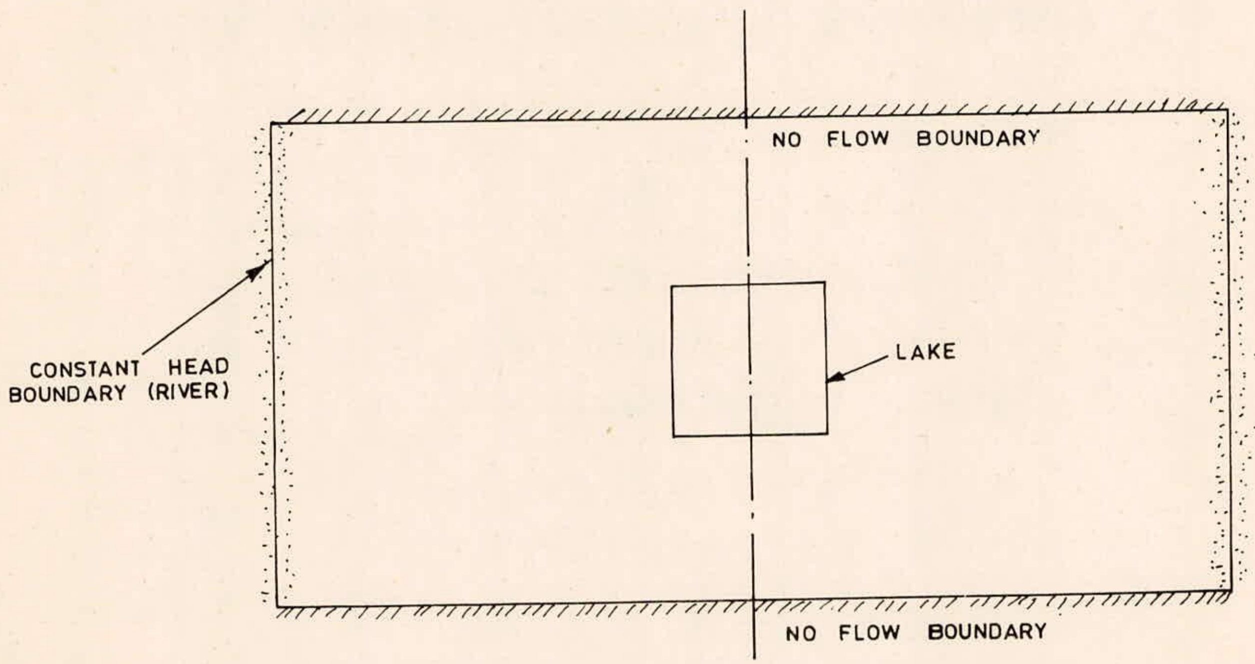

FIG. 2 - PLAN VIEW OF THE PROBLEM OF LAKE-RIVER-AQUIFER INTERACTION.

boundary (river) has been assumed and on the lateral side no flow boundary has been considered (vide fig. 2). The study aims at developing type curves between the non-dimensional time and non dimensional drawdown from the lake. For the present analysis, the aquifer has been considered to be homogeneous with Kh/Kv = 500 (Kh and Kv are the hydraulic conductivity in horizontal and vertical direction respectively) and the lake was simulated to be of square cross-section in the plan having uniform depth. A three dimensional finite difference model has been used to simulate the lake aquifer river interaction.

1.3 Further Scope

The further study is required to be done to examine the influence of various other parameters on recharge rate from lake and are given below.

1. To study the influence of variation: Kh/Kv ; Kh and Kv are the hydraulic conductrivity in horizontal and vertical direction respectively.
2. The lake boundary should be so assumed as to fit the natural shape of the lake.
3. Modelling of clay layer present in the aquifer.
4. To study the effect of the hydrograph of lake water level fluctuation on the rate of recharge from lake.

2.0 REVIEW

Among the early investigators who stressed upon the groundwater flow pattern around a lake, are Mayboon (1966, 67), Williams (1968) and Janquet (1976). Bergstrom and Hansen (1962), Skinner and Borman (1973) and Cartwrite et al. (1979) have computed the ground water flow from the lake Michigan, Bergstorm et al. calculated the groundwater component as residual term. Skinner and Borman (1973) also estimated the groundwater component as the residual of their water balance equation but cartwrite et al. (1979) made direct measurement of hydraulic gradients in the southern part of the lake. The wide variations between their estimates in seepage rates indicate the need of refined methods for determining the groundwater flux from a large lake.

Allred et al. (1971), Manson et al. (1968), Solan (1972) and Mc-Bride (1969) measured the groundwater flow from lake with the help of water level observation wells. Some insight into the groundwater regime of discharge estimates was provided by Winter (1976, 1978), who used two and three dimensional steady state models applied to hypothetical groundwater lake systems.

Janquet did a study of Snake Lake, which was classified as flow through type i.e. a lake which receives groundwater through part of the lake basin and recharges to the groundwater system over the rest of the lake basin. His data indicated that in the spring of 1973, the formation of groundwater mound on the downgradient side of the lake caused a reversal in flow direction. If the seasonal formation of ground water mounds and stagnation points along with prolonged seasonal reversals in the flow occurs commonly at flow through lakes, it would be important to study seasonal changes in head around a lake before taking certain type of planning decisions related to lake management.

McBride and Pfannkuch (1975) used a numerical model to evaluate the vertical component of groundwater flow into one side of a lake for a number of hypothetical settings. Winter (1976) used numerical simulation of vertical groundwater flow to examine the hydrological factors that control the interaction of lakes and groundwater along the entire lake bottom for a wide variety of hypothetical settings. He showed that the movement of groundwater to and from a lake depends on the continuity of the boundary separating the local groundwater flow system associated with lake, from intermediate and regional flow systems passing at depth beneath the lake. Based on his simulation study, he suggested the field methods and new approaches to the study of the interaction of lakes and groundwater along with the critique of commonly used approaches. Studies in which one or many wells are placed near a lake to determine the interaction of lakes and groundwater, must be scrutinized carefully, because placement and constr-uction of wells are critical to a proper understanding of the interrelationship of lakes and groundwater.

Rinaldo lee and Anderson (1980) conducted investig-ation into the cause of high levels for Bass Lake during the early 1970's in a groundwater dominant lake in North Western Wisconsin. A groundwater flow model was used to determine whether increased recharge rates of the magnitude that probably occurred as a result of average precipitation in the early.1970's would be sufficient to account for the observed rise in the lake level or whether regulation of the water level in the reservoir would be expected to effect lake level. The model was used to investigate (i) the expected magnitude of the change in lake level in response to increased recharge rates brought about by high precipitation and (ii) the effect on lake level of changes in the level of reservoir The result of the simulation suggests that increased recharge brought about by above average precipitation in 1975 was sufficient to account for observed increase in lake level.

Anderson and Munter (1981) studied the development of groundwater mound near Snake Lake by means of two dimensional transient groundwater flow models. The areal flow model was used to demonstrate the effect of stagnation zone on the water budget of lake. The ground water component of lakes water budget as measured by difference between total inflow of groundwater through the bottom of the lake and total outflow of groundwater through the lake bottom varied from -4.0×10^{-4} m^3/s at the beginning of the simulation of $+0.21$ ft^3/s, 36 days later Thus, the seasonal formation of stagnation zone can have a marked effect on the groundwater component of lakes water budget. They found that the primary factors governing the formation of mound and stagnation zone at snake lake are

1. Location of lake on ground weater divide
2. The suspected occurrence of zones of low permeability northwest of the lake and/or locally high recharge from leaky storm severs in that area. Stross and Spangler (1980) reported the occurance of deposit of low permeability on the downgradient side of several small lakes in Florida. They suggested that the mechanism for concentration is the flushing of fine-grained particles from the lake sediments into the aquifer downgradient of the lake If this is the case, then the presence of low permeability of Snake Lake would not be unexpected.

A seasonally high rate of recharge is primary trigger the formation of the mound while other factors such as the reduced thickness of the aquifer beneath the lake and a low regional hydraulic gradient also contribute to the development of the mound.

They concluded that the existance of a stagnation point is an indication that the ground water seeking into the lake over most of the lake basin and reflects a change in the ground water flow pattern around a flow through lake. Such a change in ground water regime can have a marked effect on a lakes water and nutrient budgets. However, more field observations of seasonal variations in ground water potential around flow through lakes are needed before an assessment of the frequency with which seasonal stagnation points form at flow through lakes can be made.

Munter and Anderson (1982) showed that two and three dimensional groundwater flow models provide flexible and effective means of calculating flow rates in well defined but complex natural flow systems around lakes. The general conclusion drawn from his study are:

1. The Bass Lake model showed that the ratio of horizontal to vertical hydreaulic conductivity Kh/Kv of the aquifer has significant effect on the simulated head distribution around Bass Lake, and on the magnitude and distribution of seepage from the lake. Kh/Kv can be estimated by model calibration of field measurements of the vertical hydraulic conductivity of littoral lake bed sediments exerts a strong influence on computed seepage rates. It is recommended that field studies of lake include mapping the thickness and distribution of different types of lake

bed sediments, particularly in littoral areas, and estimating their vertical hydraulic conductivity.

2. The models constructed for Nepco Lake shows that two dimensional profile models are not always adequate or applicable for simulating flow systems around lakes. A three dimensional model, however although larger and more difficult to use, is a realistic alternative where a complex three dimensional flow system actually exists. For lake seepage rate estimation, both the two and three dimensional models are improvement over one dimensional Darcy's Law Method because of their ability to incorporate the heterogeneity and anisotropy of the aquifer as well as hydraulic gradient data in more than one dimension, hence, it is suggested that at many lakes a combination of two-and/or three dimensional models could be used to estimate lake seepage rates. This may include the application of a two dimensionasl areal model (e.g. see Anderson and Munter, 1981).

3. Two dimensional models are appropriate when the geology of a lake is well known and where vertical head data indicate the presence of a two dimensional flow system around part or all of a lake. Two dimensional models are also useful in a three dimensional flow system where a less accurate simulation is desired or where a sensitivity analysis is to be performed. The results of a two dimensional sensitivity analysis can be transferred readily to a similarly posed three dimensional model.

4. Three dimensional modelling should be considered when geometry of surface water bodies and/or geological condition create a

truely, three dimensional flow system, and where surfficient geological data and vertical and areal hydraulic head data are available to describe the flow system.

From the critical review of the past studies, it is observed that the interference of the various variables involved,on the recharge rate from depression storage have not yet been clearly understood and so far, no guidelines or methods have been evolved for the assessment of recharge from large depression storage. Hence,there is a great need to study the influence of various parameters on the recharge rate, which will be useful for the assessment of the quantity of seepage from lake

3.0 PROBLEM DEFINITION

For the analysis of the lake-aquifer-river interaction, a lake of square cross section, 300 m x 300 m has been considered. It was also assumed that the lake has constant depth of 9 m over its section. The aquifer was assumed to be homogenous with a constant head boundary (i.e. river) and a no flow boundary at either side of the lake, at a distances of 2.5 km. from the lake side. Detail description of the problem is shown in Fig.3. For the present problem Kh/Kv was assumed to be 500.

The present problem is to study the lake-aquifer-river interaction for a hypothetical setting (described earlier) It was also intended to develop a type curve between non dimensional time and non dimensional rise of water table level, using which the rate of recharge from the lake can be determined at any instant of time, provided a record of water level fluctuations in an observation well, situated anywhere within the influence area of lake, is known and vice-versa. The proper location of the observation well has also been suggested.

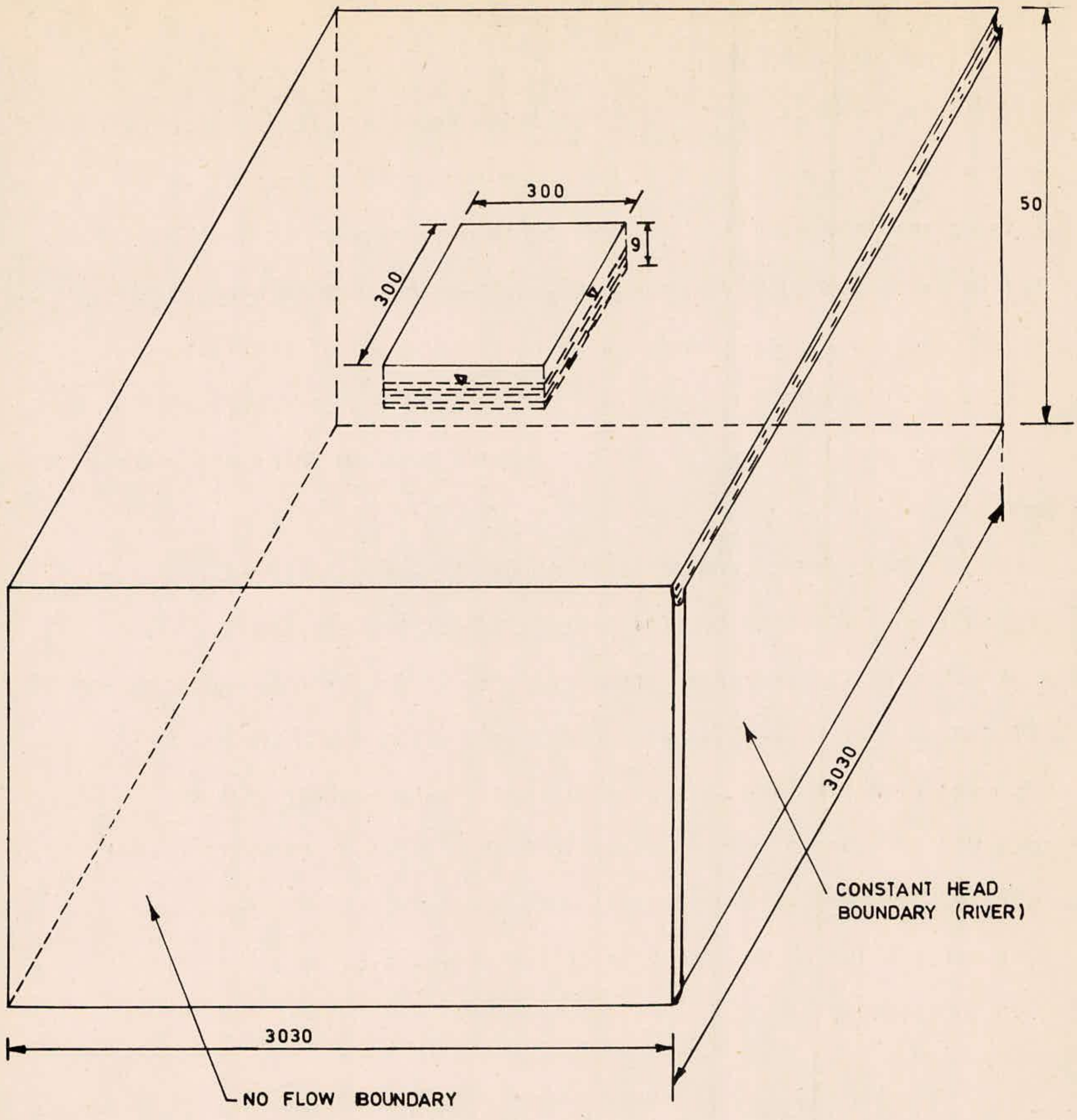

FIG. 3 - DESCRIPTION OF THE PROBLEM.

4.0 MODEL DESCRIPTION

4.1 Mathematical Model

The three dimensional movement of incompressible groundwater through heterogenous and anisotropic medium may be described by the partial difference equation

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = Ss\frac{\partial h}{\partial t} \quad \ldots (1)$$

where,

x, y and z are the cartesian cordinates alined along the major axes of hydraulic conductivity K_{xx}, K_{yy} and K_{zz},

h is the potenti-ometric head (L)

W is the volumetric flux per unit volume and represents sources and/or sinks of water (L^{-1}) ;

Ss is the specific storage of the porous material (L^{-1}) ;

t is the time (t)

The above equation when combined with specification of flow and boundary conditions of an aquifer system along with initial conditions forms a mathematical model of three dimensional groundwater flow.

Ananlytical solution of Eq. 1 is not possible for very complex systems, hence, numerical methods are employed to obtain approximate solutions. Finite difference approach is one of such numerical methods, in which, the continuous derivatives are replaced by differences between functional values at discrete points. Thus, this process will lead to a set of simultaneous algebraic differential equations, the solution of which yield the values of head at specific points and time. These values will be an approximation to the time varying head distribution that would be given by an analytical solution of the partial differential equation of flow.

4.2 Formulation of Finite Difference Equations

The basis of the finite difference equation is the use of continuity equation of flow, which implies that the sum of all flows into and out of a cell is equal to the change in the storage within the cell, therefore, the continuity equation for a cell is given by eq. 2, provided density of groundwater is constant.

$$Q_i = Ss \frac{\Delta h}{\Delta t} \Delta V \quad \ldots (2)$$

where,

Q_i = is a flow rate into the cell (L^3T^{-1});

Ss = is the specific storage defined as the ratio of the volume of water which can be injected per unit volume of aquifer material per unit change of head (L^{-1})

Δh is the change of head over a time interval of length t;

ΔV is the volume of the cell (L^3)

The flow rate into a cell (i,j,k) will be the algebraic sum of flow rate from all the six adjoining cells and flow from each adjoining cell can be represented by Darcy's law. The flow from outside the aquifer may be dependent on the head in the receiving cell but independent of all other heads in the aquifer or they may be entirely independent of the head in the receiving cell, thus, the flow from outside the aquifer may be represented by the equation 3.

$$QS_{i,j,k} = \sum_{n=1}^{N} a_{i,j,k,n}.$$

$$= \sum_{n=1}^{N} p_{i,j,k,n} h_{i,j,k} + \sum_{n=1}^{N} q_{i,j,k,n}$$

$$= P_{i,j,k} h_{i,j,k} + Q_{i,j,k} \qquad \ldots (3)$$

where,

(i,j,k) is array convention describing row, column and layer respectively

$Q_{i,j,k,n}$ represents the flow from nth external source into cell (i,j,k) (L^3t^{-1}); and

$P_{i,j,k,n}$ and $Q_{i,j,k,n}$ are constant (L^2T^{-1} and L^3T^{-1} respectively)

and $P_{i,j,k} = \sum_{n=1}^{N} P_{i,j,k,n}$; and

$Q_{i,j,k} = \sum_{n=1}^{N} q_{i,k,k,n}$;

N is the total number of external sources or stresses affecting a single cell.

Substituting in eq. 2, the sum of the flows from adjoining cells (obtained by the use of Darcy's law) and the flow from all external sources into the cell i,j,k one gets the finite difference equation (Eq. 4) for a particular cell after rearranging it.

$$CV_{i,j,k-\frac{1}{2}} h^m_{i,j,k-1} + CC_{i-\frac{1}{2},j,k} h^m_{i,j,k}$$
$$+ CR_{i,j-\frac{1}{2},k} h^m_{i,j-\frac{1}{2},k} + (-CV_{i,j,k-\frac{1}{2}} - CC_{i-\frac{1}{2},j,k}$$
$$- CR_{i,j-\frac{1}{2},k} - CR_{i,j+\frac{1}{2},k} - CC_{i+\frac{1}{2},j,k} - CV_{i,j,k+\frac{1}{2}}$$
$$+ HCOF_{i,j,k}) h^m_{i,j,k} + CR_{i,j+\frac{1}{2},k} h^m_{i,j+1,k}$$
$$+ CC_{i+\frac{1}{2},j,k} h^m_{i+1,j,k} + CV_{i,j,k+\frac{1}{2}} h^m_{i,j,k+1} = RHS_{i,j,k} \quad \ldots (4)$$

where,

$HCOF_{i,j,k} = P_{i,j,k} - SCI_{i,j,k}/(t_m - t_{m-1})$

$RHS_{i,j,k} = -Q_{i,j,k} - SC1_{i,j,k} h^{m-1}_{i,j,k} / (t_m - t_{m-1})$; and

$SC1_{i,j,k} = SS_{i,j,k} (\Delta r_j \Delta c_i \Delta v_k)$,

r, c and v are the spacing in row, column and vertical direction

CV, CR and CC are the conductance in vertical, row and column direction respectively between the two cells. Conductance is the product of hydraulic conductivity and cross sectional area of flow divided by the length of flow path (i.e. the distance between the nodes). Suffix $j-\frac{1}{2}$ represents the average value of the parameters between j and j-1.

Equation 4 can be written in backward difference form by specifying flow terms at time t_m, the end of the time interval, and approximating the time derivatives of head over the interval t_{m-1} to t_m ; i.e.

$$CR_{i,j-\frac{1}{2},k} (h^m_{i,j-1,k} - h^m_{i,j,k}) +$$

$$CR_{i,j+\frac{1}{2},k} (h^m_{i,j+1,k} - h^m_{i,j,k}) +$$

$$CC_{i-\frac{1}{2},j,k} (h^m_{i-1,j,k} - h^m_{i,j,k}) +$$

$$CC_{i+\frac{1}{2},j,k} (h^m_{i+1,j,k} - h^m_{i,j,k}) + CV_{i,j,k-\frac{1}{2}} (h^m_{i,j,k-1} - h^m_{i,j,k}) +$$

$$CV_{i,j,k+\frac{1}{2}} (h^m_{i,j,k+1} - h^m_{i,j,k}) +$$

$$P_{i,j,k} h^m_{i,j,k} + Q_{i,j,k} = SS_{i,j,k} (\Delta r_j \Delta C_i \Delta v_k)$$

$$= (h^m_{i,j,k} - h^{m-1}_{i,j,k}) / (t_m - t_{m-1}) \quad \ldots (5)$$

An equation of the above type may be written for each of the cells in the system. Thus,one have n equations in n unknowns for a system of n cells, and such a set of equations can be solved simultaneously to get the head value of each cell (specifically at nodes) of a system at different time. For the solution of these equations storongly implicit procedure has been adopted. An equation of the form of eq. 5 is required for each variable head cell. 'Constant head cells' are those in which head remains constant with time and, hence, do not require an equation. The equations for adjacent variable head cells, however, will contain non zero conductance term representing flow from constant head cell. 'No flow cells' are those to which there are no flow from adjacent cells and there is no equation for a no flow cell as well as for adjacent cells.

4.3 Discretization of Space

For the formulation of finite difference equations, the spacial discretization of the system, shown in fig.3, is given in fig.4 in which the discretization for only an quadrant block is shown. The whole system is discretized into 32 rows and 32 columns and single layer, forming 32 x 32 x 1 cells/ or nodes. The spacing of rows and columns was kept symmetrical about the axes passing through the centre of the lake. Due to greater variation of water table near the lake, the logrithmic increase in the spacings of rows/columns away from the lake was assumed. Table 1, shows the spacing of rows/columns, away from the centre of the lake, Fig. 4 shows the plan of discretized space.

Table 1

Spacing of Row/Column Away from the Lake

Spacing (m)	30	30	30	30	30	40	50	60	80	100	140	240	360	480	600	700

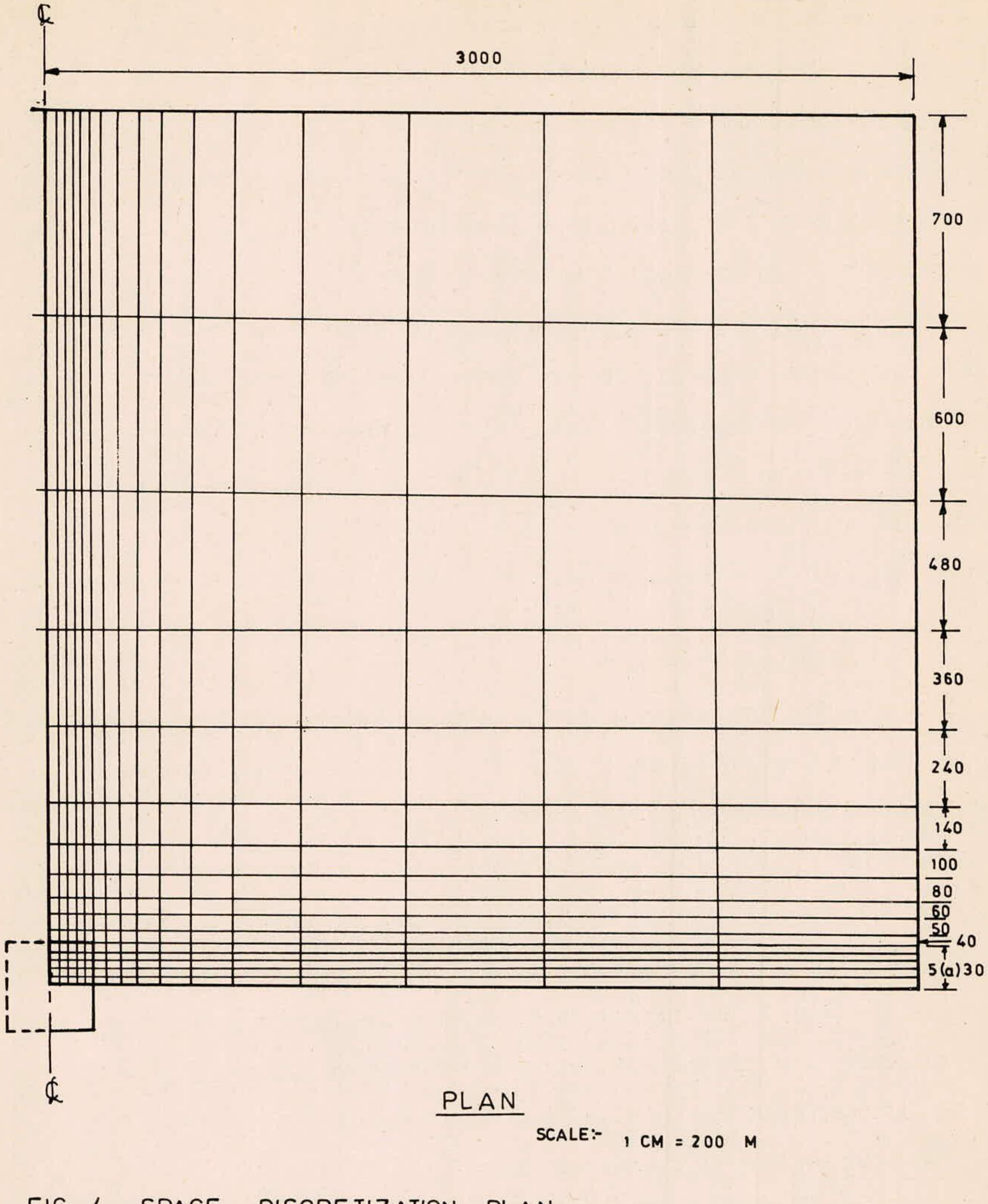

FIG. 4 — SPACE DISCRETIZATION PLAN

4.4 Discretization of Time

The total period of simulation was taken to be 364 days, consisting of 52 time step of uniform span of seven days each. The volumetric budget was obtained at each time step and the distribution of head within the aquifer was obtained at every fourth time step.

4.5 Simulation of Leakage from Lake Bed

To simulate the effect of lake-leakage in the model, terms representing the leakage are added to the ground water flow equation, i.e.,eq. 5. The lake is divided into reaches, each of which is completely contained in a single cell. For the present study the leakage through a reach of lake bed is approximated by Darcy's law as

$$Q_{RIV} = KLW\ (HRIV - HAQ)/M \quad \dots (6)$$

where,

Q_{RIV} is the leakage through the reach of the river bed (L^3t^{-1})

K is the hydraulic conductivity of the river bed (Lt^{-1})

L is the length of the reach

W is the width of the river bed

M is the thickness of the river bed (L);

HAQ is the head on the aquifer side of the lake bed (L); and

HRIV is the head on the river side of the lake bed.

5.0 ANALYSIS OF RESULTS

Simulation of the transient seepage through lake into the aquifer and subsequent drainage into an adjoining river, for the hypothetical problem described earlier, was carried out. The head distribution in the aquifer system at discrete nodes over a period of time, also discretized into different time steps, was obtained (discretization of space and time have already been discussed in the previous chapters). The simulation was repeated for different values of head difference between the water level in the lake and river strage (ΔH) i.e., 9m, 7m, 5m, and 3m. For convenience, the river stage and the water table level in the aquifer was kept at a constrant level at the start of the simulation while the water level in the lake was kept varying for different simulations. The hydraulic conductivity of the aquifer was considered as 10.7 m/day, while the specific yield(s) was taken as 15%. The ratio Kh/Kv was kept equal to 500 and the bottom and sides of the lake was considered to have low permeability values due to sediment diposition and were taken as 0.038 times hydraulic conductivity of the aquifer. The difference between the water table level at the discrete points (nodes), due to seepage occuring from the lake and the river stage was computed and designated as Δh. Plots were drawn between X/L and $\Delta h/\Delta H$ for discrete time interval for first layer for each ΔH. Fig. 5 shows such variation for ΔH = 3m. Where X is the perpendicular distance from the lake to the observation well location and L is the perpendicular distance between lake and river. Values of L for the present discretization is 2500 m. Table 2 shows the difference values of X/L for present discretization.

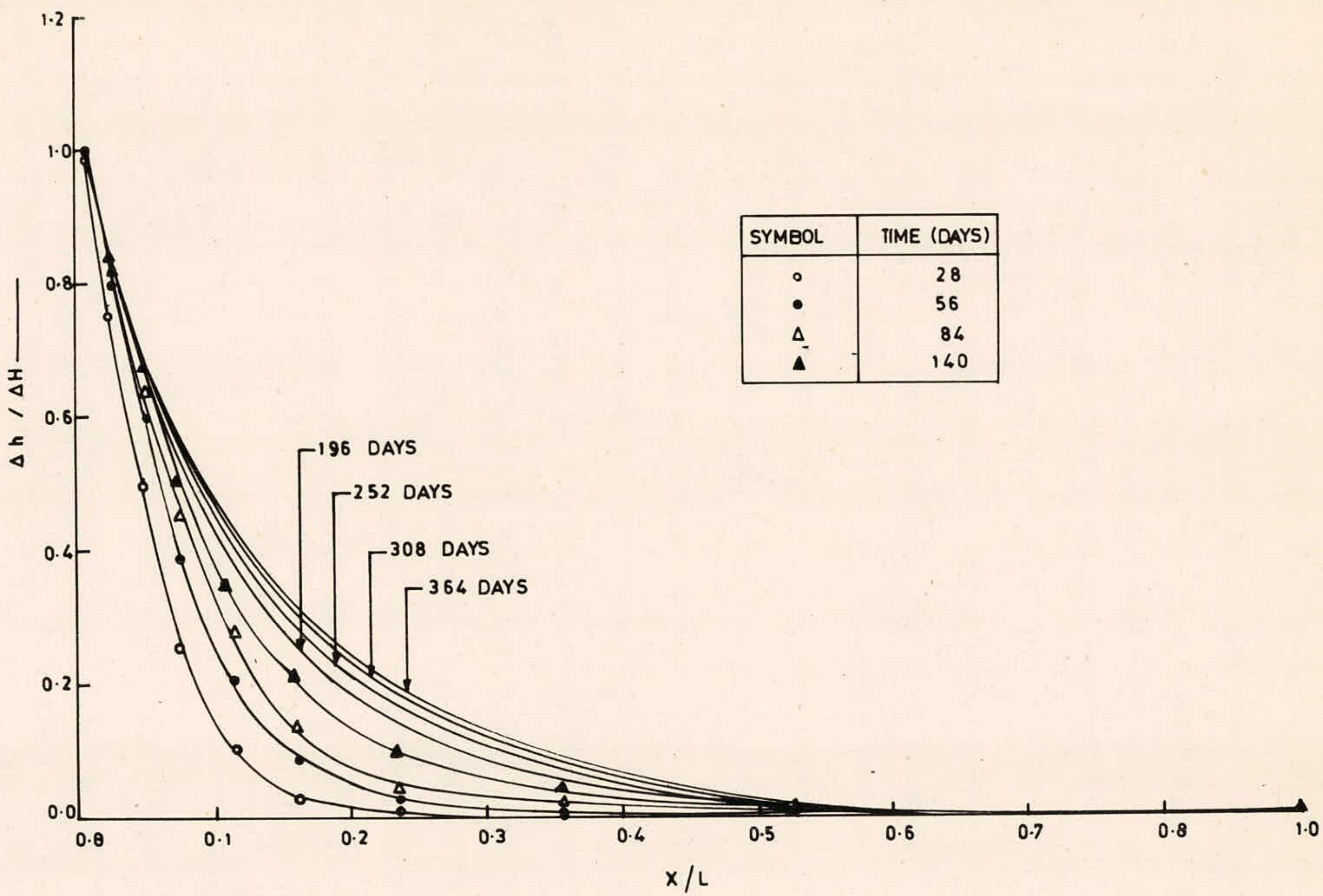

FIG. 5 - VARIATION OF $\Delta h/\Delta H$ WITH X/L AND t ($\Delta H = 3 \cdot 0$ m)

Table 2

Values of X/L for present discretization

X	20	65	120	190	280	400	590	890	1310	1850	2500
X/L	0.008	0.026	0.048	0.076	0.112	0.160	0.236	0.356	0.524	0.740	1.00

From the plots, it can be seen that $\Delta h/\Delta H$ increases with increase in time for a particular X/L, though the rate of increase, decreases with increase in time, suggesting that a steady state situation can be reached.

Variation of $((\Delta h)_s - (\Delta h))/\Delta H$ with X/L and t was observed by plotting the graph between $((\Delta h)_s-(\Delta h))/\Delta H$ vs X/L for all value of t for each simulation (i.e., ΔH = 9.0, 7.0, 5.0 & 3.0m). Figs. 6 and 7 show such variation for ΔH = 5.0 m and 3.0 m respectively which indicate that with increasing time, the maxima of the curve shifts in a positive X-direction and it is observed that the shifting of the maxima for all time is ranged between X/L = 0.15 to 0.25 (this range of X/L was found to be the same for differen value of ΔH). Hence, it can be said that within the above range, if a observation well is located, it will observe a comparatively rapid change of head and thus it will be sensitive. Therefore, it can be concluded that the location of the observation well within X/L of 0.15 to 0.25 will be ideal.

The further analysis of results have shown that the parameters $X^2S/T.t$ and $T.\Delta h/Q_R$ are uniquely related and the relation has found to be the same for different values ΔH, where T is the transmissivity of aquifer and t is the time from the commencement of simulation, and Q_R is the seepage rate from lake. Fig. 8 shows the relation between the above parameters which includes the data from all simulation (i.e.ΔH = 9.0m,

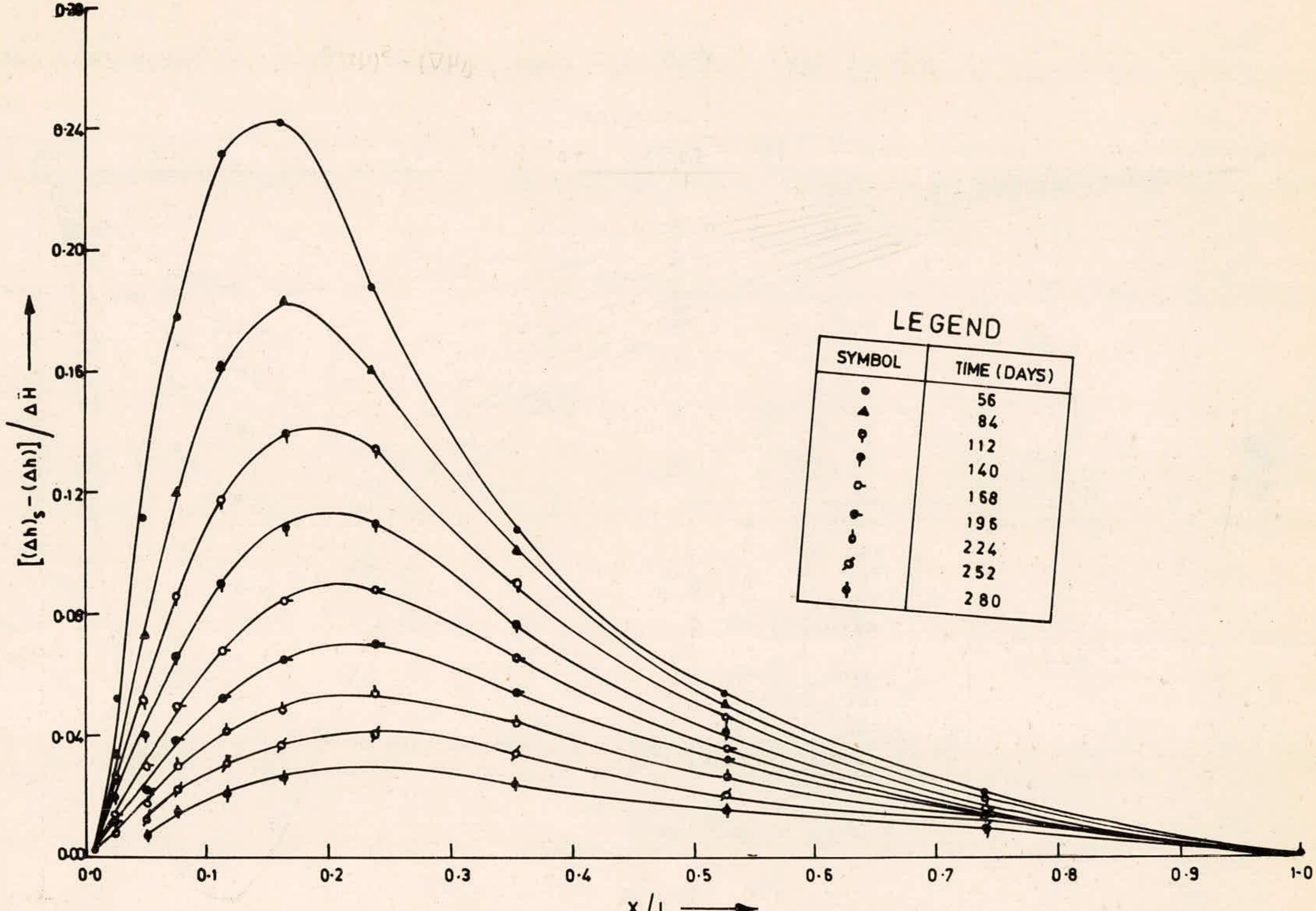

FIG. 6 – VARIATION OF $[(\Delta h)_S - (\Delta h)]/\Delta H$ WITH X/L AND t ($\Delta H = 5{\cdot}0$ m)

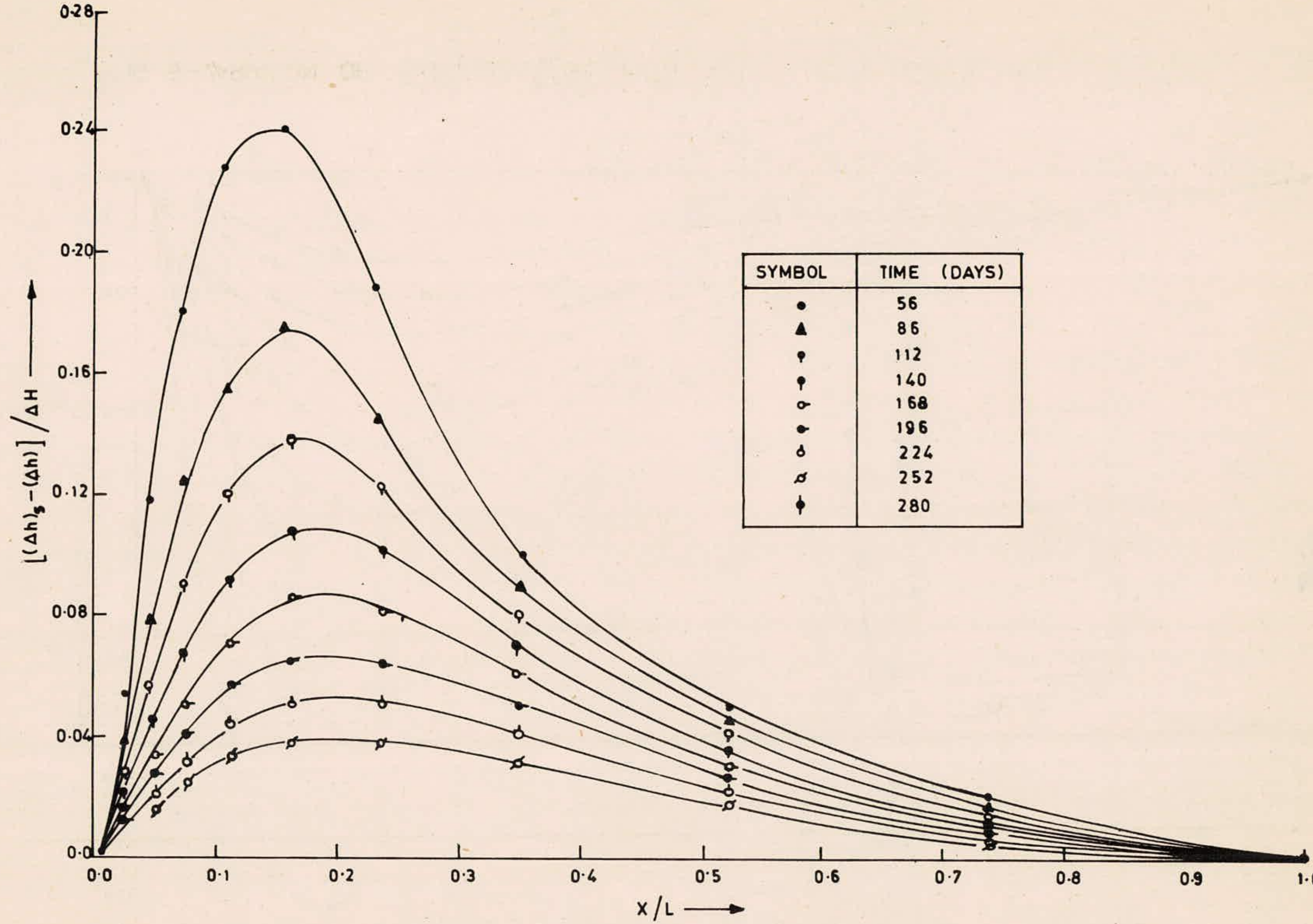

FIG. 7 – VARIATION OF $[(\Delta h)_S - (\Delta h)] / \Delta H$ WITH X/L AND t (ΔH = 3·0 m)

7.0m, 5.0m, 3.0m) and the type curve has been drawn. The variation of the above parameters has been ginve in tabular form in the Appendix.

The type curve shown in Fig. 8 can be used to estimate the recharge Q_R from the lake, knowing Δh at a certain value of X/L and t (while S & T are kept constrant for the simulation).

This type curve (Fig. 8) is applicable only for Kh/Kv = 500 and l/L = 0.12, where l is the length of the side of the lake. In order to establish the applicability of the type curve for other valaue of Kh, Kv and l/L, further study needs to be carried out.

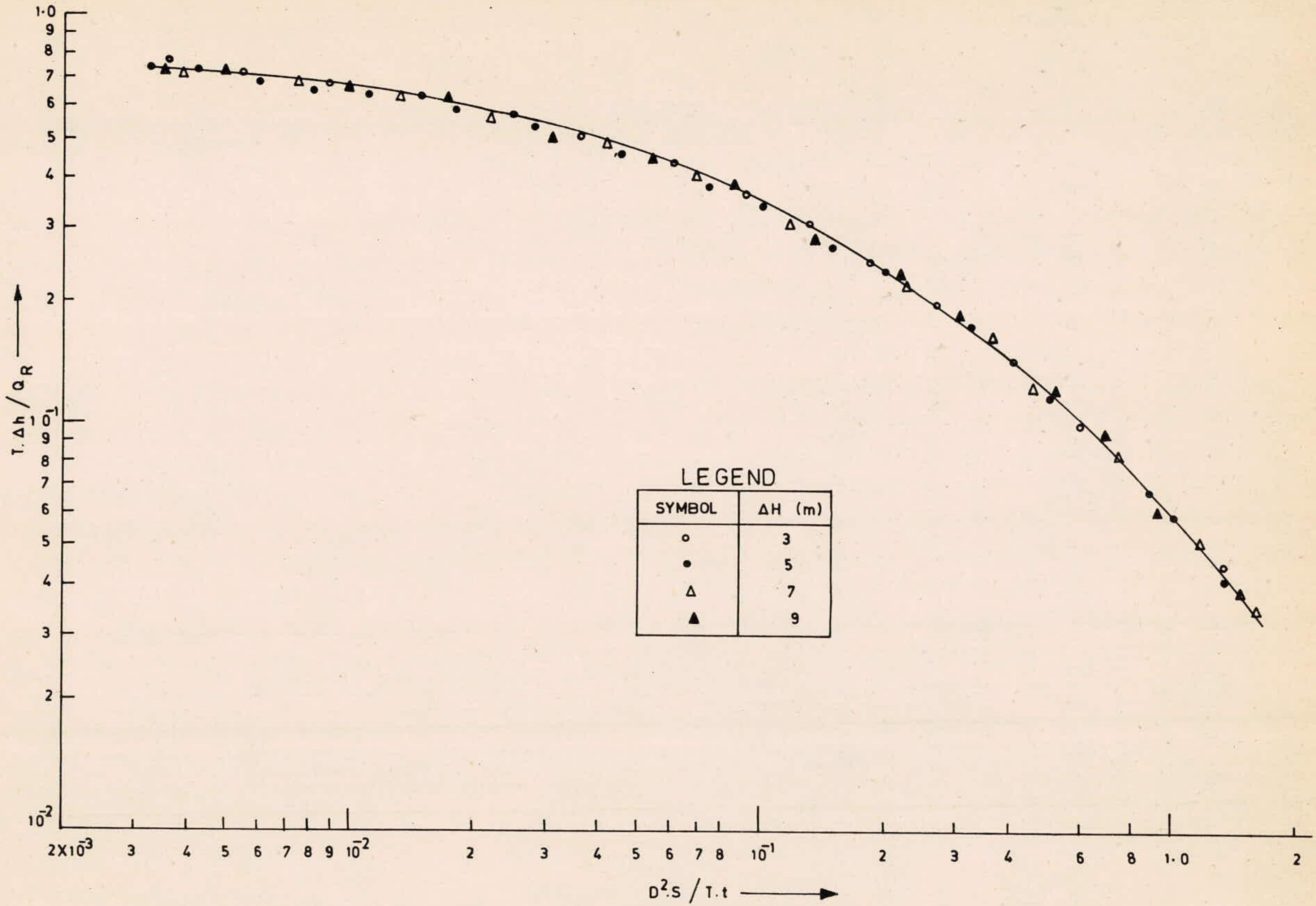

FIG. 8 - RELATION BETWEEN $X^2S/T.t$ AND $T.\Delta h/Q_R$ (FOR $K_h/K_v = 500$ AND $l/L = 0.12$)

6.0 CONCLUSIONS

Three dimensional model study of the seepage from the lake of square cross section and uniform depth in a homogeneous aquifer ($Kh/Kv = 500$) with constant head boundary on lateral sides and no flow boundary on the other sides of the lake,each at a distance of 2500m from the side of the lake, has been carried out and the results have been analysed. The conclusions dreawn from the study are as given below:

1. The parameter $X^2S/T.t$ is found to be uniquely related with the parameter $T.\Delta h/Q_R$ irrespective of the value of ΔH, while Kh/Kv and l/L are kept constant. A type curve has been developed between the parameter $X^2S/T.t$ and $T.\Delta h/Q_R$ for the specific values of $Kh/Kv = 500$ and $l/L = 0.12$.
2. The proper location of the observation well has also been suggested, a method for estimation of recharge Q_R from the lake, using the water level fluctuation in the observation well, has also been presented.

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APPENDIX - A-I

Variation of different parameters for H = 5.0 m

TABLE-A-I-1

River Leakage = Q_R in m^3/d

Storage = Q_s in m^3/d

Constant head = Q_R(out flow) in m^3/d

$T = 535\ m^2/d$

$1 = h$, $2 = \Delta h/\Delta H$, $3 = T.\Delta h/Q_R$

Sl. No.	X	X/L	Time step - 4, $Q_h = 67.24$, $Q_R = 5050.4$			Time step = 8, $Q_h = 109.86$, $Q_R = 4163.1$				Time step = 12, $Q_h = 154.71$, $Q_R = 3816.0$			Time step = 16, $Q_h = 202.09$, $Q_R = 3018.4$			Time step = 20, $Q_h = 251.82$, $Q_R = 3487.0$			Time step = 24, $Q_h = 303.53$, $Q_R = 3391.2$		
	m		1	2	3	1	2	3/	3	1	2	3	1	2	3	1	2	3	1	2	3
1.	20	0.008	4.94	0.998	0.523	4.95	0.990		0.636	4.95	0.990	0.694	4.95	0.9909	0.732	4.96	0.992	0.761	4.96	0.992	0.782
2.	65	0.026	3.85	0.770	0.408	4.10	0.820		0.527	4.19	0.838	0.587	4.23	0.846	0.625	4.26	0.852	0.654	4.29	0.858	0.677
3.	120	0.048	2.65	0.930	0.281	3.15	0.630		0.405	3.34	0.668	0.468	3.45	0.690	0.510	3.51	0.702	0.539	3.56	0.712	0.562
4.	190	0.076	1.45	0.290	0.154	2.14	0.428		0.275	2.43	0.486	0.341	2.60	0.520	0.384	2.70	0.540	0.414	2.78	0.556	0.438
5.	280	0.112	0.60	0.120	0.064	1.20	0.240		0.154	1.55	0.310	0.217	1.77	0.354	0.262	1.91	0.382	0.293	2.02	0.404	0.319
6.	400	0.160	0.19	0.038	0.020	0.52	0.104		0.0668	0.81	0.162	0.114	1.03	0.206	0.152	1.19	0.238	0.1826	1.31	0.262	0.207
7.	590	0.236	0.06	0.012	6.355×10^{-3}	0.16	0.032		0.0206	0.30	0.060	0.042	0.43	0.086	0.0636	0.55	0.110	0.0844	0.66	0.132	0.104
8.	890	0.356	0.02	4.0×10^{-3}	2.11×10^{-3}	0.06	0.012		7.71×10^{-3}	0.10	0.020	0.014	0.15	0.030	0.022	0.21	0.042	0.032	0.27	0.054	0.043
9.	1310	0.524	0.01	2.0×10^{-3}	1.06×10^{-3}	0.02	4×10^{-3}		2.57×10^{-3}	0.04	8×10^{-3}	5.6×10^{-3}	0.06	0.012	8.87×10^{-3}	0.08	0.016	0.012	0.11	0.022	0.0174
10.	1850	0.740	0.0	0.0	0.0	0.01	2×10^{-3}		1.29×10^{-3}	0.01	2×10^{-3}	1.4×10^{-3}	0.02	4×10^{-3}	2.96×10^{-3}	0.03	6×10^{-3}	4.6×10^{-3}	0.04	8×10^{-3}	6.31×10^{-3}
11.	2500	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A-I-2

River leakage = Q_R

Storage = Q_s

Constant head = Q_R

T = 535 m^2/d

Sl.	X	X/L	Time step = 28 Q_h = 356.84 Q_R = 3317.1			Time step = 32 Q_h = 411.37 Q_R = 3257.4			Time step = 36 Q_R = 466.76 Q_R = 3207.9			Time step = 40 Q_h = 522.66 Q_R = 3165.7			Time step = 44 Q_h = 578.80 Q_R = 3129.1			Time step = 48 Q_h = 634.89 Q_R = 3096.9		
	m		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1.	20	0.008	4.96	0.992	0.80	4.96	0.992	0.815	4.96	0.992	0.827	4.96	0.992	0.838	4.96	0.992	0.848	4.96	0.992	0.857
2.	65	0.026	4.30	0.860	0.694	4.32	0.864	0.710	4.33	0.865	0.722	4.34	0.868	0.733	4.35	0.870	0.743	4.35	0.870	0.751
3.	120	0.048	3.60	0.720	0.581	3.62	0.724	0.595	3.65	0.730	0.608	3.67	6.734	0.620	3.69	0.738	0.630	3.70	0.740	0.639
4.	190	0.076	2.84	0.568	0.458	2.88	0.576	0.473	2.92	0.584	0.487	2.95	0.590	0.447	2.98	0.596	0.560	3.01	0.602	0.520
5.	280	0.112	2.10	0.420	0.339	2.16	0.432	0.355	2.21	0.442	0.369	2.26	0.452	0.382	2.30	0.460	0.393	2.33	0.466	0.4025
6.	400	0.160	1.41	0.282	0.227	1.49	0.298	0.245	1.55	0.310	0.289	1.60	0.320	0.270	1.65	0.330	0.282	1.69	0.338	0.292
7.	590	0.236	0.75	0.150	0.121	0.83	0.166	0.136	0.90	0.180	0.150	0.96	0.192	0.162	1.01	0.202	0.173	1.06	0.212	0.183
8.	890	0.356	0.33	0.066	0.0532	0.38	0.676	0.0624	0.43	0.086	0.072	0.,48	0.096	0.081	0.52	0.104	0.089	0.56	0.112	0.0967
9.	1310	0.524	0.13	0.026	0.021	0.16	0.032	0.026	0.19	0.038	0.036	0.21	0.042	0.035	0.24	0.048	0.041	0.27	0.054	0.0466
10.	1850	0.740	0.05	0.01	8.06×10^{-3}	0.06	0.012	9.85×10^{-3}		0.07	0.012	0.08	0.016	0.014	4.09	0.018	0.0154	0.10	0.020	0.017
11.	2500	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.0.	0000	0.0	0.0

Table -A-I-3

River Leakage = Q_R

Storage = Q_s

Constant head = Q_R

Sl.	X	X/L	Time step = 52 Q_h = 690.71 Q_R = 3068.2		
	m		1	2	3
1.	20	0.008	4.96	0.992	0.865
2.	65	0.026	4.36	0.72	0.760
3.	120	0.048	3.71	0.742	0.647
4.	190	0.076	3.03	0.606	0.528
5.	280	0.112	2.36	0.472	0.411
6.	400	0.160	1.73	0.346	0.302
7.	590	0.236	1.10	0.220	0.192
8.	890	0.356	0.60	0.120	0.105
9.	1310	0.524	0.29	0.058	0.0506
10.	1850	0.740	0.12	0.024	0.021
11.	2500	1.000	0.0 0.0	0.0	0.0

APPENDIX - A-II

VARIATION OF DIFFERENT PARAMETERS FOR $\Delta H = 3.0m$
River Leakage = Q_R

Storage = Q_s

Constant head = Q_h

$T = 535 m^2/d$

$1 = \Delta h$, $2 = \Delta h/\Delta H$, $3 = T\Delta h/Q_R$

Sl. No.	X	X/L	Time step=4 Q_h = 38.266 Q_R = 2814.3			Time step = 8 Q_h = 63.311 Q_R = 2258.4			Time step = 12 Q_h = 87.212 Q_R = 2127.1			Time step = 16 Q_h = 134.29 Q_R = 1961.0			Time step = 20 Q_h = 141.32 Q_R = 1945.1			Time step = 24 Q_h = 177.4 Q_R = 1881.0		
	m																			
1.	20	0.008	2.96	0.986	0.562	2.97	0.99	0.703	2.97	0.99	0.747	2.97	0.99	0.810	2.98	0.993	0.819	2.98	0.993	0.847
2.	65	0.026	2.25	0.75	0.427	2.41	0.807	0.570	2.46	0.827	0.618	2.50	0.837	0.682	2.52	0.836	0.693	2.53	0.843	0.719
3.	120	0.048	1.49	0.496	0.283	1.80	0.60	0.426	1.92	0.64	0.482	1.99	0.663	0.542	2.03	0.676	0.558	2.06	0.686	0.585
4.	190	0.076	0.78	0.26	0.148	1.18	0.393	0.279	1.35	0.453	0.339	1.46	0.486	0.398	1.57	0.506	0.420	<.58	0.526	0.449
5.	280	0.112	0.31	0.103	0.058	0.64	0.213	0.151	0.84	0.28	0.211	0.97	0.323	0.264	1.06	0.35	0.291	1.12	0.373	0.318
6.	400	0.160	0.10	0.037	0.019	0.28	0.093	0.066	0.43	0.147	0.108	0.55	0.183	0.150	0.60	0.213	0.165	0.71	0.236	0.201
7.	590	0.236	0.03	0.01	0.0057	0.09	0.03	9.921	0.16	0.053	0.040	0.23	0.076	0.062	0.30	0.10	0.082	0.36	0.12	0.102
8.	890	0.356	0.01	0.0	0.0019	0.03	0.01	0.007	0.06	0.02	0.015	0.09	0.03	0.024	0.12	0.02	0.033	0.15	0.05	0.042
9.	1310	0.524	0.00	0.0	0.0	0.01	0.003	0.002	0.02	0.006	0.005	0.03	0.01	0.008	0.05	0.016	0.013	0.06	0.02	0.017
10.	1850	0.740	0.00	0.0	0.0	0.0	0.0	0.0	0.01	0.003	0.002	0.01	0.003	0.002	0.02	0.006	0.005	0.02	0.006	0.005
11.	2500	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table - A-II-2

River leakage = Q_R

Storage = Q_s

Constant head = Q_h

1 = Δh, 2 = $\Delta h / \Delta H$, 3 = $T.\Delta h / Q_R$

Sl. No.	X	X/L	Time step = 28, Q_h = 199.75, Q = 1851.4			Time step = 32, Q_h = 230.06, Q_R = 1818.5			Time step = 36, Q_h = 260.84, Q_R = 1791.3			Time step = 40, Q_h = 291.89, Q = 1768.1			Time step = 44, Q_h = 323.07, Q = 1748.1			Time step = 48, Q_h = 354.21, Q = 1730.4		
	m		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1.	20	0.008	2.98	0.997	0.86	2.98	0.993	0.876	2.98	0.993	0.890	2.98	0.943	0.901	2.98	0.993	0.912	2.98	0.007	0.921
2.	65	0.026	2 54	0.846	0.733	2.55	0.85	0.750	2.56	0.853	0.764	2.57	0.856	0.777	2.57	0.856	0.786	2.57	0.86	0.794
3.	120	0.048	2.08	0.697	0.601	2.10	0.703	0.617	2.11	0.706	0.630	2.17	0.71	0.64	2.14	0.713	0.654	2.15	0.716	0.664
4.	190	0.076	1.61	0.536	0.465	1.60	0.546	0.482	1.66	0.556	0.495	1.69	0.567	0.511	1.7	0.566	0.520	1.71	0.573	0.528
5.	280	0.112	1.16	0.386	0.335	1.20	0.4	0.353	1.23	0.413	0.367	1.26	0.42	0.381	1.23	0.43	0.391	1.30	0.476	0.401
6.	400	0.160	0.77	0.256	0.222	0.81	0.27	0.238	0.85	0.283	0.257	0.88	0.293	0.266	0.91	0.303	0.278	0.93	0.317	0.287
7.	590	0.236	0.41	0.136	0.118	0.45	0.15	0.132	0.49	0.163	0.146	0.52	0.171	0.157	0.55	0.183	0.168	0.58	0.193	0.179
8.	890	0.356	0.18	0.06	0.052	0.21	0.07	0.061	0.20	0.08	0.071	0.26	0.086	0.078	0.29	0.096	0.088	0.31	0.107	0.095
9.	1310	0.524	0.07	0.023	0.020	0.09	0.03	0.026	0.10	0.037	0.029	0.12	0.04	0.036	0.13	0.043	0.39	0.15	0.05	0.046
10.	1850	0.740	0.03	0.01	0.008	0.03	0.01	0.008	0.04	0.013	0.011	0.05	0.016	0.015	0.05	0.016	0.015	0.06	0.02	0.018
11.	2500	1.000	0.0	0.0	0.0	0.0	0.0	0.0 0.0	0.0	0.0	0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table - A-II-3

River Leakage = Q_R

Storage = Q_s

Constant head = Q_h

SL. No.	X	X/L	Time step = 52 Q_h = 385.20 Q_R = 1714.7		
	m		1	2	3
1.	20	0.008	2.98	0.997	0.929
2.	65	0.026	2.58	0.86	0.804
3.	120	0.048	2.16	0.72	0.673
4.	190	0.076	1.73	0.576	0.539
5.	280	0.112	1.33	0.44	0.414
6.	400	0.160	0.90	0.32	0.299
7.	590	0.236	0.60	0.20	0.187
8.	890	0.356	0.33	0.11	0.102
9.	1310	0.524	0.16	0.053	0.049
10.	1850	0.740	0.06	0.02	0.018
11.	2500	1.000	0.0	0.0	0.0

APPENDIX - B

Values of $X^2.S/T.t$

Sl. No.	X/L m	Time step 4	Time step 8	Time step 12	Time step 16	Time step 20	Time step 24	Time step 28	Time step 32	Time step 36	Time step 40	Time step 44	Time step 48	Time step 52
1.	20	400×10^{-3}	1.99×10^{-3}	1.22×10^{-3}	2.00×10^{-3}	8.00×10^{-4}	6.67×10^{-4}	5.72×10^{-4}	5.00×10^{-4}	4.45×10^{-4}	4.00×10^{-4}	3.64×10^{-4}	3.34×10^{-4}	3.079×10^{-4}
2.	65	0.042	0.0211	0.0141	0.0105	8.43×10^{-3}	7.04×10^{-3}	6.03×10^{-3}	5.28×10^{-3}	4.69×10^{-3}	4.22×10^{-3}	3.83×10^{-3}	3.506×10^{-3}	3.24×10^{-3}
3.	120	0.145	0.072	0.0481	0.0361	0.0289	0.0241	0.0206	0.0181	0.0161	0.0145	0.0131	0.0120	0.011
4.	190	0.362	0.176	0.1206	0.0904	0.0723	0.0603	0.0516	0.045	0.040	0.036	0.0329	0.030	0.0278
5.	280	0.785	0.390	0.2613	0.1960	0.1570	0.1306	0.112	0.098	0.087	0.0785	0.0714	0.0654	0.0604
6.	400	1.602	0.804	0.534	0.400	0.3204	0.2664	0.229	0.1998	0.178	0.1602	0.1458	0.133	0.1233
7.	590	3.487	1.746	1.162	0.867	0.697	0.582	0.503	0.435	0.387	0.349	0.317	0.289	0.2677
8.	890	7.929	3.971	2.639	1.983	1.586	1.322	1.131	0.992	0.881	0.793	0.720	0.661	0.609
9.	1310	17.178	8.595	5.730	4.285	3.436	2.65	2.45	1.910	1.722	1.558	1.431	1.321	
10.	1850	34.306	17.153	11.435	8.545	6.86	5.718	4.90	4.285	3.808	3.430	3.116	2.59	2.63
11.	2500	62.58	31.29	20.86	15.645	12.516	10.43	8.92	7.829	6.95	6.258	5.693	5.215	4.83